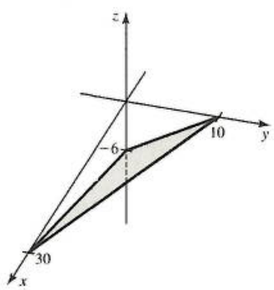
Answers

CHAPTER 1

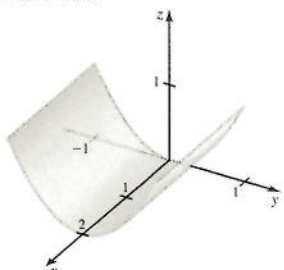
Section 1.1 Exercises, pp. 14–17

1. A point and a normal vector **3.** $x = -6, y = -4, z = 3$
5. z-axis; x-axis; y-axis **7.** Intersection of the surface with a plane parallel to one of the coordinate planes **9.** Ellipsoid
11. $x + y - z = 4$ **13.** $-x + 2y - 3z = 4$
15. $2x + y - 2z = -2$ **17.** $7x + 2y + z = 10$
19. $4x + 27y + 10z = 21$ **21.** Intercepts $x = 2, y = -3, z = 6$; $3x - 2y = 6, z = 0$; $-2y + z = 6, x = 0$; and $3x + z = 6, y = 0$

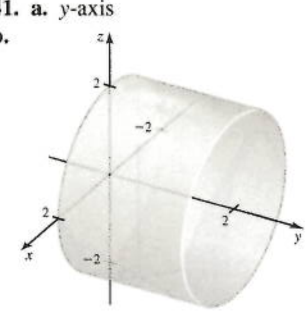
23. Intercepts $x = 30, y = 10, z = -6$; $x + 3y = 30, z = 0$; $x - 5z = 30, y = 0$; and $3y - 5z = 30, x = 0$



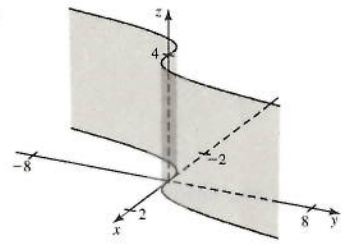
25. Orthogonal **27.** Neither **29.** Q and T are identical; Q, R, and T are parallel; S is orthogonal to Q, R, and T.
31. $-x + 2y - 4z = -17$ **33.** $4x + 3y - 2z = -5$
35. $x = t, y = 1 + 2t, z = -1 - 3t$
37. $x = \frac{7}{5} + 2t, y = \frac{9}{5} + t, z = -t$
39. a. x-axis
b.



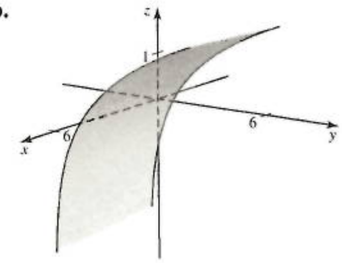
41. a. y-axis
b.



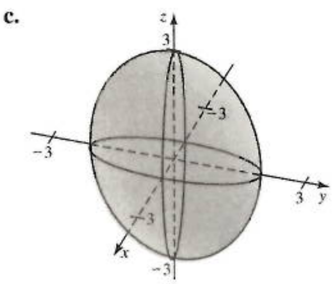
43. a. z-axis **b.**



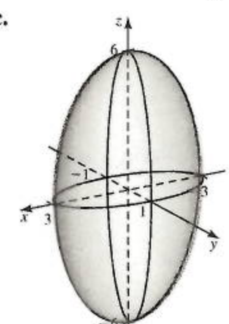
45. a. x-axis **b.**



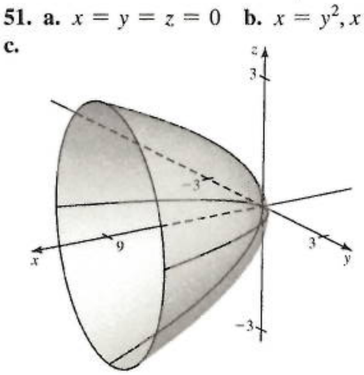
47. a. $x = \pm 1, y = \pm 2, z = \pm 3$ **b.** $x^2 + \frac{y^2}{4} = 1, x^2 + \frac{z^2}{9} = 1, \frac{y^2}{4} + \frac{z^2}{9} = 1$ **c.**



49. a. $x = \pm 3, y = \pm 1, z = \pm 6$ **b.** $\frac{x^2}{3} + 3y^2 = 3, \frac{x^2}{3} + \frac{z^2}{12} = 3, 3y^2 + \frac{z^2}{12} = 3$ **c.**

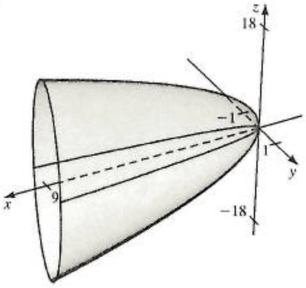


51. a. $x = y = z = 0$ **b.** $x = y^2, x = z^2$, origin
c.



53. a. $x = y = z = 0$ **b.** Origin, $x - 9y^2 = 0, 9x - \frac{z^2}{4} = 0$

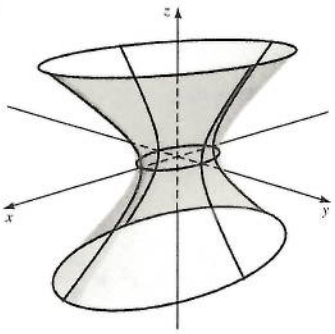
c.



55. a. $x = \pm 5, y = \pm 3$, no z-intercept

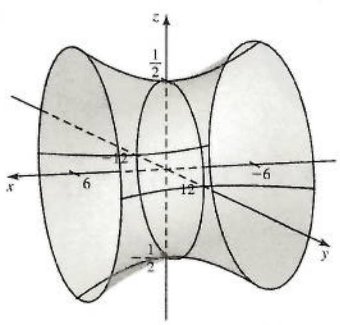
b. $\frac{x^2}{25} + \frac{y^2}{9} = 1, \frac{x^2}{25} - z^2 = 1, \frac{y^2}{9} - z^2 = 1$

c.



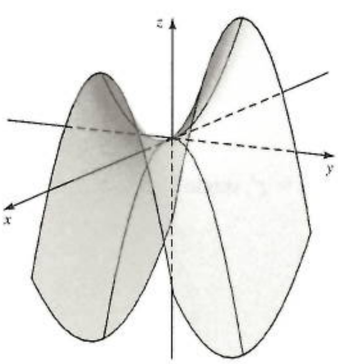
57. a. No x-intercept, $y = \pm 12, z = \pm\frac{1}{2}$ **b.** $-\frac{x^2}{4} + \frac{y^2}{16} = 9$, $-\frac{x^2}{4} + 36z^2 = 9, \frac{y^2}{16} + 36z^2 = 9$

c.



59. a. $x = y = z = 0$ **b.** $\frac{x^2}{9} - y^2 = 0, z = \frac{x^2}{9}, z = -y^2$

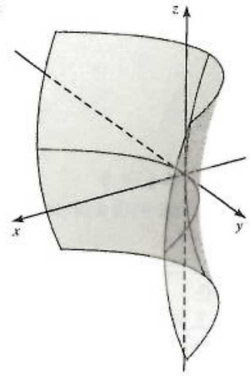
c.



61. a. $x = y = z = 0$

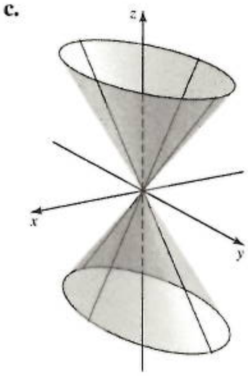
b. $5x - \frac{y^2}{5} = 0, 5x + \frac{z^2}{20} = 0, -\frac{y^2}{5} + \frac{z^2}{20} = 0$

c.



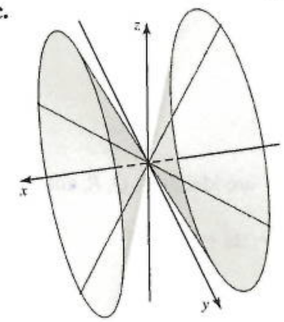
63. a. $x = y = z = 0$ **b.** Origin, $\frac{y^2}{4} = z^2$, $x^2 = z^2$

c.



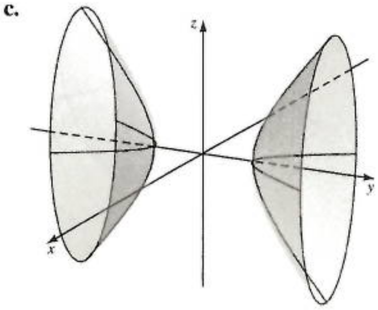
65. a. $x = y = z = 0$ **b.** $\frac{y^2}{18} = 2x^2, \frac{z^2}{32} = 2x^2$, origin

c.

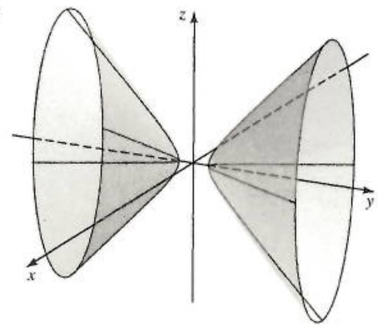


67. a. No x-intercept, $y = \pm 2$, no z-intercept **b.** $-x^2 + \frac{y^2}{4} = 1$, no xz-trace, $\frac{y^2}{4} - \frac{z^2}{9} = 1$

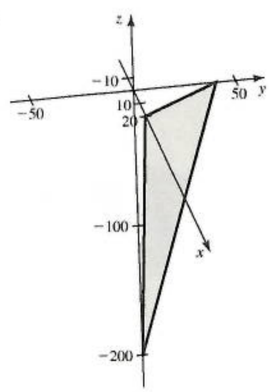
c.



69. a. No x-intercept, $y = \pm\frac{\sqrt{3}}{3}$, no z-intercept
b. $-\frac{x^2}{3} + 3y^2 = 1$, no xz-trace, $3y^2 - \frac{z^2}{12} = 1$
c.



71. a. True **b.** False **c.** False **d.** True **e.** False **f.** False **g.** False **73.** $\mathbf{r}(t) = \langle 2 + 2t, 1 - 4t, 3 + t\rangle$ **75.** $6x - 4y + z = d$ **77.** The planes intersect in the point $(3, 6, 0)$. **79. a.** D **b.** A **c.** E **d.** F **e.** B **f.** C **81.** Hyperbolic paraboloid **83.** Elliptic paraboloid **85.** Hyperboloid of one sheet **87.** Hyperbolic cylinder **89.** Hyperboloid of two sheets **91.** $(3, 9, 27)$ and $(-5, 25, 75)$
93. $\left(\frac{6\sqrt{10}}{5}, \frac{2\sqrt{10}}{5}, \frac{3\sqrt{10}}{10}\right)$ and $\left(-\frac{6\sqrt{10}}{5}, -\frac{2\sqrt{10}}{5}, -\frac{3\sqrt{10}}{10}\right)$
95. $\theta = \cos^{-1}\left(-\frac{\sqrt{105}}{14}\right) \approx 2.392$ rad; $137°$ **97.** All except the hyperbolic paraboloid **99. a.**

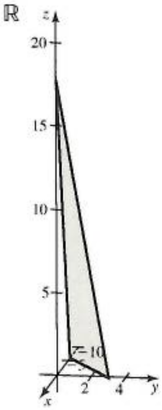


b. Positive **c.** $2x + y = 40$, line in the xy-plane **101. a.** $z = cy$ **b.** $\theta = \tan^{-1} c$ **103. a.** The length of the orthogonal projection of $\overrightarrow{PQ}$ onto the normal vector $\mathbf{n}$ is the magnitude of the scalar component of $\overrightarrow{PQ}$ in the direction of $\mathbf{n}$, which is $\frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{|\mathbf{n}|}$. **b.** $\frac{13}{\sqrt{14}}$

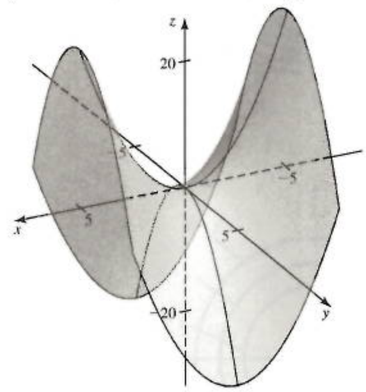
Section 1.2 Exercises, pp. 26–29

1. Independent: x and y; dependent: z
3. $D = \{(x, y): x \neq 0 \text{ and } y \neq 0\}$ **5.** Three **7.** Circles **9.** $n = 6$
11. $\mathbb{R}^2$ **13.** $\{(x, y): x^2 + y^2 \leq 25\}$ **15.** $D = \{(x, y): y \neq 0\}$
17. $D = \{(x, y): y < x^2\}$
19. $D = \{(x, y): xy \geq 0, (x, y) \neq (0, 0)\}$

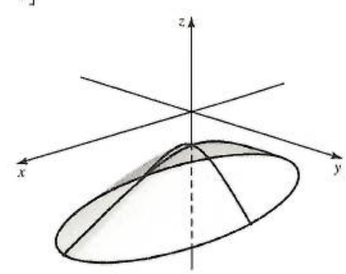
21. Plane; domain $= \mathbb{R}^2$, range $= \mathbb{R}$



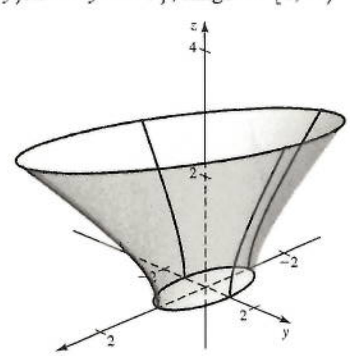
23. Hyperbolic paraboloid; domain $= \mathbb{R}^2$, range $= \mathbb{R}$



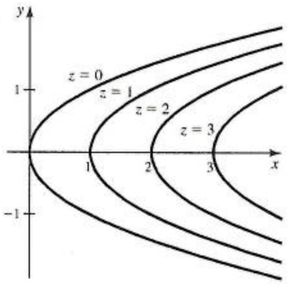
25. Lower part of a hyperboloid of two sheets; domain $= \mathbb{R}^2$, range $= (-\infty, -1]$



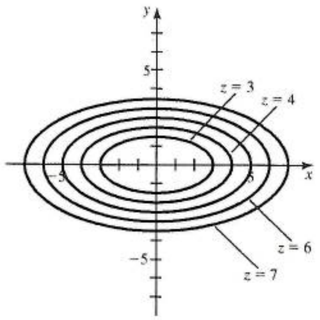
27. Upper half of a hyperboloid of one sheet; domain $= \{(x, y): x^2 + y^2 \geq 1\}$, range $= [0, \infty)$



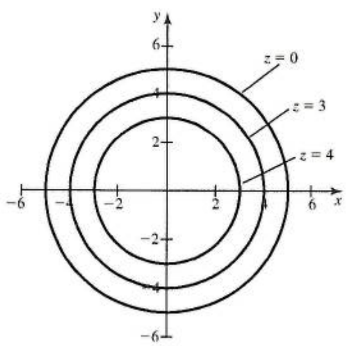
29. a. A **b.** D **c.** B **d.** C **31.**



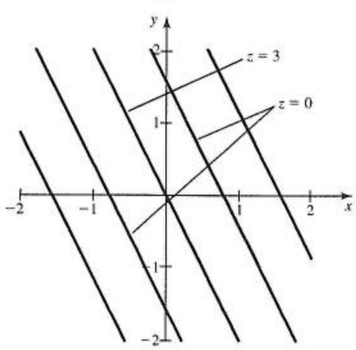
33.



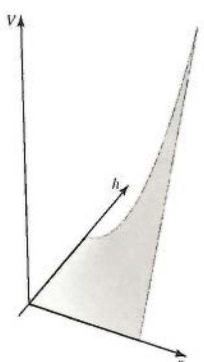
35.



37.

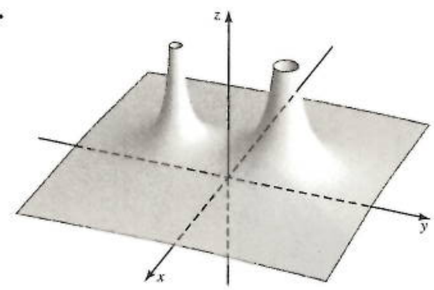


39. a.



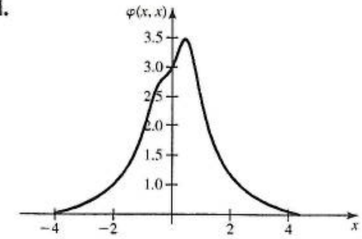
b. $D = \{(r, h): r > 0, h > 0\}$ **c.** $h = 300/(\pi r^2)$

41. a.

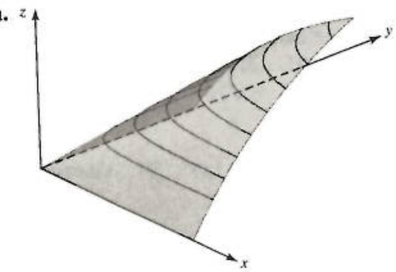


b. $\mathbb{R}^2$ without the points $(0, 1)$ and $(0, -1)$

c. $\varphi(2, 3)$ is greater. **d.**

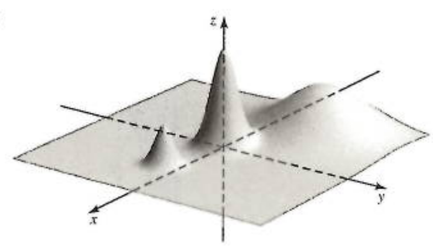


43. a.

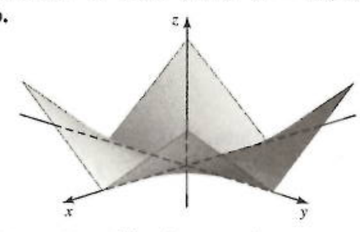


b. $R(10, 10) = 5$ **c.** $R(x, y) = R(y, x)$

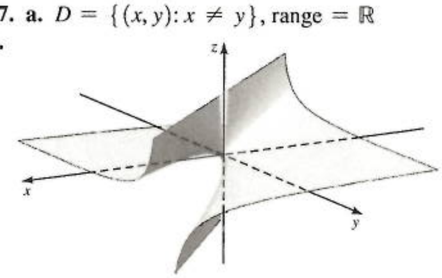
45. a.



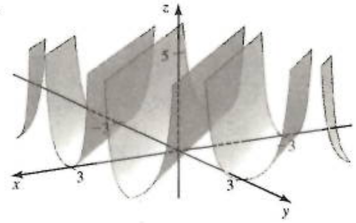
b. $(0, 0), (-5, 3), (4, -1)$
c. $f(0, 0) = 10.17$, $f(-5, 3) = 5.00$, $f(4, -1) = 4.00$
47. $D = \{(x, y, z): x \neq z\}$; all points not on the plane $x = z$
49. $D = \{(x, y, z): y \geq z\}$; all points on or below the plane $y = z$
51. $D = \{(x, y, z): x^2 \leq y\}$; all points on the side of the vertical cylinder $y = x^2$ that contains the positive y-axis **53. a.** False **b.** False **c.** True **55. a.** $D = \mathbb{R}^2$, range $= [0, \infty)$
b.



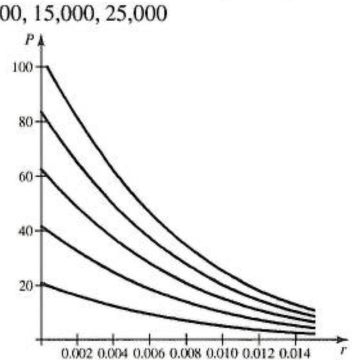
57. a. $D = \{(x, y): x \neq y\}$, range $= \mathbb{R}$
b.



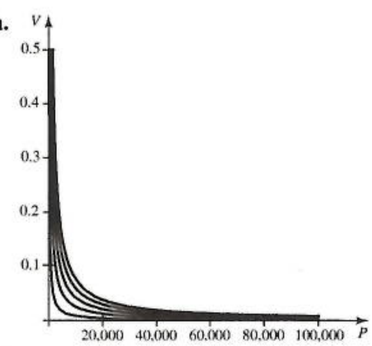
59. a. $D = \{(x, y): y \neq x + \pi/2 + n\pi \text{ for any integer } n\}$, range $= [0, \infty)$ **b.**



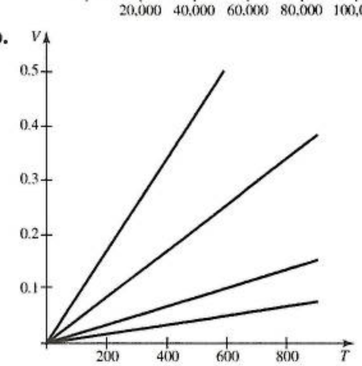
61. Peak at the origin **63.** Depression at $(1, 0)$ **65.** The level curves are $ax + by = d - cz_0$, where z_0 is a constant, which are lines with slope $-a/b$ if $b \neq 0$ or vertical lines if $b = 0$.
67. $z = x^2 + y^2 - C$; paraboloids with vertices at $(0, 0, -C)$
69. $x^2 + 2z^2 = C$; elliptic cylinders parallel to the y-axis
71. a. $P = \dfrac{20{,}000r}{(1 + r)^{240} - 1}$ **b.** $P = \dfrac{Br}{(1 + r)^{240} - 1}$, with $B = 5000, 10{,}000, 15{,}000, 25{,}000$



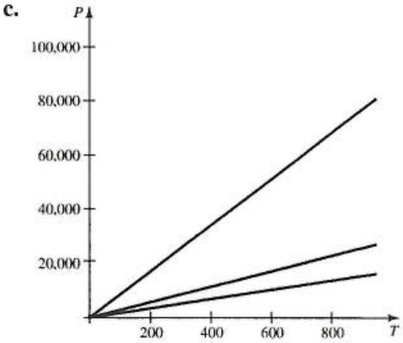
73. a.



b.



c.



75. $D = \{(x, y): x - 1 \leq y \leq x + 1\}$
77. $D = \{(x, y, z): (x \leq z \text{ and } y \geq -z) \text{ or } (x \geq z \text{ and } y \leq -z)\}$

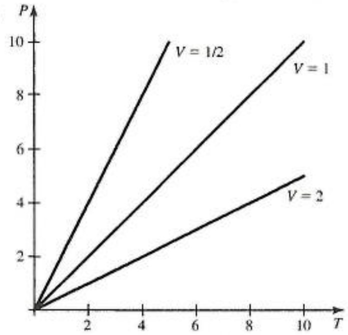
Section 1.3 Exercises, pp. 36–38

1. The values of $f(x, y)$ are arbitrarily close to L for all (x, y) sufficiently close to (a, b). **3.** Because polynomials of n variables are continuous on all of $\mathbb{R}^n$, limits of polynomials can be evaluated with direct substitution. **5.** If the function approaches different values along different paths, the limit does not exist. **7.** f must be defined, the limit must exist, and the limit must equal the function value.
9. At any point where the denominator is nonzero **11.** 101 **13.** 27
15. $1/(2\pi)$ **17.** 2 **19.** 6 **21.** -1 **23.** 2 **25.** $1/(2\sqrt{2}) = \sqrt{2}/4$
27. $L = 1$ along $y = 0$, and $L = -1$ along $x = 0$ **29.** $L = 1$ along $x = 0$, and $L = -2$ along $y = 0$ **31.** $L = 2$ along $y = x$, and $L = 0$ along $y = -x$ **33.** $\mathbb{R}^2$ **35.** All points except $(0, 0)$
37. $\{(x, y): x \neq 0\}$ **39.** All points except $(0, 0)$ **41.** $\mathbb{R}^2$
43. $\mathbb{R}^2$ **45.** $\mathbb{R}^2$ **47.** All points except $(0, 0)$ **49.** $\mathbb{R}^2$ **51.** $\mathbb{R}^2$
53. 6 **55.** -1 **57.** 2 **59. a.** False **b.** False **c.** True **d.** False
61. $\frac{1}{2}$ **63.** 0 **65.** Does not exist **67.** $\frac{1}{4}$ **69.** 0 **71.** 1 **73.** $b = 1$
77. 1 **79.** 1 **81.** 0

Section 1.4 Exercises, pp. 48–51

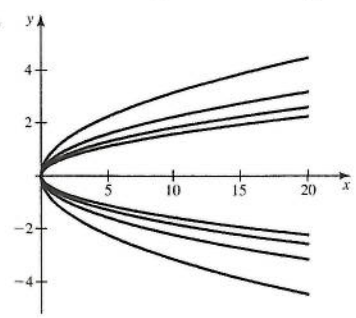
1. $f_x(a, b)$ is the slope of the surface in the direction parallel to the positive x-axis, $f_y(a, b)$ is the slope of the surface in the direction parallel to the positive y-axis, both taken at (a, b).
3. $f_x(x, y) = \cos xy - xy \sin xy$; $f_y(x, y) = -x^2 \sin xy$
5. Think of x and y as being fixed, and take the derivative with respect to the variable z. **7.** $f_x(x, y) = 5y$; $f_y(x, y) = 5x$
9. $f_x(x, y) = \dfrac{1}{y}$; $f_y(x, y) = -\dfrac{x}{y^2}$ **11.** $f_x(x, y) = 6x$; $f_y(x, y) = 12y^2$
13. $f_x(x, y) = 6xy$; $f_y(x, y) = 3x^2$ **15.** $f_x(x, y) = e^y$; $f_y(x, y) = xe^y$
17. $g_x(x, y) = -2y \sin 2xy$; $g_y(x, y) = -2x \sin 2xy$
19. $f_x(x, y) = 2xye^{x^2y}$; $f_y(x, y) = x^2e^{x^2y}$ **21.** $f_w(w, z) = \dfrac{z^2 - w^2}{(w^2 + z^2)^2}$; $f_z(w, z) = -\dfrac{2wz}{(w^2 + z^2)^2}$ **23.** $s_y(y, z) = z^3 \sec^2 yz$; $s_z(y, z) = 2z \tan yz + yz^2 \sec^2 yz$ **25.** $G_s(s, t) = \dfrac{\sqrt{st}(t - s)}{2s(s + t)^2}$; $G_t(s, t) = \dfrac{\sqrt{st}(s - t)}{2t(s + t)^2}$ **27.** $f_x(x, y) = 2yx^{2y-1}$; $f_y(x, y) = 2x^{2y} \ln x$
29. $h_{xx}(x, y) = 6x$; $h_{xy}(x, y) = 2y$; $h_{yx}(x, y) = 2y$; $h_{yy}(x, y) = 2x$
31. $f_{xx}(x, y) = 2y^3$; $f_{xy}(x, y) = 6xy^2$; $f_{yx}(x, y) = 6xy^2$; $f_{yy}(x, y) = 6x^2y$
33. $f_{xx}(x, y) = -16y^3 \sin 4x$; $f_{xy}(x, y) = 12y^2 \cos 4x$; $f_{yx}(x, y) = 12y^2 \cos 4x$; $f_{yy}(x, y) = 6y \sin 4x$
35. $p_{uu}(u, v) = \dfrac{-2u^2 + 2v^2 + 8}{(u^2 + v^2 + 4)^2}$; $p_{uv}(u, v) = -\dfrac{4uv}{(u^2 + v^2 + 4)^2}$; $p_{vu}(u, v) = -\dfrac{4uv}{(u^2 + v^2 + 4)^2}$; $p_{vv}(u, v) = \dfrac{2u^2 - 2v^2 + 8}{(u^2 + v^2 + 4)^2}$

37. $F_{rr}(r, s) = 0$; $F_{rs}(r, s) = e^s$; $F_{sr}(r, s) = e^s$; $F_{ss}(r, s) = re^s$
39. $f_{xy} = 0 = f_{yx}$ **41.** $f_{xy} = -(xy \cos xy + \sin xy) = f_{yx}$
43. $f_{xy} = e^{x+y} = f_{yx}$ **45.** $f_x(x, y, z) = y + z$; $f_y(x, y, z) = x + z$; $f_z(x, y, z) = x + y$ **47.** $h_x(x, y, z) = h_y(x, y, z) = h_z(x, y, z) = -\sin(x + y + z)$
49. $F_u(u, v, w) = \dfrac{1}{v + w}$; $F_v(u, v, w) = F_w(u, v, w) = -\dfrac{u}{(v + w)^2}$
51. $f_w(w, x, y, z) = 2wxy^2$; $f_x(w, x, y, z) = w^2y^2 + y^3z^2$; $f_y(w, x, y, z) = 2w^2xy + 3xy^2z^2$; $f_z(w, x, y, z) = 2xy^3z$
53. $h_w(w, x, y, z) = \dfrac{z}{xy}$; $h_x(w, x, y, z) = -\dfrac{wz}{x^2y}$; $h_y(w, x, y, z) = -\dfrac{wz}{xy^2}$; $h_z(w, x, y, z) = \dfrac{w}{xy}$ **55. a.** $\dfrac{\partial V}{\partial P} = -\dfrac{kT}{P^2}$; volume decreases with pressure at fixed temperature **b.** $\dfrac{\partial V}{\partial T} = \dfrac{k}{P}$; volume increases with temperature at fixed pressure
c.



57. a. No **b.** No **c.** $f_x(0, 0) = f_y(0, 0) = 0$ **d.** f_x and f_y are not continuous at $(0, 0)$. **59. a.** False **b.** False **c.** True
61. 1.41 **63.** 1.55 (answer will vary)
65. $f_x(x, y) = -\dfrac{2x}{1 + (x^2 + y^2)^2}$; $f_y(x, y) = -\dfrac{2y}{1 + (x^2 + y^2)^2}$
67. $h_x(x, y, z) = z(1 + x + 2y)^{z-1}$; $h_y(x, y, z) = 2z(1 + x + 2y)^{z-1}$; $h_z(x, y, z) = (1 + x + 2y)^z \ln(1 + x + 2y)$
69. a. $z_x(x, y) = \dfrac{1}{y^2}$; $z_y(x, y) = -\dfrac{2x}{y^3}$

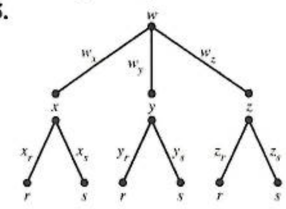
b.



