Frequent mistakes (MATH2173)

I'll list here some of the most frequent mistakes done in quizzes, so you can avoid them in the future. For some of the mistakes listed, people still did not lose points, though. If things go very bad in the some of the next quizzes, I'll just update this file (but I hope not to do this).

Figure 1: Your incompetent TA seeing the same mistakes almost 90 times in a row.

Quiz 2

- When you're asked to look for absolute maximum and minimum of a function f of two variables over a region R (with boundary, such as a triangle or rectangle), the usual strategy is
  - Compute $\nabla f(x,y)$, set $\nabla f(x,y) = (0,0)$ and solve for x and y to find the critical points. However, you need to check if the critical points found **actually lie inside** R. Most people did not check this! You have to ignore

such points, as they are not in the domain of discussion and thus are not legitimate candidates!

  - Parametrize the "sides" of the boundary of R to look for more candidates, which will reduce the analysis of the boundary to three or four Calculus 1 problems. **It is not enough** to just verify the "sharp corners" of the boundary, as **there still might be more candidates** in the "interior of the sides"! Most people did not parametrize the sides, and almost all of you who did, only checked the endpoints.

- Note that I use the word "candidates" instead of "critical points" when talking about the boundary. By definition, a critical point of f is a point (x_0, y_0) such that $\nabla f(x_0, y_0) = (0, 0)$, and nothing else! For example, if R is a triangle with one of its sides lying on the line $y = x$, one considers the function of one variable $g(x) = f(x, x)$. A value x_0 for which $g'(x_0) = 0$ need not be a critical point of f! That is to say, $g'(x_0) = 0$ **does not imply** that $\nabla f(x_0, x_0) = (0, 0)$. Namely, the actual relation is

$$g'(x) = \frac{\partial f}{\partial x}(x, x) + \frac{\partial f}{\partial y}(x, x),$$

by the chain rule, and this does not imply that each of the derivatives being added above are zero.

- Continuing the point above, three things about notation:

  - You should use a different letter for naming the single variable function considered. In class, when I wrote that you had to look at $f(x, x)$ for analyzing the corresponding side of the triangle, it is understood that this is some $g(x) = f(x, x)$. **It is wrong** to call this f again, after all, in the setting of the problem, f is a function of two variables, and not a function of one variable. That is to say, you should avoid writing that $f(x) = f(x, x)$, as this makes no sense (for example, a computer could not parse this).
  - If $g(x) = f(x, x)$ and you want to compute $g'(x)$, **you should not write** $f'(x, x)$. Prime notation is, in this course[1], ambiguous. That is the whole reason why we have the notation $\partial f/\partial x$, $\partial f/\partial y$, f_x and f_y instead. Also, do not mix prime notation with those notations, i.e., f'_x and f'_y are nonsense for us here.
  - I am not sure if this is lack of attention or just laziness, but **you should keep track of the point where the function is evaluated**. More precisely, write

$$f(x, y) = 2x + y^2 \text{ instead of } f = 2x + y^2,$$

[1] There is a precise definition of what $f'(p)$ means if $p \in \mathbb{R}^n$, but this is done in a much more advanced context than what we will do in this course. We won't even come close to that here. Ask me more about this during office hours if you want to be traumatized, I won't say anything about this in class (I swear).

also

$$f_y(x, y) = 2y \text{ instead of } f_y = 2y,$$

etc. This is useful to avoid confusion. I have seen a lot of people writing things like

$$f = 2x + y^2 = 2(0) + 1^2 = 1.$$

In this particular case it is easy to see that the person actually meant to compute $f(0, 1)$, but one **should not mix** $f(x, y)$ with $f(0, 1)$ in the same computation. This might sound harsh, but it is your job to use clear notation, not my job to figure out what you mean. As I used to tell my Calculus 1 students, "don't make me reason for you". Along the same lines, this being a math class does not mean that you're not allowed to use words. The key to doing mathematics is communication, and words are an important tool for that – find your balance.

