MATH1149 — Trigonometry — Math. Dept.

Recitation Instructor: Ivo Terek

Solution for quizzes — SP20

Quiz 1

Question 1 (1.0 point). Convert $5\pi/18$ radians to *decimal degrees*. Round your answer to the nearest degree if necessary.

Solution. Just recall that $180\,\text{deg} = \pi\,\text{rad}$, so

$$\frac{5\pi}{18}\,\text{rad} = \frac{5\pi}{18}\frac{180}{\pi}\,\text{deg} = 50\,\text{deg}.$$

Question 2 (4.0 points). An energy-efficient hard drive has a 2.5-in. diameter and spins at 4600 rpm. Give *exact* values for:

(a) the angular speed of the hard drive (in $1/\text{min}$).

(b) the linear speed of a point on the edge of the hard drive (in $\text{in.}/\text{min}$).

Solution.

(a) Note that $\text{rpm} = 2\pi/\text{min}$, so

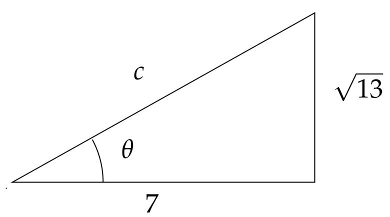
$$\omega = 4600\,\text{rpm} = 4600\frac{2\pi}{\text{min}} = \frac{9200\pi}{\text{min}}.$$

(b) The linear speed is given by $v = r\omega$, where the radius here is $r = 2.5\,\text{in}/2 = 1.25\,\text{in}$ (which also equals $(5/4)\text{in}$), so we get that

$$v = \frac{5}{4}\,\text{in}\frac{9200\pi}{\text{min}} = 11500\pi\frac{\text{in}}{\text{min}}.$$

Question 3 (5.0 points). Assume that θ is an acute angle with $\tan\theta = \sqrt{13}/7$. By drawing a right triangle with angle θ (as we have done in recitation) and finding the lengths of all sides, determine the value of $\cos\theta$.

Solution. Since $\tan\theta = \text{opp}/\text{adj}$, we may draw a right triangle whose opposite side to θ is $\sqrt{13}$ and the adjacent is 7:



To find c, use the Pythagorean Theorem to write $c^2 = 7^2 + (\sqrt{13})^2 = 49 + 13 = 62 \implies c = \sqrt{62}$, since $c > 0$. So $\cos\theta = \text{adj}/\text{hyp} = 7/\sqrt{62} = 7\sqrt{62}/62$.

Quiz 2

Question 1 (4.0 points). Given an angle θ, and knowing that the point $(-3, 2)$ lies on the terminal side of θ, compute $\sin\theta$, $\cos\theta$ and $\tan\theta$. Round your answers to 2 decimal places.

Solution. The distance of this point to the origin is $r = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$. So we obtain

$$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{13}} \approx 0.55, \quad \cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{13}} \approx -0.83 \quad \text{and} \quad \tan\theta = \frac{y}{x} = \frac{2}{-3} \approx -0.67.$$

(Note that $-0.666\ldots$ is rounded to -0.67 because the third decimal place is bigger than 5.)

Question 2 (2.0 points). Give, in radians, the reference angle for $\theta = -7\pi/8$.

Solution. This angle is in the third quadrant (draw a picture), so the reference angle is just $\pi/8$. Alternatively, work with the coterminal angle $-7\pi/8 + 2\pi = 9\pi/8$ and compute the reference angle as $9\pi/8 - \pi = \pi/8$.

Question 3 (4.0 points). Write $\cos t$ in terms of $\sin t$ for t in the third quadrant of the plane.

Solution. Since $\cos^2 t + \sin^2 t = 1$, we have that $\cos^2 t = 1 - \sin^2 t$ and so $\cos t = \pm\sqrt{1 - \sin^2 t}$. The choice of sign $\pm$ depends on the quadrant. If t is in the third quadrant, we have $\cos t = -\sqrt{1 - \sin^2 t}$.

QUIZ 3

Question 1 (2.0 points)**.** Give the exact value for $\arcsin\left(\sin\frac{7\pi}{4}\right)$.

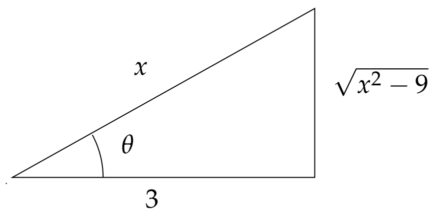
Solution. Directly compute

$$\arcsin\left(\sin\frac{7\pi}{4}\right) = \arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}.$$

Observe that you cannot "cancel" arcsin and sin to say that the answer is $7\pi/4$, because arcsin is only defined once a choice of interval on which sin is one-to-one has been made. The default choice is $\arcsin\colon [-1,1] \to [-\pi/2, \pi/2]$, so your answer should be in this latter interval.

Question 2 (4.0 points)**.** Write $\tan\left(\arccos\frac{3}{x}\right)$ (valid for $x > 3$) as an algebraic expression.

Solution. If we write $\theta = \arccos(3/x)$, then $\cos\theta = 3/x$. We may draw the following right triangle:



The opposite side $\sqrt{x^2-9}$ was found using the Pythagorean Theorem again. This implies that

$$\tan\left(\arccos\frac{3}{x}\right) = \frac{\sqrt{x^2-9}}{3}.$$

Question 3 (4.0 points)**.** Write $\dfrac{\sin^2 x + 1}{\cos^2 x} + 1$ in terms of $\sec x$ only.

Solution. Use the identity $1 + \tan^2 x = \sec^2 x$ given in the first page of the quiz (as usual) to write

$$\frac{\sin^2 x + 1}{\cos^2 x} + 1 = \tan^2 x + \sec^2 x + 1 = (\tan^2 x + 1) + \sec^2 x = (\sec^2 x) + \sec^2 x = 2\sec^2 x.$$