

WEEK 1 DISCUSSION WORKSHEET: PROPOSITIONAL LOGIC

Problem 1: Consider the following pieces of identification a person might have in order to apply for a credit card:

B: Applicant presents a birth certificate.

D: Applicant presents a driver's license.

M: Applicant presents a marriage license.

$$p \vee q : p \text{ or } q$$
~~$$p \wedge q : p \text{ and } q$$~~

Write a logical expression for the requirements under the following conditions:

(a): The applicant must present either a birth certificate, a driver's license or a marriage license.

$$B \text{ or } D \text{ or } M$$

$$\underline{B \vee D \vee M}$$

(b): The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license

$$(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$$

(c): Applicant must present either a birth certificate or both a driver's license and a marriage license.

$$B \vee (D \wedge M)$$

Problem 2: If $p = T$, $q = F$, $r = T$, and $s = F$, what is the truth value of $(p \wedge q) \leftrightarrow (r \vee s)$?

Problem 3: Write truth tables for the following expressions:

(a): $(p \oplus \neg q)$

p	q	$\neg q$	$p \oplus \neg q$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

$$\underline{p \oplus \neg q}$$

$$(b): (p \vee q) \wedge \neg r$$

p	q	r	$p \vee q$	$\neg r$	$(p \vee q) \wedge \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	F	T	F

$$(c): (p \rightarrow q) \rightarrow r$$

$$(d): (p \vee q) \oplus (p \vee \neg r)$$

$$(e): \neg p \wedge ((p \rightarrow q) \vee (\neg p \rightarrow r))$$

p	q	r	$p \vee q$	$\neg r$	$p \vee \neg r$	$(p \vee q) \oplus (p \vee \neg r)$

10 min

Problem 4: Use a truth table to prove that the following two expressions are logically equivalent, then explain in words why they should be equivalent: $p \wedge (p \rightarrow q)$ and $p \wedge q$.

Problem 5: Prove the logical equivalence of the following using the laws of propositional logic:

(a): $p \wedge (\neg p \rightarrow q)$ and p

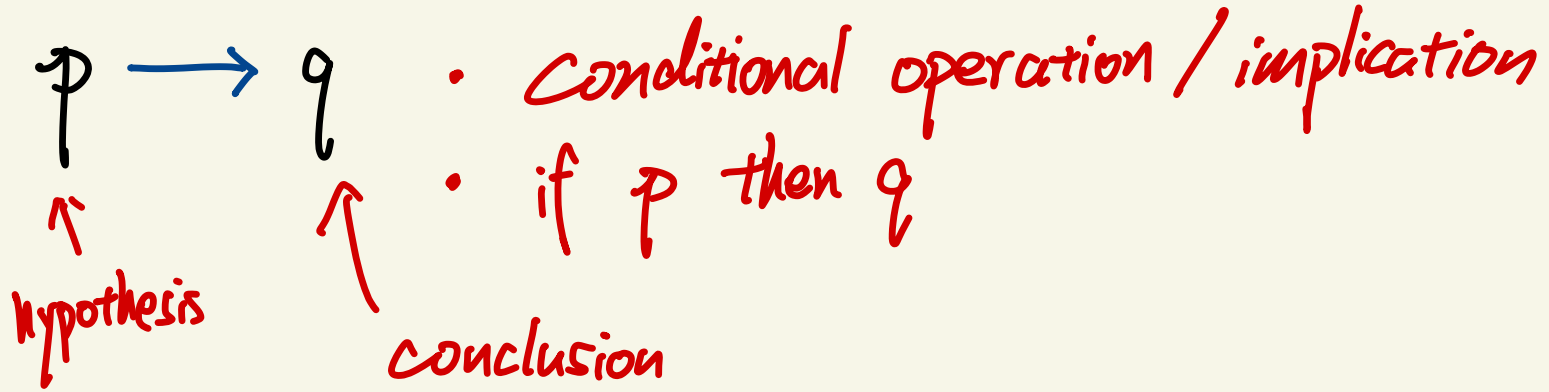
(b): $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$

(c): $p \leftrightarrow (p \wedge r)$ and $\neg p \vee r$

Problem 6: Show that the two sentences below are logically equivalent. Express each pair of sentences using a logical expression. Then prove whether the two expressions are logically equivalent. Note: you can assume that x and y are real numbers, so if x is not irrational, then x is rational, and if x is not rational, then x is an irrational number.

1. If x is a rational number and y is an irrational number then $x-y$ is an irrational number.
2. If x is a rational number and $x-y$ is a rational number then y is a rational number.

1.3 Conditional statements (implication)



Truth table for $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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\wedge

`\wedge`

\vee

`\vee`

\neg

`\neg`

\oplus

`\oplus`

\rightarrow

`\rightarrow`

Problem 3(c)

$$(p \rightarrow q) \rightarrow r$$

p	q	r	$p \rightarrow q$	$(p \rightarrow q) \rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

$$3(e) \quad \neg p \wedge ((p \rightarrow q) \vee (\neg p \rightarrow r))$$