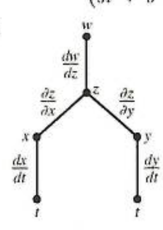
c. z increases as x increases. **d.** z increases as y increases when $y < 0$, z is undefined for $y = 0$, and z decreases as y increases for $y > 0$. **71. a.** $\dfrac{\partial c}{\partial a} = \dfrac{2a - b}{2\sqrt{a^2 + b^2 - ab}}$; $\dfrac{\partial c}{\partial b} = \dfrac{2b - a}{2\sqrt{a^2 + b^2 - ab}}$
b. $\dfrac{\partial c}{\partial a} = \dfrac{2a - b}{2c}$; $\dfrac{\partial c}{\partial b} = \dfrac{2b - a}{2c}$ **c.** $a > \frac{1}{2}b$
73. a. $\varphi_x(x, y) = -\dfrac{2x}{(x^2 + (y - 1)^2)^{3/2}} - \dfrac{x}{(x^2 + (y + 1)^2)^{3/2}}$;
$\varphi_y(x, y) = -\dfrac{2(y - 1)}{(x^2 + (y - 1)^2)^{3/2}} - \dfrac{y + 1}{(x^2 + (y + 1)^2)^{3/2}}$
b. They both approach zero. **c.** $\varphi_x(0, y) = 0$
d. $\varphi_y(x, 0) = \dfrac{1}{(x^2 + 1)^{3/2}}$
75. a. $\dfrac{\partial R}{\partial R_1} = \dfrac{R_2^2}{(R_1 + R_2)^2}$; $\dfrac{\partial R}{\partial R_2} = \dfrac{R_1^2}{(R_1 + R_2)^2}$
b. $\dfrac{\partial R}{\partial R_1} = \dfrac{R^2}{R_1^2}$; $\dfrac{\partial R}{\partial R_2} = \dfrac{R^2}{R_2^2}$ **c.** Increase **d.** Decrease
77. $\dfrac{\partial^2 u}{\partial t^2} = -4c^2 \cos(2(x + ct)) = c^2 \dfrac{\partial^2 u}{\partial x^2}$
79. $\dfrac{\partial^2 u}{\partial t^2} = c^2Af''(x + ct) + c^2Bg''(x - ct) = c^2 \dfrac{\partial^2 u}{\partial x^2}$
81. $u_{xx} = 6x$; $u_{yy} = -6x$
83. $u_{xx} = \dfrac{2(x - 1)y}{((x - 1)^2 + y^2)^2} - \dfrac{2(x + 1)y}{((x + 1)^2 + y^2)^2}$;
$u_{yy} = -\dfrac{2(x - 1)y}{((x - 1)^2 + y^2)^2} + \dfrac{2(x + 1)y}{((x + 1)^2 + y^2)^2}$
85. $u_t = -16e^{-4t} \cos 2x = u_{xx}$ **87.** $u_t = -a^2Ae^{-a^2t} \cos ax = u_{xx}$
89. $\varepsilon_1 = \Delta y$, $\varepsilon_2 = 0$ or $\varepsilon_1 = 0$, $\varepsilon_2 = \Delta x$ **91. a.** f is continuous at $(0, 0)$. **b.** f is not differentiable at $(0, 0)$. **c.** $f_x(0, 0) = f_y(0, 0) = 0$ **d.** f_x and f_y are not continuous at $(0, 0)$. **e.** Theorem 13.5 does not apply because f_x and f_y are not continuous at $(0, 0)$; Theorem 13.6 does not apply because f is not differentiable at $(0, 0)$. **93. a.** $f_x(x, y) = -h(x)$; $f_y(x, y) = h(y)$
b. $f_x(x, y) = yh(xy)$; $f_y(x, y) = xh(xy)$

Section 1.5 Exercises, pp. 58–61

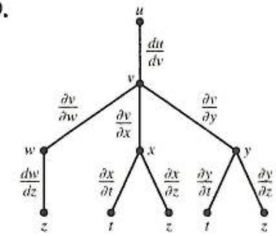
1. One dependent, two intermediate, and one independent variable
3. Multiply each of the partial derivatives of w by the t-derivative of the corresponding function and add all these expressions.
5.



7. $4t^3 + 3t^2$ **9.** $z'(t) = 2t \sin 4t^3 + 12t^4 \cos 4t^3$
11. $w'(t) = -\sin t \sin 3t^4 + 12t^3 \cos t \cos 3t^4$
13. $w'(t) = 20t^4 \sin(t + 1) + 4t^5 \cos(t + 1)$
15. $U'(t) = \dfrac{1 + 2t + 3t^2}{t + t^2 + t^3}$
17. a. $V'(t) = 2\pi r(t)h(t)r'(t) + \pi r(t)^2h'(t)$ **b.** $V'(t) = 0$
c. The volume remains constant. **19.** $z_s = 2(s - t) \sin t^2$; $z_t = 2(s - t)(t(s - t) \cos t^2 - \sin t^2)$
21. $z_s = 2s - 3s^2 - 2st + t^2$; $z_t = -s^2 - 2t + 2st + 3t^2$
23. $z_s = (t + 1)e^{st+s+t}$; $z_t = (s + 1)e^{st+s+t}$
25. $w_s = -\dfrac{2t(t + 1)}{(st + s - t)^2}$; $w_t = \dfrac{2s}{(st + s - t)^2}$
27.



$$\frac{dw}{dt} = \frac{dw}{dz}\left(\frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}\right)$$

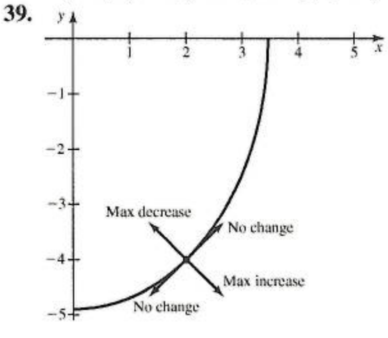
29.  $\dfrac{\partial u}{\partial z} = \dfrac{du}{dv}\left(\dfrac{\partial v}{\partial w}\dfrac{dw}{dz} + \dfrac{\partial v}{\partial x}\dfrac{\partial x}{\partial z} + \dfrac{\partial v}{\partial y}\dfrac{\partial y}{\partial z}\right)$

31. $\dfrac{dy}{dx} = \dfrac{x}{2y}$ **33.** $\dfrac{dy}{dx} = -\dfrac{y}{x}$ **35.** $\dfrac{dy}{dx} = -\dfrac{x+y}{2y^3+x}$
37. $\dfrac{\partial s}{\partial x} = \dfrac{2x}{\sqrt{x^2+y^2}}; \dfrac{\partial s}{\partial y} = \dfrac{2y}{\sqrt{x^2+y^2}}$ **39.** $f_{ss} = 2(3s+t)$; $f_{st} = 2(s-t)$; $f_{tt} = -2(s+3t)$ **41.** $f_{ss} = \dfrac{4t^2(-3s^2+t^2)}{(s^2+t^2)^3}$; $f_{st} = \dfrac{8st(s^2-t^2)}{(s^2+t^2)^3}$; $f_{tt} = -\dfrac{4(s^4-3s^2t^2)}{(s^2+t^2)^3}$
43. $f''(s) = 4\left(\dfrac{6}{s^4} - \dfrac{2}{s^3} - 1 - 9s + 9s^2\right)$ **45. a.** False **b.** False
47. $z'(t) = -\dfrac{2t+2}{(t+2t)} - \dfrac{3t^2}{(t^3-2)}$ **49.** $w'(t) = 0$
51. $\dfrac{\partial z}{\partial x} = -\dfrac{z^2}{x^2}$ **53. a.** $w'(t) = af_x + bf_y + cf_z$
b. $w'(t) = ayz + bxz + cxy = 3abct^2$
c. $w'(t) = \sqrt{a^2+b^2+c^2}\,\dfrac{t}{|t|}$
d. $w''(t) = a^2f_{xx} + b^2f_{yy} + c^2f_{zz} + 2abf_{xy} + 2acf_{xz} + 2bcf_{yz}$
55. $\dfrac{\partial z}{\partial x} = -\dfrac{y+z}{x+y}; \dfrac{\partial z}{\partial y} = -\dfrac{x+z}{x+y}$ **57.** $\dfrac{\partial z}{\partial x} = -\dfrac{yz+1}{xy-1}; \dfrac{\partial z}{\partial y} = -\dfrac{xz+1}{xy-1}$
59. a. $z'(t) = -2x\sin t + 8y\cos t = 3\sin 2t$ **b.** $0 < t < \pi/2$ and $\pi < t < 3\pi/2$ **61. a.** $z'(t) = \dfrac{(x+y)e^{-t}}{\sqrt{1-x^2-y^2}} = \dfrac{2e^{-2t}}{\sqrt{1-2e^{-2t}}}$
b. All $t \ge \frac{1}{2}\ln 2$ **63.** $E'(t) = mx'x'' + my'y'' + mgy' = 0$
65. a. The volume increases. **b.** The volume decreases.
67. a. $\dfrac{\partial P}{\partial V} = -\dfrac{P}{V}; \dfrac{\partial T}{\partial P} = \dfrac{V}{k}; \dfrac{\partial V}{\partial T} = \dfrac{k}{P}$ **b.** Follows directly from part (a)
69. a. $w'(t) = \dfrac{2t(t^2+1)\cos 2t - (t^2-1)\sin 2t}{2(t^2+1)^2}$
b. Max value of $t \approx 0.838$, $(x, y, z) \approx (0.669, 0.743, 0.838)$
71. a. $z_x = \dfrac{x}{r}z_r - \dfrac{y}{r^2}z_\theta$; $z_y = \dfrac{y}{r}z_r + \dfrac{x}{r^2}z_\theta$
b. $z_{xx} = \dfrac{x^2}{r^2}z_{rr} + \dfrac{y^2}{r^4}z_{\theta\theta} - \dfrac{2xy}{r^3}z_{r\theta} + \dfrac{y^2}{r^3}z_r + \dfrac{2xy}{r^4}z_\theta$
c. $z_{yy} = \dfrac{y^2}{r^2}z_{rr} + \dfrac{x^2}{r^4}z_{\theta\theta} + \dfrac{2xy}{r^3}z_{r\theta} + \dfrac{x^2}{r^3}z_r - \dfrac{2xy}{r^4}z_\theta$
d. Add the results from (b) and (c). **73. a.** $\left(\dfrac{\partial z}{\partial x}\right)_y = -\dfrac{F_x}{F_z}$
b. $\left(\dfrac{\partial y}{\partial z}\right)_x = -\dfrac{F_z}{F_y}; \left(\dfrac{\partial x}{\partial y}\right)_z = -\dfrac{F_y}{F_x}$ **c.** Follows from (a) and (b) by multiplication **d.** $\left(\dfrac{\partial w}{\partial x}\right)_{y,z}\left(\dfrac{\partial z}{\partial w}\right)_{x,y}\left(\dfrac{\partial y}{\partial z}\right)_{x,w}\left(\dfrac{\partial x}{\partial y}\right)_{z,w} = 1$
75. a. $\left(\dfrac{\partial w}{\partial x}\right)_y = f_x + f_z\dfrac{dz}{dx} = 18$ **b.** $\left(\dfrac{\partial w}{\partial x}\right)_z = f_x + f_y\dfrac{dy}{dx} = 8$
d. $\left(\dfrac{\partial w}{\partial y}\right)_x = -5; \left(\dfrac{\partial w}{\partial y}\right)_z = 4; \left(\dfrac{\partial w}{\partial z}\right)_x = \dfrac{5}{2}; \left(\dfrac{\partial w}{\partial z}\right)_y = \dfrac{9}{2}$

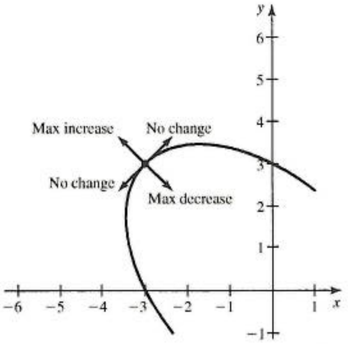
Section 1.6 Exercises, pp. 70–73

1. Form the dot product between the unit direction vector **u** and the gradient of the function. **3.** Direction of steepest ascent **5.** The gradient is orthogonal to the level curves of f.
7. a.

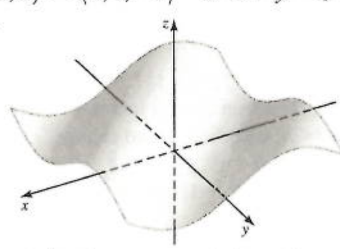
	$(a,b) = (2,0)$	$(a,b) = (0,2)$	$(a,b) = (1,1)$
$\mathbf{u} = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle$	$-\sqrt{2}$	$-2\sqrt{2}$	$-3\sqrt{2}/2$
$\mathbf{v} = \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle$	$\sqrt{2}$	$-2\sqrt{2}$	$-\sqrt{2}/2$
$\mathbf{w} = \langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\rangle$	$\sqrt{2}$	$2\sqrt{2}$	$3\sqrt{2}/2$

b. The function is decreasing at $(2, 0)$ in the direction of **u** and increasing at $(2, 0)$ in the directions of **v** and **w**.
9. $\nabla f(x, y) = \langle 6x, -10y\rangle$, $\nabla f(2, -1) = \langle 12, 10\rangle$
11. $\nabla g(x, y) = \langle 2(x - 4xy - 4y^2), -4x(x + 4y)\rangle$, $\nabla g(-1, 2) = \langle -18, 28\rangle$ **13.** $\nabla f(x, y) = e^{2xy}\langle 1 + 2xy, 2x^2\rangle$, $\nabla f(1, 0) = \langle 1, 2\rangle$ **15.** $\nabla F(x, y) = -2e^{-x^2-2y^2}\langle x, 2y\rangle$, $\nabla F(-1, 2) = 2e^{-9}\langle 1, -4\rangle$ **17.** -6 **19.** $\frac{27}{2} - 6\sqrt{3}$
21. $-\dfrac{2}{\sqrt{5}}$ **23.** -2 **25.** 0 **27. a.** Direction of steepest ascent: $\dfrac{1}{\sqrt{65}}\langle 1, 8\rangle$; direction of steepest descent: $-\dfrac{1}{\sqrt{65}}\langle 1, 8\rangle$
b. $\langle -8, 1\rangle$ **29. a.** Direction of steepest ascent: $\dfrac{1}{\sqrt{5}}\langle -2, 1\rangle$; direction of steepest descent: $\dfrac{1}{\sqrt{5}}\langle 2, -1\rangle$ **b.** $\langle 1, 2\rangle$
31. a. Direction of steepest ascent: $\dfrac{1}{\sqrt{2}}\langle 1, -1\rangle$; direction of steepest descent: $\dfrac{1}{\sqrt{2}}\langle -1, 1\rangle$ **b.** $\langle 1, 1\rangle$
33. a. $\nabla f(3, 2) = -12\mathbf{i} - 12\mathbf{j}$ **b.** Direction of max increase, $\theta = \dfrac{5\pi}{4}$; direction of max decrease, $\theta = \dfrac{\pi}{4}$; directions of no change, $\theta = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$ **c.** $g(\theta) = -12\cos\theta - 12\sin\theta$ **d.** $\theta = \frac{5\pi}{4}$, $g\left(\frac{5\pi}{4}\right) = 12\sqrt{2}$ **e.** $\nabla f(3, 2) = 12\sqrt{2}\,\langle \cos\frac{5\pi}{4}, \sin\frac{5\pi}{4}\rangle$, $|\nabla f(3, 2)| = 12\sqrt{2}$ **35. a.** $\nabla f(\sqrt{3}, 1) = \frac{\sqrt{6}}{6}\langle \sqrt{3}, 1\rangle$
b. Direction of max increase, $\theta = \frac{\pi}{6}$; direction of max decrease, $\theta = \frac{7\pi}{6}$; directions of no change, $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$
c. $g(\theta) = \frac{\sqrt{2}}{2}\cos\theta + \frac{\sqrt{6}}{6}\sin\theta$ **d.** $\theta = \frac{\pi}{6}, g\left(\frac{\pi}{6}\right) = \frac{\sqrt{6}}{3}$
e. $\nabla f(\sqrt{3}, 1) = \frac{\sqrt{6}}{3}\langle \cos\frac{\pi}{6}, \sin\frac{\pi}{6}\rangle$, $|\nabla f(\sqrt{3}, 1)| = \frac{\sqrt{6}}{3}$
37. a. $\nabla F(-1, 0) = \frac{2}{e}\mathbf{i}$ **b.** Direction of max increase, $\theta = 0$; direction of max decrease, $\theta = \pi$; directions of no change, $\theta = \pm\frac{\pi}{2}$ **c.** $g(\theta) = \frac{2}{e}\cos\theta$ **d.** $\theta = 0, g(0) = \frac{2}{e}$
e. $\nabla F(-1, 0) = \frac{2}{e}\langle \cos 0, \sin 0\rangle$, $|\nabla F(-1, 0)| = \frac{2}{e}$
39. 

41.



43. $y' = 0$ **45.** Vertical tangent **47.** $y' = -2/\sqrt{3}$ **49.** Vertical tangent **51. a.** $\nabla f = \langle 1, 0\rangle$ **b.** $x = 4 - t, y = 4, t \ge 0$ **c.** $x = 4 - t$, $y = 4$, $z = 8 - t$, for $t \ge 0$
53. a. $\nabla f = \langle -2x, -4y\rangle$ **b.** $y = x^2, x \ge 1$ **c.** $\mathbf{r}(t) = \langle t, t^2, 4 - t^2 - 2t^4\rangle$, for $t \ge 1$
55. a. $\nabla f(x, y, z) = 2x\mathbf{i} + 4y\mathbf{j} + 8z\mathbf{k}$, $\nabla f(1, 0, 4) = 2\mathbf{i} + 32\mathbf{k}$ **b.** $\frac{1}{\sqrt{257}}(\mathbf{i} + 16\mathbf{k})$ **c.** $2\sqrt{257}$ **d.** $17\sqrt{2}$ **57. a.** $\nabla f(x, y, z) = 4yz\mathbf{i} + 4xz\mathbf{j} + 4xy\mathbf{k}$, $\nabla f(1, -1, -1) = 4\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$ **b.** $\frac{1}{\sqrt{3}}(\mathbf{i} - \mathbf{j} - \mathbf{k})$ **c.** $4\sqrt{3}$ **d.** $\frac{4}{\sqrt{3}}$
59. a. $\nabla f(x, y, z) = \cos(x + 2y - z)(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$, $\nabla f\left(\frac{\pi}{6}, \frac{\pi}{6}, -\frac{\pi}{6}\right) = -\frac{1}{2}\mathbf{i} - \mathbf{j} + \frac{1}{2}\mathbf{k}$ **b.** $\frac{1}{\sqrt{6}}(-\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ **c.** $\sqrt{6}/2$ **d.** $-\frac{1}{2}$
61. a. $\nabla f(x, y, z) = \frac{2}{1 + x^2 + y^2 + z^2}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$, $\nabla f(1, 1, -1) = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} - \frac{1}{2}\mathbf{k}$ **b.** $\frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} - \mathbf{k})$ **c.** $\frac{\sqrt{3}}{2}$ **d.** $\frac{5}{6}$ **63. a.** False **b.** False **c.** False **d.** True **65.** $\pm\frac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$
67. $\pm\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$ **69.** $x = x_0 + at, y = y_0 + bt$
71. a. $\nabla f(x, y, z) = \langle 2x, 2y, 2z\rangle$, $\nabla f(1, 1, 1) = \langle 2, 2, 2\rangle$ **b.** $x + y + z = 3$ **73. a.** $\nabla f(x, y, z) = e^{x+y-z}\langle 1, 1, -1\rangle$, $\nabla f(1, 1, 2) = \langle 1, 1, -1\rangle$ **b.** $x + y - z = 0$
75. a.



b. $\mathbf{v} = \pm\langle 1, 1\rangle$ **c.** $\mathbf{v} = \pm\langle 1, -1\rangle$
79. $\langle u, v\rangle = \langle \pi\cos\pi x \sin 2\pi y, 2\pi \sin\pi x \cos 2\pi y\rangle$
83. $\nabla f(x, y) = \frac{1}{(x^2 + y^2)^2}\langle y^2 - x^2 - 2xy, x^2 - y^2 - 2xy\rangle$
85. $\nabla f(x, y, z) = -\frac{1}{\sqrt{25 - x^2 - y^2 - z^2}}\langle x, y, z\rangle$
87. $\nabla f(x, y, z) = \frac{(y + xz)\langle 1, z, y\rangle - (x + yz)\langle z, 1, x\rangle}{(y + xz)^2} = \frac{1}{(y + xz)^2}\langle y(1 - z^2), x(z^2 - 1), y^2 - x^2\rangle$

Section 1.7 Exercises, pp. 80–83

1. The gradient of f is a multiple of $\mathbf{n}$.
3. $F_x(a, b, c)(x - a) + F_y(a, b, c)(y - b) + F_z(a, b, c)(z - c) = 0$
5. Multiply the change in x by $f_x(a, b)$ and the change in y by $f_y(a, b)$, and add both terms to f. **7.** $dz = f_x(x, y)\,dx + f_y(x, y)\,dy$
9. $2x + y + z = 4$; $4x + y + z = 7$
11. $x + y + z = 6$; $3x + 4y + z = 12$
13. $x + \frac{1}{2}y + \sqrt{3}z = 2 + \frac{\sqrt{3}\pi}{6}$ and $\frac{1}{2}x + y + \sqrt{3}z = \frac{5\sqrt{3}\pi}{6} - 2$
15. $\frac{1}{2}x + \frac{2}{3}y + 2\sqrt{3}z = -2$ and $x - 2y + 2\sqrt{14}z = 2$
17. $z = -8x - 4y + 16$ and $z = 4x + 2y + 7$
19. $z = y + 1$ and $z = x + 1$ **21.** $z = 8x - 4y - 4$ and $z = -x - y - 1$ **23.** $z = \frac{7}{25}x - \frac{1}{25}y - \frac{2}{5}$ and $z = -\frac{7}{25}x + \frac{1}{25}y + \frac{6}{5}$
25. a. $L(x, y) = 4x + y - 6$ **b.** $L(2.1, 2.99) = 5.39$
27. a. $L(x, y) = -6x - 4y + 7$ **b.** $L(3.1, -1.04) = -7.44$
29. a. $L(x, y, z) = x + y + 2z$ **b.** $L(0.1, -0.2, 0.2) = 0.3$
31. $dz = -6dx - 5dy = -0.1$ **33.** $dz = dx + dy = 0.05$
35. a. The surface area decreases. **b.** Impossible to say **c.** $dS \approx 53.3$ **d.** $dS = 33.95$ **e.** $RdR = rdr$ **37.** $\frac{dA}{A} = 3.5\%$
39. $dw = (y^2 + 2xz)\,dx + (2xy + z^2)\,dy + (x^2 + 2yz)\,dz$
41. $dw = \frac{dx}{y + z} - \frac{u + x}{(y + z)^2}dy - \frac{u + x}{(y + z)^2}dz + \frac{du}{y + z}$
43. a. $dc = 0.035$ **b.** When $\theta = \frac{\pi}{20}$ **45. a.** True **b.** True **c.** False **47.** $z = \frac{1}{2}x + \frac{1}{2}y + \frac{\pi}{4} - 1$
49. $\frac{1}{6}(x - \pi) + \frac{\pi}{6}(y - 1) + \pi\left(z - \frac{1}{6}\right) = 0$ **51.** $(1, -1, 1)$ and $(1, -1, -1)$ **53.** Points with $x = 0, \pm\frac{\pi}{2}, \pm\pi$ and $y = \pm\frac{\pi}{2}$, or points with $x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$ and $y = 0, \pm\pi$ **55. a.** $dS = 0.749$ **b.** More sensitive to changes in r **57. a.** $dA = \frac{2}{1225} = 0.00163$ **b.** No. The batting average increases more if he gets a hit than it would decrease if he fails to get a hit. **c.** Yes. The answer depends on whether A is less than 0.500 or greater than 0.500.
59. a. $dV = \frac{21}{5000} = 0.0042$ **b.** $\frac{dV}{V} = -4\%$ **c.** $2p\%$
61. a. $f_r = n(1 - r)^{n-1}, f_n = -(1 - r)^n \ln(1 - r)$ **b.** $\Delta P \approx 0.027$ **c.** $\Delta P \approx 2 \times 10^{-20}$ **63.** $dR = 7/540 \approx 0.013\Omega$
65. a. Apply the Chain Rule. **b.** Follows directly from (a) **c.** $d(\ln(xy)) = \frac{dx}{x} + \frac{dy}{y}$ **d.** $d(\ln(x/y)) = \frac{dx}{x} - \frac{dy}{y}$ **e.** $\frac{df}{f} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2} + \cdots + \frac{dx_n}{x_n}$

Section 1.8 Exercises, pp. 94–96

1. It is locally the highest point on the surface; you cannot get to a higher point in any direction. **3.** The partial derivatives are both zero or do not exist. **5.** The discriminant is a determinant; it is defined as $D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}^2(a, b)$. **7.** f has an absolute minimum value on R at (a, b) if $f(a, b) \le f(x, y)$ for all (x, y) in R.
9. $(0, 0)$ **11.** $(\frac{2}{3}, 4)$ **13.** $(0, 0), (2, 2)$, and $(-2, -2)$
15. $(0, 2), (\pm 1, 2)$ **17.** $(-3, 0)$ **19.** Local min at $(0, 0)$
21. Saddle point at $(0, 0)$ **23.** Saddle point at $(0, 0)$; local min at $(1, 1)$ and at $(-1, -1)$ **25.** Local min at $(2, 0)$ **27.** Saddle point at $(0, 0)$; local max at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$; local min at

$\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ **29.** Local min: $(-1, 0)$; local max: $(1, 0)$ **31.** Saddle point: $(0, 1)$; local min: $(\pm 2, 0)$ **33.** Saddle point at $(0, 0)$ **35.** Height $= 32$ in, base is 16 in $\times$ 16 in; volume is 8192 in^3 **37.** 2 m $\times$ 2 m $\times$ 1 m **39.** Critical point at $(0, 0)$, $D(0, 0) = 0$, absolute min **41.** Critical points along the x- and y-axes, all absolute min **43.** Absolute min: $0 = f(0, 1)$; absolute max: $9 = f(0, -2)$ **45.** Absolute min: $4 = f(0, 0)$; absolute max: $7 = f(\pm 1, \pm 1)$ **47.** Absolute min: $0 = f(1, 0)$; absolute max: $3 = f(1, 1) = f(1, -1)$ **49.** Absolute min: $1 = f(1, -2) = f(1, 0)$; absolute max: $4 = f(1, -1)$ **51.** Absolute min: $0 = f(0, 0)$; absolute max: $\frac{7}{8} = f\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$ **53.** Absolute min: $-4 = f(0, 0)$; no absolute max on R **55.** Absolute max: $2 = f(0, 0)$; no absolute min on R **57.** $P\left(-\frac{5}{3}, \frac{4}{3}, \frac{13}{3}\right)$ **59.** $\left(\frac{1}{2}, \frac{1}{4}\right)$; $\left(\frac{7}{8}, -\frac{1}{8}\right)$ **61. a.** True **b.** False **c.** True **d.** True **63.** Local min at $(0.3, -0.3)$; saddle point at $(0, 0)$ **65.** $P\left(\frac{4}{3}, \frac{2}{3}, \frac{4}{3}\right)$ **67. a.–d.** $x = y = z = \frac{200}{3}$

69. a. $P\left(1, \frac{1}{3}\right)$ **b.** $P\left(\frac{1}{3}(x_1 + x_2 + x_3), \frac{1}{3}(y_1 + y_2 + y_3)\right)$
c. $P(\bar{x}, \bar{y})$, where $\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k$ and $\bar{y} = \frac{1}{n}\sum_{k=1}^{n} y_k$
d. $d(x, y) = \sqrt{x^2 + y^2} + \sqrt{(x - 2)^2 + y^2} + \sqrt{(x - 1)^2 + (y - 1)^2}$. The absolute min of this function is $1 + \sqrt{3} = f\left(1, \frac{1}{\sqrt{3}}\right)$. **73.** $y = \frac{22}{13}x + \frac{46}{13}$ **75.** $a = b = c = 3$

77. a. $\nabla d_1(x, y) = \frac{x - x_1}{d_1(x, y)}\mathbf{i} + \frac{y - y_1}{d_1(x, y)}\mathbf{j}$
b. $\nabla d_2(x, y) = \frac{x - x_2}{d_2(x, y)}\mathbf{i} + \frac{y - y_2}{d_2(x, y)}\mathbf{j}$;
$\nabla d_3(x, y) = \frac{x - x_3}{d_3(x, y)}\mathbf{i} + \frac{y - y_3}{d_3(x, y)}\mathbf{j}$
c. Follows from $\nabla f = \nabla d_1 + \nabla d_2 + \nabla d_3$ **d.** Three unit vectors add to zero. **e.** P is the vertex at the large angle. **f.** $P(0.255457, 0.304504)$
79. a. Local max at $(1, 0)$, $(-1, 0)$ **b.** $(1, 0)$ and $(-1, 0)$

Section 1.9 Exercises, pp. 102–104

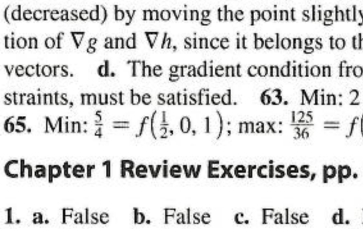
1. The level curve of f must be tangent to the curve $g = 0$ at the optimal point; therefore, the gradients are parallel. **3.** $2x = 2\lambda$, $2y = 3\lambda$, $2z = -5\lambda$, $2x + 3y - 5z + 4 = 0$ **5.** Min: $-2\sqrt{5}$ at $\left(-\frac{2}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right)$; max: $2\sqrt{5}$ at $\left(\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$ **7.** Min: -2 at $(-1, -1)$; max: 2 at $(1, 1)$ **9.** Min: -3 at $(-\sqrt{3}, \sqrt{3})$ and $(\sqrt{3}, -\sqrt{3})$; max: 9 at $(3, 3)$ and $(-3, -3)$ **11.** Min: e^{-16} at $(2\sqrt{2}, -2\sqrt{2})$ and $(-2\sqrt{2}, 2\sqrt{2})$; max: e^{16} at $(-2\sqrt{2}, -2\sqrt{2})$ and $(2\sqrt{2}, 2\sqrt{2})$ **13.** Min: -16 at $(\pm 2, 0)$; max: 2 at $(0, \pm\sqrt{2})$ **15.** Min: $-2\sqrt{11}$ at $\left(-\frac{2}{\sqrt{11}}, -\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}\right)$; max: $2\sqrt{11}$ at $\left(\frac{2}{\sqrt{11}}, \frac{6}{\sqrt{11}}, -\frac{2}{\sqrt{11}}\right)$ **17.** Min: $-\frac{\sqrt{5}}{2}$ at $\left(-\frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right)$; max: $\frac{\sqrt{5}}{2}$ at $\left(\frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right)$ **19.** Min: $\frac{1}{3}$ at $\left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, 0\right)$ and $\left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, 0\right)$; max: 1 at $(0, 0, \pm 1)$
21. Min: -10 at $(-5, 0, 0)$; max: $\frac{29}{2}$ at $\left(2, 0, \pm\sqrt{\frac{21}{2}}\right)$
23. Min: $6\sqrt[3]{2} = f(\pm\sqrt[3]{4}, \pm\sqrt[3]{4}, \pm\sqrt[3]{4})$; no max
25. 18 in $\times$ 18 in $\times$ 36 in **27.** Min: 0.6731; max: 1.1230

29. 2×1 **31.** $\left(-\frac{3}{17}, \frac{29}{17}, -3\right)$ **33.** Min: $\sqrt{38 - 6\sqrt{29}}$ (or $\sqrt{29} - 3$); max: $\sqrt{38 + 6\sqrt{29}}$ (or $\sqrt{29} + 3$) **35.** $\ell = 3$ and $g = \frac{3}{2}$; $U = 15\sqrt{2}$ **37.** $\ell = \frac{16}{5}$ and $g = 1$; $U = 20.287$
39. a. True **b.** False **41.** $\frac{\sqrt{6}}{3}$ m $\times$ $\frac{\sqrt{6}}{3}$ m $\times$ $\frac{\sqrt{6}}{6}$ m
43. $2 \times 1 \times \frac{2}{3}$ **45.** $P\left(\frac{4}{3}, \frac{2}{3}, \frac{4}{3}\right)$ **47.** Min: $-\frac{7 + \sqrt{661}}{2}$; max: $\frac{\sqrt{661} - 7}{2}$ **49.** Min: 0; max: $6 + 4\sqrt{2}$ **51.** Min: 1; max: 8
53. $K = 7.5$ and $L = 5$ **55.** $K = aB/p$ and $L = (1 - a)B/q$
57. Max: 8 **59.** Max: $\sqrt{a_1^2 + a_2^2 + a_3^2 + \cdots + a_n^2}$
61. a. Gradients are perpendicular to level surfaces. **b.** If the gradient was not in the plane spanned by ∇g and ∇h, f could be increased (decreased) by moving the point slightly. **c.** ∇f is a linear combination of ∇g and ∇h, since it belongs to the plane spanned by these two vectors. **d.** The gradient condition from part (c), as well as the constraints, must be satisfied. **63.** Min: $2 - 4\sqrt{2}$; max: $2 + 4\sqrt{2}$
65. Min: $\frac{5}{4} = f\left(\frac{1}{2}, 0, 1\right)$; max: $\frac{125}{36} = f\left(-\frac{5}{6}, 0, \frac{5}{3}\right)$

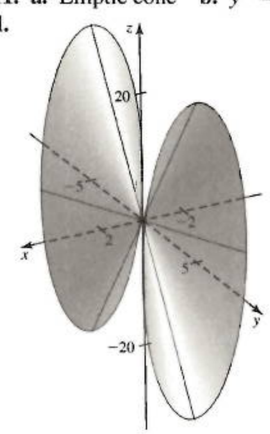
Chapter 1 Review Exercises, pp. 104–108

1. a. False **b.** False **c.** False **d.** False **e.** True
3. a. $18x - 9y + 2z = 6$ **b.** $x = \frac{1}{3}$, $y = -\frac{2}{3}$, $z = 3$
c.

