WEEK 1 DISCUSSION WORKSHEET: PROPOSITIONAL LOGIC

Problem 1: Consider the following pieces of identification a person might have in order to apply for a credit card:

B: Applicant presents a birth certificate.
D: Applicant presents a driver's license.
M: Applicant presents a marriage license.

$p \vee q : p$ or q
$p \wedge q : p$ and q

Write a logical expression for the requirements under the following conditions:

(a): The applicant must present either a birth certificate, a driver's license or a marriage license.

B or D or M $\underline{B \vee D \vee M}$

(b): The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license

$(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$

3 min

(c): Applicant must present either a birth certificate or both a driver's license and a marriage license.

$B \vee (D \wedge M)$

Problem 2: If $p = T$, $q = F$, $r = T$, and $s = F$, what is the truth value of $(p \wedge q) \leftrightarrow (r \vee s)$?

$p \wedge q \equiv T \wedge F \equiv F$ $r \vee s \equiv T \vee F \equiv T$

$\equiv F \leftrightarrow T \equiv F$

Problem 3: Write truth tables for the following expressions:
(a): $(p \oplus \neg q)$

$s \oplus t$

p	q	$\neg q$	$p \oplus \neg q$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

1

(b): $(p \lor q) \land \neg r$

p	q	r	$p \lor q$	$\neg r$	$(p \lor q) \land \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	F	T	F

10 min

(c): $(p \to q) \to r$

5 min

p	q	r	$p \to q$	$(p \to q) \to r$
T	~	~	T	T
T	t	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

(d): $(p \lor q) \oplus (p \lor \neg r)$

(e): $\neg p \land ((p \to q) \lor (\neg p \to r))$

p	q	r	$p \lor q$	$p \lor \neg r$	$(p \lor q) \oplus (p \lor \neg r)$
T	T	T	T	T	F
T	T	F	T	T	F
T	F	F	T	T	F
T	F	T	T	T	F
F	T	T	T	F	F
F	F	T	F	F	F
F	F	F	F	T	T
F	F	F	F	T	T

Problem 4: Use a truth table to prove that the following two expressions are logically equivalent, then explain in words why they should be equivalent: $p \wedge (p \to q)$ and $p \wedge q$.

Problem 5: Prove the logical equivalence of the following using the laws of propositional logic:

(a): $p \wedge (\neg p \to q)$ and p

$$p \to q \equiv \neg p \vee q.$$

$$p \wedge (\neg p \to q) = p \wedge ((\neg p) \to q)$$
$$\equiv p \wedge ((\neg \neg p) \vee q) \equiv p \wedge (p \vee q)$$
$$\equiv p$$

$$p = T \checkmark \quad p = F$$

(b): $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$

$$\equiv (\neg p \vee r) \vee (\neg q \vee r)$$
$$\equiv \neg p \vee r \vee \neg q \vee r$$
$$\equiv \neg p \vee \neg q \vee r$$
$$\equiv \neg (p \wedge q) \vee r \equiv (p \wedge q) \to r.$$

10min

(c): $p \leftrightarrow (p \wedge r)$ and $\neg p \vee r$

$$\equiv (p \to (p \wedge r)) \wedge ((p \wedge r) \to p)$$
$$\equiv (\neg p \vee (p \wedge r)) \wedge (\neg (p \wedge r) \vee p)$$
$$\equiv ((\neg p \vee p) \wedge (\neg p \vee r)) \wedge ((\neg p \vee \neg r) \vee p)$$
$$\equiv (\neg p \vee p) \wedge (\neg p \vee r) \equiv \neg p \vee r.$$

Problem 6: Show that the two sentences below are logically equivalent. Express each pair of sentences using a logical expression. Then prove whether the two expressions are logically equivalent. Note: you can assume that x and y are real numbers, so if x is not irrational, then x is rational, and if x is not rational, then x is an irrational number.

1. If x is a rational number and y is an irrational number then x-y is an irrational number.

2. If x is a rational number and x-y is a rational number then y is a rational number.

1.3 Conditional statements (implication)

$$p \longrightarrow q$$

- Conditional operation / implication
- if p then q

p — hypothesis

q — conclusion
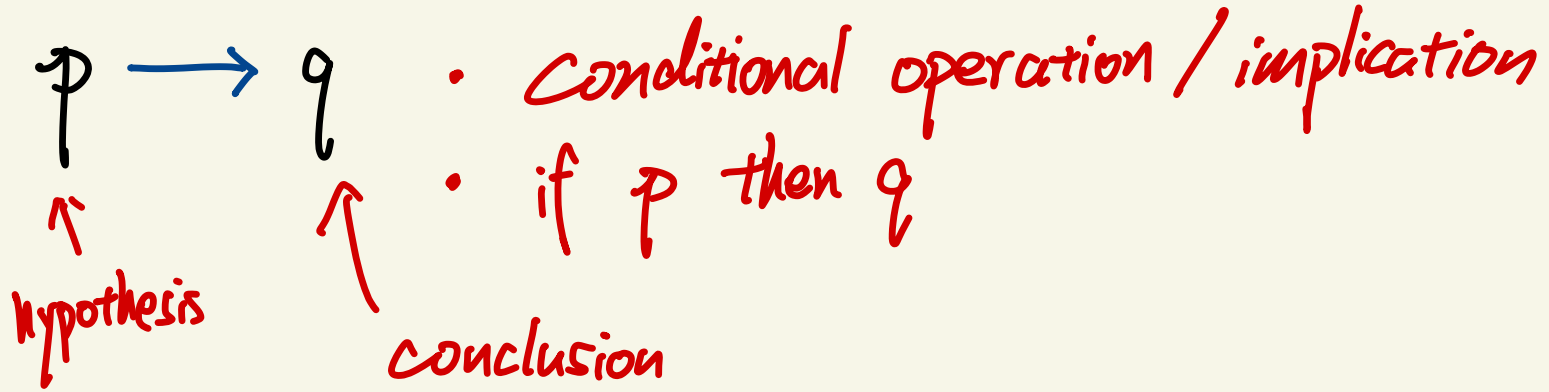
Truth table for $P \rightarrow q$

P	q	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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$\wedge$	\wedge
$\vee$	\vee
$\neg$	\neg
$\oplus$	\oplus
$\rightarrow$	\rightarrow

Problem 3(c)

$(p \rightarrow q) \rightarrow r$

P	q	r	p→q	(p→q)→r
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

3(e) $\neg p \wedge ((p \rightarrow q) \vee (\neg p \rightarrow r))$