**WEEK 1 DISCUSSION WORKSHEET: PROPOSITIONAL LOGIC**

*Problem 1:* Consider the following pieces of identification a person might have in order to apply for a credit card:

B: Applicant presents a birth certificate.
D: Applicant presents a driver's license.
M: Applicant presents a marriage license.

Write a logical expression for the requirements under the following conditions:
*(a):* The applicant must present either a birth certificate, a driver's license or a marriage license.

*(b):* The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license

*(c):* Applicant must present either a birth certificate or both a driver's license and a marriage license.

*Problem 2:* If $p = T$, $q = F$, $r = T$, and $s = F$, what is the truth value of $(p \wedge q) \leftrightarrow (r \vee s)$?

*Problem 3:* Write truth tables for the following expressions:
*(a):* $(p \oplus \neg q)$

*(b):* $(p \vee q) \wedge \neg r$

*(c):* $(p \to q) \to r$

*(d):* $(p \vee q) \oplus (p \vee \neg r)$

*(e):* $\neg p \wedge ((p \to q) \vee (\neg p \to r))$

*Problem 4:* Use a truth table to prove that the following two expressions are logically equivalent, then explain in words why they should be equivalent: $p \wedge (p \to q)$ and $p \wedge q$.

*Problem 5:* Prove the logical equivalence of the following using the laws of propositional logic:

*(a):* $p \wedge (\neg p \to q)$ and $p$

*(b):* $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$

*(c):* $p \leftrightarrow (p \wedge r)$ and $\neg p \vee r$

*Problem 6:* Show that the two sentences below are logically equivalent. Express each pair of sentences using a logical expression. Then prove whether the two expressions are logically equivalent. Note: you can assume that x and y are real numbers, so if x is not irrational, then x is rational, and if x is not rational, then x is an irrational number.

1. If x is a rational number and y is an irrational number then x-y is an irrational number.
2. If x is a rational number and x-y is a rational number then y is a rational number.