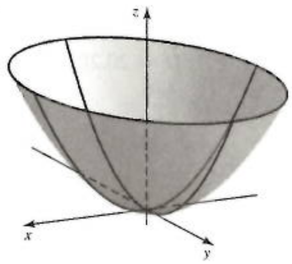
5. $x = t$, $y = 12 - 9t$, $z = -6 + 6t$ **7.** $3x + y + 7z = 4$
9. a. Hyperbolic paraboloid **b.** $y^2 = 4x^2$, $z = \frac{x^2}{36}$, $z = -\frac{y^2}{144}$
c. $x = y = z = 0$
d.



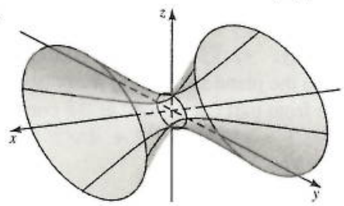
11. a. Elliptic cone **b.** $y^2 = 4x^2$, origin, $y^2 = \frac{z^2}{25}$ **c.** Origin
d.



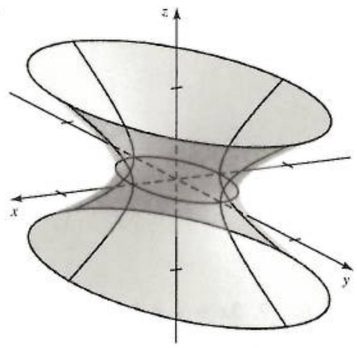
13. a. Elliptic paraboloid **b.** Origin, $z = \frac{x^2}{16}, z = \frac{y^2}{36}$ **c.** Origin
d.



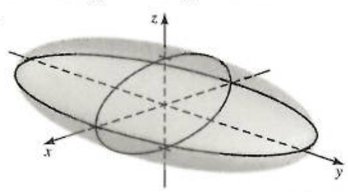
15. a. Hyperboloid of one sheet **b.** $y^2 - 2x^2 = 1, 4z^2 - 2x^2 = 1, y^2 + 4z^2 = 1$ **c.** No x-intercept, $y = \pm 1, z = \pm \frac{1}{2}$
d.



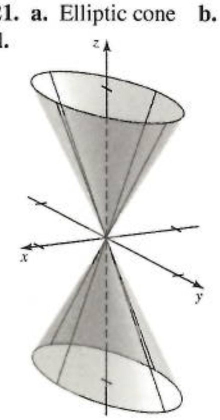
17. a. Hyperboloid of one sheet
b. $\frac{x^2}{4} + \frac{y^2}{16} = 4, \frac{x^2}{4} - z^2 = 4, \frac{y^2}{16} - z^2 = 4$
c. $x = \pm 4, y = \pm 8$, no z-intercept
d.



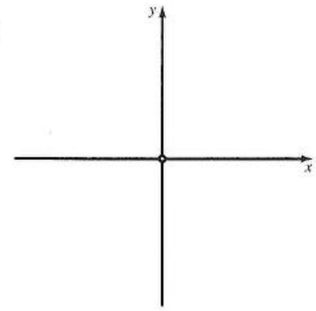
19. a. Ellipsoid **b.** $\frac{x^2}{4} + \frac{y^2}{16} = 4, \frac{x^2}{4} + z^2 = 4, \frac{y^2}{16} + z^2 = 4$
c. $x = \pm 4, y = \pm 8, z = \pm 2$
d.



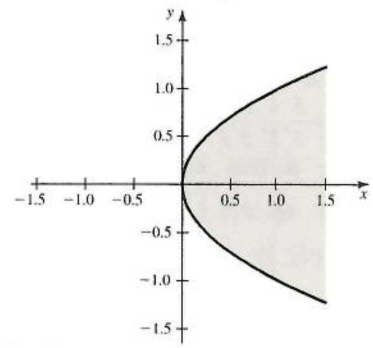
21. a. Elliptic cone **b.** Origin, $\frac{x^2}{9} = \frac{z^2}{64}, \frac{y^2}{49} = \frac{z^2}{64}$. **c.** Origin
d.



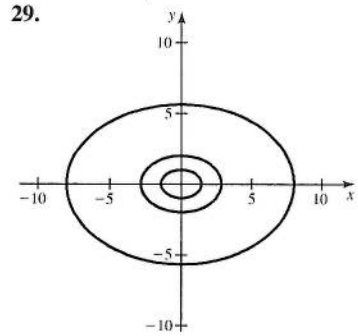
23. $D = \{(x, y): (x, y) \neq (0, 0)\}$



25. $D = \{(x, y): x \geq y^2\}$



27. a. A **b.** D **c.** C **d.** B **29.**



31. 2 **33.** Does not exist **35.** $\frac{2}{3}$ **37.** 4
39. $f_x = 6xy^5; f_y = 15x^2y^4$ **41.** $f_x = \frac{2xy^2}{(x^2 + y^2)^2}; f_y = -\frac{2x^2y}{(x^2 + y^2)^2}$
43. $f_x = y(1 + xy)e^{xy}; f_y = x(1 + xy)e^{xy}$ **45.** $f_x = e^{x+2y+3z}$; $f_y = 2e^{x+2y+3z}; f_z = 3e^{x+2y+3z}$ **47.** $\frac{\partial^2 u}{\partial x^2} = 6y = -\frac{\partial^2 u}{\partial y^2}$ **49. a.** V increases with R if r is fixed, $V_R > 0$; V decreases if r increases and R is fixed, $V_r < 0$. **b.** $V_r = -4\pi r^2; V_R = 4\pi R^2$ **c.** The volume increases more if R is increased. **51.** $w'(t) = -\frac{\cos t \sin t}{\sqrt{1 + \cos^2 t}}$
53. $w_r = \frac{3r + s}{r(r + s)}; w_s = \frac{r + 3s}{s(r + s)}; w_t = \frac{1}{t}$
55. $\frac{dy}{dx} = -\frac{2xy}{2y^2 + (x^2 + y^2)\ln(x^2 + y^2)}$
57. a. $z'(t) = -24 \sin t \cos t = -12 \sin 2t$
b. $z'(t) > 0$ for $\frac{\pi}{2} < t < \pi$ and $\frac{3\pi}{2} < t < 2\pi$
59. a.

	$(a, b) = (0, 0)$	$(a, b) = (2, 0)$	$(a, b) = (1, 1)$
$\mathbf{u} = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$	0	$4\sqrt{2}$	$-2\sqrt{2}$
$\mathbf{v} = \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$	0	$-4\sqrt{2}$	$-6\sqrt{2}$
$\mathbf{w} = \langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$	0	$-4\sqrt{2}$	$2\sqrt{2}$

b. The function is increasing at $(2, 0)$ in the direction of $\mathbf{u}$ and decreasing at $(2, 0)$in the directions of $\mathbf{v}$ and $\mathbf{w}$.
61. $\nabla g = \langle 2xy^3, 3x^2y^2 \rangle$; $\nabla g(-1, 1) = \langle -2, 3 \rangle$; $D_{\mathbf{u}}\, g(-1, 1) = 2$
63. $\nabla h = \left\langle \dfrac{x}{\sqrt{2 + x^2 + 2y^2}}, \dfrac{2y}{\sqrt{2 + x^2 + 2y^2}} \right\rangle$;
$\nabla h(2, 1) = \left\langle \dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2} \right\rangle$; $D_{\mathbf{u}}\, h(2, 1) = \dfrac{7\sqrt{2}}{10}$
65. $\nabla f = \langle \cos(x + 2y - z)$, $2\cos(x + 2y - z), -\cos(x + 2y - z) \rangle$;
$\nabla f\left(\dfrac{\pi}{6}, \dfrac{\pi}{6}, -\dfrac{\pi}{6}\right) = \left\langle -\dfrac{1}{2}, -1, \dfrac{1}{2} \right\rangle$; $D_{\mathbf{u}} f\left(\dfrac{\pi}{6}, \dfrac{\pi}{6}, -\dfrac{\pi}{6}\right) = -\dfrac{1}{2}$
67. a. Direction of steepest ascent: $\mathbf{u} = \dfrac{\sqrt{2}}{2}\mathbf{i} - \dfrac{\sqrt{2}}{2}\mathbf{j}$;
direction of steepest descent: $\mathbf{u} = -\dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j}$
b. No change: $\mathbf{u} = \pm\left(\dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j}\right)$
69. Tangent line is vertical; $\nabla f(2, 0) = -8\mathbf{i}$
71. $E = \dfrac{kx}{x^2 + y^2}\mathbf{i} + \dfrac{ky}{x^2 + y^2}\mathbf{j}$
73. $y = 2$ and $12x + 3y - 2z = 12$
75. $16x + 2y + z - 8 = 0$ and $8x + y + 8z + 16 = 0$
77. $z = \ln 3 + \dfrac{2}{3}(x - 1) + \dfrac{1}{3}(y - 2)$;
$z = \ln 3 - \dfrac{1}{3}(x + 2) - \dfrac{2}{3}(y + 1)$ **79. a.** $L(x, y) = x + 5y$
b. $L(1.95, 0.05) = 2.2$ **81.** -4% **83. a.** $dV = -0.1\pi\ \text{m}^3$
b. $dS = -0.05\pi\ \text{m}^2$ **85.** Saddle point at $(0, 0)$; local min at $(2, -2)$
87. Saddle points at $(0, 0)$ and $(-2, 2)$; local max at $(0, 2)$; local min at $(-2, 0)$ **89.** Absolute min: $-1 = f(1, 1) = f(-1, -1)$; absolute max: $49 = f(2, -2) = f(-2, 2)$ **91.** Absolute min: $-\dfrac{1}{2} = f\left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$; absolute max: $\dfrac{1}{2} = f\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$
93. Max: $\dfrac{29}{2} = f\left(\dfrac{5}{3}, \dfrac{7}{6}\right)$; min: $\dfrac{23}{2} = f\left(\dfrac{1}{3}, \dfrac{5}{6}\right)$
95. Max: $f\left(\dfrac{\sqrt{6}}{6}, \dfrac{\sqrt{6}}{3}, -\dfrac{\sqrt{6}}{6}\right) = \sqrt{6}$;
min: $f\left(-\dfrac{\sqrt{6}}{6}, -\dfrac{\sqrt{6}}{3}, \dfrac{\sqrt{6}}{6}\right) = -\sqrt{6}$
97. $\dfrac{2a^2}{\sqrt{a^2 + b^2}}$ by $\dfrac{2b^2}{\sqrt{a^2 + b^2}}$
99. $x = \dfrac{1}{2} + \dfrac{\sqrt{10}}{20}, y = \dfrac{3}{2} + \dfrac{3\sqrt{10}}{20} = 3x, z = \dfrac{1}{2} + \dfrac{\sqrt{10}}{2} = \sqrt{10}x$

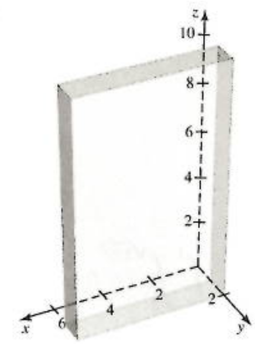
CHAPTER 2

Section 2.1 Exercises, pp. 116–119

1. $\displaystyle\int_0^2 \int_1^3 xy\, dy\, dx$ or $\displaystyle\int_1^3 \int_0^2 xy\, dx\, dy$ **3.** $\displaystyle\int_{-2}^4 \int_1^5 f(x, y)\, dy\, dx$ or $\displaystyle\int_1^5 \int_{-2}^4 f(x, y)\, dx\, dy$ **5.** 4 **7.** $\dfrac{32}{3}$ **9.** 4 **11.** $\dfrac{224}{9}$ **13.** 7
15. $10 - 2e$ **17.** $\dfrac{117}{2}$ **19.** 15 **21.** $\dfrac{4}{3}$ **23.** $\dfrac{9 - e^2}{2}$ **25.** $\dfrac{4}{11}$
27. $e^2 - 3$ **29.** $e^{16} - 17$ **31.** $\ln\dfrac{5}{3}$ **33.** $\dfrac{1}{2\ln 2}$ **35.** $\dfrac{8}{3}$

37. a. True **b.** False **c.** True **39. a.** 1475 **b.** The sum of products of population densities and areas is a Riemann sum.

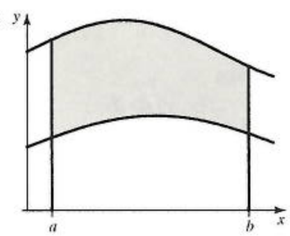
41. 60



43. $\frac{1}{2}$ **45.** $10\sqrt{5} - 4\sqrt{2} - 14$ **47.** 3 **49.** 136 **51.** $a = \pi/6, 5\pi/6$
53. $a = \sqrt{6}$ **55. a.** $\frac{1}{2}\pi^2 + \pi$ **b.** $\frac{1}{2}\pi^2 + \pi$ **c.** $\frac{1}{2}\pi^2 + 2$
57. $\int_c^d \int_a^b f(x)\, dy\, dx = (c - d) \int_a^b f(x)\, dx$. The integral is the area of the cross section of S. **59.** $f(a, b) - f(a, 0) - f(0, b) + f(0, 0)$
61. Use substitution ($u = x^r y^s$ and then $v = x^r$).

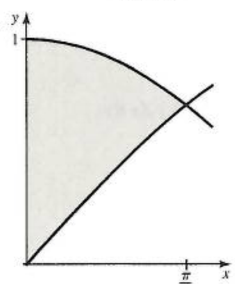
Section 2.2 Exercises, pp. 126–130

1.



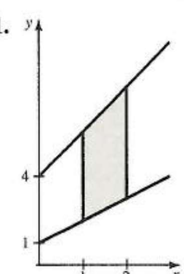
3. $dx\, dy$ **5.** $\displaystyle\int_0^1 \int_{x^2}^{\sqrt{x}} f(x, y)\, dy\, dx$ **7.** $\displaystyle\int_0^2 \int_{x^3}^{4x} f(x, y)\, dy\, dx$

9.



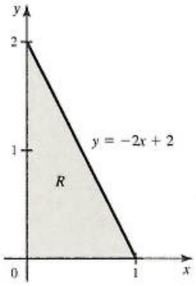
$\displaystyle\int_0^{\pi/4} \int_{\sin x}^{\cos x} f(x, y)\, dy\, dx$

11.



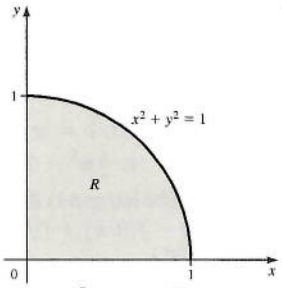
$\displaystyle\int_1^2 \int_{x+1}^{2x+4} f(x, y)\, dy\, dx$

13.



$$\int_0^1 \int_0^{-2x+2} f(x, y)\, dy\, dx$$

15.

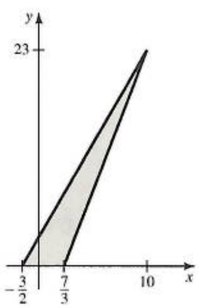


$$\int_0^1 \int_0^{\sqrt{1-x^2}} f(x, y)\, dy\, dx$$

17. 2 **19.** $\frac{8}{3}$ **21.** $\sqrt{2}$ **23.** 0 **25.** $e - 1$ **27.** 2 **29.** 12

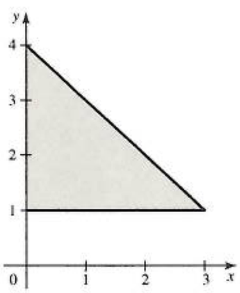
31. $\displaystyle\int_0^{18} \int_{y/2}^{(y+9)/3} f(x, y)\, dx\, dy$

33.



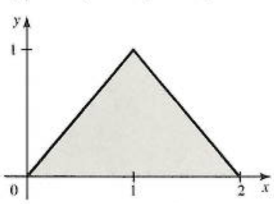
$$\int_0^{23} \int_{(y-3)/2}^{(y+7)/3} f(x, y)\, dx\, dy$$

35.



$$\int_1^4 \int_0^{4-y} f(x, y)\, dx\, dy$$

37.



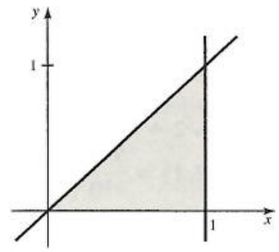
$$\int_0^1 \int_y^{2-y} f(x, y)\, dx\, dy$$

39. 9 **41.** 0 **43.** $\dfrac{\ln^3 2}{6}$ **45.** 2 **47.** 5 **49.** 14 **51.** 32 **53.** $\dfrac{32}{3}$

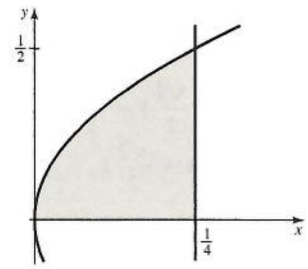
55. 12π **57.** $\displaystyle\int_0^4 \int_{y/2}^{\sqrt{y}} f(x, y)\, dx\, dy$ **59.** $\displaystyle\int_0^{\ln 2} \int_{1/2}^{e^{-x}} f(x, y)\, dy\, dx$

61. $\displaystyle\int_0^{\pi/2} \int_0^{\cos x} f(x, y)\, dy\, dx$

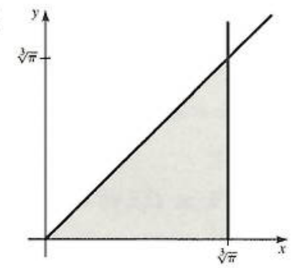
63. $\frac{1}{2}(e - 1)$



65. 0

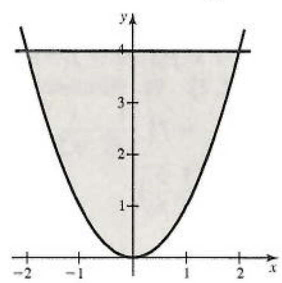


67. $\frac{2}{3}$

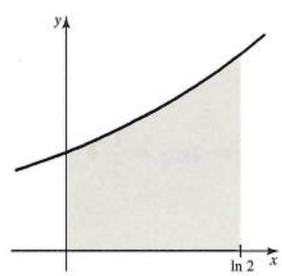


69. $\frac{2}{3}$ **71.** $\frac{81\pi}{2}$ **73.** $\frac{43}{6}$

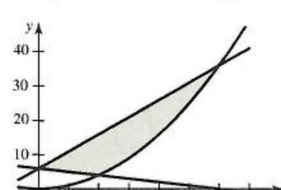
75. $\frac{32}{3}$



77. 1

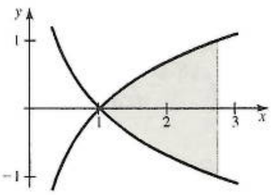


79. $\frac{140}{3}$

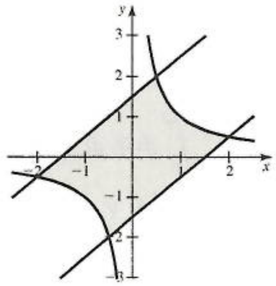


81. a. False **b.** False **c.** False **83.** $\frac{9}{8}$ **85.** $\frac{1}{4}\ln 2$

87. $\displaystyle\int_1^e \int_{-\ln x}^{\ln x} f(x, y)\, dy\, dx$



89. $\dfrac{a}{3}$ **91. a.**



b. $\dfrac{15}{4} + 4 \ln 2$

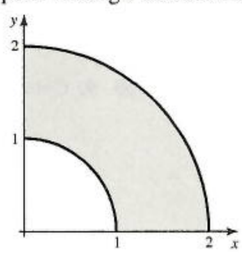
c. $2 \ln 2 - \dfrac{5}{64}$

93. $\dfrac{3}{8e^2}$ **95.** 1 **97.** 30 **99.** 16 **101.** $4a\pi$

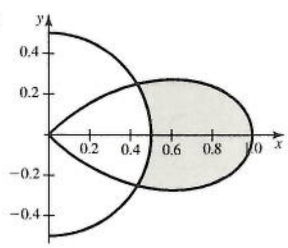
103. The integral over R_1

Section 2.3 Exercises, pp. 137–140

1. It is called a polar rectangle because each of r and θ vary between two constants.

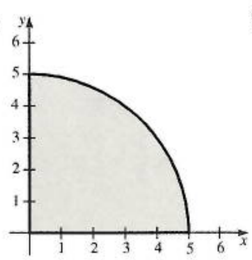


3.

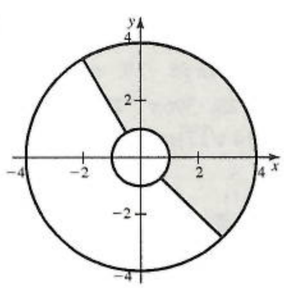


5. Evaluate the integral $\displaystyle\int_\alpha^\beta \int_{g(\theta)}^{h(\theta)} r\, dr\, d\theta$.

7.

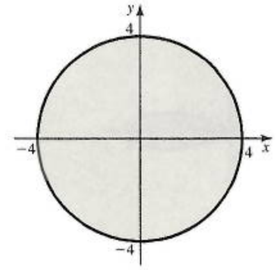


9.

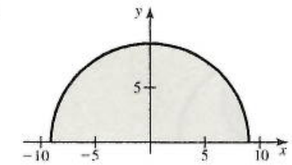


11. $\dfrac{7\pi}{2}$ **13.** $\dfrac{9\pi}{2}$ **15.** $\dfrac{62 - 10\sqrt{5}}{3}\pi$ **17.** $\dfrac{37\pi}{3}$ **19.** π

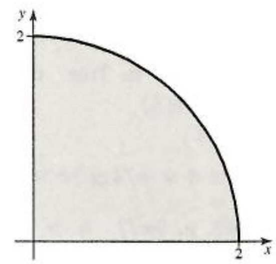
21. $\pi/2$ **23.** 128π



25. 0

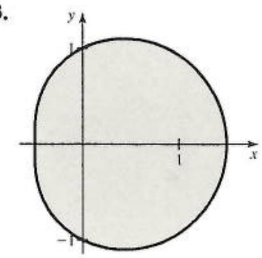


27. $(2 - \sqrt{3})\pi$



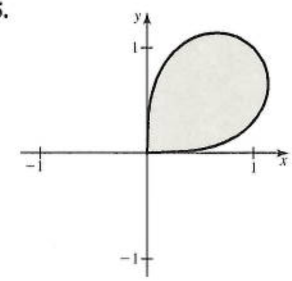
29. $(8 - 24e^{-2})\pi$ **31.** $\dfrac{15{,}625\pi}{3}$

33.



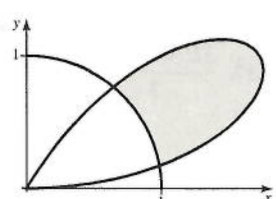
$\displaystyle\int_0^{2\pi} \int_0^{1 + \frac{1}{2}\cos\theta} g(r, \theta) r\, dr\, d\theta$

35.



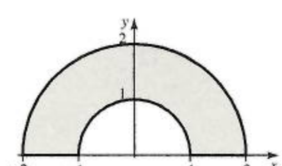
$\displaystyle\int_0^{\pi/2} \int_0^{\sqrt{2 \sin 2\theta}} g(r, \theta) r\, dr\, d\theta$

37.

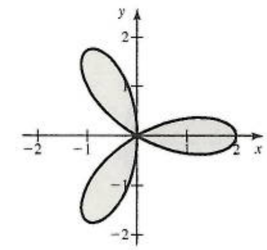


$\displaystyle\int_{\pi/18}^{5\pi/18} \int_1^{2 \sin 3\theta} g(r, \theta) r\, dr\, d\theta$

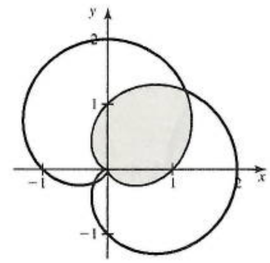
39. $3\pi/2$



41. π



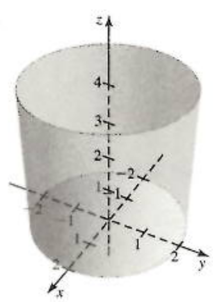
43. $\frac{3\pi}{2} - 2\sqrt{2}$



45. $2a/3$ **47.** $5/2$ **49. a.** False **b.** True **c.** True **51.** $2\pi/5$
53. $\frac{1}{3}$ **55.** $\frac{14\pi}{3}$ **57.** $2\pi\left(1 - 2\ln\frac{3}{2}\right)$
59. The hyperboloid $\left(V = \frac{112\pi}{3}\right)$
61. a. $R = \{(r, \theta): -\pi/4 \le \theta \le \pi/4 \text{ or } 3\pi/4 \le \theta \le 5\pi/4\}$
b. $\frac{a^4}{4}$ **63.** 1 **65.** $\pi/4$ **67. a.** $9\pi/2$ **b.** $\pi + 3\sqrt{3}$
c. $\pi - 3\sqrt{3}/2$ **69.** $30\pi + 42$ **71. b.** $\sqrt{\pi}/2$, $1/2$, and $\sqrt{\pi}/4$
73. a. $I = \frac{\sqrt{2}}{2}\tan^{-1}\frac{\sqrt{2}}{2}$
b. $I = \frac{\sqrt{2}}{4}\tan^{-1}\frac{\sqrt{2}a}{2} + \frac{a}{2\sqrt{a^2+1}}\tan^{-1}\frac{1}{\sqrt{a^2+1}}$ **c.** $\frac{\sqrt{2}\pi}{8}$

Section 2.4 Exercises, pp. 148–152

1.



