Exercise 1. Describe the region of integration and rewrite the integral

$$\int_{-2}^{2}\int_{0}^{4-y^2}\int_{0}^{y+2} f(x,y,z)\,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y :$$

(a) in the order $\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}y$.

(b) as a sum of two integrals in the order $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$.

Exercise 2. Use integration with spherical coordinates (i.e., necessarily use a triple integral[1]) to prove that volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

Exercise 3. Compute the integral

$$\iint_R (x+y)\,\mathrm{d}A,$$

where R is the ellipse described by the relation $\left(\frac{x-1}{2}\right)^2 + \left(\frac{y-2}{3}\right)^2 \leq 1$.

Hint. Modify polar coordinates.

Exercise 4. Compute the integral

$$\iint_D xyz\,\mathrm{d}V,$$

where D is described by $(y+1)^2 + (z-3)^2 \leq 4$ and $-1 \leq x \leq 1$.

Hint. Modify cylindrical coordinates.

[1]Or course, there are other ways (even easier) to justify such formula. The point of the exercise is to practice spherical coordinates to recover a well-known fact.

Answers:

1. (a) $\displaystyle\int_{-2}^{2}\int_{0}^{y+2}\int_{0}^{4-y^2} f(x,y,z)\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}y.$

   (b) $\displaystyle\int_{0}^{2}\int_{-2}^{2}\int_{0}^{4-y^2} f(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z + \int_{2}^{4}\int_{z-2}^{2}\int_{0}^{4-y^2} f(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

2. —

3. 18π.

4. 0. Reality check: is there any symmetry making this result reasonable to expect?