WEEK 6 DISCUSSION THURSDAY WORKSHEET

(1) Prove that for any integer n, 2 divides $n(n+1)$.

(2) Prove or disprove that if $a|bc$, where a, b, and c are positive integers and $a \neq 0$, then $a|b$ or $a|c$.

(3) Prove that if a is a positive integer, then 4 does not divide $a^2 + 2$.

(4) Suppose that a and b are integers, $a \equiv 11 (mod 19)$, and $b \equiv 3 (mod 19)$. Find the integer c with $0 \leq c \leq 18$ such that

   (a) $c \equiv 13a (mod 19)$
   (b) $c \equiv 8b (mod 19)$
   (c) $c \equiv a - b (mod 19)$

(5) Evaluate these quantities
   (a) $-17 mod 2$
   (b) $-101 mod 13$
   (c) $144 mod 7$
   (d) $199 mod 19$

(6) Show that if n is an integer then $n2 \equiv 0$ or $1 (mod 4)$.

(7) Prove that if n is an odd positive integer, then $n^2 \equiv 1 (mod 8)$.

(8) Use problem 5 to show that the equation $x^2 + y^2 = 3443$ has no integer solutions.

(9) Given a nonzero integer a, find $\gcd(a, 0)$ and $\gcd(a, 1)$

(10) Find:
   (a) Find $\gcd(4567, 91837)$ and $\mathrm{lcm}(4567, 91837)$
   (b) Find integers x, y such that $\gcd(4567, 91837) = 4567x + 91837y$
   (c) Solve the congruence $91837x \equiv 1 (mod 4567)$