(c) the function that assigns the number of bits leftover when a bit string is split into bytes (which are blocks of 8 bits)

$$11101011 \mid 10110111 \mid \underbrace{011} \longmapsto 3$$

$$\longrightarrow \{0, 1, 2, \ldots, 7\}.$$

WEEK 3 DISCUSSION WORKSHEET: BOOLEAN ALGEBRA

(1) Evaluate the logical expressions with $x = y = 1$ and $w = z = 0$
   (a) $xy\overline{wz}$
   (b) $x\overline{y} + z(\overline{w + z})$
   (c) $\overline{z}y\overline{x}(1 + w)$
   (d) $xy\overline{z} + z\overline{w}$
   (e) $\overline{(z + y)(w + x)}$

(2) Use the laws of Boolean algebra to show that the two Boolean expressions in each pair are equivalent.
   (a) $xy + x\overline{y} = x$
   (b) $x + xy = x$
   (c) $x(\overline{y} + x) = x$
   (d) $\overline{x + \overline{y}} + \overline{xy} = \overline{x}$

(3) A function f is defined by

x	y	z	$f(x, y, z)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

   (a) The function g is defined as $g(x, y, z) = \overline{xy}z + \overline{x}yz + xyz$ Give a set of values for the variables x, y, and z, for which the functions f and g have different output values
   (b) The function g is defined as $h(x, y, z) = \overline{xy}z + \overline{x}y\overline{z} + \overline{x}yz + xy\overline{z}$ Give a set of values for the variables x, y, and z, for which the functions f and h have different output values

(4) For each expression below, give an equivalent expression that uses only the NAND operation. Then give an equivalent expression that uses only the NOR operation.
   (a) $\overline{x} + y$
   (b) $\overline{xy}$
   (c) $(x + y)z$

(1) Evaluate the logical expressions with $x = y = 1$ and $w = z = 0$

(a) $xy\overline{wz}$

3min

(b) $x\overline{y} + z(\overline{w+z})$

(c) $\overline{z}y\overline{x}(1+w)$

(d) $xy\overline{z} + z\overline{w}$

(e) $\overline{(z+y)(w+x)}$

(a) $xy\overline{wz} = 1\cdot1\cdot\overline{(0\cdot0)} = 1\cdot1\cdot\overline{0} = 1\cdot1\cdot1 = 1$

(b) $1\cdot\overline{1} + 0\cdot\overline{(0+0)} = 1\cdot0 + 0\cdot\overline{0} = 1\cdot0+0\cdot1 = 0+0 = 0$

(c) $\overline{0}\cdot1\cdot\overline{1}\cdot(1+0) = 1\cdot1\cdot0\cdot1 = 0$

(d) $1\cdot1\cdot\overline{0} + 0\cdot\overline{0} = 1\cdot1\cdot1 + 0\cdot1 = 1 + 0 = 1$

(e) $\overline{(0+1)\cdot(0+1)} = \overline{1\cdot1} = \overline{1} = 0$

(2) Use the laws of Boolean algebra to show that the two Boolean expressions in each pair are equivalent.

(a) $xy + x\overline{y} = x$

Gmin

(b) $x + xy = x$

(c) $x(\overline{y} + x) = x$

(d) $\overline{x + \overline{y}} + \cancel{\overline{x}\,\overline{y}} = \overline{x}$

$\overline{x} \cdot \overline{y}$

Idempotent laws:	x + x = x	x • x = x
Associative laws:	(x + y) + z = x + (y + z)	(xy)z = x(yz)
Commutative laws:	x + y = y + x	xy = yx
Distributive laws:	x + yz =(x + y)(x + z)	x(y + z) =xy + xz
Identity laws:	x + 0 = x	x • 1 = x
Domination laws:	x + 1 = 1	x • 0 = 0
Double complement law:	$\overline{\overline{x}} = x$	
Complement laws:	$x + \overline{x} = 1$ $\overline{0} = 1$	$x\overline{x} = 0$ $\overline{1} = 0$
De Morgan's laws:	$\overline{x + y} = \overline{x}\,\overline{y}$	$\overline{xy} = \overline{x} + \overline{y}$
Absorption laws:	x + (xy) = x	x(x + y) = x

(a)

$xy + x\overline{y} = x(y + \overline{y})$ (Distribution)

$= x \cdot 1$ (Complement)

$= x$ (Identity)

(b) $x + xy = x$ (Absorption laws).

$$x + xy = x \cdot 1 + xy \quad \text{Identity}$$

$$= x(1+y) \quad \text{Distr.}$$

$$= x \cdot 1 \quad \text{Domination}$$

$$= x \quad \text{Identity}$$

(c) $x(\bar{y} + x) = x$ Absorption Laws.

(d) $\overline{x + \bar{y}} + \bar{x} \cdot \bar{y} = \bar{x} \cdot \bar{\bar{y}} + \bar{x} \cdot \bar{y}$ (De Morgan)

$$= \bar{x}(y + \bar{y}) \quad \text{(Double Complement + Dist.)}$$

$$= \bar{x} \cdot 1 = \bar{x}$$

(c) $x(\bar{y}+x) = x\cdot\bar{y} + x\cdot x$ (Dist.)

$= x\cdot\bar{y} + x$ (Idempotent)

$= x\cdot\bar{y} + x\cdot 1$ (Identity)

$= x(\bar{y}+1)$ (Dist.)

