WEEK 1 DISCUSSION WORKSHEET: PREDICATE LOGIC

Problem 1: In this problem, the domain is the set of all integers. Which statements are true? If an existential statement is true, give an example. If a universal statement is false, give a counterexample.

(a): $\exists x(x + x = 1)$ $\quad 2x = 1 \quad x = \frac{1}{2}$

F

2min

(b): $\forall x(x^2 - x \neq 1)$ $\quad x^2 - x - 1 = 0 \quad x^2 - bx + c = 0$

$\exists x(x^2 - x = 1)$ $\quad \Delta = 1^2 + 4 \times 1 = 5$ $\quad \frac{b \pm \sqrt{b^2 - 4c}}{2}$ $\to \Delta$

$\boxed{\frac{1 \pm \sqrt{5}}{2}}$

(c): $\forall x(x^2 - x \neq 0)$

$\neg$: $\exists x(x^2 - x = 0)$

$x = 0$ or $x = 1$

5min

Problem 2: $P(x)$ is a predicate and the domain for the variable x is $\{1, 2, 3, 4\}$. Give an equivalent logical expression to $\forall x P(x)$ that does not use quantifiers

Problem 3: Determine the truth value of each expression below. The domain is the set of all real numbers.

(a): $\forall x \exists y(xy > 0)$

F $\quad x = 0 \quad xy = 0$

(b): $\forall x \forall y(xy = yx)$

T

5 min

(c): $\forall x \forall y \exists z(z = (x - y)/3)$

T

(d): $\exists x \exists y (x^2 = y^2 \wedge |x| \neq |y|)$

Problem 4: A student club holds an election for officers. Before the voting, members can nominate each other. It is also possible for a member to nominate himself or herself. Some of the members are new members. Some of the members are currently officers. The domain is the set of members of the club. One of the members of the club is named Sam. Define the following predicates.

$N(x, y)$: person x nominated person y for a position.

$W(x)$: person x is a new member.

$O(x)$: person x is currently an officer. Give a quantified expression that is logically equivalent to each of the following statements.

(a): All the new members nominated all the officers.

(b): One of the current officers did not nominate anyone.

9 min

(c): Everyone nominated someone.

(d): Everyone nominated someone other than themselves.