- The second derivative test, with the Hessian matrix, is used to find out the **local** nature of critical points **inside** the region R. **When looking for absolute maximum and minimum of f over R, the local nature of critical points does not matter**. The **critical points are just candidates**, so you will have to evaluate f at such points in the end, and compare with the other candidates in the list. For example, one could use the second derivative test to say that a critical point inside R is a local maximum, but then find a point in the boundary which gives a higher value. Meaning you just lost precious time computing second order derivatives for nothing. The take away here is that local maxima (resp. minima) need not be global maxima (resp. minima).

Quiz 5

- $\cos 0$ is 1, and **not** 0. Also, $\cos(1) \neq 0$. In fact, $\cos(1)$ is **just a real number**. We don't know (and don't care about) its exact value. It is something strictly between 0 and 1. Usually the only exact values for $\sin\theta$ and $\cos\theta$ we can say quickly are when θ is a "notable" angle such as $\pi/6$, $\pi/3$, $\pi/4$, $\pi/2$, etc., etc..

- The area of an annulus described by $r_1^2 \leq x^2 + y^2 \leq r_2^2$ is equal to the area of the bigger circle minus the area of the smaller one: $\pi r_2^2 - \pi r_1^2 = \pi(r_2^2 - r_1^2)$. In the particular case we had, $1 \leq x^2 + y^2 \leq 2$, **almost everyone incorrectly identified the radii** as 1 and 2, when they're actually 1 and $\sqrt{2}$, so that the area is $\pi \cdot (\sqrt{2})^2 - \pi \cdot 1^2 = \pi$ instead of 3π. Some people computed the area of the annulus via a double integral: although conceptually correct, it was a ridiculous waste of time (a luxury you do not have).

- Few people correctly recognized the region in the first problem as the right hald of a circle. People seemed to be under the impression that no matter the region in the plane you're integrating over, we'll have $0 \leq \theta \leq 2\pi$ and $0 \leq r \leq 1$. **This is not the case. Not all things are perfect circles**. Review

MATH1172 if you need to. If you try to memorize things like this instead of understanding the geometry behind it, **you will fail**.

- The area of a rectangle is the product of the sides, when such lengths are described in **cartesian coordinates only**. Rectangles are poorly described in polar coordinates.
- $\mathrm{d}x\,\mathrm{d}y = r\,\mathrm{d}r\,\mathrm{d}\theta$. Repeat after me: "I will **never** forget this damn r again". Every time you forget it, a kitten dies.
- The distance between a point (x, y) and the origin is $\sqrt{x^2+y^2} = \sqrt{r^2} = r$, and not r^2. The latter is the **squared** distance, unrelated[2] to the problem at hand. I saw people trying to integrate weird powers such as $r^{3/2}$, r^3, etc..
- When trying to find the volume bounded by two surfaces described by the equations $z = f(x, y)$ and $z = g(x, y)$, with $f \geq g$, one computes

  $$\iint_R f(x,y) - g(x,y)\,\mathrm{d}A,$$

  where R is the interior of the curve in the plane described by the equation $f(x, y) = g(x, y)$. In our particular case, $12 - 2x^2 - y^2 = x^2 + 2y^2$ implies that $12 = 3x^2 + 3y^2$, and then $x^2 + y^2 = 4$. This says that the region R is $\{(x, y) \mid x^2 + y^2 \leq 4\}$, begging for the use of polar coordinates. Some people:

  - switched the order of top and bottom. A quick sketch would have prevented that. In the same way that things like $y = ax^2$ $(a \neq 0)$ are parabolas in the plane, things like $z = ax^2 + by^2 + c$ (with $ab > 0$, c anything) are paraboloids in space, you should know that by now.
  - didn't even try to use polar coordinates here. **It was a quiz about integration on polar coordinates**. Come on.
  - got negative results for the volume of the solid. **You should immediately recognize that this is impossible** and redo your calculations. Such **sanity-checks are useful for detecting errors** (for example, if you're integrating a positive function and got a negative result, you messed up).

  Bottom line: use your common sense.

[2] Again, if $\overline{f}$ denotes the average of f over some fixed region R, in general $\overline{f^2} \neq \overline{f}^2$. And there's no clear relation between them.