$= x\cdot 1$ (Domination)

$= x$ (Identity)

$x(x+y) = x$

$\rightsquigarrow$ $x(y+x) = x$

$x(\bar{y}+x) = x$

(3) A function f is defined by

x	y	z	$f(x,y,z)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

$g(x,y,z) = \overline{x}\overline{y}z + \overline{x}yz + xyz$

$g(x,y,z) = 1.$

$\Longleftrightarrow \quad \overline{x}\overline{y}z \quad o \quad \overline{x}yz \quad or \quad xyz = 1.$

001 011 111

001
011
101

(a) The function g is defined as $g(x, y, z) = \overline{x}\overline{y}z + \overline{x}yz + xyz$ Give a set of values for the variables x, y, and z, for which the functions f and g have different output values

2min

(b) The function g is defined as $h(x, y, z) = \overline{x}\overline{y}z + \overline{x}y\overline{z} + \overline{x}yz + xy\overline{z}$ Give a set of values for the variables x, y, and z, for which the functions f and h have different output values and if have different output values

(4) For each expression below, give an equivalent expression that uses only the NAND operation. Then give an equivalent expression that uses only the NOR operation.

Sketch

(a) $\overline{x} + y$

(b) $\overline{xy}$

(c) $(x + y)z$

$x \uparrow y = \overline{xy}$ $x \downarrow y = \overline{x+y}$

$x \uparrow y = \overline{xy}$

$x = x + 1$

$(x+1) \uparrow y$

$= \overline{(x+1)y}$

$x \uparrow x = \overline{x \cdot x} = \overline{x}$ ① $\overline{x} = x \uparrow x$

② $xy = \overline{\overline{xy}} = \overline{x \uparrow y} = (x \uparrow y) \uparrow (x \uparrow y)$

③ $x + y = \overline{\overline{x} \cdot \overline{y}} = (\overline{x} \cdot \overline{y}) \uparrow (\overline{x} \cdot \overline{y})$

$= ((\overline{x} \uparrow \overline{y}) \uparrow (\overline{x} \uparrow \overline{y})) \uparrow ((\overline{x} \uparrow \overline{y}) \uparrow (\overline{x} \uparrow \overline{y}))$

(a) $\bar{x}+y = \overline{\overline{\bar{x}+y}} = \overline{\bar{\bar{x}}\cdot\bar{y}} = \overline{x\cdot\bar{y}}$

$= (x\cdot\bar{y})\uparrow(x\cdot\bar{y})$

$\overline{\overline{\bar{x}+y}} = \overline{\bar{\bar{x}}\cdot\bar{y}}$

$= \overline{x\cdot\bar{y}}$

$= \big((x\uparrow\bar{y})\uparrow(x\uparrow\bar{y})\big)\uparrow\big((x\uparrow\bar{y})\uparrow(x\uparrow\bar{y})\big)$

$x\uparrow\bar{y} = x\uparrow(y\uparrow y)$

$= \Big(\big(x\uparrow(y\uparrow y)\big)\uparrow\big(x\uparrow(y\uparrow y)\big)\Big)\uparrow\Big(\big(x\uparrow(y\uparrow y)\big)\uparrow\big(x\uparrow(y\uparrow y)\big)\Big)$

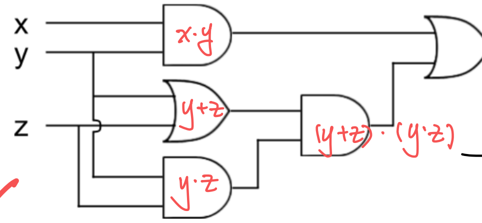
$\overline{A} \stackrel{?}{=} A\uparrow A \qquad A = x\cdot\bar{y}$

$x = 1$

$(x+1)^2 = x^2+2x+1 \qquad x = z^3 \qquad x = y+z$

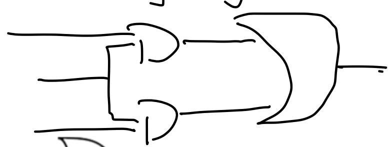
$(y+z)\cdot y = y$

(5) Give an equivalent Boolean expression for each circuit. Then use the laws of Boolean algebra to find a simpler circuit that computes the same function.

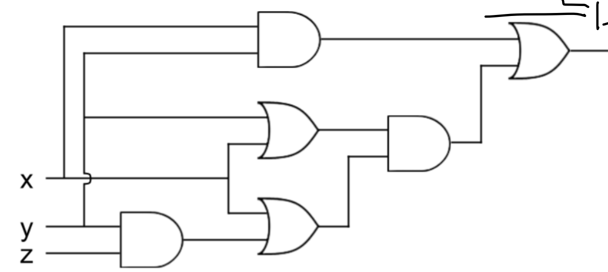


$x\cdot y + (y+z)(y\cdot z)$

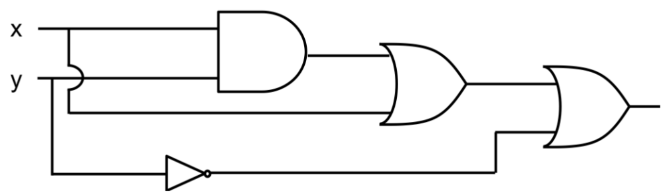
$= x\cdot y + y\cdot z$



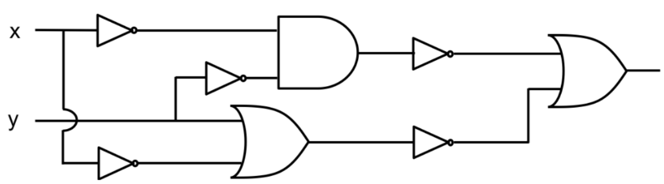
(a)



(b)



(c)



(d)