Recall the formulas for *angular speed* and *linear speed* of a spinning disk:

$$\omega = \frac{\theta}{t} \quad \text{and} \quad v = r\omega,$$

where:

- ω (lowercase greek letter "omega", not a "w") denotes angular speed;
- θ is how much whatever is spinning has actually turned in time t;
- v denotes the linear speed of some point in the edge of disk.
- r is the radius of the disk.

The SI units for ω are just s^{-1} instead of rad/s, as rad is a purely geometric unit and hence dimensionless. Let's see some examples:

Exercise (#89, p. 505)**.** A circular paddle wheel of radius 3 ft. is lowered into a flowing river. The current causes the wheel to rotate at a speed of 12 rpm. To 1 decimal place:

(a) What is the angular speed?

(b) Find the speed of the current in ft./min.

(c) Find the speed of the current in mph.

Solution:

(a) If the wheel is rotating at 12 rpm, this means 12 full rotations (which, in other words, is $\theta = 12 \times 2\pi = 24\pi$ rad) in $t = 1$ min. So the angular speed is

$$\omega = \frac{\theta}{t} = 24\pi\,\mathrm{min}^{-1} \simeq 75.4\,\mathrm{min}^{-1}.$$

(b) The speed of the current is the linear speed of any point in the edge of the paddle wheel. The radius of the wheel is $r = 3$ ft., so

$$v = r\omega = 3 \times 24\pi \frac{\text{ft.}}{\text{min}} = 72\pi \frac{\text{ft.}}{\text{min}} \simeq 226.2 \frac{\text{ft.}}{\text{min}}.$$

(c) We just have to convert the result from (b) to mph. This is done by directly manipulating units, as follows:

- $1\,\mathrm{h} = 60\,\mathrm{min} \implies \frac{1}{\mathrm{h}} = \frac{1}{60\,\mathrm{min}} \implies \frac{60}{\mathrm{h}} = \frac{1}{\mathrm{min}}$
- $1\,\text{mile} = 5280\,\text{ft} \implies \text{ft} = \frac{\text{mile}}{5280}$

Now plug this in the expression for v to get

$$v = 72\pi \frac{\text{ft.}}{\text{min}} = 72\pi \times \text{ft.} \times \frac{1}{\text{min}} = 72\pi \times \frac{\text{mile}}{5280} \times \frac{60}{\mathrm{h}} = \frac{9\pi}{11}\,\text{mph} \simeq 2.6\,\text{mph}.$$

$\square$

Exercise (#92, p.505)**.** On a weed-cutting device, a thick nylon line rotates on a spindle at 3000 rpm.

(a) Determine the angular speed.

(b) Determine the linear speed (to the nearest inch per minute) of a point on the tip of the line if the line is 5 in.

Solution:

(a) If the line rotates at 3000 rpm, this means 3000 full rotations (so $\theta = 3000 \times 2\pi = 6000\pi$ rad) in $t = 1$ min. So we have

$$\omega = \frac{\theta}{t} = 6000\pi\,\mathrm{min}^{-1}.$$

(b) Again, we're given $r = 5$ in, so

$$v = r\omega = 5\mathrm{in.} \times 6000\pi\,\mathrm{min}^{-1} = 30000\pi\frac{\mathrm{in.}}{\mathrm{min}} \simeq 94,248\,\frac{\mathrm{in.}}{\mathrm{min}}.$$

$\square$