

## Week 2 Discussion Worksheet: Proofs, Sets and Functors

June 27, 2022

Problem 1: (Prove by contrapositive): If  $3n + 2$  is even, then  $n$  is even.

Problem 2: Use a direct proof to show that every odd integer is the difference of two squares. (Hint: Find the difference of the squares of  $k + 1$  and  $k$ , where  $k$  is a positive integer.)

Problem 3: Prove that if  $n$  is an integer, then  $n$  is even if and only if  $7n + 4$  is even.

Problem 4: Prove: If  $x$  and  $y$  are real numbers, then  $\max(x, y) + \min(x, y) = x + y$ .  
(Hint: break it into cases.) - Note: if  $x = y$ , then  $\max(x, y) = \min(x, y) = x = y$ .

Problem 5: Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

- a.  $\{3, 6, 9, 12, \dots\}$
- b.  $\{-3, -1, 1, 3, 5, 7, 9\}$

Problem 6: Determine whether each statement is true or false for any two sets  $A$  and  $B$ . If the statement is false, explain why.

- a. If  $A \subseteq B$ , then  $A \subset B$ .
- b. If  $A \subset B$ , then  $A \subseteq B$ .
- c. If  $A = B$ , then  $A \subseteq B$ .
- d. If  $A = B$ , then  $A \subset B$ .
- e. If  $A \subset B$ , then  $A \neq B$ .

Problem 7: Show that if  $A$  and  $B$  are sets, then

a.  $A - B = A \cap \bar{B}$

b.  $(A \cap B) \cup (A \cap \bar{B}) = A$

You can do this by showing that each side is contained in the other, or by using setbuilder notation and logical equivalences. Drawing a Venn diagram may help your intuition, but does NOT constitute a proof.