1: For each of the sequences given, indicate whether it is increasing, non-increasing, decreasing, or non-decreasing. Any of these sequences may have multiple properties!
(a) $a_n = n^2 - 2n$ for $n \geq 1$.

(b) $a_n = n^2 - 3n$ for $n \geq 1$.

(c) $a_n = 2^n - n!$ for $n \geq 1$.

2: Under what conditions on r, the common ratio, and a, the initial value, is the geometric sequence $a_n = ar^n$ increasing? Decreasing?

3: Suppose someone takes out a home improvement loan for \$30,000. The annual interest on the loan is 6% and is compounded monthly. The monthly payment is \$600. Let a_n denote the amount owed at the end of the nth month. The payments start in the first month and are due the last day of every month.
(a) Give a recurrence relation for a_n.

(b) Suppose the borrower wants a lower monthly payment. How large does the monthly payment need to be in order to ensure the amount owed decreases each month?

4: Calculate the following:
(a) $\sum_{j=2}^{5}(2j-1)$

(b) $\sum_{j=0}^{2}(j+1)^2$

(c) $\sum_{k=0}^{100}(3+5k)$

5: **(a)** Rewrite the sum $\sum_{k=0}^{m+2}(k^2-4k+1)$ so that it ends on the mth term.

(b) Rewrite the sum $\sum_{k=0}^{q-1}(k^2+4k+3)$ so that it ends on the qth term.

(c) Rewrite the sum $\sum_{k=0}^{n+1}(j^2-2j+1)$ so that the index variable is $i = j-1$.

6: Prove by induction that for any positive integer n,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$$

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$= n(n-2)$

5min

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(a) $a_n = n^2 - 2n$

1. $a_{n+1} - a_n$

$= [(n+1)^2 - 2(n+1)] - [n^2 - 2n]$

$= (n+1)^2 - n^2 - 2n - 2 + 2n$

$= 2n + 1 - 2 = 2n - 1 \geq 1 > 0$

2. $\dfrac{a_{n+1}}{a_n} = \dfrac{(n+1)(n-1)}{n(n-2)} = \dfrac{n+1}{n} \cdot \dfrac{n-1}{n-2} > 1$

provided $n \geq 3$, $n = 1, 2$. compare term by terms

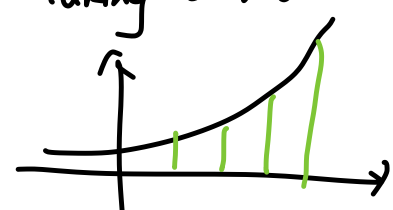
1. $a_{n+1} - a_n$ vs 0

2. If $a_n > 0$ or $a_n < 0$

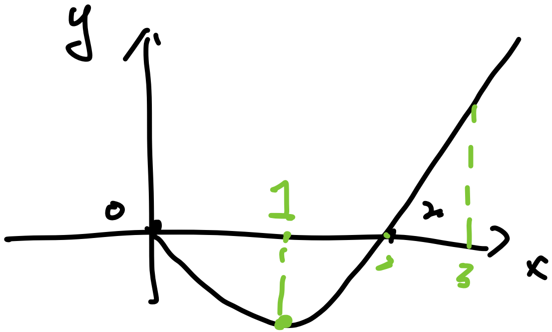
$\dfrac{a_{n+1}}{a_n}$ vs 1

3. $a_n = f(n)$, where $f(x)$ function

Taking derivative



3. $f(x) = x^2 - 2x = x(x-2)$



$f(x)' = 2x - 2 \geq 0$ when $x \geq 1$.

≥ 0 when $x > 1$

3min **2:** Under what conditions on r, the common ratio, and a, the initial value, is the geometric sequence $a_n = ar^n$ increasing? Decreasing?

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Quick

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5min

6: Prove by induction that for any positive integer n,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$$

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