3. $\int_{-9}^{9}\int_{-\sqrt{81-x^2}}^{\sqrt{81-x^2}}\int_{-\sqrt{81-x^2-y^2}}^{\sqrt{81-x^2-y^2}} f(x, y, z)\, dz\, dy\, dx$

5. $\int_0^1\int_0^{\sqrt{1-z^2}}\int_0^{\sqrt{1-z^2-x^2}} f(x, y, z)\, dy\, dx\, dz$

7. 24 **9.** 8 **11.** $\frac{2}{\pi}$ **13.** 0 **15.** 8 **17.** $\frac{32(\sqrt{2} - 1)}{3}\pi$

19. $\frac{16}{3}$ **21.** $\frac{2\pi(1 + 19\sqrt{19} - 20\sqrt{10})}{3}$

23. 12π **25.** $\frac{2}{3}$ **27.** 128π **29.** $(10\sqrt{10} - 1)\frac{\pi}{6}$

31. $\frac{3\ln 2}{2} + \frac{e}{16} - 1$ **33.** $\frac{256}{9}$ **35.** $\frac{5}{12}$

37. 8 **39.** $\int_0^4\int_{y/4-1}^{0}\int_0^5 dz\, dx\, dy = 10$

41. $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2}} dz\, dy\, dx = \frac{2}{3}$ **43.** $\frac{7}{\ln^3 2}$ **45.** $\frac{10}{3}$ **47.** $\frac{3}{2}$

49. a. False **b.** False **c.** False **51.** 1 **53.** $\frac{16}{3}$ **55.** 2

57. $\int_0^1\int_0^2\int_0^{1-y} dz\, dx\, dy$,

$\int_0^2\int_0^1\int_0^{1-z} dy\, dz\, dx$, $\int_0^1\int_0^2\int_0^{1-z} dy\, dx\, dz$,

$\int_0^1\int_0^{1-y}\int_0^2 dx\, dz\, dy$, $\int_0^1\int_0^{1-z}\int_0^2 dx\, dy\, dz$

59. $\frac{224}{3}$ and $\frac{160}{3}$ **61.** $V = \frac{\pi r^2 h}{3}$ **63.** $V = \frac{\pi h^2}{3}(3R - h)$

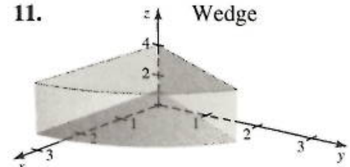
65. $V = \frac{4\pi abc}{3}$ **67.** $\frac{1}{24}$

Section 2.5 Exercises, pp. 165–169

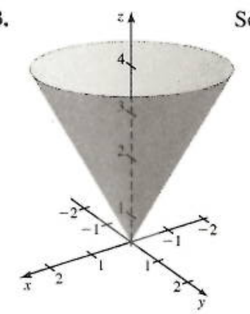
1. r measures the distance from the point to the z axis, θ is the angle that the segment from the point to the z-axis makes with the positive xz-plane, and z is the directed distance from the point to the xy-plane. **3.** A cone **5.** It approximates the volume of the cylindrical wedge formed by the changes Δr, $\Delta\theta$, and Δz.

7. $\int_\alpha^\beta\int_{g(\theta)}^{h(\theta)}\int_{G(r,\theta)}^{H(r,\theta)} w(r, \theta, z)\, r\, dz\, dr\, d\theta$ **9.** Cylindrical coordinates

11. Wedge



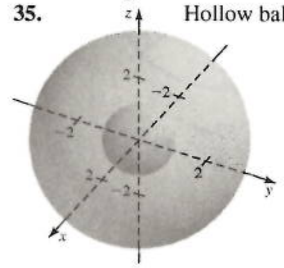
13. Solid bounded by cone and plane

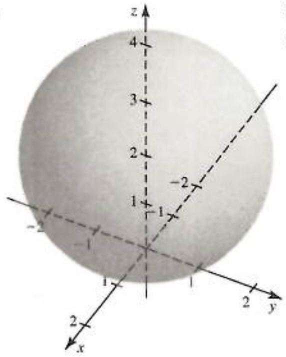


15. 2π **17.** $4\pi/5$ **19.** $\pi(1 - e^{-1})/2$ **21.** $9\pi/4$
23. 560π **25.** 396π **27.** The paraboloid $(V = 44\pi/3)$

29. $\frac{(2 + 14\sqrt{17})\pi}{3}$ **31.** $\frac{(16 + 17\sqrt{29})\pi}{3}$ **33.** $\frac{1}{3}$

35. Hollow ball



37. 

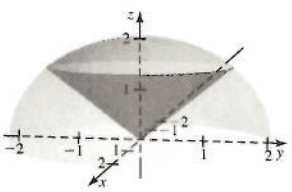
Sphere of radius $r = 2$, centered at $(0, 0, 2)$

39. $\frac{\pi}{2}$ **41.** $4\pi \ln 2$ **43.** $\pi\left(\frac{188}{9} - \frac{32\sqrt{3}}{3}\right)$ **45.** $\frac{32\pi\sqrt{3}}{9}$

47. $\frac{5\pi}{12}$ **49.** $\frac{8\pi}{3}$ **51.** $\frac{8\pi}{3}(9\sqrt{3} - 11)$ **53. a.** True **b.** True

55. $z = \sqrt{x^2 + y^2 - 1}$; upper half of a hyperboloid of one sheet

57. $\frac{8\pi}{3}(1 - e^{-512}) \approx \frac{8\pi}{3}$ **59.** 32π

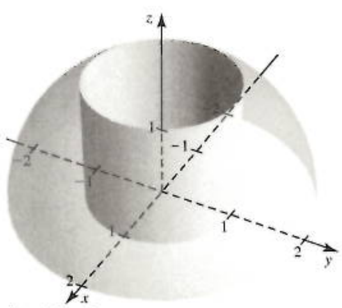
61. 

$$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} g(r, \theta, z)\, r\, dz\, dr\, d\theta,$$

$$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_0^{z} g(r, \theta, z)\, r\, dr\, dz\, d\theta$$

$$+\int_0^{2\pi}\int_{\sqrt{2}}^{2}\int_0^{\sqrt{4-z^2}} g(r, \theta, z)\, r\, dr\, dz\, d\theta,$$

$$\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}}\int_0^{2\pi} g(r, \theta, z)\, r\, d\theta\, dz\, dr$$

63. 

$$\int_{\pi/6}^{\pi/2}\int_0^{2\pi}\int_{\csc\varphi}^{2} g(\rho, \varphi, \theta)\rho^2 \sin\varphi\, d\rho\, d\theta\, d\varphi,$$

$$\int_{\pi/6}^{\pi/2}\int_{\csc\varphi}^{2}\int_0^{2\pi} g(\rho, \varphi, \theta)\, \rho^2 \sin\varphi\, d\theta\, d\rho\, d\varphi$$

65. $32\sqrt{3}\pi/9$ **67.** $2\sqrt{2}/3$ **69.** $7\pi/2$ **71.** $16/3$ **73.** 95.6036

77. $V = \frac{\pi r^2 h}{3}$ **79.** $V = \frac{\pi}{3}(R^2 + rR + r^2)h$

81. $V = \frac{\pi R^3(8r - 3R)}{12r}$

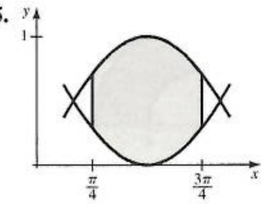
Section 2.6 Exercises, pp. 177–179

1. The pivot should be located at the center of mass of the system.
3. Use a double integral. Integrate the density function over the region occupied by the plate. **5.** Use a triple integral to find the mass of the object and the three moments.

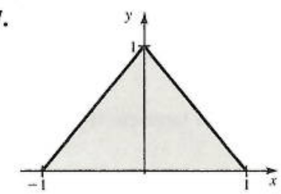
7. 

$\frac{27}{13}$ **9.** Mass is $2 + \pi$; $\bar{x} = \frac{\pi}{2}$

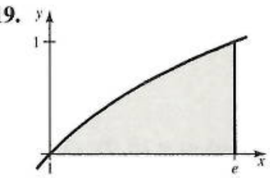
11. Mass is $\frac{20}{3}$; $\bar{x} = \frac{9}{5}$ **13.** Mass is 10; $\bar{x} = \frac{8}{3}$

15. 

$\left(\frac{\pi}{2}, \frac{1}{2}\right)$

17. 

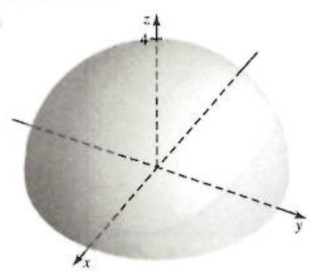
$\left(0, \frac{1}{3}\right)$

19. 

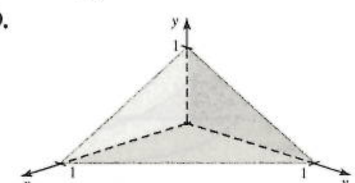
$\left(\frac{1}{4}(e^2 + 1), \frac{e}{2} - 1\right) \approx (2.10, 0.36)$

21. $\left(\frac{7}{3}, 1\right)$; density increases to the right. **23.** $\left(\frac{16}{11}, \frac{16}{11}\right)$; density increases toward the hypotenuse of the triangle.

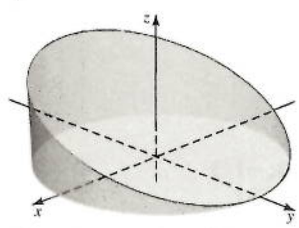
25. $\left(0, \frac{16 + 3\pi}{16 + 12\pi}\right) \approx (0, 0.4735)$; density increases away from the x-axis.

27. 

$\left(0, 0, \frac{3}{2}\right)$

29. 

$\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

31. 

$\left(0, -\frac{1}{4}, \frac{5}{8}\right)$

33. $\left(\frac{7}{3}, \frac{1}{2}, \frac{1}{2}\right)$ **35.** $\left(0, 0, \frac{198}{85}\right)$ **37.** $\left(\frac{2}{3}, \frac{7}{3}, \frac{1}{3}\right)$ **39. a.** False **b.** True

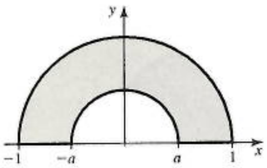
c. False **d.** False **41.** $\bar{x} = \dfrac{\ln(1 + L^2)}{2\tan^{-1} L}$, $\lim_{L\to\infty} \bar{x} = \infty$

43. $\left(0, \frac{8}{9}\right)$ **45.** $\left(0, \frac{8}{3\pi}\right)$ **47.** $\left(\frac{5}{6}, 0\right)$ **49.** $\left(\frac{128}{105\pi}, \frac{128}{105\pi}\right)$

51. On the line of symmetry, $2a/\pi$ units above the diameter

53. $\left(\dfrac{2a}{3(4-\pi)}, \dfrac{2a}{3(4-\pi)}\right)$ **55.** $h/4$ units **57.** $h/3$ units, where h is the height of the triangle **59.** $3a/8$ units

61. a. $\left(0, \dfrac{4(1 + a + a^2)}{3(1 + a)\pi}\right)$



b. $a = \dfrac{1}{2}\left(-1 + \sqrt{1 + \dfrac{16}{3\pi - 4}}\right) \approx 0.4937$

63. Depth $= \dfrac{40\sqrt{10} - 4}{333}$ cm ≈ 0.3678 cm

65. a. $(\bar{x}, \bar{y}) = \left(\dfrac{-r^2}{R + r}, 0\right)$ (origin at center of large circle);

$(\bar{x}, \bar{y}) = \left(\dfrac{R^2 + Rr + r^2}{R + r}, 0\right)$ (origin at common point of the circles)

b. *Hint:* Solve $\bar{x} = R - 2r$.

Section 2.7 Exercises, pp. 189–191

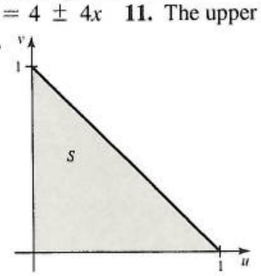
1. The image of S is the 2×2 square with vertices at $(0, 0)$, $(2, 0)$, $(2, 2)$, and $(0, 2)$. **3.** $\displaystyle\int_0^1\int_0^1 f(u + v, u - v)\, 2\, du\, dv$

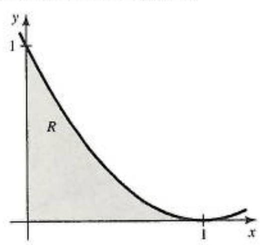
5. The rectangle with vertices at $(0, 0)$, $(2, 0)$, $\left(2, \frac{1}{2}\right)$, and $\left(0, \frac{1}{2}\right)$

7. The square with vertices at $(0, 0)$, $\left(\frac{1}{2}, \frac{1}{2}\right)$, $(1, 0)$, and $\left(\frac{1}{2}, -\frac{1}{2}\right)$

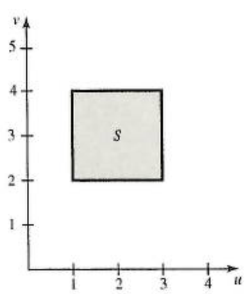
9. The region above the x-axis and bounded by the curves $y^2 = 4 \pm 4x$ **11.** The upper half of the unit circle

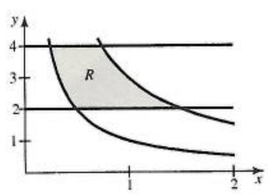
13.





15.



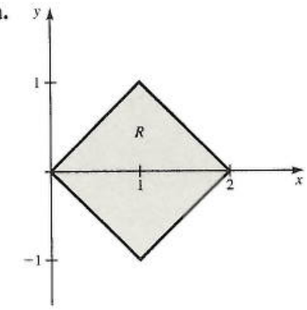


17. -9 **19.** $-4(u^2 + v^2)$ **21.** -1

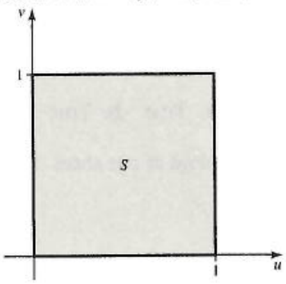
23. $x = (u + v)/3, y = (2u - v)/3; -1/3$

25. $x = -(u + 3v), y = -(u + 2v); -1$

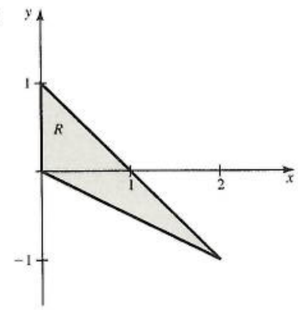
27. a.



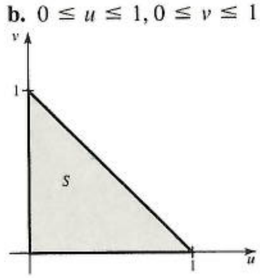
b. $0 \le u \le 1, 0 \le v \le 1$ **c.** $J(u, v) = -2$ **d.** 0



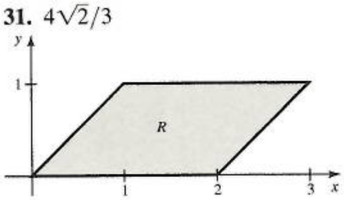
29. a.

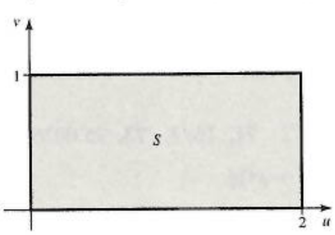


b. $0 \le u \le 1, 0 \le v \le 1 - u$ **c.** $J(u, v) = 2$ **d.** $256\sqrt{2}/945$

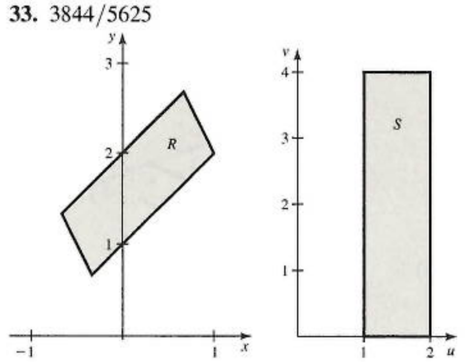


31. $4\sqrt{2}/3$

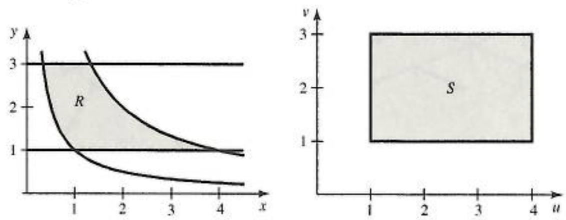




33. 3844/5625



35. $\dfrac{15\ln 3}{2}$



37. 2 **39.** $2w(u^2 - v^2)$ **41.** 5 **43.** $1024\pi/3$

45. a. True **b.** True **c.** True

47. *Hint:* $J(\rho, \varphi, \theta) = \begin{vmatrix} \sin\varphi\cos\theta & \rho\cos\varphi\cos\theta & -\rho\sin\varphi\sin\theta \\ \sin\varphi\sin\theta & \rho\cos\varphi\sin\theta & \rho\sin\varphi\cos\theta \\ \cos\varphi & -\rho\sin\varphi & 0 \end{vmatrix}$

49. $a^2b^2/2$ **51.** $(a^2 + b^2)/4$ **53.** $4\pi abc/3$

55. $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \dfrac{3c}{8}\right)$ **57. a.** $x = a^2 - \dfrac{y^2}{4a^2}$

b. $x = \dfrac{y^2}{4b^2} - b^2$ **c.** $J(u, v) = 4(u^2 + v^2)$ **d.** $\dfrac{80}{3}$ **e.** 160

f. Vertical lines become parabolas opening downward with vertices on the positive y-axis, and horizontal lines become parabolas opening upward with vertices on the negative y-axis. **59. a.** S is stretched in the positive u- and v-directions but not in the w-direction. The amount of stretching increases with u and v. **b.** $J(u, v, w) = ad$

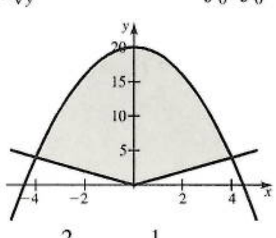
c. Volume $= ad$ **d.** $\left(\dfrac{a+b+c}{2}, \dfrac{d+e}{2}, \dfrac{1}{2}\right)$

Chapter 2 Review Exercises, pp. 192–195

1. a. False **b.** True **c.** False **d.** False **3.** $\frac{26}{3}$

5. $\displaystyle\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y)\, dx\, dy$ **7.** $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} f(x, y)\, dy\, dx$

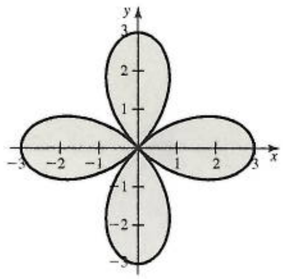
9. $\dfrac{304}{3}$



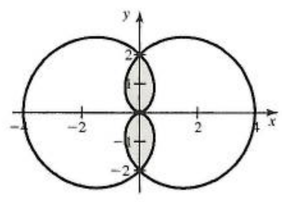
11. $\dfrac{\sqrt{17} - \sqrt{2}}{2}$

13. 8π **15.** $\dfrac{2}{7\pi^2}$ **17.** $\dfrac{1}{5}$

19. $\dfrac{9\pi}{2}$



21. $6\pi - 16$



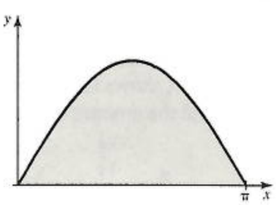
23. 2 **25.** $\displaystyle\int_0^1 \int_{2y}^{2} \int_0^{\sqrt{z^2-4y^2}/2} f(x, y, z)\, dx\, dz\, dy$ **27.** $\pi - \dfrac{4}{3}$

29. $8\sin^2 2 = 4(1 - \cos 4)$ **31.** $\dfrac{848}{9}$ **33.** $\dfrac{16}{3}$ **35.** $\dfrac{128}{3}$

37. $\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}$ **39. a.** $\dfrac{512}{15}$ **b.** Five **c.** $\dfrac{2^{pq+q+1}}{q(p+1)^2 + p + 1}$

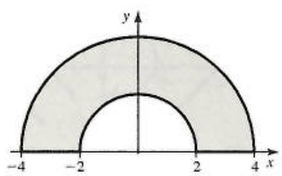
41. $\frac{1}{3}$ **43.** π **45.** 4π **47.** $\dfrac{28\pi}{3}$ **49.** $\dfrac{2048\pi}{105}$

51.



$(\bar{x}, \bar{y}) = \left(\dfrac{\pi}{2}, \dfrac{\pi}{8}\right)$

53.



$(\bar{x}, \bar{y}) = \left(0, \dfrac{56}{9\pi}\right)$

55. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 24)$ **57.** $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \dfrac{63}{10}\right)$ **59.** $\dfrac{h}{3}$

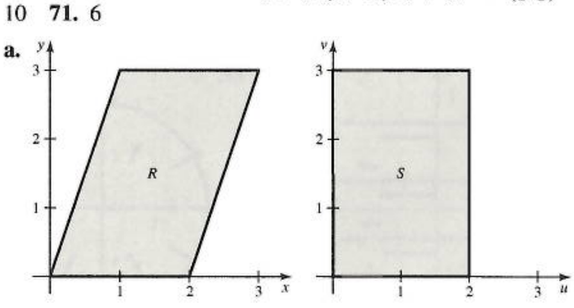
61. $\dfrac{1}{6}\sqrt{4s^2 - b^2} = \dfrac{h}{3}$, where h is the height of the triangle.

63. a. $\dfrac{4\pi}{3}$ **b.** $\dfrac{16Q}{3}$ **65.** $R = \{0 \le x \le 1, 0 \le y \le 1\}$

67. The square with vertices at $(0, 0)$, $\left(\frac{1}{2}, -\frac{1}{2}\right)$, $(1, 0)$, and $\left(\frac{1}{2}, \frac{1}{2}\right)$.

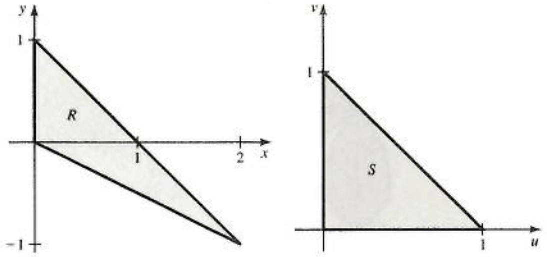
69. 10 **71.** 6

73. a.



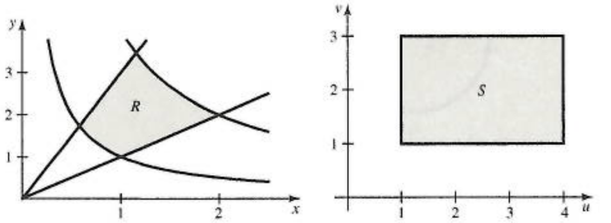
b. $0 \le u \le 2, 0 \le v \le 3$ **c.** $J(u, v) = 1$ **d.** $\frac{63}{2}$

75. a.



b. $0 \le u \le 1, 0 \le v \le 1 - u$ **c.** $J(u, v) = 2$ **d.** $\dfrac{256\sqrt{2}}{945}$

77. 42



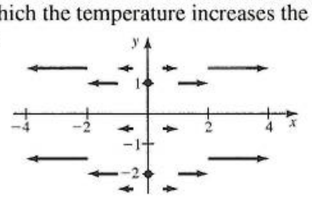
79. $-\dfrac{7}{16}$

CHAPTER 3

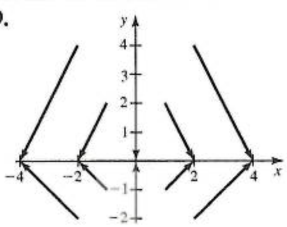
Section 3.1 Exercises, pp. 203–206

1. $\mathbf{F} = \langle f, g, h \rangle$ evaluated at (x, y, z) is the velocity vector of an air particle at (x, y, z) at a fixed point in time. **3.** At selected points (a, b), plot the vector $\langle f(a, b), g(a, b) \rangle$. **5.** It shows the direction in which the temperature increases the fastest and the amount of increase.

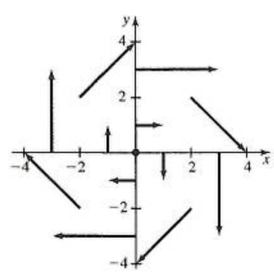
7.



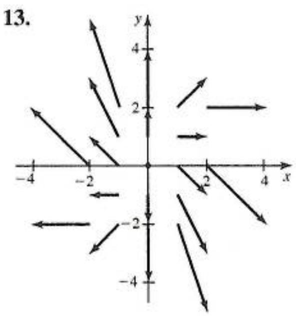
9.



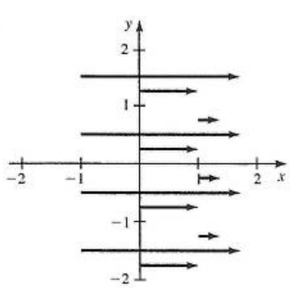
11.



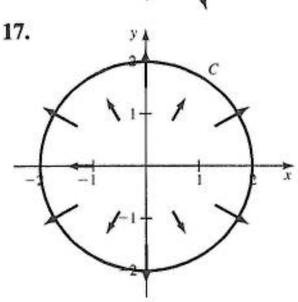
13.



15.

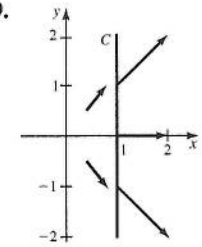


17.



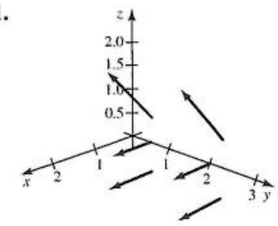
Normal at all points of C

19.

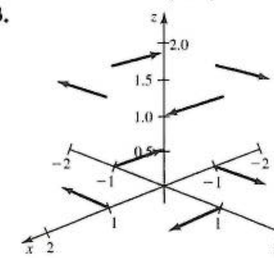


Normal to C at $(1, 0)$

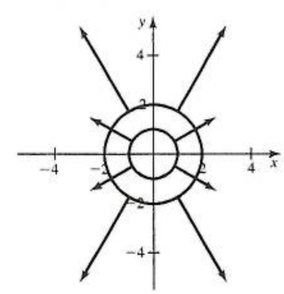
21.



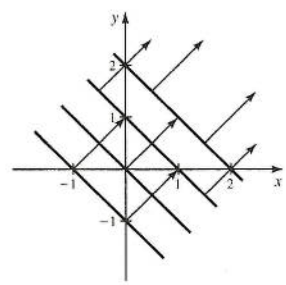
23.



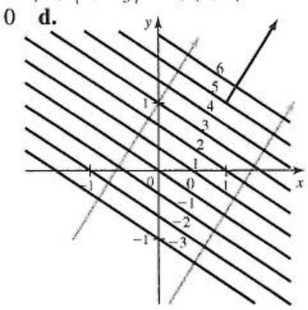
25. $\nabla\varphi(x, y) = 2\langle x, y \rangle$



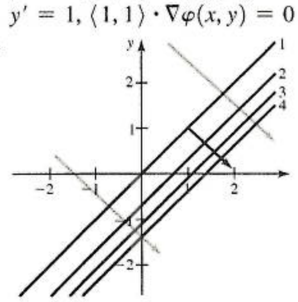
27. $\nabla\varphi = \langle 1, 1 \rangle$



29. $\nabla\varphi(x, y) = \langle 2xy - y^2, x^2 - 2xy \rangle$
31. $\nabla\varphi(x, y) = \langle 1/y, -x/y^2 \rangle$ **33.** $\nabla\varphi(x, y, z) = \langle x, y, z \rangle = \mathbf{r}$
35. $\nabla\varphi(x, y, z) = -(x^2 + y^2 + z^2)^{-3/2}\langle x, y, z \rangle = -\dfrac{\mathbf{r}}{|\mathbf{r}|^3}$
37. a. $\nabla\varphi(x, y) = \langle 2, 3 \rangle$ **b.** $y' = -2/3$, $\langle 1, -\frac{2}{3} \rangle \cdot \nabla\varphi(1, 1) = 0$
c. $y' = -2/3$, $\langle 1, -\frac{2}{3} \rangle \cdot \nabla\varphi(x, y) = 0$ **d.**

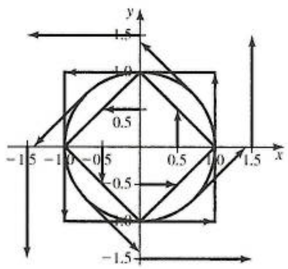


39. a. $\nabla\varphi(x, y) = \langle e^{x-y}, -e^{x-y} \rangle = e^{x-y}\langle 1, -1 \rangle$
b. $y' = 1$, $\langle 1, 1 \rangle \cdot \nabla\varphi(1, 1) = 0$
c. $y' = 1$, $\langle 1, 1 \rangle \cdot \nabla\varphi(x, y) = 0$
d.



41. a. True **b.** False **c.** True

43.



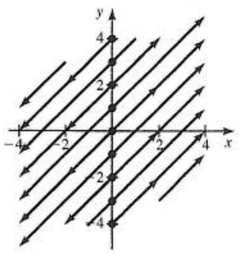
a. For S and D, the vectors with maximum magnitude occur at the vertices; on C, all vectors on the boundary have the same maximum magnitude ($|\mathbf{F}| = 1$). **b.** For S and D, the field is directed out of the region on line segments between any vertex and the midpoint of the boundary line when proceeding in a counterclockwise direction; on C, the vector field is tangent to the boundary curve everywhere. **45.** $\mathbf{F} = \langle -y, x \rangle$ or $\mathbf{F} = \langle -1, 1 \rangle$

47. $\mathbf{F}(x, y) = \dfrac{\langle x, y \rangle}{\sqrt{x^2 + y^2}} = \dfrac{\mathbf{r}}{|\mathbf{r}|}, \mathbf{F}(0, 0) = \mathbf{0}$

49. a. $\mathbf{E} = \dfrac{c}{x^2 + y^2}\langle x, y \rangle$ **b.** $|\mathbf{E}| = \left|\dfrac{c}{|\mathbf{r}|^2}\mathbf{r}\right| = \dfrac{c}{r}$

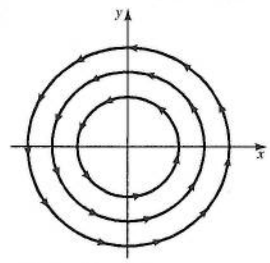
c. *Hint:* The equipotential curves are circles centered at the origin. **51.** The slope of the streamline at (x, y) is $y'(x)$, which equals the slope of the vector $\mathbf{F}(x, y)$, which is g/f. Therefore, $y'(x) = g/f$.

