1: Prove by induction that for each positive integer n, 3 divides $2^{2n} - 1$.

2: Prove that if $c_0 = 5$ and $c_n = (c_{n-1})^2$ for $n \geq 1$, then for all $n \geq 0$, $c_n = 5^{(2^n)}$.

3: Prove by strong induction that if f_n denotes the nth Fibonacci number, then $f_n \leq 2^n$. Remember, the recursive construction of the Fibonacci sequence is $f_0 = 1$, $f_1 = 1$, $f_n = f_{n-1} + f_{n-2}$.

4: Use the well-ordering principle to prove that any amount of postage worth 8 cents or more can be made from some combination of 3-cent and 5-cent stamps.

5: Use the given loop invariant to show that if the pre-condition is true before the loop, then the post-condition is true after the loop.

While ($j < n$)
prod:= prod + m
j := j + 1
End-while

6: Here is a recursive definition of the set of *properly nested* strings of the characters (, {, },).

() and {} are properly nested.
If X is a properly nested string, then (X) and $\{Y\}$ are properly nested, and if X, Y are properly nested strings, then XY is a properly nested string.

Give three examples of properly nested strings.

7: Here is a recursive definition of *perfect binary trees.*

The single node with no edges is a perfect binary tree. The node itself is called the root of the tree.
If X is a perfect binary tree, then the new graph constructed by copying X, creating a new node, and connecting that node to the root of each copy of X is a perfect binary tree.

Give three examples of perfect binary trees.

8: The function $v(T)$ maps a perfect binary tree to the number of vertices it has. Give a recursive definition of the function $v(T)$.