WEEK 1 DISCUSSION WORKSHEET: PROPOSITIONAL LOGIC

Problem 1: Consider the following pieces of identification a person might have in order to apply for a credit card:

B: Applicant presents a birth certificate.
D: Applicant presents a driver's license.
M: Applicant presents a marriage license.

$p \vee q : p$ or q
$p \wedge q : p$ and q

Write a logical expression for the requirements under the following conditions:

(a): The applicant must present either a birth certificate, a driver's license or a marriage license.

B or D or M $\qquad$ $\underline{B \vee D \vee M}$

(b): The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license

3 min

$(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$

(c): Applicant must present either a birth certificate or both a driver's license and a marriage license.

$B \vee (D \wedge M)$

Problem 2: If $p = T$, $q = F$, $r = T$, and $s = F$, what is the truth value of $(p \wedge q) \leftrightarrow (r \vee s)$?

Problem 3: Write truth tables for the following expressions:
(a): $(p \oplus \neg q)$

$\underline{s \oplus t}$

p	q	$\neg q$	$p \oplus \neg q$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

(b): $(p \lor q) \land \neg r$

P	q	r	p∨q	¬r	(p∨q)∧¬r
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	F	T	F

(c): $(p \to q) \to r$

10 min

(d): $(p \lor q) \oplus (p \lor \neg r)$

P	q	r	p∨q	¬r	p∨¬r	(p∨q)⊕(p∨¬r)

(e): $\neg p \land ((p \to q) \lor (\neg p \to r))$

Problem 4: Use a truth table to prove that the following two expressions are logically equivalent, then explain in words why they should be equivalent: $p \wedge (p \to q)$ and $p \wedge q$.

Problem 5: Prove the logical equivalence of the following using the laws of propositional logic:

(a): $p \wedge (\neg p \to q)$ and p

(b): $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$

(c): $p \leftrightarrow (p \wedge r)$ and $\neg p \vee r$

Problem 6: Show that the two sentences below are logically equivalent. Express each pair of sentences using a logical expression. Then prove whether the two expressions are logically equivalent. Note: you can assume that x and y are real numbers, so if x is not irrational, then x is rational, and if x is not rational, then x is an irrational number.

1. If x is a rational number and y is an irrational number then x-y is an irrational number.

2. If x is a rational number and x-y is a rational number then y is a rational number.

1.3 Conditional statements (implication)

$$p \longrightarrow q$$

- Conditional operation / implication
- if p then q

$\uparrow$ hypothesis

$\uparrow$ conclusion
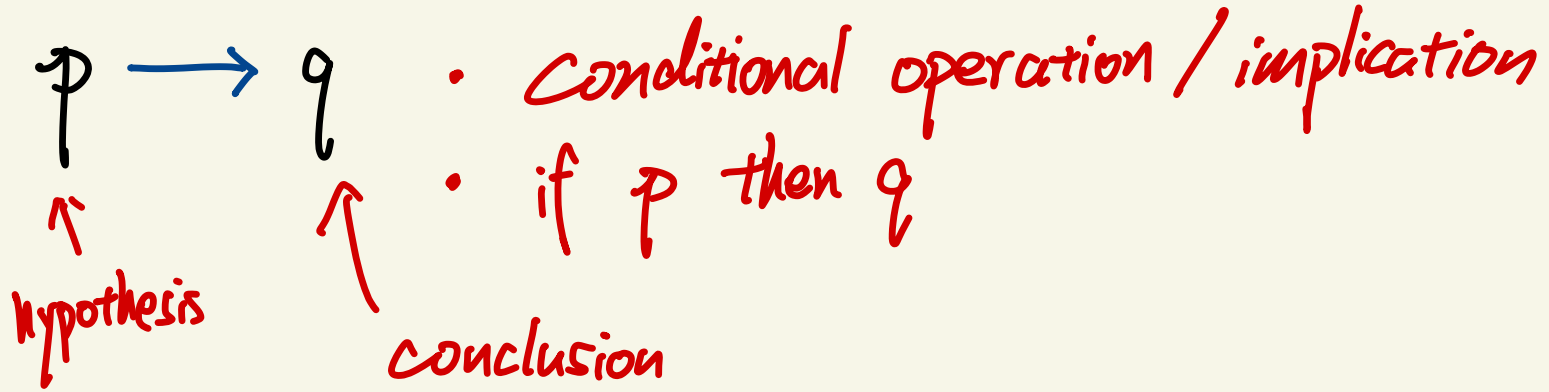
Truth table for P →q

P	q	P →q
T	T	T
T	F	F
F	T	T
F	F	T

Latex & Markdown

$\wedge$	\wedge
$\vee$	\vee
$\neg$	\neg
$\oplus$	\oplus
$\rightarrow$	\rightarrow

Problem 3(c)

$(p \rightarrow q) \rightarrow r$

P	q	r	p→q	(p→q)→r
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

3(e) $\neg p \land ((p \to q) \lor (\neg p \to r))$