53.



$y = x + C$

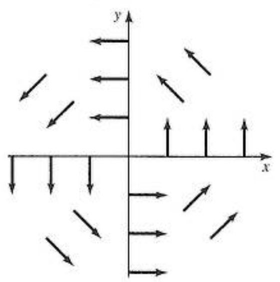
55.



$x^2 + y^2 = C$

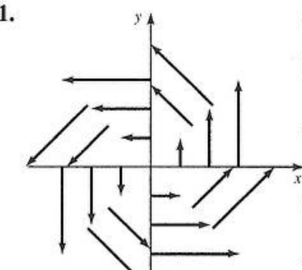
57. For $\theta = 0$: $\mathbf{u}_r = \mathbf{i}$ and $\mathbf{u}_\theta = \mathbf{j}$
for $\theta = \frac{\pi}{2}$: $\mathbf{u}_r = \mathbf{j}$ and $\mathbf{u}_\theta = -\mathbf{i}$
for $\theta = \pi$: $\mathbf{u}_r = -\mathbf{i}$ and $\mathbf{u}_\theta = -\mathbf{j}$
for $\theta = \frac{3\pi}{2}$: $\mathbf{u}_r = -\mathbf{j}$ and $\mathbf{u}_\theta = \mathbf{i}$

59.



$\mathbf{F} = \dfrac{1}{\sqrt{x^2 + y^2}}\langle -y, x \rangle$

61.



$\mathbf{F} = r\,\mathbf{u}_\theta$

Section 3.2 Exercises, pp. 221–224

1. A line integral is taken along a curve; an ordinary single-variable integral is taken along an interval. **3.** $\sqrt{1 + 4t^2}$ **5.** The integrand of the alternative form is a dot product of $\mathbf{F}$ and $\mathbf{T}\,ds$. **7.** Take the line integral of $\mathbf{F} \cdot \mathbf{T}$ along the curve with arc length as the parameter.
9. Take the line integral of $\mathbf{F} \cdot \mathbf{n}$ along the curve with arc length as the parameter, where $\mathbf{n}$ is the outward normal vector of the curve.
11. 0 **13.** $-\frac{32}{3}$ **15. a.** $\mathbf{r}(t) = \langle 4\cos t, 4\sin t \rangle, 0 \le t \le 2\pi$ **b.** $|\mathbf{r}'(t)| = 4$ **c.** 128π **17. a.** $\mathbf{r}(t) = \langle t, t \rangle, 1 \le t \le 10$ **b.** $|\mathbf{r}'(t)| = \sqrt{2}$ **c.** $\dfrac{\sqrt{2}}{2}\ln 10$ **19. a.** $\mathbf{r}(t) = \langle 2\cos t, 4\sin t \rangle$, $0 \le t \le \dfrac{\pi}{2}$ **b.** $|\mathbf{r}'(t)| = 2\sqrt{1 + 3\cos^2 t}$ **c.** $\dfrac{112}{9}$

21. $\dfrac{15}{2}$ **23.** $\dfrac{1431}{268}$ **25.** 0 **27.** $\dfrac{3\sqrt{14}}{2}$ **29.** $-2\pi^2\sqrt{10}$

31. $\sqrt{101}$ **33.** $\dfrac{17}{2}$ **35.** 49 **37.** $\dfrac{3}{4\sqrt{10}}$ **39.** 0 **41.** 16

43. 0 **45.** $\dfrac{3\sqrt{3}}{10}$ **47. b.** 0 **49. a.** Negative **b.** -4π
51. a. True **b.** True **c.** True **d.** True **53. a.** Both paths require the same work: $W = 28{,}200$. **b.** Both paths require the same work: $W = 28{,}200$. **55. a.** $\dfrac{5\sqrt{5} - 1}{12}$ **b.** $\dfrac{5\sqrt{5} - 1}{12}$
c. The results are identical.

57. *Hint:* Show that $\displaystyle\int_C \mathbf{F} \cdot \mathbf{T}\,ds = \pi r^2(c - b)$.

59. *Hint:* Show that $\displaystyle\int_C \mathbf{F} \cdot \mathbf{n}\,ds = \pi r^2(a + d)$.

61. The work equals zero for all three paths. **63.** 409.5 **65. a.** $\ln a$ **b.** No **c.** $\dfrac{1}{6}\left(1 - \dfrac{1}{a^2}\right)$ **d.** Yes **e.** $W = \dfrac{3^{1-p/2}}{2 - p}(a^{2-p} - 1)$, for $p \ne 2$; otherwise, $W = \ln a$. **f.** $p > 2$ **67.** ab

Section 3.3 Exercises, pp. 231–233

1. A simple curve has no self-intersections; the initial and terminal points of a closed curve are identical. **3.** Test for equality of partial derivatives as given in Theorem 15.3. **5.** Integrate f with respect to x and make the constant of integration a function of y to obtain $\varphi = \int f\,dx + h(y)$; finally, set $\frac{\partial \varphi}{\partial y} = g$ to determine h. **7.** 0
9. Conservative **11.** Conservative **13.** Conservative
15. $\varphi(x, y) = \frac{1}{2}(x^2 + y^2)$ **17.** Not conservative
19. $\varphi(x, y) = \sqrt{x^2 + y^2}$ **21.** $\varphi(x, y, z) = xz + y$
23. $\varphi(x, y, z) = xy + yz + zx$ **25.** $\varphi(x, y) = \sqrt{x^2 + y^2 + z^2}$
27. a, b. 0 **29. a, b.** 4 **31. a, b.** 2 **33.** 0 **35.** 0 **37.** 0
39. a. False **b.** True **c.** True **d.** True **e.** True **41.** $-\frac{1}{2}$
43. 0 **45.** 10 **47.** 25 **49.** C_1 negative, C_2 positive
53. a. Compare partial derivatives.

b. $\varphi(x, y, z) = \dfrac{GMm}{\sqrt{x^2 + y^2 + z^2}} = \dfrac{GMm}{|\mathbf{r}|}$

c. $\varphi(B) - \varphi(A) = GMm\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right)$ **d.** No

55. a. $\dfrac{\partial}{\partial y}\left(\dfrac{-y}{(x^2 + y^2)^{p/2}}\right) = \dfrac{-x^2 + (p - 1)y^2}{(x^2 + y^2)^{1+p/2}}$ and

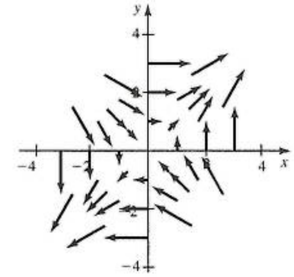
$\dfrac{\partial}{\partial x}\left(\dfrac{x}{(x^2 + y^2)^{p/2}}\right) = \dfrac{-(p - 1)x^2 + y^2}{(x^2 + y^2)^{1+p/2}}$

b. The two partial derivatives in (a) are equal if $p = 2$.
c. $\varphi(x, y) = \tan^{-1}(y/x)$ **59.** $\varphi(x, y) = \frac{1}{2}(x^2 + y^2)$
61. $\varphi(x, y) = \frac{1}{2}(x^4 + x^2y^2 + y^4)$

Section 3.4 Exercises, pp. 244–247

1. In both forms, the integral of a *derivative* is computed from boundary data. **3.** y^2 **5.** Area $= \frac{1}{2}\oint_C (x\,dy - y\,dx)$, where C encloses the region **7.** The integral in the flux form of Green's Theorem vanishes.

9. $\mathbf{F} = \langle y, x \rangle$



11. a. 0 **b.** Both integrals equal zero. **c.** Yes **13. a.** -4 **b.** Both integrals equal -8. **c.** No **15. a.** 0 **b.** Both integrals equal zero. **c.** Yes **17.** 25π **19.** 16π **21.** 32 **23. a.** 2 **b.** Both integrals equal 8π. **c.** No **25. a.** 0 **b.** Both integrals equal zero. **c.** Yes **27. a.** 0 **b.** Both integrals equal zero. **c.** Yes **29.** 6 **31.** $\frac{8}{3}$ **33.** $8 - \frac{\pi}{2}$ **35. a.** 0 **b.** 3π **37. a.** 0 **b.** $-\frac{15\pi}{2}$ **39. a.** True **b.** False **c.** True **41. a.** 0 **b.** 2π **43. a.** 5702.4 **b.** 0

45. Note: $\frac{\partial f}{\partial y} = 0 = \frac{\partial g}{\partial x}$ **47.** The integral becomes $\iint_R 2\,dA$.

49. a. $f_x = g_y = 0$ **b.** $\psi(x, y) = -2x + 4y$

51. a. $f_x = e^{-x} \sin y = -g_y$ **b.** $\psi(x, y) = e^{-x} \cos y$

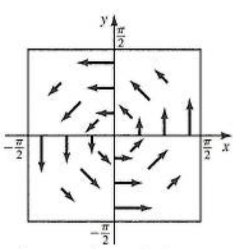
53. a. *Hint:* $f_x = e^x \cos y$, $f_y = -e^x \sin y$, $g_x = -e^x \sin y$, $g_y = -e^x \cos y$

b. $\varphi(x, y) = e^x \cos y$, $\psi(x, y) = e^x \sin y$

55. a. *Hint:* $f_x = -\frac{y}{x^2 + y^2}$, $f_y = \frac{x}{x^2 + y^2}$, $g_x = \frac{x}{x^2 + y^2}$, $g_y = \frac{y}{x^2 + y^2}$

b. $\varphi(x, y) = x \tan^{-1} \frac{y}{x} + \frac{y}{2} \ln(x^2 + y^2) - y$, $\psi(x, y) = y \tan^{-1} \frac{y}{x} - \frac{x}{2} \ln(x^2 + y^2) + x$

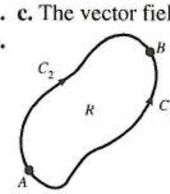
57. a.



$\mathbf{F} = \langle -4 \cos x \sin y, 4 \sin x \cos y \rangle$ **b.** Yes, the divergence equals zero. **c.** No, the two-dimensional curl equals $8 \cos x \cos y$. **d.** 0 **e.** 32

61. c. The vector field is undefined at the origin.

63.



Basic ideas: Let C_1 and C_2 be two smooth simple curves from A to B.

$$\int_{C_1} \mathbf{F} \cdot \mathbf{n}\, ds - \int_{C_2} \mathbf{F} \cdot \mathbf{n}\, ds = \oint_C \mathbf{F} \cdot \mathbf{n}\, ds = \iint_R \operatorname{div} \mathbf{F}\, dA = 0$$

and $\int_{C_1} \mathbf{F} \cdot \mathbf{n}\, ds = \int_{C_1} \psi_x\, dx + \psi_y\, dy = \int_{C_1} d\psi = \psi(B) - \psi(A)$

65. Use $\nabla\varphi \cdot \nabla\psi = \langle f, g \rangle \cdot \langle -g, f \rangle = 0$

Section 3.5 Exercises, pp. 254–257

1. Compute $f_x + g_y + h_z$. **3.** There is no source or sink. **5.** It indicates the axis and the angular speed of the circulation at a point. **7.** 0 **9.** 3 **11.** 0 **13.** $2(x + y + z)$

15. $\frac{x^2 + y^2 + 3}{(1 + x^2 + y^2)^2}$ **17.** $\frac{1}{|\mathbf{r}|^2}$ **19.** $-\frac{1}{|\mathbf{r}|^4}$ **21. a.** Positive for both points **b.** $\operatorname{div} \mathbf{F} = 2$ **c.** Outward everywhere **d.** Positive **23. a.** $\operatorname{curl} \mathbf{F} = 2\,\mathbf{i}$ **b.** $|\operatorname{curl} \mathbf{F}| = 2$ **25. a.** $\operatorname{curl} \mathbf{F} = 2\,\mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k}$ **b.** $|\operatorname{curl} \mathbf{F}| = 2\sqrt{3}$ **27.** $3y\,\mathbf{k}$ **29.** $-4z\,\mathbf{j}$ **31. 0** **33. 0**

35. Follows from partial differentiation of $\frac{1}{(x^2 + y^2 + z^2)^{3/2}}$

37. Combine Exercise 36 with Theorem 15.10. **39. a.** False **b.** False **c.** False **d.** False **e.** False **41. a.** No **b.** No **c.** Yes, scalar function **d.** No **e.** No **f.** No **g.** Yes, vector field **h.** No **i.** Yes, vector field **43. a.** At $(0, 1, 1)$, $\mathbf{F}$ points in the positive x-direction; at $(1, 1, 0)$, $\mathbf{F}$ points in the negative z-direction; at $(0, 1, -1)$, $\mathbf{F}$ points in the negative x-direction; and at $(-1, 1, 0)$, $\mathbf{F}$ points in the positive z-direction. These vectors circle the y-axis in the counterclockwise direction looking along $\mathbf{a}$ from head to tail. **b.** The argument in part (a) can be repeated in any plane perpendicular to the y-axis to show that the vectors of $\mathbf{F}$ circle the y-axis in the counterclockwise direction looking along $\mathbf{a}$ from head to tail. Alternatively, computing the cross product, we find that $\mathbf{F} = \mathbf{a} \times \mathbf{r} = \langle z, 0, -x \rangle$, which is a rotation field in any plane perpendicular to $\mathbf{a}$.

45. Compute an explicit expression for $\mathbf{a} \times \mathbf{r}$ and then take the required partial derivatives. **47.** div $\mathbf{F}$ has a maximum value of 6 at $(1, 1, 1)$, $(1, -1, -1)$, $(-1, 1, 1)$, and $(-1, -1, -1)$.

49. $\mathbf{n} = \langle a, b, 2a + b \rangle$, where a and b are real numbers

51. $\mathbf{F} = \frac{1}{2}(y^2 + z^2)\,\mathbf{i}$; no **53. a.** The wheel does not spin. **b.** Clockwise, looking in the positive y-direction **c.** The wheel does not spin. **55.** $\omega = \frac{10}{\sqrt{3}}$, or $\frac{5}{\sqrt{3}\pi} \approx 0.9189$ revolutions per unit time

57. $\mathbf{F} = -200ke^{-x^2+y^2+z^2}(-x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k})$

$\nabla \cdot \mathbf{F} = -200k(1 + 2(x^2 + y^2 + z^2))e^{-x^2+y^2+z^2}$

59. a. $\mathbf{F} = -\frac{GMm\mathbf{r}}{|\mathbf{r}|^3}$ **b.** See Theorem 15.11.

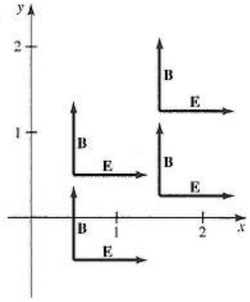
61.

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

63. a. Use $\nabla \times \mathbf{B} = -Ak\cos(kz - \omega t)\,\mathbf{i}$ and $\frac{\partial \mathbf{E}}{\partial t} = -A\omega\cos(kz - \omega t)\,\mathbf{i}$. **b.**



Section 3.6 Exercises, pp. 270–273

1. $\mathbf{r}(u, v) = \langle a\cos u, a\sin u, v\rangle, 0 \le u \le 2\pi, 0 \le v \le h$ **3.** $\mathbf{r}(u, v) = \langle a\sin u\cos v, a\sin u\sin v, a\cos u\rangle, 0 \le u \le \pi,$ $0 \le v \le 2\pi$ **5.** Use the parameterization from Exercise 3 and compute $\int_0^{\pi}\int_0^{2\pi} f(a\sin u\cos v, a\sin u\sin v, a\cos u)\,a^2\sin u\,dv\,du$. **7.** Use the parametrization from Exercise 3 and compute $\int_0^{\pi}\int_0^{2\pi} a^2\sin u\,(f\sin u\cos v + g\sin u\sin v + h\cos u)\,dv\,du$. **9.** The normal vectors point outward. **11.** $\langle u, v, \frac{1}{3}(16 - 2u + 4v)\rangle$, $|u| < \infty, |v| < \infty$ **13.** $\langle v\cos u, v\sin u, v\rangle, 0 \le u \le 2\pi,$ $2 \le v \le 8$ **15.** $\langle 3\cos u, 3\sin u, v\rangle, 0 \le u \le \frac{\pi}{2}, 0 \le v \le 3$ **17.** The plane $z = 2x + 3y - 1$ **19.** Part of the upper half of the cone $z^2 = 16x^2 + 16y^2$ of height 12 and radius 3 (with $y \ge 0$) **21.** 28π **23.** $16\sqrt{3}$ **25.** $\pi r\sqrt{r^2 + h^2}$ **27.** 1728π **29.** 0 **31.** $4\pi\sqrt{5}$ **33.** $8\sqrt{17} + 2\ln(\sqrt{17} + 4) = 37.1743$ **35.** $\frac{2\sqrt{3}}{3}$ **37.** $\frac{1250\pi}{3}$ **39.** $\frac{1}{48}(e - e^{-5} - e^{-7} + e^{-13})$ **41.** $\frac{1}{4\pi}$ **43.** -8 **45.** 0 **47.** 4π **49. a.** True **b.** False **c.** True **d.** True **51.** $8\pi(4\sqrt{17} + \ln(\sqrt{17} + 4))$ **53.** $8\pi a$ **55. a.** 8 **b.** $4\pi - 8$ **57. a.** 0 **b.** 0; the flow is tangent to the surface (radial flow). **59.** $2\pi ah$ **61.** $-400\left(e - \frac{1}{e}\right)^2$ **63.** $8\pi a$ **65. a.** $4\pi(b^3 - a^3)$ **b.** The net flux is zero. **67.** $\left(0, 0, \frac{2}{3}h\right)$ **69.** $\left(0, 0, \frac{7}{6}\right)$ **73.** Flux $= \iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iint_R dA$

Section 3.7 Exercises, pp. 280–282

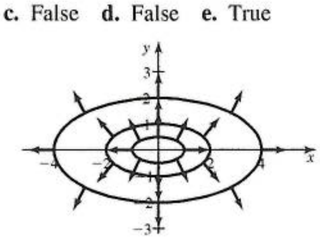
1. The integral measures the circulation along the closed curve C. **3.** Under certain conditions, the accumulated rotation of the vector field over the surface S equals the net circulation on the boundary of S. **5.** Both integrals equal -2π. **7.** Both integrals equal zero. **9.** Both integrals equal -18π. **11.** -24π **13.** $-\frac{128}{3}$ **15.** 15π **17.** 0 **19.** 0 **21.** $\nabla \times \mathbf{v} = \langle 1, 0, 0\rangle$; a paddle wheel with its axis aligned with the x-axis will spin with maximum angular speed counterclockwise (looking in the negative x-direction) at all points. **23.** $\nabla \times \mathbf{v} = \langle 0, -2, 0\rangle$; a paddle wheel with its axis aligned with the y-axis will spin with maximum angular speed clockwise (looking in the negative y-direction) at all points. **25. a.** False **b.** False **c.** True **d.** True **27.** 0 **29.** 0 **31.** 2π **33.** $\pi(\cos\varphi - \sin\varphi)$; maximum for $\varphi = 0$ **35.** The circulation is 48π; it depends on the radius of the circle but not on the center. **37. a.** The normal vectors point toward the z-axis on the curved surface of S and in the direction of $\langle 0, 1, 0\rangle$ on the flat surface of S. **b.** 2π **c.** 2π **39.** The integral is π for all a. **41. a.** 0 **b.** 0 **43. b.** 2π for any circle of radius r centered at the origin **c.** $\mathbf{F}$ is not differentiable along the z-axis. **45.** Apply the Chain Rule. **47.** $\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_R \left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z}\right) dA$

Section 3.8 Exercises, pp. 291–294

1. The surface integral measures the flow across the boundary. **3.** The flux across the boundary equals the cumulative expansion or contraction of the vector field inside the region. **5.** 32π **7.** The outward fluxes are equal. **9.** Both integrals equal 96π. **11.** Both integrals equal zero. **13.** 0 **15.** 0 **17.** $16\sqrt{6}\pi$ **19.** $\frac{2}{3}$ **21.** $-\frac{128}{3}\pi$ **23.** 24π **25.** -224π **27.** 12π **29.** 20 **31. a.** False **b.** False **c.** True **33.** 0 **35.** $\frac{3}{2}$ **37. b.** The net flux between the two spheres is $4\pi(a^2 - \varepsilon^2)$. **39. b.** Use $\nabla\cdot\mathbf{E} = 0$. **c.** The flux across S is the sum of the contributions from the individual charges. **d.** For an arbitrary volume, we find $\frac{1}{\varepsilon_0}\iiint_D q(x, y, z)\,dV = \iint_S \mathbf{E}\cdot\mathbf{n}\,dS = \iiint \nabla\cdot\mathbf{E}\,dV$. **e.** Use $\nabla^2\varphi = \nabla\cdot\nabla\varphi$. **41.** 0 **43.** $e^{-1} - 1$ **45.** $800\pi a^3 e^{-a^2}$

Chapter 3 Review Exercises, pp. 294–297

1. a. False **b.** True **c.** False **d.** False **e.** True **3.** $\nabla\varphi = \langle 2x, 8y\rangle$



5. $-\frac{\mathbf{r}}{|\mathbf{r}|^3}$ **7. a.** $\mathbf{n} = \frac{1}{2}\langle x, y\rangle$ **b.** 0 **c.** $\frac{1}{2}$ **9.** $\frac{\sqrt{46}}{4}(e^{6(\ln 8)^2} - 1)$ **11.** Both integrals equal zero. **13.** 0 **15.** The circulation is -4π; the outward flux is zero. **17.** The circulation is zero; the outward flux is 2π. **19.** $\frac{4v_0L^3}{3}$ **21.** $\varphi(x, y, z) = xy + yz^2$ **23.** $\varphi(x, y, z) = xye^z$ **25.** 0 for both methods **27. a.** $-\pi$ **b.** $\mathbf{F}$ is not conservative. **29.** 0 **31.** $\frac{20}{3}$ **33.** 8π **35.** The circulation is zero; the outward flux equals 2π. **37. a.** $b = c$ **b.** $a = -d$ **c.** $a = -d$ and $b = c$ **39.** $\nabla\cdot\mathbf{F} = 4\sqrt{x^2 + y^2 + z^2} = 4|\mathbf{r}|, \nabla\times\mathbf{F} = \mathbf{0}, \nabla\cdot\mathbf{F} \ne 0$; irrotational but not source free **41.** $\nabla\cdot\mathbf{F} = 2y + 12xz^2, \nabla\times\mathbf{F} = \mathbf{0}$, $\nabla\cdot\mathbf{F} \ne 0$; irrotational but not source free **43. a.** -1 and 0 **b.** $\mathbf{n} = \frac{1}{\sqrt{3}}\langle -1, 1, 1\rangle$ **45.** 18π **47.** $4\sqrt{3}$ **49.** $\frac{8\sqrt{3}}{3}$ **51.** 8π **53.** $4\pi a^2$ **55. a.** Use $x = y = 0$ to confirm the highest point; use $z = 0$ to confirm the base. **b.** The hemisphere S has the greater surface area—$2\pi a^2$ for S versus $\frac{5\sqrt{5} - 1}{6}\pi a^2$ for T. **57.** 0 **59.** 99π **61.** 0 **63.** $\frac{972}{5}\pi$ **65.** $\frac{124}{5}\pi$ **67.** $\frac{32}{3}$

CHAPTER 4

Exercises 4.1, p. 310

1. Linear

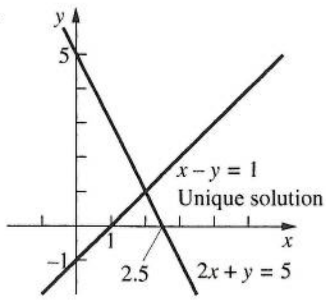
3. Linear

5. Nonlinear

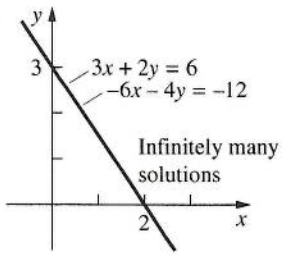
7. $\begin{aligned} x_1 + 3x_2 &= 7 \\ 4x_1 - x_2 &= 2 \end{aligned}$

9. $\begin{aligned} x_1 + x_2 &= 0 \\ 3x_1 + 4x_2 &= -1 \\ -x_1 + 2x_2 &= -3 \end{aligned}$

11.



13.



17. $\begin{aligned} x_1 &= -3t + 4 \\ x_2 &= 2t - 1 \\ x_3 &= t \end{aligned}$

19. $A = \begin{bmatrix} 2 & 1 & 6 \\ 4 & 3 & 8 \end{bmatrix}$

21. $Q = \begin{bmatrix} 1 & 4 & -3 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$

23. $\begin{aligned} 2x_1 + x_2 &= 6 \\ 4x_1 + 3x_2 &= 8 \end{aligned}$ and $\begin{aligned} x_1 + 4x_2 &= -3 \\ 2x_1 + x_2 &= 1 \\ 3x_1 + 2x_2 &= 1 \end{aligned}$

25. $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 2 & 0 & -1 & 1 \end{bmatrix}$

27. $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & -1 \\ -1 & 1 & 1 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 1 & 2 & 6 \\ 3 & 4 & -1 & 5 \\ -1 & 1 & 1 & 2 \end{bmatrix}$

29. $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 2 \\ 1 & -1 & 3 & 2 \end{bmatrix}$

31. $\begin{aligned} x_1 + 2x_2 - x_3 &= 1 \\ -x_2 + 3x_3 &= 1 \\ 5x_2 - 2x_3 &= 6 \end{aligned}$

33. $\begin{aligned} x_1 + x_2 &= 9 \\ -2x_2 &= -2 \\ -2x_2 &= -21 \end{aligned}$

35. $\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 1 \\ x_2 + x_3 - x_4 &= 3 \\ 3x_2 + 6x_3 &= 1 \end{aligned}$

Exercises 4.2, p. 324

1. a) The matrix is in echelon form.

b) The operation $R_1 - 2R_2$ yields reduced echelon form $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

3. a) The operations $R_2 - 2R_1$, $(1/2)R_1$, $R_2 - 4R_1$, $(1/5)R_2$ yield echelon form

$$\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 2/5 \end{bmatrix}.$$

5. a) The operations $R_1 \leftrightarrow R_2$, $(1/2)R_1$, $(1/2)R_2$ yield echelon form

$$\begin{bmatrix} 1 & 0 & 1/2 & 2 \\ 0 & 0 & 1 & 3/2 \end{bmatrix}.$$

7. a) The matrix is in echelon form.

b) The operations $R_1 - 2R_3$, $R_2 - 4R_3$, $R_1 - 3R_2$ yield reduced echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

9. a) The operation $(1/2)R_2$ yields echelon form

$$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & -1 & -3/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

11. $x_1 = 0$, $x_2 = 0$

13. $x_1 = -2 + 5x_3$, $x_2 = 1 - 3x_3$, x_3 arbitrary

15. The system is inconsistent.

17. $x_1 = x_3 = x_4 = 0$; x_2 arbitrary

19. The system is inconsistent.

21. $x_1 = -1 - (1/2)x_2 + (1/2)x_4$, $x_3 = 1 - x_4$, x_2 and x_4 arbitrary, $x_5 = 0$

23. Inconsistent

25. $x_1 = 2 - x_2$, x_2 arbitrary

27. $x_1 = 2 - x_2 + x_3$, x_2 and x_3 arbitrary

29. $x_1 = 3 - 2x_3$, $x_2 = -2 + 3x_3$, x_3 arbitrary

31. $x_1 = 3 - (7x_4 - 16x_5)/2$, $x_2 = (x_4 + 2x_5)/2$, $x_3 = -2 + (5x_4 - 12x_5)/2$, x_4 and x_5 arbitrary

33. Inconsistent

35. Inconsistent

37. All values of a except $a = 8$

39. $a = 3$ or $a = -3$

41. $\alpha = \pi/3$ or $\alpha = 5\pi/3$; $\beta = \pi/6$ or $\beta = 5\pi/6$

45. $\begin{bmatrix} 1 & \times & \times \\ 0 & 1 & \times \end{bmatrix}, \begin{bmatrix} 1 & \times & \times \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & \times & \times \\ 0 & 0 & 0 \end{bmatrix},$
$\begin{bmatrix} 0 & 1 & \times \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & \times \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$
$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

47. The operations $R_2 - 2R_1$, $R_1 + 2R_2$, $-R_2$ transform B to I. The operations $R_2 - 3R_1$, $R_1 + R_2$, $(-1/2)R_2$ reduce C to I, so the operations $-2R_2$, $R_1 - R_2$, $R_2 + 3R_1$ transform I to C. Thus the operations $R_2 - 2R_1$, $R_1 + 2R_2$, $-R_2$, $-2R_2$, $R_1 - R_2$, $R_2 + 3R_1$ transform B to C.

49. $N = 135$

51. The amounts were \$39, \$21, and \$12.

53. Let A denote the number of adults, S the number of students, and C the number of children. Possible solutions are: $A = 5k$, $S = 67 - 11k$, $C = 12 + 6k$, where $k = 0, 1, \ldots, 6$.

55. $n(n+1)/2$

57. $n(n+1)(2n+1)(3n^2+3n-1)/30$

Exercises 4.3, p. 335

1. $\begin{bmatrix} 1 & 1 & 0 & 5/6 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $n = 3$, $r = 2$, x_2

3. $\begin{bmatrix} 1 & 0 & 4 & 0 & 13/2 \\ 0 & 1 & -1 & 0 & -3/2 \\ 0 & 0 & 0 & 1 & 1/2 \end{bmatrix}$ $n = 4$, $r = 3$, x_3

5. $r = 2$, $r = 1$, $r = 0$

7. Infinitely many solutions

9. Infinitely many solutions, a unique solution, or no solution

11. A unique solution or infinitely many solutions

13. Infinitely many solutions

15. A unique solution or infinitely many solutions

17. Infinitely many solutions

19. There are nontrivial solutions.

21. There is only the trivial solution.

23. $a = 1$

25. a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

27. $7x + 2y - 30 = 0$

29. $-3x^2 + 3xy + y^2 - 54y + 113 = 0$

Exercises 4.4, p. 342

1. a)
$$\begin{aligned} x_1 \qquad\quad + x_4 &= 1200 \\ x_1 + x_2 \qquad\quad &= 1000 \\ x_3 + x_4 &= 600 \\ x_2 + x_3 \quad\; &= 400 \end{aligned}$$

b) $x_1 = 1100$, $x_2 = -100$, $x_3 = 500$;

c) The minimum value is $x_1 = 600$ and the maximum value is $x_1 = 1000$.

3. $x_2 = 800$, $x_3 = 400$, $x_4 = 200$

5. $I_1 = 0.05$, $I_2 = 0.6$, $I_3 = 0.55$

7. $I_1 = 35/13$, $I_2 = 20/13$, $I_3 = 15/13$

Exercises 4.5, p. 356

1. a) $\begin{bmatrix} 2 & 0 \\ 2 & 6 \end{bmatrix}$; **b)** $\begin{bmatrix} 0 & 4 \\ 2 & 4 \end{bmatrix}$;

c) $\begin{bmatrix} 0 & -6 \\ 6 & 18 \end{bmatrix}$; **d)** $\begin{bmatrix} -6 & 8 \\ 4 & 6 \end{bmatrix}$

3. $\begin{bmatrix} -2 & -2 \\ 0 & 0 \end{bmatrix}$ **5.** $\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}$

7. a) $\begin{bmatrix} 3 \\ -3 \end{bmatrix}$; **b)** $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$; **c)** $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

9. a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$; **b)** $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$; **c)** $\begin{bmatrix} 17 \\ 14 \end{bmatrix}$

11. a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$; **b)** $\begin{bmatrix} 20 \\ 16 \end{bmatrix}$

13. $a_1 = 11/3$, $a_2 = -4/3$

15. $a_1 = -2$, $a_2 = 0$ **17.** No solution

19. $a_1 = 4$, $a_2 = -3/2$ **21.** $\mathbf{w}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

23. $\mathbf{w}_3 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ **25.** $\begin{bmatrix} -4 & 6 \\ 2 & 12 \end{bmatrix}$

27. $\begin{bmatrix} 4 & 12 \\ 4 & 10 \end{bmatrix}$ **29.** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

31. $AB = \begin{bmatrix} 5 & 16 \\ 5 & 18 \end{bmatrix}$, $BA = \begin{bmatrix} 4 & 11 \\ 6 & 19 \end{bmatrix}$

33. $A\mathbf{u} = \begin{bmatrix} 11 \\ 13 \end{bmatrix}$, $\mathbf{v}A = [8, 22]$

35. 66 **37.** $\begin{bmatrix} 5 & 10 \\ 8 & 12 \\ 15 & 20 \\ 8 & 17 \end{bmatrix}$ **39.** $\begin{bmatrix} 27 \\ 28 \\ 43 \\ 47 \end{bmatrix}$

41. $(BA)\mathbf{u} = B(A\mathbf{u}) = \begin{bmatrix} 37 \\ 63 \end{bmatrix}$

43. $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

45. $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

47. $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} +$

$x_5 \begin{bmatrix} 3 \\ -4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

49. $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}$

51.

$C(A(B\mathbf{u}))$	$(CA)(B\mathbf{u})$	$(C(AB))\mathbf{u}$	$C((AB)\mathbf{u})$
12	16	20	16

53. a) AB is (2×4); BA is undefined.
b) Neither is defined.
c) AB is undefined; BA is (6×7).
d) AB is (2×2); BA is (3×3).
e) AB is (3×1); BA is undefined.
f) Both are (2×4).
g) AB is (4×4), BA is (1×1).

61. a) For (i), $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$,

$\mathbf{b} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$; for (ii), $A = \begin{bmatrix} 1 & -3 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$,

$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

b) For (i), $x_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$;

for (ii), $x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} +$

$x_3 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

c) For (i), $\mathbf{b} = 2\mathbf{A}_1 + \mathbf{A}_2$;
for (ii), $\mathbf{b} = 2\mathbf{A}_1 + \mathbf{A}_2 + 2\mathbf{A}_3$.

63. $B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

65. a) $B = \begin{bmatrix} -1 & 6 \\ 1 & 0 \end{bmatrix}$; **b)** Not possible;

c) $B = \begin{bmatrix} -2a & -2b \\ a & b \end{bmatrix}$, a and b arbitrary

Exercises 4.6, p. 367

1. $(DE)F = D(EF) = \begin{bmatrix} 23 & 23 \\ 29 & 29 \end{bmatrix}$

3. $DE = \begin{bmatrix} 8 & 15 \\ 11 & 18 \end{bmatrix}$, $ED = \begin{bmatrix} 12 & 27 \\ 7 & 14 \end{bmatrix}$

5. $F\mathbf{u} = F\mathbf{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ **7.** $\begin{bmatrix} 3 & 4 & 2 \\ 1 & 7 & 6 \end{bmatrix}$

9. $\begin{bmatrix} 5 & 5 \\ 9 & 9 \end{bmatrix}$ **11.** $[0\ \ 0]$ **13.** -6

15. 36 **17.** 2 **19.** $\sqrt{2}$

21. $\sqrt{29}$ **23.** 0 **25.** $2\sqrt{5}$

29. D and F are symmetric.

31. AB is symmetric if and only if $AB = BA$.

33. $\mathbf{x}^T D\mathbf{x} = x_1^2 + 3x_2^2 + (x_1 + x_2)^2 > 0$

35. $\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}$ **37.** $\begin{bmatrix} -27 & -9 \\ 27 & 9 \end{bmatrix}$

39. $\begin{bmatrix} -12 & 18 & 24 \\ 18 & -27 & -36 \\ 24 & -36 & -48 \end{bmatrix}$

41. a) $\mathbf{x} = \begin{bmatrix} -10 \\ 8 \end{bmatrix}$; **b)** $\mathbf{x} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

57. $n = 5$, $m = 7$
59. $n = 4$, $m = 6$
61. $n = 5$, $m = 5$

Exercises 4.7, p. 376

1. Linearly independent
3. Linearly dependent, $\mathbf{v}_5 = 3\mathbf{v}_1$
5. Linearly dependent, $\mathbf{v}_3 = 2\mathbf{v}_1$
7. Linearly dependent, $\mathbf{u}_4 = 4\mathbf{u}_5$
9. Linearly independent
11. Linearly dependent, $\mathbf{u}_4 = 4\mathbf{u}_5$

13. Linearly dependent, $\mathbf{u}_4 = \frac{16}{5}\mathbf{u}_0 + \frac{12}{5}\mathbf{u}_1 - \frac{4}{5}\mathbf{u}_2$

15. Those in Exercises 5, 6, 13, and 14

17. Singular; $x_1 = -2x_2$ **19.** Singular; $x_1 = -2x_2$

21. Singular; $x_1 = x_2 = 0$, x_3 arbitrary

23. Nonsingular

25. Singular; $x_2 = x_3 = 0$, x_1 arbitrary

27. Nonsingular **29.** $a = 6$

31. $b(a-2) = 4$ **33.** $c - ab = 0$

35. $\mathbf{v}_3 = \mathbf{A}_2$ **37.** $\mathbf{v}_2 = (\mathbf{C}_1 + \mathbf{C}_2)/2$

39. $\mathbf{u}_3 = (-8\mathbf{F}_1 - 2\mathbf{F}_2 + 9\mathbf{F}_3)/3$

41. $\mathbf{b} = -11\mathbf{v}_1 + 7\mathbf{v}_2$ **43.** $\mathbf{b} = 0\mathbf{v}_1 + 0\mathbf{v}_2$

45. $\mathbf{b} = -3\mathbf{v}_1 + 2\mathbf{v}_2$

47. a) Any value a **b)** Any value a

Exercises 4.8, p. 388

1. $p(t) = (-1/2)t^2 + (9/2)t - 1$

3. $p(t) = 2t + 3$

5. $p(t) = 2t^3 - 2t^2 + 3t + 1$

7. $y = 2e^{2x} + e^{3x}$

9. $y = 3e^{-x} + 4e^x + e^{2x}$

11. $\int_0^{3h} f(t)\,dt \approx \frac{3h}{2}[f(h) + f(2h)]$

13. $\int_0^{3h} f(t)\,dt$

$\approx \frac{3h}{8}[f(0) + 3f(h) + 3f(2h) + f(3h)]$

15. $\int_0^{h} f(t)\,dt \approx \frac{h}{2}[-f(-h) + 3f(0)]$

17. $f'(0) \approx [-f(0) + f(h)]/h$

19. $f'(0) \approx [-3f(0) + 4f(h) - f(2h)]/(2h)$

21. $f''(0) \approx [f(-h) - 2f(0) + f(h)]/h^2$

27. $p(t) = t^3 + 2t^2 + 3t + 2$

29. $p(t) = t^3 + t^2 + 4t + 3$

35. $f'(a) \approx \frac{1}{12h}$

$\times [f(a-2h) - 8f(a-h) + 8f(a+h) - f(a+2h)]$

Exercises 4.9, p. 400

5. $x_1 = -3$, $x_2 = 1.5$

7. $x_1 = 14$, $x_2 = -20$, $x_3 = 8$

9. If $B = (b_{ij})$ is a (3×3) matrix such that $AB = I$, then $0b_{11} + 0b_{21} + 0b_{31} = 1$. Since this is impossible, no such matrix exists.

13. $\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$

15. $\begin{bmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix}$ **17.** $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & -4 & 1 \end{bmatrix}$

19. $\begin{bmatrix} 1 & -2 & 0 \\ 3 & -3 & -1 \\ -6 & 7 & 2 \end{bmatrix}$

21. $\begin{bmatrix} -1/2 & -2/3 & -1/6 & 7/6 \\ 1 & 1/3 & 1/3 & -4/3 \\ 0 & -1/3 & -1/3 & 1/3 \\ -1/2 & 1 & 1/2 & 1/2 \end{bmatrix}$

23. $A^{-1} = (1/10)\begin{bmatrix} 3 & 2 \\ -2 & 2 \end{bmatrix}$

25. A has no inverse **27.** $\lambda = 2$ and $\lambda = -2$

29. $x_1 = 6$, $x_2 = -8$ **31.** $x_1 = 18$, $x_2 = 13$

33. $x_1 = 5/2$, $x_2 = 5/2$

35. $Q^{-1} = C^{-1}A^{-1} = \begin{bmatrix} -3 & 1 \\ 3 & 5 \end{bmatrix}$

37. $Q^{-1} = (A^{-1})^T = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$

39. $Q^{-1} = (A^{-1})^T(C^{-1})^T = \begin{bmatrix} -3 & 3 \\ 1 & 5 \end{bmatrix}$

41. $Q^{-1} = BC^{-1} = \begin{bmatrix} 1 & 5 \\ -1 & 4 \end{bmatrix}$

43. $Q^{-1} = (1/2)A^{-1} = \begin{bmatrix} 3/2 & 1/2 \\ 0 & 1 \end{bmatrix}$

45. $Q^{-1} = B(C^{-1}A^{-1}) = \begin{bmatrix} 3 & 11 \\ -3 & 7 \end{bmatrix}$

47. $B = \begin{bmatrix} 1 & 10 \\ 15 & 12 \\ 3 & 3 \end{bmatrix}$; $C = \begin{bmatrix} 13 & 12 & 8 \\ 2 & 3 & 5 \end{bmatrix}$

49. $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 2 & 35 & 1 \\ 14 & 35 & 34 \\ 23 & 12 & 70 \end{bmatrix}$,

$(3A)^{-1} = \frac{1}{3}A^{-1} = \begin{bmatrix} 1/3 & 2/3 & 5/3 \\ 1 & 1/3 & 2 \\ 2/3 & 8/3 & 1/3 \end{bmatrix}$,

$(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 8 \\ 5 & 6 & 1 \end{bmatrix}$

63. $b_0 = -5$, $b_1 = 2$

64. $b_0 = -7$, $b_1 = 0$

CHAPTER 5

Section 5.1, pp. 420–423

1. The order of a differential equation is the order of the highest order derivative in the equation. **3.** A differential equation of the form $y''(t) + p(t)y'(t) + q(t)y(t) = f(t)$ is homogeneous if $f(t) = 0$ on the interval of interest. **5.** If one function is a nonzero constant multiple of the other on the interval, then the two functions are linearly dependent. **7.** Find two linearly independent solutions, y_1 and y_2, of the corresponding homogeneous differential equation. Find any particular solution, y_p, of the original differential equation. The general solution is then $y = c_1y_1 + c_2y_2 + y_p$, where c_1 and c_2 are arbitrary constants.

9. order 2; linear; nonhomogeneous **11.** order 2; nonlinear; nonhomogeneous

13. $$\begin{aligned} y'' - 4y &= (3e^{2t} - 5e^{-2t})'' - 4(3e^{2t} - 5e^{-2t}) \\ &= 12e^{2t} - 20e^{-2t} - 12e^{2t} + 20e^{-2t} \\ &= 0 \end{aligned}$$

15. $$\begin{aligned} y'' - 9y &= (4e^{3t} + 3e^{-3t} - 2t)'' - 9(4e^{3t} + 3e^{-3t} - 2t) \\ &= 36e^{3t} + 27e^{-3t} - 36e^{3t} - 27e^{-3t} + 18t \\ &= 18t \end{aligned}$$

17. $$\begin{aligned} y'' - y' - 2y &= (c_1e^{-t} + c_2e^{2t})'' - (c_1e^{-t} + c_2e^{2t})' \\ &\quad - 2(c_1e^{-t} + c_2e^{2t}) \\ &= c_1e^{-t} + 4c_2e^{2t} - (-c_1e^{-t} + 2c_2e^{2t}) \\ &\quad - 2c_1e^{-t} - 2c_2e^{2t} \\ &= 0 \end{aligned}$$

19. $$\begin{aligned} &y'' + 6y' + 25y \\ &= (c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t)'' \\ &\quad + 6(c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t)' \\ &\quad + 25(c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t) \\ &= (-7c_1e^{-3t}\sin 4t - 24c_1e^{-3t}\cos 4t + 24c_2e^{-3t}\sin 4t - 7c_2e^{-3t}\cos 4t) \\ &\quad + 6(-3c_1e^{-3t}\sin 4t + 4c_1e^{-3t}\cos 4t - 4c_2e^{-3t}\sin 4t \\ &\quad - 3c_2e^{-3t}\cos 4t) + 25(c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t) \\ &= (-7 - 18 + 25)c_1e^{-3t}\sin 4t + (-24 + 24)c_1e^{-3t}\cos 4t \\ &\quad + (24 - 24)c_2e^{-3t}\sin 4t + (-7 - 18 + 25)c_2e^{-3t}\cos 4t \\ &= 0 \end{aligned}$$

21. $$\begin{aligned} &ty'' - (t+1)y' + y = t(c_1e^t + c_2(t+1))'' - (t+1)\cdot \\ &\quad (c_1e^t + c_2(t+1))' + (c_1e^t + c_2(t+1)) \\ &= t(c_1e^t) - (t+1)(c_1e^t + c_2) + (c_1e^t + c_2(t+1)) \\ &= tc_1e^t - tc_1e^t - c_1e^t - (t+1)c_2 + c_1e^t + c_2(t+1) \\ &= 0 \end{aligned}$$

23. $y = c_1e^{-6t} + c_2e^{6t}$

25. $y = c_1e^{-t} + c_2te^{-t}$

27. $y'' - y = (e^{-3t})'' - e^{-3t} = 9e^{-3t} - e^{-3t} = 8e^{-3t}$

29. $$\begin{aligned} y'' - 4y' + 4y &= (t^2e^{2t})'' - 4(t^2e^{2t})' + 4t^2e^{2t} \\ &= (4t^2e^{2t} + 8te^{2t} + 2e^{2t}) - 4(2t^2e^{2t} + 2te^{2t}) + 4t^2e^{2t} \\ &= (4 - 8 + 4)t^2e^{2t} + (8 - 8)te^{2t} + 2e^{2t} \\ &= 2e^{2t} \end{aligned}$$

31. Let $y_1 = \dfrac{e^{-t}}{2}$, $y_2 = \dfrac{e^{-t}}{2} + 3e^{7t}$, and $y_3 = y_2 - y_1 = 3e^{7t}$. Then

$$y_1'' - 49y_1 = \left(\frac{e^{-t}}{2}\right)'' - 49\left(\frac{e^{-t}}{2}\right) = \frac{e^{-t}}{2} - \frac{49e^{-t}}{2} = -24e^{-t}$$

$$\begin{aligned} y_2'' - 49y_2 &= \left(\frac{e^{-t}}{2} + 3e^{7t}\right)'' - 49\left(\frac{e^{-t}}{2} + 3e^{7t}\right) \\ &= \frac{e^{-t}}{2} + 147e^{7t} - \frac{49e^{-t}}{2} - 147e^{7t} = -24e^{-t} \end{aligned}$$

$$y_3'' - 49y_3 = (3e^{7t})'' - 49(3e^{7t}) = 147e^{7t} - 147e^{7t} = 0$$

33. Let $y_1 = -e^t$, $y_2 = 6e^{4t} - e^t$, and $y_3 = y_2 - y_1 = 6e^{4t} - e^t - (-e^t) = 6e^{4t}$. Then

$$\begin{aligned} y_1'' - y_1' - 12y_1 &= (-e^t)'' - (-e^t)' - 12(-e^t) \\ &= -e^t + e^t + 12e^t = 12e^t \end{aligned}$$

$$\begin{aligned} &y_2'' - y_2' - 12y_2 \\ &= (6e^{4t} - e^t)'' - (6e^{4t} - e^t)' - 12(6e^{4t} - e^t) \\ &= (96e^{4t} - e^t) - (24e^{4t} - e^t) - 72e^{4t} + 12e^t = 12e^t \end{aligned}$$

$$\begin{aligned} y_3'' - y_3' - 12y_3 &= (6e^{4t})'' - (6e^{4t})' - 12(6e^{4t}) \\ &= 96e^{4t} - 24e^{4t} - 72e^{4t} = 0 \end{aligned}$$

35. homogeneous solutions: $\sin\sqrt{2}t$, $\cos\sqrt{2}t$
general solution: $y = c_1\sin\sqrt{2}t + c_2\cos\sqrt{2}t + e^t$

37. homogeneous solutions: $e^{3t/2}\sin 4t$, $e^{3t/2}\cos 4t$
general solution: $y = c_1e^{3t/2}\sin 4t + c_2e^{3t/2}\cos 4t + 100t + 48$

39. $y = 4\cos 3t$ **41.** $y = -e^{5t} - 2e^{-4t}$

43. $y = \dfrac{1}{16}e^{4t} + \dfrac{1}{16}e^{-4t} - t^2 - \dfrac{1}{8}$ **45.** $y = \dfrac{3}{4}t^{-2} + \dfrac{1}{4}t^2$

47. a. False **b.** True **c.** False **d.** False **e.** False

49. $$\begin{aligned} &y'' - 12y' + 36y = (c_1e^{6t} + c_2te^{6t} + t^2e^{6t})'' \\ &\quad - 12(c_1e^{6t} + c_2te^{6t} + t^2e^{6t})' + 36(c_1e^{6t} + c_2te^{6t} + t^2e^{6t}) \\ &= (36c_1e^{6t} + 12c_2e^{6t} + 36c_2te^{6t} + 2e^{6t} + 24te^{6t} + 36t^2e^{6t}) \\ &\quad - 12(6c_1e^{6t} + c_2e^{6t} + 6c_2te^{6t} + 2te^{6t} + 6t^2e^{6t}) \\ &\quad + 36(c_1e^{6t} + c_2te^{6t} + t^2e^{6t}) \\ &= (36 - 72 + 36)c_1e^{6t} + (12 - 12)c_2e^{6t} + (36 - 72 + 36)c_2te^{6t} \\ &\quad + 2e^{6t} + (24 - 24)te^{6t} + (36 - 72 + 36)t^2e^{6t} \\ &= 2e^{6t} \end{aligned}$$

51. $$\begin{aligned} &t^2y'' - 3ty' + 4y \\ &= t^2(c_1t^2 + c_2t^2\ln t)'' - 3t(c_1t^2 + c_2t^2\ln t)' + 4(c_1t^2 + c_2t^2\ln t) \\ &= t^2(2c_1 + 3c_2 + 2c_2\ln t) - 3t(2c_1t + c_2t + 2c_2t\ln t) \\ &\quad + 4(c_1t^2 + c_2t^2\ln t) = (2c_1 + 3c_2 - 6c_1 - 3c_2 + 4c_1)t^2 \\ &\quad + (2c_2 - 6c_2 + 4c_2)t^2\ln t \\ &= 0 \end{aligned}$$

53. $$\begin{aligned} &t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right)y \\ &= t^2(c_1t^{-1/2}\cos t + c_2t^{-1/2}\sin t)'' + t(c_1t^{-1/2}\cos t + c_2t^{-1/2}\sin t)' \\ &\quad + \left(t^2 - \frac{1}{4}\right)(c_1t^{-1/2}\cos t + c_2t^{-1/2}\sin t) \\ &= t^2\left(\frac{3}{4}c_1t^{-5/2}\cos t + \frac{3}{4}c_2t^{-5/2}\sin t - c_2t^{-3/2}\cos t + c_1t^{-3/2}\sin t\right. \\ &\quad \left. - c_1t^{-1/2}\cos t - c_2t^{-1/2}\sin t\right) \\ &\quad + t\left(-c_1t^{-1/2}\sin t + c_2t^{-1/2}\cos t - \frac{1}{2}c_1t^{-3/2}\cos t - \frac{1}{2}c_2t^{-3/2}\sin t\right) \\ &\quad + \left(t^2 - \frac{1}{4}\right)(c_1t^{-1/2}\cos t + c_2t^{-1/2}\sin t) \\ &= \left(\frac{3}{4} - \frac{1}{2} - \frac{1}{4}\right)c_1t^{-1/2}\cos t + \left(\frac{3}{4} - \frac{1}{2} - \frac{1}{4}\right)c_2t^{-1/2}\sin t \\ &\quad + (-1 + 1)c_2t^{1/2}\cos t + (1 - 1)c_1t^{1/2}\sin t \\ &\quad + (-1 + 1)c_1t^{3/2}\cos t + (-1 + 1)c_2t^{3/2}\sin t \\ &= 0 \end{aligned}$$

55. a. $y'' - y = (e^t)'' - e^t = e^t - e^t = 0$
$y'' - y = (e^{-t})'' - e^{-t} = e^{-t} - e^{-t} = 0$
The functions are not multiples of each other and so are a linearly independent set of solutions. **b.** $y = \sinh t$ and $y = \cosh t$ are linear combinations of the solutions e^{-t} and e^t and so must be solutions themselves by the Superposition Principle. They are independent since one is not a multiple of the other. **c.** $(\sinh t)' = \dfrac{e^t + e^{-t}}{2} = \cosh t$ and $(\cosh t)' = \dfrac{e^t - e^{-t}}{2} = \sinh t$ so $(\sinh t)'' - \sinh t =$
$(\cosh t)' - \sinh t = \sinh t - \sinh t = 0$
$(\cosh t)'' - \cosh t = (\sinh t)' - \cosh t = \cosh t - \cosh t = 0$

d. $y = c_1e^{-t} + c_2e^{t}$, $y = c_3 \sinh t + c_4 \cosh t$ **e.** For $y = e^{kt}$, $y'' - k^2y = (e^{kt})'' - k^2e^{kt} = (ke^{kt})' - k^2e^{kt} = k^2e^{kt} - k^2e^{kt} = 0$. For $y = e^{kt}$, $y'' - k^2y = (e^{-kt})'' - k^2e^{-kt} = (-ke^{-kt})' - k^2e^{-kt} = k^2e^{-kt} - k^2e^{-kt} = 0$. **f.** $y = c_1e^{-kt} + c_2e^{kt}$, $y = c_3 \sinh kt + c_4 \cosh kt$

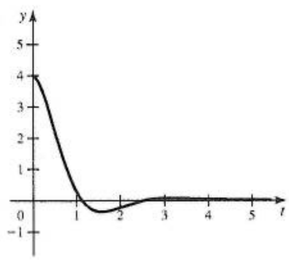
57. $y^{(4)} - 16y$
$= (c_1e^{-2t} + c_2e^{2t} + c_3 \sin 2t + c_4 \cos 2t)^{(4)}$
$\quad - 16(c_1e^{-2t} + c_2e^{2t} + c_3 \sin 2t + c_4 \cos 2t)$
$= (16c_1e^{-2t} + 16c_2e^{2t} + 16c_3 \sin 2t + 16c_4 \cos 2t)$
$\quad - 16(c_1e^{-2t} + c_2e^{2t} + c_3 \sin 2t + c_4 \cos 2t)$
$= 0$

59. a. By the chain rule $\frac{d}{dt}(y'(t)^2) = 2y'(t)\frac{d}{dt}y'(t) = 2y'(t)y''(t)$
b. $y''y' = 1 \Rightarrow 2y''y' = 2 \Rightarrow (y'(t)^2)' = 2$ using part (a).
c. $(y'(t)^2)' = 2 \Rightarrow y'(t)^2 = \int 2\,dt = 2t + c_1$ so $y'(t) = \pm\sqrt{2t + c_1}$ **d.** From part (c), $y(t) = \int \pm\sqrt{2t + c_1}\,dt = \pm\left(\frac{1}{2}\right)\frac{2}{3}(2t + c_1)^{3/2} + c_2 = c_2 \pm \frac{1}{3}(2t + c_1)^{3/2}$

61. $y = c_1e^{3t} + c_2 - \frac{4}{3}t$ **63.** $y = -\frac{1}{\sqrt{c_1}}\tan^{-1}\frac{t}{\sqrt{c_1}} + c_2$ if $c_1 > 0$; $y = -\frac{1}{2\sqrt{|c_1|}}\ln\left|\frac{t - \sqrt{|c_1|}}{t + \sqrt{|c_1|}}\right| + c_2$ if $c_1 < 0$

65. a. $y'' + 3y' + \frac{25}{4}y$
$= \left(e^{-3t/2}(c_1 \sin 2t + c_2 \cos 2t)\right)'' + 3\left(e^{-3t/2}(c_1 \sin 2t + c_2 \cos 2t)\right)'$
$\quad + \frac{25}{4}\left(e^{-3t/2}(c_1 \sin 2t + c_2 \cos 2t)\right)$
$= \left(6c_2 - \frac{7}{4}c_1\right)e^{-3t/2}\sin 2t + \left(-6c_1 - \frac{7}{4}c_2\right)e^{-3t/2}\cos 2t$
$\quad + \left(-6c_2 - \frac{9}{2}c_1\right)e^{-3t/2}\sin 2t + \left(6c_1 - \frac{9}{2}c_2\right)e^{-3t/2}\cos 2t$
$\quad + \frac{25}{4}c_1e^{-3t/2}\sin 2t + \frac{25}{4}c_2e^{-3t/2}\cos 2t$
$= 0$

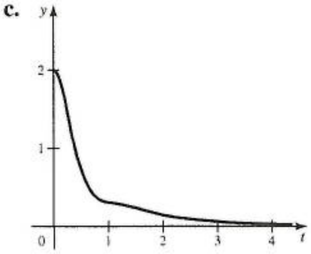
b. $y = e^{-3t/2}(3 \sin 2t + 4 \cos 2t)$ **c.**



67. a. $y'' + 6y' + 25y$
$= \left(e^{-3t}(c_1 \sin 4t + c_2 \cos 4t)\right)'' + 6\left(e^{-3t}(c_1 \sin 4t + c_2 \cos 4t)\right)'$
$\quad + 25\left(e^{-3t}(c_1 \sin 4t + c_2 \cos 4t)\right)$
$= (24c_2 - 7c_1)e^{-3t}\sin 4t + (-24c_1 - 7c_2)e^{-3t}\cos 4t + e^{-t}$
$\quad + (-24c_2 - 18c_1)e^{-3t}\sin 4t + (24c_1 - 18c_2)e^{-3t}\cos 4t - 6e^{-t}$
$\quad + 25c_1e^{-3t}\sin 4t + 25c_2e^{-3t}\cos 4t + 25e^{-t}$
$= 20e^{-t}$

b. $y = e^{-3t}(\sin 4t + \cos 4t) + e^{-t}$

c.



69. a. By the Chain Rule $\frac{d}{dt}\varphi(x) = \frac{d\varphi}{dx}\frac{dx}{dt} = -F(x)x'(t)$, then
$\frac{d}{dt}\left(\frac{1}{2}m(x'(t))^2 + \varphi(x)\right) = 2\left(\frac{1}{2}m\right)x'(t)x''(t) - F(x)x'(t)$
$\qquad = mx'(t)x''(t) - F(x)x'(t)$
$\qquad = x'(t)(mx''(t) - F(x))$
$\qquad = x'(t)(0) = 0$

Since $v = x'(t)$, this relation can be written $\frac{d}{dt}\left(\frac{1}{2}mv^2 + \varphi(x)\right) = 0$.
b. By part (a), the derivative of $E(t)$ is identically 0, so $E(t)$ is constant.

Section 5.2, pp. 433–435

1. $y = e^{rt}$
3. Case 1: real distinct roots
Case 2: real repeated roots
Case 3: complex roots
5. $y = c_1e^{rt} + c_2te^{rt}$ **7.** $y = c_1e^{-2t}\sin 3t + c_2e^{-2t}\cos 3t$
9. $y = c_1e^{-5t} + c_2e^{5t}$ **11.** $y = c_1e^{3t} + c_2$
13. $y = c_1e^{-5t} + c_2e^{2t}$ **15.** $y = c_1e^{-6t} + c_2e^{6t}$; $y = \frac{3}{2}e^{-6t} + \frac{3}{2}e^{6t}$
17. $y = c_1e^{-3t} + c_2e^{6t}$; $y = -\frac{4}{9}e^{-3t} + \frac{4}{9}e^{6t}$
19. $y = c_1e^{-t/2} + c_2e^{5t/2}$; $y = \frac{5}{2}e^{-t/2} + \frac{1}{2}e^{5t/2}$
21. $y = c_1e^{t} + c_2te^{t}$; $y = 4e^{t} - 4te^{t}$
23. $y = c_1e^{t/2} + c_2te^{t/2}$; $y = e^{t/2} + \frac{3}{2}te^{t/2}$
25. $y = c_1e^{-2t} + c_2te^{-2t}$; $y = e^{-2t} + 2te^{-2t}$
27. $y = c_1 \sin 3t + c_2 \cos 3t$; $y = -\frac{8}{3}\sin 3t + 8 \cos 3t$
29. $y = c_1e^{t}\sin 2t + c_2e^{t}\cos 2t$; $y = -e^{t}\sin 2t + e^{t}\cos 2t$
31. $y = c_1e^{-3t}\sin t + c_2e^{-3t}\cos t$; $y = 6e^{-3t}\sin t$
33. $y = t^{-1} + t$ **35.** $y = \frac{31}{8}t^{-3} + \frac{17}{8}t^{5}$ **37.** $y = 4t^{-3} - 4t^{-2}$
39. a. False **b.** False **c.** False **d.** True **e.** False
41. $A\cos(\omega t + \varphi) = A\cos\omega t\cos\varphi - A\sin\omega t\sin\varphi$
$\qquad = c_1 \sin \omega t + c_2 \cos \omega t$
$\Rightarrow c_1 = -A\sin\varphi,\ c_2 = A\cos\varphi$
$\Rightarrow \sqrt{c_1^2 + c_2^2} = \sqrt{A^2\sin^2\varphi + A^2\cos^2\varphi} = \sqrt{A^2(1)} = A$
$\Rightarrow -\frac{c_1}{c_2} = \frac{A\sin\varphi}{A\cos\varphi} = \tan\varphi$

43. $y = 3\sqrt{2}\sin\left(4t + \frac{3\pi}{4}\right)$ **45.** $y = 2\sin\left(2t + \frac{2\pi}{3}\right)$
47. $y = c_1e^{-2t} + c_2e^{3t} + c_3$ **49.** $y = c_1e^{2t} + c_2e^{4t} + c_3$
51. $y = c_1 \sin t + c_2 \cos t + c_3 \sin 2t + c_4 \cos 2t$
53. $y = c_1t^{-1} + c_2t^{-1}\ln t$ **55.** $y = c_1t^{-3} + c_2t^{-3}\ln t$
57. $y = c_1t^{-3}\sin(4\ln t) + c_2t^{-3}\cos(4\ln t)$
59. $y = \sqrt{t}\left(c_1 \sin\frac{\ln t}{2} + c_2\cos\frac{\ln t}{2}\right)$
61. $y = c_1 \sin kt + c_2 \cos kt + c_3t + c_4$
63. a. As seen in the text, the associated quadratic equation is $p^2 + (a-1)p + b = 0$. If there is a repeated root, the discriminant in the quadratic formula is 0, and the root is $p = \frac{-(a-1) \pm \sqrt{0}}{2} = \frac{1-a}{2}$. **b.** If $y = t^pv(t)$ then

$$
\begin{aligned}
&t^2y'' + aty' + by \\
&= t^2(t^pv)'' + at(t^pv)' + b(t^pv) \\
&= t^2(t^pv'' + 2pt^{p-1}v' + p(p-1)t^{p-2}v) + at(t^pv' + pt^{p-1}v) + bt^pv \\
&= t^p\left(t^2v'' + (2p+a)tv' + (p^2 + (a-1)p + b)v\right) \\
&= t^p\left(t^2v'' + (1)tv' + (0)v\right) \quad \text{(using part (a) and the fact that } p \text{ satisfies } p^2 + (a-1)p + b = 0) \\
&= t^p(t^2v'' + tv') = 0
\end{aligned}
$$

so $t^2v'' + tv' = 0$.
c. If $w = v'$ then $t^2v'' + tv' = 0 \Rightarrow t^2w' + tw = 0$ $\Rightarrow t^2\frac{dw}{dt} + tw = 0 \Rightarrow \frac{dw}{w} = -\frac{dt}{t}$. Integrating both sides gives

$\ln w = -\ln t + c \Rightarrow e^{\ln w} = w = e^{-\ln t + c} = e^ce^{-\ln t} = c_1\left(\frac{1}{t}\right)$

so $w = \frac{c_1}{t}$. **d.** Integrating both sides gives: $w = v' = \frac{c_1}{t} \Rightarrow$ $v = c_1 \ln t + c_2$. Then $y = t^pv(t) = t^p(c_1\ln t + c_2) =$ $c_1t^p\ln t + c_2t^p$ is the general solution. **65. a.** u has the form of the general solution with $c_1 = \frac{1}{r_1 - r_2}$ and $c_2 = -\frac{1}{r_1 - r_2}$. **b.** Apply l'Hôpital's rule, treating r_1 as a variable: $\lim_{r_1 \to r_2} \frac{e^{r_1t} - e^{r_2t}}{r_1 - r_2} =$ $\lim_{r_1 \to r_2} \frac{te^{r_1t}}{1} = te^{r_2t}$. If $r_1 = r_2$ the linearly independent solutions are e^{r_1t} and te^{r_1t}.

Section 5.3, pp. 443–444

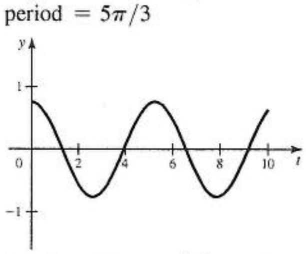
1. Find two linearly independent solutions, y_1 and y_2, of the corresponding homogeneous equation. Find a particular solution, y_p, of the equation. Form the general solution, $y = c_1y_1 + c_2y_2 + y_p$.
3. $y_p = Ae^{-4t}$ **5.** $y_p = Ae^{-t}\sin 4t + Be^{-t}\cos 4t$
7. $y_p = (At^2 + Bt + C)e^{-t}$ **9.** $y_p = -\frac{2}{9}t - \frac{1}{9}$
11. $y_p = -\frac{1}{9}t^4 - \frac{2}{27}t^3 - \frac{1}{18}t^2 - \frac{7}{162}t - \frac{13}{972}$ **13.** $y_p = \frac{1}{3}e^{-2t}$
15. $y_p = -\frac{6}{11}e^{-3t}$ **17.** $y_p = -\frac{3}{5}\sin 2t$ **19.** $y_p = -\frac{1}{5}\sin t - \frac{2}{5}\cos t$
21. $y_p = -\frac{2}{3}e^t + \frac{1}{4}$ **23.** $y_p = \left(-\frac{1}{6}t + \frac{1}{72}\right)e^{-t}$
25. $y_p = \frac{1}{36}\sin 2t + \frac{1}{12}t\cos 2t$
27. $y_p = \left(\frac{1}{5}t + \frac{13}{50}\right)\sin t - \left(\frac{2}{5}t - \frac{9}{50}\right)\cos t$ **29.** $y_p = \frac{3}{2}te^t$
31. $y_p = -\frac{4}{5}te^{-3t}$ **33.** $y_p = 2te^{-2t}$ **35. a.** False **b.** False **c.** True
37. $y = c_1\sin t + c_2\cos t - \frac{4}{3}\sin 2t$; $y = \frac{8}{3}\sin t + \cos t - \frac{4}{3}\sin 2t$
39. $y = c_1e^{-2t}\sin t + c_2e^{-2t}\cos t + \frac{12}{5}$;
$y = -\frac{19}{5}e^{-2t}\sin t - \frac{7}{5}e^{-2t}\cos t + \frac{12}{5}$
41. $y = c_1 \sin 3t + c_2\cos 3t + t\sin 3t$; $y = t\sin 3t$
43. $y_p = -3t^4 - 9t^3 - 63t^2 - 181t - 307$
45. $y_p = \left(-\frac{25}{13}t - \frac{1550}{1521}\right)e^{-t}\sin 3t + \left(\frac{50}{39}t - \frac{150}{169}\right)e^{-t}\cos 3t$
47. $y_p = -e^t$ **49.** $y_p = \left(t - \frac{10}{3}\right)e^{2t}$
51. Assume $y_{p1}'' + py_{p1}' + qy_{p1} = f(t)$ and $y_{p2}'' + py_{p2}' + qy_{p2} = g(t)$. Then

$$
\begin{aligned}
&(y_{p1} + y_{p2})'' + p(y_{p1} + y_{p2})' + q(y_{p1} + y_{p2}) \\
&= (y_{p1}'' + y_{p2}'') + p(y_{p1}' + y_{p2}') + q(y_{p1} + y_{p2}) \\
&= y_{p1}'' + y_{p2}'' + py_{p1}' + py_{p2}' + qy_{p1} + qy_{p2} \\
&= (y_{p1}'' + py_{p1}' + qy_{p1}) + (y_{p2}'' + py_{p2}' + qy_{p2}) \\
&= f(t) + g(t)
\end{aligned}
$$

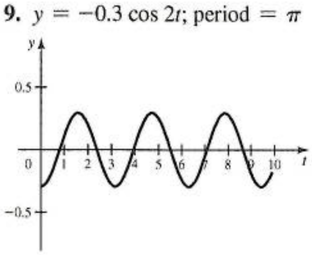
Section 5.4, pp. 457–462

1. A system is damped if the object in motion encounters resistance to the movement; otherwise the motion is undamped. The motion is free if no external forces act on the object; otherwise, the motion is forced.
3. Beats occur when the natural frequency of the oscillator ω_0 is close in value to the forcing frequency ω.
5. Find a solution y_h to the homogeneous equation. Find a particular solution y_p, form the general solution $y_h + y_p$, and evaluate constants using the initial conditions.

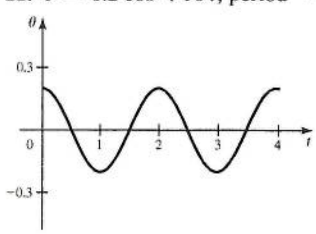
7. $y = 0.75\cos 1.2t$; period $= 5\pi/3$



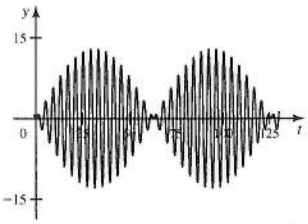
9. $y = -0.3\cos 2t$; period $= \pi$



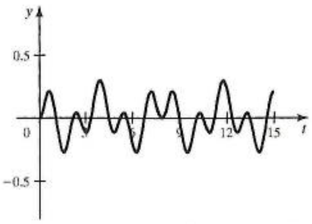
11. $\theta = 0.2\cos\sqrt{10}\,t$; period $= 2\pi/\sqrt{10}$



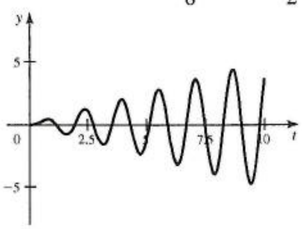
13. a. $\omega = 1.5$: $y = \frac{200}{31}\cos\frac{3}{2}t - \frac{200}{31}\cos\frac{8}{5}t$. This solution has beats.



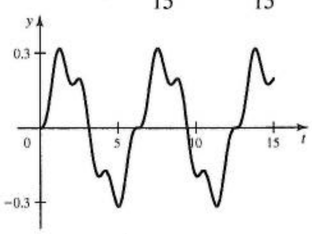
b. $\omega = 4$: $y = \frac{25}{168}\cos\frac{8}{5}t - \frac{25}{168}\cos 4t$



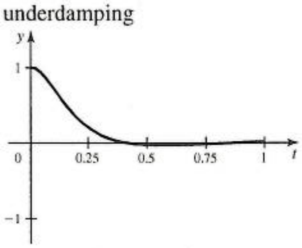
15. a. $\omega = 4$: $y = \frac{1}{8}\sin 4t - \frac{1}{2}t\cos 4t$; resonance case



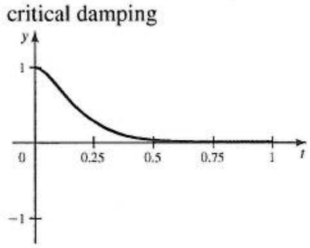
b. $\omega = 1$: $y = \frac{4}{15}\sin t - \frac{1}{15}\sin 4t$



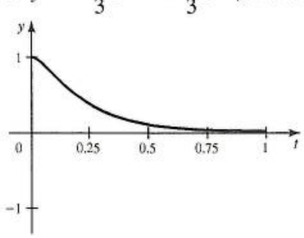
17. a. $y = \frac{4}{3}e^{-8t}\sin 6t + e^{-8t}\cos 6t$; underdamping



b. $y = e^{-10t} + 10te^{-10t}$; critical damping

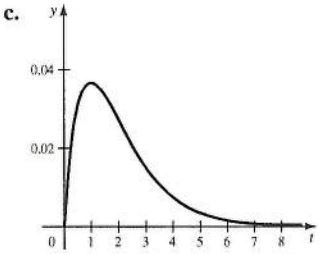


c. $y = -\frac{1}{3}e^{-20t} + \frac{4}{3}e^{-5t}$; overdamping



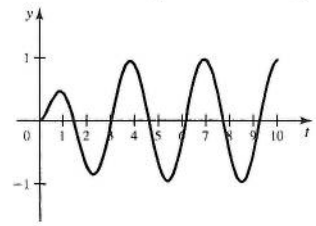
19. a. $k = 250\ \mathrm{N/m}$ **b.** $y = \frac{1}{10}te^{-t}$

c.



The maximum displacement is $\frac{1}{10}e^{-1} \approx 0.037$ m. **d.** With an increase in k of 50%, the motion becomes underdamped, maximum displacement decreases to 0.034 m. With an increase in k of 100%, the motion becomes underdamped; maximum displacement decreases to 0.032 m. **e.** With a decrease in k of 50%, the motion becomes overdamped; maximum displacement increases to 0.041 m.

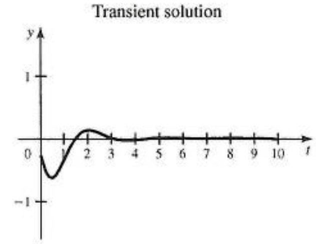
21. a. $y = -\frac{18}{17}e^{-t}\sin 2t - \frac{4}{17}e^{-t}\cos 2t + \frac{16}{17}\sin 2t + \frac{4}{17}\cos 2t$



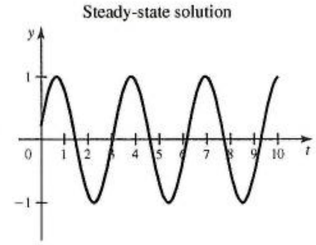
b. transient solution: $y = -\frac{18}{17}e^{-t}\sin 2t - \frac{4}{17}e^{-t}\cos 2t$

steady-state: $y = \frac{16}{17}\sin 2t + \frac{4}{17}\cos 2t$

Transient solution



Steady-state solution



c. After approximately 3 seconds.

23. a. $y = c_1e^{-t/2}\sin t + c_2e^{-t/2}\cos t + \frac{32\omega}{16\omega^4 - 24\omega^2 + 25}\sin\omega t - \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25}\cos\omega t$

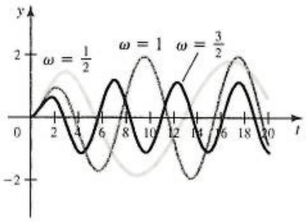
b. $y = -\frac{16\omega^2 + 20}{16\omega^4 - 24\omega^2 + 25}e^{-t/2}\sin t + \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25}e^{-t/2}\cos t + \frac{32\omega}{16\omega^4 - 24\omega^2 + 25}\sin\omega t - \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25}\cos\omega t$

c. transient solution:

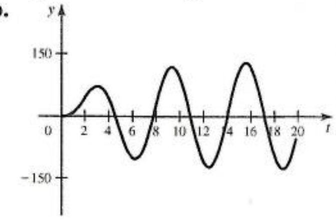
$$y = -\frac{16\omega^2 + 20}{16\omega^4 - 24\omega^2 + 25}e^{-t/2}\sin t + \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25}e^{-t/2}\cos t$$

steady-state solution:

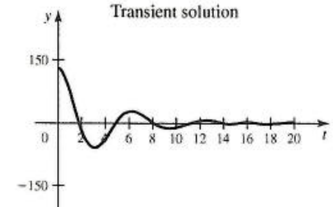
$$\frac{32\omega}{16\omega^4 - 24\omega^2 + 25}\sin \omega t - \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25}\cos \omega t$$



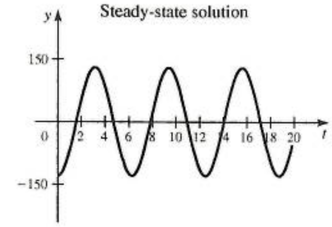
25. When $L = 0$, the equation $LQ'' + RQ' + \frac{1}{C}Q = E(t)$ becomes $RQ' + \frac{1}{C}Q = E(t)$. Solution is $Q = -\frac{1}{20}e^{-20t} + \frac{1}{20}$ and it approaches a steady-state value of $\frac{1}{20}$. **27. a.** $y = 50e^{-40t} - 50e^{-60t}$
b. transient current: $y = 50e^{-40t} - 50e^{-60t}$; steady-state current: $y = 0$
29. a. $y = \frac{40\sqrt{15}}{3}e^{-80t}\sin 2\sqrt{15}t$ **b.** transient current: $y = \frac{40\sqrt{15}}{3}e^{-80t}\sin 2\sqrt{15}t$; steady-state current: $y = 0$
31. $Q = \frac{12}{5} - \frac{4}{5}e^{-4t}(3\cos 3t + 4\sin 3t)$; $I = 20e^{-4t}\sin 3t$
33. a. True **b.** True **c.** True **d.** False **e.** False **f.** True
35. a. $y = 16e^{-t/4}\sin t + 128e^{-t/4}\cos t + 16\sin t - 128\cos t$
transient solution: $y = 16e^{-t/4}\sin t + 128e^{-t/4}\cos t$
steady-state solution: $y = 16\sin t - 128\cos t$
b.



Transient solution

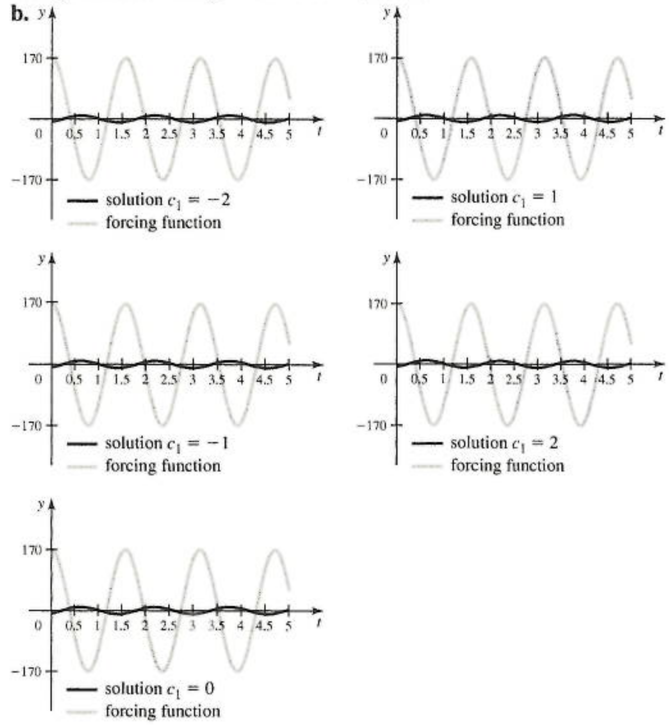


Steady-state solution



c. Around $t = 20$; $\lim_{t\to\infty}(16e^{-t/4}\sin t + 128e^{-t/4}\cos t) = 0$ so the solution approaches the steady-state solution.

37. a. $y = c_1e^{-2t} + c_2e^{-t} + 6\sin 4t - 7\cos 4t$
transient solution: $y = c_1e^{-2t} + c_2e^{-t}$
steady-state solution: $y = 6\sin 4t - 7\cos 4t$
b.

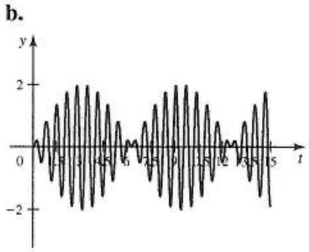


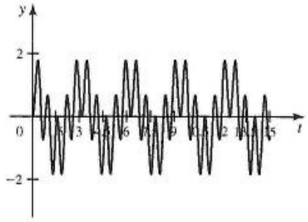
c. In all cases, the solution approaches the steady-state solution quickly. The steady-state solution has smaller magnitude than the external force function and is shifted.

39. a.

$$\cos \omega t - \cos \omega_0 t = 2\sin\left(\frac{\omega_0 t - \omega t}{2}\right)\sin\left(\frac{\omega_0 t + \omega t}{2}\right) = 2\sin\frac{(\omega_0 - \omega)t}{2}\sin\frac{(\omega_0 + \omega)t}{2}$$

b.





41. $C^* = \dfrac{F_0}{\sqrt{c^2\omega^2 + m^2(\omega_0^2 - \omega^2)^2}} = \dfrac{\omega E_0}{\sqrt{R^2\omega^2 + L^2\left(\dfrac{1}{CL} - \omega^2\right)^2}}$

$= \dfrac{\omega E_0}{\sqrt{R^2\omega^2 + \omega^2\dfrac{L^2}{\omega^2}\left(\dfrac{1}{CL} - \omega^2\right)^2}}$

$= \dfrac{\omega E_0}{\sqrt{R^2\omega^2 + \omega^2\left(\dfrac{L}{\omega}\left(\dfrac{1}{CL} - \omega^2\right)\right)^2}} = \dfrac{\omega E_0}{\omega\sqrt{R^2 + \left(\dfrac{L}{\omega}\left(\dfrac{1}{CL} - \omega^2\right)\right)^2}}$

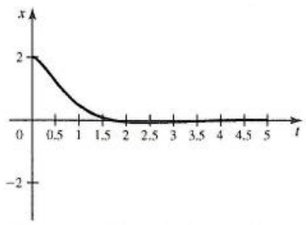
$= \dfrac{E_0}{\sqrt{R^2 + \left(\dfrac{1}{C\omega} - L\omega\right)^2}}$

$= \dfrac{E_0}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

$R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2$ is smallest when $\omega L = \dfrac{1}{\omega C}$ or $\omega^2 = \dfrac{1}{CL}$.

43. a. $x = \dfrac{4\sqrt{7}}{7}e^{-3t/2}\sin\dfrac{\sqrt{7}}{2}t + 2e^{-3t/2}\cos\dfrac{\sqrt{7}}{2}t$

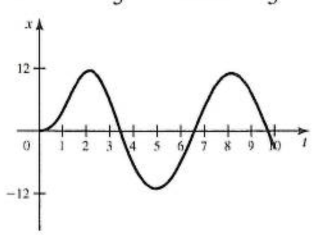
b.



c. The motion is underdamped.

45. a. $x = -\dfrac{22}{5}e^{-t/2}\sin 2t + \dfrac{16}{5}e^{-t/2}\cos 2t + \dfrac{52}{5}\sin t - \dfrac{16}{5}\cos t$

b.



c. The motion approaches a steady-state solution of $x = \dfrac{52}{5}\sin t - \dfrac{16}{5}\cos t$.

47. a. There are two forces acting: inertia, given by $ma = ms''(t) = m\ell\theta''(t)$, and weight of the bob. The component of weight in the direction of motion is $mg\sin\theta(t)$. By Newton's law, the sum of the forces is 0 or $m\ell\theta''(t) + mg\sin\theta(t) = 0 \Rightarrow m\ell\theta''(t) = -mg\sin\theta(t)$.

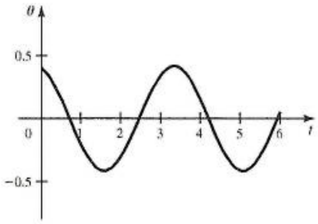
b. $m\ell\theta''(t) = -mg\sin\theta(t) \Rightarrow \theta''(t) = -\dfrac{g}{\ell}\sin\theta(t)$

$= -\omega_0^2\sin\theta(t)$ or $\theta'' + \omega_0^2\sin\theta = 0$

c. Using $\sin\theta \approx \theta$ in the equation from (c) gives $\theta'' + \omega_0^2\theta = 0$.

d. The frequency is given by $\omega_0^2 = \dfrac{\ell}{g}$ or $\omega_0 = \sqrt{\dfrac{g}{\ell}}$. The period is $T = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{\sqrt{\dfrac{g}{\ell}}} = 2\pi\sqrt{\dfrac{\ell}{g}}$. Doubling the length increases the period by a factor of $\sqrt{2}$.

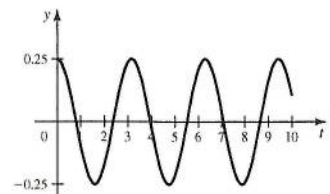
49. a. $\theta(t) = -\dfrac{\sqrt{15}}{35}\sin\dfrac{7\sqrt{15}}{15}t + \dfrac{2}{5}\cos\dfrac{7\sqrt{15}}{15}t$



b. $\theta(t) = \dfrac{\sqrt{211}}{35}\sin\left(\dfrac{7\sqrt{15}}{15}t + \varphi\right)$, where $\tan\varphi = -\dfrac{14\sqrt{15}}{15}$ or $\theta(t) \approx 0.415\sin(1.807t + 1.841)$.

c. $T = 2\pi\sqrt{\dfrac{\ell}{g}} = 2\pi\sqrt{\dfrac{3}{9.8}} \approx 3.476$ seconds.

51. a. $y = 0.25\cos 2t$



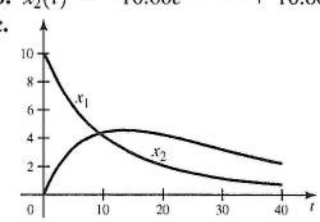
b. $y = 0.25\cos 2t$ **c.** $T = \pi$

53. a. $x_1' = -k_1x_1 + k_2x_2 + f(t) \Rightarrow x_1'' = -k_1x_1' + k_2x_2' + f'(t)$ $\Rightarrow k_2x_2' = x_1'' + k_1x_1' - f'(t)$. Eliminate x_2 from the given first-order system by multiplying the equation for x_1' by $(k_2 + k_3)$ and the equation for x_2' by k_2 and adding. This results in $(k_2 + k_3)x_1' + k_2x_2' = -k_1k_3x_1 + (k_2 + k_3)f(t)$. Substitute $k_2x_2' = x_1'' + k_1x_1' - f'(t)$ and solve for x_1'' to get $x_1'' = -(k_1 + k_2 + k_3)x_1' - k_1k_3x_1 + (k_2 + k_3)f(t) + f'(t)$. Replace x_1 with x. **b.** Because x is another name for x_1, $x_1(0) = A$ is the same as $x(0) = A$. Using the first DE gives $x'(0) = x_1'(0)$ $= -k_1x_1(0) + k_2x_2(0) + f(0) = -k_1x_1(0) + f(0)$.

55. a. $x_1(t) = 6.59e^{-0.1322t} + 3.41e^{-0.0378t}$

b. $x_2(t) = -10.60e^{-0.1322t} + 10.60e^{-0.0378t}$

c.

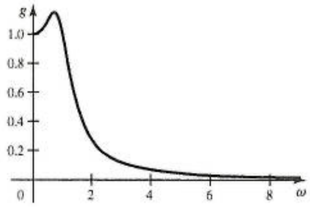


Section 5.5, pp. 471–473

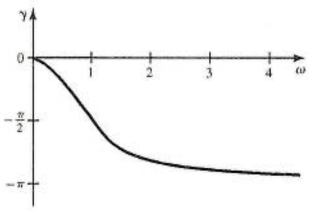
1. $H(\omega) = g(\omega)e^{i\gamma(\omega)}$ **3.** The phase lag function gives the phase of the output relative to the input. The output lags the input.

5. a. $g(\omega) = \dfrac{1}{\sqrt{(1-\omega^2)^2+\omega^2}}$;

$\tan\gamma(\omega) = \dfrac{\omega}{\omega^2-1}$

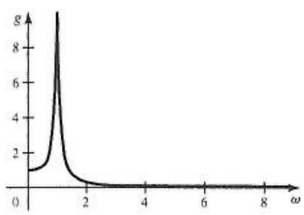


b. Local maximum of the gain function at $\omega = \dfrac{\sqrt{2}}{2}$. Weak damping.

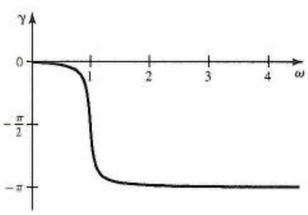


7. a. $g(\omega) = \dfrac{1}{\sqrt{(1-\omega^2)^2+\dfrac{\omega^2}{100}}}$;

$\tan\gamma(\omega) = \dfrac{\omega}{10(\omega^2-1)}$



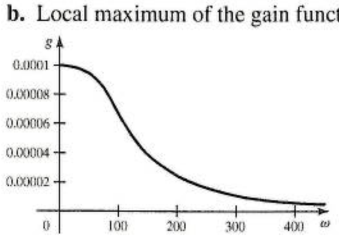
b. Local maximum of the gain function at $\omega = \dfrac{\sqrt{398}}{20}$. Weak damping.

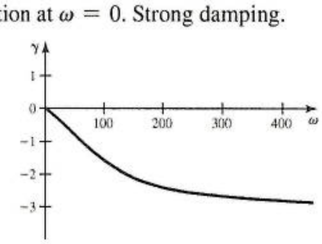


9. a. $g(\omega) = \dfrac{1}{\sqrt{(10{,}000-\omega^2)^2+22{,}500\omega^2}}$;

$\tan\gamma(\omega) = \dfrac{150\omega}{\omega^2-10{,}000}$

b. Local maximum of the gain function at $\omega = 0$. Strong damping.



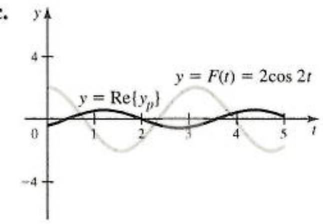


11. a. See Exercise 5a.

b. $\mathrm{Re}\{y_p\} = \dfrac{2}{\sqrt{13}}\cos\left(2t + \tan^{-1}\dfrac{2}{3} - \pi\right)$

$\approx 0.555\cos(2t - 2.554)$

c.

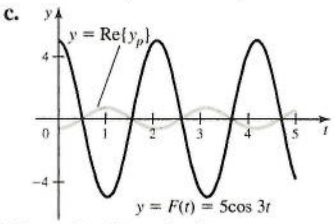


13. a. See Exercise 7a.

b. $\mathrm{Re}\{y_p\} = \dfrac{50}{\sqrt{6409}}\cos\left(3t + \tan^{-1}\dfrac{3}{80} - \pi\right)$

$\approx 0.625\cos(3t - 3.104)$

c.

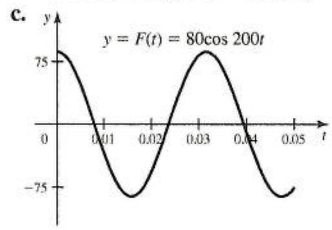


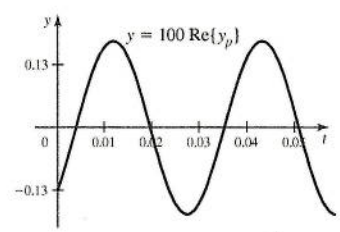
15. a. See Exercise 9a.

b. $\mathrm{Re}\{y_p\} = \dfrac{\sqrt{2}}{750}\cos(200t + \tan^{-1}1 - \pi)$

$\approx 0.00189\cos(200t - 2.356)$

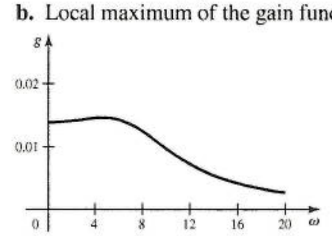
c.



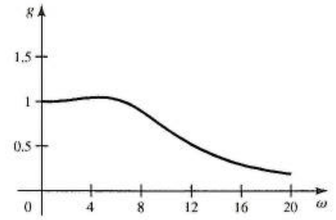


17. a. $g(\omega) = \dfrac{1}{\sqrt{(72-\omega^2)^2+100\omega^2}}$

b. Local maximum of the gain function at $\omega = \sqrt{22}$. **c.** $(\sqrt{22}, \infty)$

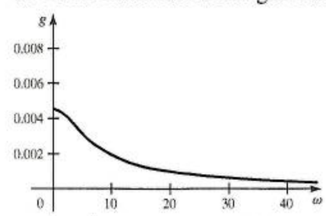


d. $\left|\dfrac{v_{\text{out}}}{v_{\text{in}}}\right| = \dfrac{72}{\sqrt{(72 - \omega^2)^2 + 100\omega^2}}$

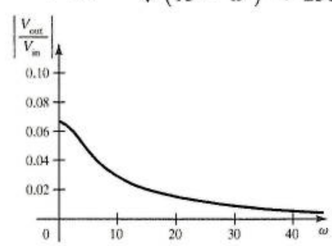


19. a. $g(\omega) = \dfrac{1}{\sqrt{(15 - \omega^2)^2 + 2500\,\omega^2}}$

b. Local maximum of the gain function at $\omega = 0$. **c.** $[0, \infty)$



d. $\left|\dfrac{v_{\text{out}}}{v_{\text{in}}}\right| = \dfrac{15}{\sqrt{(15 - \omega^2)^2 + 2500\,\omega^2}}$



21. a. True **b.** True

23. a. With 1 in the numerator, the smaller the denominator, the larger the value of $g(\omega)$. **b.** A square root will be smallest when the quantity under the square root is smallest.

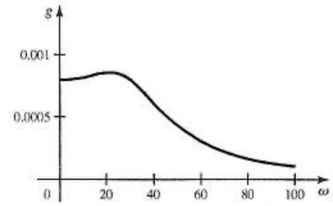
c. $h'(\omega) = \dfrac{d}{d\omega}\left((\omega_0^2 - \omega^2)^2 + b^2\omega^2\right) = 2(\omega_0^2 - \omega^2)(-2\omega) + 2b^2\omega$
$= 2\,\omega(-2\,(\omega_0^2 - \omega^2) + b^2) = 2\,\omega(b^2 - 2\,\omega_0^2 + 2\,\omega^2)$

d. From part (c), $h'(\omega) = 2\omega(b^2 - 2\omega_0^2 + 2\omega^2)$. If $b < \sqrt{2}\omega_0$, then $h'(\omega) = 0$ has a real solution: $b^2 - 2\omega_0^2 + 2\omega^2 = 0$
$\Rightarrow 2\omega^2 = 2\omega_0^2 - b^2 \Rightarrow \omega = \sqrt{\dfrac{2\omega_0^2 - b^2}{2}} = \sqrt{\omega_0^2 - \dfrac{b^2}{2}}$.
That this gives a local minimum for h, and hence a local maximum for g, can be verified by the First or Second Derivative Test. The maximum value of g is $\dfrac{\sqrt{2}}{\sqrt{b^2 + b^2(2\omega_0^2 - b^2)}}$.

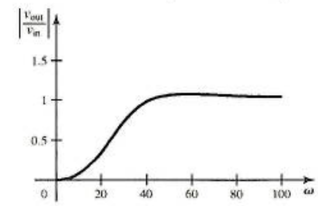
e. If $b \geq \sqrt{2}\omega_0$, then $b^2 \geq 2\omega_0^2$ and $b^2 - 2\omega_0^2 + 2\omega^2 \geq 0$. Hence $h'(\omega) = 2\omega(b^2 - 2\omega_0^2 + 2\omega^2) \geq 0$ for $\omega \geq 0$ and h is an increasing function. It follows that $g = \dfrac{1}{\sqrt{h}}$ is decreasing for $\omega \geq 0$ and so has a local maximum at $\omega = 0$. **f.** Part (e); the gain function is maximized at 0.

25. a. $g(\omega) = \dfrac{1}{\sqrt{(\omega^2 - 1250)^2 + 1600\omega^2}}$

b. Increases until $\omega = 15\sqrt{2}$ then decreases.

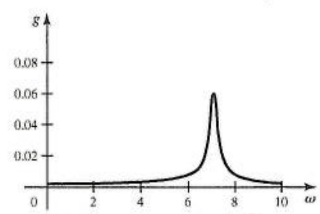


c. $\left|\dfrac{v_{\text{out}}}{v_{\text{in}}}\right| = \dfrac{\omega^2}{\sqrt{(\omega^2 - 1250)^2 + 1600\omega^2}}$

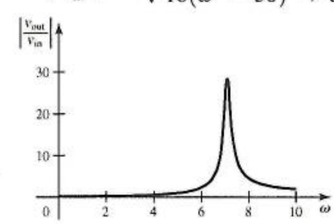


27. a. $g(\omega) = \dfrac{4}{\sqrt{16(\omega^2 - 50)^2 + \omega^2}}$

b. Increases until $\omega = \dfrac{\sqrt{3198}}{8} \approx 7.069$ then decreases.



c. $\left|\dfrac{v_{\text{out}}}{v_{\text{in}}}\right| = \dfrac{4\omega^2}{\sqrt{16(\omega^2 - 50)^2 + \omega^2}}$



Chapter 5 Review Exercises, pp. 473–474

1. **a.** True **b.** False **c.** False **d.** False **e.** True
3. $y = c_1e^{-2t} + c_2e^{4t}$ **5.** $y = c_1 \sin 6t + c_2 \cos 6t$
7. $y = c_1e^{-2t} + c_2te^{-2t}$ **9.** $y = c_1e^t \sin 2t + c_2e^t \cos 2t$

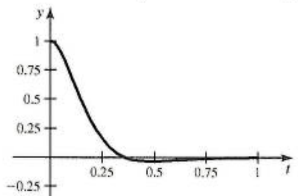
11. $y_p = \frac{1}{7}\cos 2t$ **13.** $y_p = -\frac{1}{6}t - \frac{5}{36} - \frac{1}{5}e^{-t}$

15. $y_p = \frac{1}{4}t \sin 4t$ **17.** $y = c_1 \sin 2t + c_2 \cos 2t - \frac{3}{5}\sin 3t$

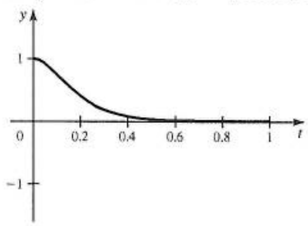
19. $y = c_1e^{-2t}\sin t + c_2e^{-2t}\cos t + \frac{1}{4}\sin t + \frac{1}{4}\cos t$

21. $y = c_1e^{-t} + c_2e^t + te^t$

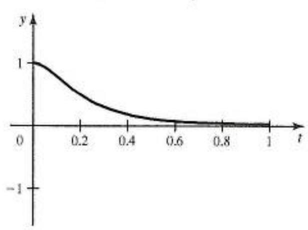
23. **a.** $y = \frac{3\sqrt{7}}{7}e^{-15t/2}\sin\frac{5\sqrt{7}}{2}t + e^{-15t/2}\cos\frac{5\sqrt{7}}{2}t$; underdamping



b. $y = e^{-10t} + 10te^{-10t}$; critical damping



c. $y = -\frac{1}{3}e^{-20t} + \frac{4}{3}e^{-5t}$; overdamping



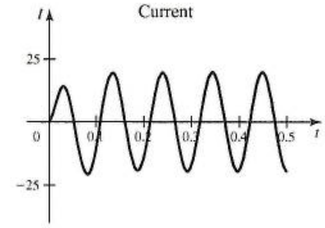
25. **a.** $I = 50e^{-60t} - \frac{600}{13}e^{-40t} + \frac{250}{13}\sin 60t - \frac{50}{13}\cos 60t$

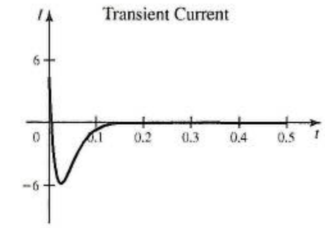
b. Transient current:

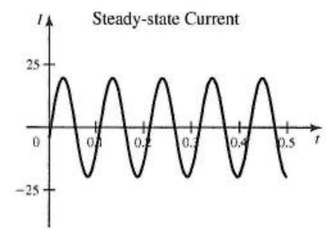
$$I = 50e^{-60t} - \frac{600}{13}e^{-40t}$$

c. Steady-state current:

$$I = \frac{250}{13}\sin 60t - \frac{50}{13}\cos 60t$$







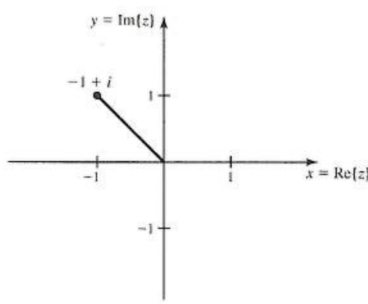
d. The solutions are the same.

APPENDIX C

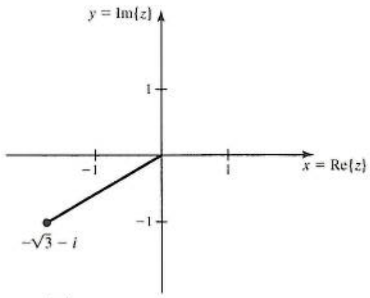
Exercises, p. 480

1. $7 - 7i$ **3.** $-21 - 20i$ **5.** $\frac{-26 + 7i}{25}$ **7.** $\sqrt{29}$

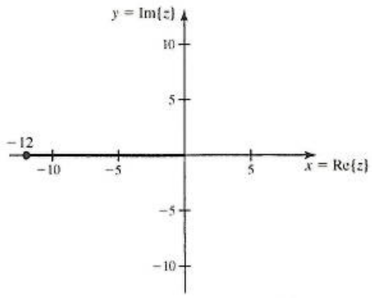
9. $|z| = \sqrt{2}$, $\arg z = \frac{3\pi}{4}$



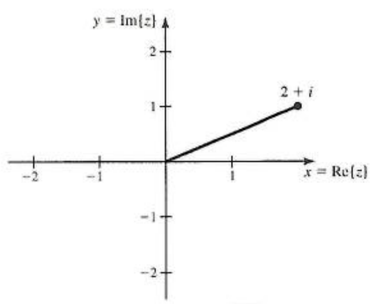
11. $|z| = 2, \arg z = \dfrac{7\pi}{6}$



13. $|z| = 12, \arg z = \pi$



15. $|z| = \sqrt{5}, \arg z = \tan^{-1}\dfrac{1}{2} \approx 0.464$



17. $-1 - 4i$ **19.** $\dfrac{1}{2}\sqrt{\dfrac{17}{29}}$ **21.** -1 **23.** $-2\sqrt{3} + 2i$ **25.** 0

27. $-\sqrt{2} - \sqrt{2}i$

31. $\sqrt{2}e^{-i\,\pi/4}$ **33.** $2e^{i\,5\pi/6}$ **35.** $2e^{i\pi}$ **37.** $\dfrac{1}{\sqrt{2}}e^{i\,\pi/4}$ **39.** $4\sqrt{2}e^{i\,7\pi/12}$

41. $4\sqrt{2}e^{-i\,7\pi/12}$ **43.** -64 **45.** $\dfrac{1}{128\sqrt{2}}e^{i\,5\pi/4}$ **47.** $\pm 2i$

49. $\sqrt{2}e^{-i\,\pi/6}, \sqrt{2}e^{i\,5\pi/6}$ **51.** $\sqrt[4]{2}e^{-i\,\pi/8}, \sqrt[4]{2}e^{i\,7\pi/8}$

53. $2e^{i\,\pi/4}, 2e^{i\,3\pi/4}, 2e^{i\,5\pi/4}, 2e^{i\,7\pi/4}$ **55.** $x = \pm 5i$ **57.** $-1 \pm i$

CHAPTER 6

Exercises 6.2, page 492

1. $y = [4/(e - e^{-1})](e^{-x} - e^{x})$ **3.** $y \equiv 0$

5. $y = e^{x} + 2x - 1$ **7.** $y = \cos x + c \sin x$; c arbitrary

9. $\lambda_n = (2n - 1)^2/4$ and $y_n = c_n \sin[(2n - 1)x/2]$, where $n = 1, 2, 3, \ldots$ and the c_n's are arbitrary.

11. $\lambda_n = n^2, n = 0, 1, 2, \ldots; y_0 = a_0$ and $y_n = a_n \cos nx + b_n \sin nx, n = 1, 2, 3, \ldots$, where a_0, a_n, and b_n are arbitrary.

13. The eigenvalues are the roots of $\tan(\sqrt{\lambda_n}\,\pi) + \sqrt{\lambda_n} = 0$, where $\lambda_n > 0$.
For n large, $\lambda_n \approx (2n - 1)^2/4$, n is a positive integer.
The eigenfunctions are
$y_n = c_n[\sin(\sqrt{\lambda_n}x) + \sqrt{\lambda_n}\cos(\sqrt{\lambda_n}x)]$,
where the c_n's are arbitrary.

15. $u(x, t) = e^{-3t}\sin x - 6e^{-48t}\sin 4x$

17. $u(x, t) = e^{-3t}\sin x - 7e^{-27t}\sin 3x + e^{-75t}\sin 5x$

19. $u(x, t) = 3\cos 6t \sin 2x + 12\cos 39t \sin 13x$

21. $u(x, t) = 6\cos 6t \sin 2x + 2\cos 18t \sin 6x + (11/27)\sin 27t \sin 9x - (14/45)\sin 45t \sin 15x$

23. $u(x, t) = \sum_{n=1}^{\infty} n^{-2}e^{-2\pi^2 n^2 t}\sin n\pi x$

25. If $K > 0$, then $T(t)$ becomes unbounded as $t \to \infty$, and so the temperature $u(x, t) = X(x)T(t)$ becomes unbounded at each position x. Since the temperature must remain bounded for all time, $K \not> 0$.

33. **(a)** $u(x) \equiv 50$ **(b)** $u(x) = 30x/L + 10$

Exercises 6.3, page 506

1. Odd **3.** Even **5.** Neither

9. $f(x) \sim \sum_{n=1}^{\infty} \dfrac{2(-1)^{n+1}}{n}\sin nx$

11. $f(x) \sim 1 + \sum_{n=1}^{\infty}\left[\dfrac{2}{\pi^2 n^2}(-1 + (-1)^n)\cos\dfrac{n\pi x}{2} + \dfrac{1}{\pi n}((-1)^{n+1} - 1)\sin\dfrac{n\pi x}{2}\right]$

13. $f(x) \sim \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2\pi^2} \cos n\pi x$

15. $f(x) \sim [(\sinh \pi)/\pi]\left(1 + 2\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{1+n^2}\cos nx + \frac{(-1)^{n+1}n}{1+n^2}\sin nx\right]\right)$

17. The 2π-periodic function $g(x)$, where

$$g(x) = \begin{cases} x & (-\pi < x < \pi), \\ 0 & (x = \pm\pi) \end{cases}$$

19. The 4-periodic function $g(x)$, where

$$g(x) = \begin{cases} 1 & (-2 < x < 0), \\ x & (0 < x < 2), \\ 1/2 & (x = 0), \\ 3/2 & (x = \pm 2) \end{cases}$$

21. The 2-periodic function $g(x)$, where

$g(x) = x^2 \quad (-1 \le x \le 1)$

23. The 2π-periodic function $g(x)$, where

$$g(x) = \begin{cases} e^x & (-\pi < x < \pi), \\ (e^{\pi} + e^{-\pi})/2 & (x = \pm\pi) \end{cases}$$

25. **(a)** $F(x) = (x^2 - \pi^2)/2$ **(b)** $F(x) = |x| - \pi$

27. $f(x) \sim \sum_{n=1}^{\infty} \frac{2}{(2n-1)\pi}\left[(-1)^{n+1}\cos\left(\frac{(2n-1)\pi x}{2}\right) + \sin\left(\frac{(2n-1)\pi x}{2}\right)\right]$

29. $a_0 = 0$; $a_1 = 3/2$; $a_2 = 0$

Exercises 6.4, page 513

1. **(a)** The π-periodic function $\tilde{f}(x)$, where

$\tilde{f}(x) = x^2 \quad (0 < x < \pi)$

(b) The 2π-periodic function $f_o(x)$, where

$$f_o(x) = \begin{cases} x^2 & (0 < x < \pi), \\ -x^2 & (-\pi < x < 0) \end{cases}$$

(c) The 2π-periodic function $f_e(x)$, where

$$f_e(x) = \begin{cases} x^2 & (0 < x < \pi), \\ x^2 & (-\pi < x < 0) \end{cases}$$

3. **(a)** The π-periodic function $\tilde{f}(x)$, where

$$\tilde{f}(x) = \begin{cases} 0 & (0 < x < \pi/2), \\ 1 & (\pi/2 < x < \pi) \end{cases}$$

(b) The 2π-periodic function $f_o(x)$, where

$$f_o(x) = \begin{cases} -1 & (-\pi < x < -\pi/2), \\ 0 & (-\pi/2 < x < 0), \\ 0 & (0 < x < \pi/2), \\ 1 & (\pi/2 < x < \pi) \end{cases}$$

(c) The 2π-periodic function $f_e(x)$, where

$$f_e(x) = \begin{cases} 1 & (-\pi < x < -\pi/2), \\ 0 & (-\pi/2 < x < 0), \\ 0 & (0 < x < \pi/2), \\ 1 & (\pi/2 < x < \pi) \end{cases}$$

5. $f(x) \sim -\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{2k-1}\sin(2k-1)\pi x$

7. $f(x) \sim \sum_{n=1}^{\infty}\left[\frac{2\pi(-1)^{n+1}}{n} + \frac{4}{\pi n^3}\left((-1)^n - 1\right)\right]\sin nx$

9. $f(x) \sim \sum_{k=0}^{\infty}\frac{8}{(2k+1)^3\pi^3}\sin(2k+1)\pi x$

11. $f(x) \sim \frac{\pi}{2} + \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\cos(2k-1)x$

13. $f(x) \sim e - 1 + 2\sum_{n=1}^{\infty}\frac{(-1)^n e - 1}{1 + \pi^2 n^2}\cos n\pi x$

15. $f(x) \sim \frac{2}{\pi} + \frac{2}{\pi}\sum_{k=1}^{\infty}\left(\frac{1}{2k+1} - \frac{1}{2k-1}\right)\cos 2kx$

17. $u(x, t) =$

$$\frac{2}{\pi}\sum_{k=1}^{\infty}\left[\frac{2}{2k-1} - \frac{1}{2k+1} - \frac{1}{2k-3}\right]e^{-5(2k-1)^2 t}\sin(2k-1)x$$

19. $u(x, t) = \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{(2k+1)^2}e^{-5(2k+1)^2 t}\sin(2k+1)x$

Exercises 6.5, page 525

1. $u(x, t) = \sum_{n=1}^{\infty}\frac{8(-1)^{n+1} - 4}{\pi^3 n^3}e^{-5\pi^2 n^2 t}\sin n\pi x$

3. $u(x, t) = \frac{\pi}{2} - \sum_{k=0}^{\infty}\frac{4}{\pi(2k+1)^2}e^{-3(2k+1)^2 t}\cos(2k+1)x$

5. $u(x, t) = \frac{2(e^{\pi} - 1)}{\pi} + \sum_{n=1}^{\infty}\frac{2e^{\pi}(-1)^n - 2}{\pi(1+n^2)}e^{-n^2 t}\cos nx$

7. $u(x, t) = 5 + \frac{5}{\pi}x - \frac{30}{\pi}e^{-2t}\sin x + \frac{5}{\pi}e^{-8t}\sin 2x + \left(1 - \frac{10}{\pi}\right)e^{-18t}\sin 3x + \frac{5}{2\pi}e^{-32t}\sin 4x - \left(1 + \frac{6}{\pi}\right)e^{-50t}\sin 5x + \sum_{n=6}^{\infty}\frac{10}{\pi n}[2(-1)^n - 1]e^{-2n^2 t}\sin nx$

9. $u(x, t) = \frac{e^{-\pi} - 1}{\pi}x - e^{-x} + 1 + \sum_{n=1}^{\infty}c_n e^{-n^2 t}\sin nx$,

where

$$c_n = \begin{cases} \frac{2e^{-\pi} - 2}{\pi n}(-1)^n + \frac{2n}{\pi(1+n^2)}[(-1)^{n+1}e^{-\pi} + 1] + \frac{2}{\pi n}[(-1)^n - 1] & (n \ne 2), \\ \frac{e^{-\pi} - 1}{\pi} + \frac{4}{5\pi}(1 - e^{-\pi}) + 1 & (n = 2) \end{cases}$$

11. $u(x, t) = \sum_{n=0}^{\infty}a_n e^{-4(n+1/2)^2 t}\cos(n + 1/2)x$, where

$f(x) = \sum_{n=0}^{\infty}a_n\cos(n + 1/2)x$

13. $u(x,t) = \frac{\pi^2}{3}x - \frac{1}{3}x^3 - 3e^{-2t}\sin x + \sum_{n=2}^{\infty} \frac{4(-1)^n}{n^3} e^{-2n^2 t}\sin nx$

15. $u(x,y,t) = e^{-52t}\cos 6x \sin 4y - 3e^{-122t}\cos x \sin 11y$

17. $u(x,y,t) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-n^2 t}\sin ny$

19. $C(x,t) = \sum_{n=1}^{\infty} c_n e^{-[L+kn^2\pi^2/a^2]t}\sin\left(\frac{n\pi x}{a}\right)$, where

$c_n = \frac{2}{a}\int_0^a f(x)\sin\left(\frac{n\pi x}{a}\right)dx$

Concentration goes to zero as $t \to +\infty$.

17. $u(x,t) = \frac{1}{2}\left[e^{-(x+\alpha t)^2} + e^{-(x-\alpha t)^2} + \frac{\cos(x-\alpha t) - \cos(x+\alpha t)}{\alpha}\right]$

21. $u(r,t) = \sum_{n=1}^{\infty} [a_n \cos(k_n \alpha t) + b_n \sin(k_n \alpha t)] J_0(k_n r)$,

where

$a_n = \frac{1}{c_n}\int_0^1 f(r)J_0(k_n r)r\,dr$, and

$b_n = \frac{1}{\alpha k_n c_n}\int_0^1 g(r)J_0(k_n r)r\,dr$, with

$c_n = \int_0^1 J_0^2(k_n r)r\,dr$

Exercises 6.6, page 536

1. $u(x,t) = \frac{1}{7\pi}\sin 7\pi t \sin 7\pi x + \sum_{k=0}^{\infty} \frac{8}{((2k+1)\pi)^3}\cos(2k+1)\pi t \sin(2k+1)\pi x$

3. $u(x,t) = \sum_{n=1}^{\infty} \frac{4}{n^3}[2(-1)^{n+1} - 1]\cos 2nt \sin nx$

5. $u(x,t) = \frac{2h_0 L^2}{\pi^2 a(L-a)} \sum_{n=1}^{\infty} \frac{1}{n^2}\sin\frac{n\pi a}{L}\sin\frac{n\pi x}{L}\cos\frac{n\pi\alpha t}{L}$

7. $u(x,t) = \cos t \sin x + \frac{5}{2}\sin 2t \sin 2x - \frac{3}{5}\sin 5t \sin 5x + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}\left[t - \frac{\sin nt}{n}\right]\sin nx$

9. $u(x,t) = \sum_{n=0}^{\infty}\left[a_n \cos\frac{(2n+1)\pi\alpha t}{2L} + b_n \sin\frac{(2n+1)\pi\alpha t}{2L}\right]\sin\frac{(2n+1)\pi x}{2L}$, where

$f(x) = \sum_{n=0}^{\infty} a_n \sin\frac{(2n+1)\pi x}{2L}$ and

$g(x) = \sum_{n=0}^{\infty} b_n \frac{(2n+1)\pi\alpha}{2L}\sin\frac{(2n+1)\pi x}{2L}$

11. $u(x,t) = \sum_{n=1}^{\infty} a_n T_n(t)\sin\frac{n\pi x}{L}$, where

$a_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$, and

$T_n(t) = e^{-t/2}\left(\cos\beta_n t + \frac{1}{2\beta_n}\sin\beta_n t\right)$, where

$\beta_n = \frac{1}{2L}\sqrt{3L^2 + 4\alpha^2\pi^2 n^2}$

13. $u(x,t) = \frac{1}{2\alpha}[\sin(x+\alpha t) - \sin(x-\alpha t)] = \frac{1}{\alpha}\sin\alpha t \cos x$

15. $u(x,t) = x + tx$

Exercises 6.7, page 548

1. $u(x,y) = \frac{4\cos 6x \sinh[6(y-1)]}{\sinh(-6)} + \frac{\cos 7x \sinh[7(y-1)]}{\sinh(-7)}$

3. $u(x,y) = \sum_{n=1}^{\infty} A_n \sin nx \sinh(ny - n\pi)$, where

$A_n = \frac{2}{\pi\sinh(-n\pi)}\int_0^{\pi} f(x)\sin nx\,dx$

5. $u(x,y) = \frac{\cos x \sinh(y-1)}{\sinh(-1)} - \frac{\cos 3x \sinh(3y-3)}{\sinh(-3)} + \frac{\cos 2x \sinh 2y}{\sinh(2)}$

7. $u(r,\theta) = \frac{\pi}{2} - \sum_{k=0}^{\infty} \frac{r^{2k+1}}{(2k+1)^2\pi 2^{2k-1}}\cos(2k+1)\theta$

9. $u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(\frac{r}{a}\right)^n (a_n \cos n\theta + b_n \sin n\theta)$, where

a_0 is arbitrary, and for $n = 1, 2, 3, \ldots$

$a_n = \frac{a}{\pi n}\int_{-\pi}^{\pi} f(\theta)\cos n\theta\,d\theta$,

$b_n = \frac{a}{n\pi}\int_{-\pi}^{\pi} f(\theta)\sin n\theta\,d\theta$

11. $u(r,\theta) = \left(\frac{1}{3}r - \frac{4}{3}r^{-1}\right)\cos\theta + \left(\frac{2}{3}r - \frac{2}{3}r^{-1}\right)\sin\theta + \left(-\frac{1}{255}r^4 + \frac{256}{255}r^{-4}\right)\sin 4\theta$

13. $u(r, \theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} r^{-n}(a_n \cos n\theta + b_n \sin n\theta)$,

where

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\cos n\theta \, d\theta \qquad (n = 0, 1, 2, \ldots)\ ,$$

and

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\sin n\theta \, d\theta \qquad (n = 1, 2, 3, \ldots)$$

15. $u(r, \theta) = r^3 \sin 3\theta$

17. $u(r, \theta) = a_0 + b_0 \ln r + \sum_{n=1}^{\infty} \left[(a_n r^n + b_n r^{-n})\cos n\theta + (c_n r^n + d_n r^{-n})\sin n\theta \right]$

where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)\, d\theta\ , \qquad b_0 = \frac{3}{2\pi}\int_{-\pi}^{\pi} g(\theta)\, d\theta\ ,$$

and for $n = 1, 2, 3, \ldots$

$$a_n + b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\cos n\theta \, d\theta\ ,$$

$$n3^{n-1}a_n - n3^{-n-1}b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(\theta)\cos n\theta \, d\theta\ ,$$

and

$$c_n + d_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\sin n\theta \, d\theta\ ,$$

$$n3^{n-1}c_n - n3^{-n-1}d_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(\theta)\sin n\theta \, d\theta$$

21. $u(r, \theta, z) = \left[I_0(r)/I_0(\pi) \right] \sin z$

23. **(c)** $\phi(x, y) = 2xy$