WEEK 6 DISCUSSION THURSDAY WORKSHEET

(1) Prove that for any integer $n, 2$ divides $n(n+1)$.

(2) Prove or disprove that if $a|bc$, where a, b, and c are positive integers and $a \neq 0$, then $a|b$ or $a|c$.

(3) Prove that if a is a positive integer, then 4 does not divide $a^2 + 2$.

(4) Suppose that a and b are integers, $a \equiv 11(mod 19)$, and $b \equiv 3(mod 19)$. Find the integer c with $0 \leq c \leq 18$ such that

   (a) $c \equiv 13a(mod 19)$
   (b) $c \equiv 8b(mod 19)$
   (c) $c \equiv a - b(mod 19)$

(5) Evaluate these quantities
   (a) $-17 mod 2$
   (b) $-101 mod 13$
   (c) $144 mod 7$
   (d) $199 mod 19$

(6) Show that if n is an integer then $n2 \equiv 0$ or $1(mod 4)$.

(7) Prove that if n is an odd positive integer, then $n^2 \equiv 1(mod 8)$.

(8) Use problem 5 to show that the equation $x^2 + y^2 = 3443$ has no integer solutions.

(9) Given a nonzero integer a, find $\gcd(a, 0)$ and $\gcd(a, 1)$

(10) Find:
   (a) Find $\gcd(4567, 91837)$ and $\operatorname{lcm}(4567, 91837)$
   (b) Find integers x, y such that $\gcd(4567, 91837) = 4567x + 91837y$
   (c) Solve the congruence $91837x \equiv 1(mod 4567)$

3min

(1) Prove that for any integer n, 2 divides $n(n+1)$.

Proof. Want to show: $n(n+1) \equiv 0 \pmod 2$

1. $n \equiv 0 \pmod 2$

$n+1 \equiv 0+1 \equiv 1 \pmod 2$

$n(n+1) \equiv 0 \cdot 1 \equiv 0 \pmod 2$ ✓

2. $n \equiv 1 \pmod 2$

$n+1 \equiv 1+1 \equiv 2 \equiv 0 \pmod 2$ ✓

Therefore, $n(n+1) \equiv 0 \pmod 2$

Hence $2 \mid n(n+1)$.

$x, y \in \mathbb{Z}$

$\bar{x}, \bar{y}$ their mod N

$\overline{x+y} = \bar{x} + \bar{y} \mod N$

$\bar{x} \cdot \bar{y} = \overline{x \cdot y} \mod N$

2min

(2) Prove or disprove that if $a|bc$, where a, b, and c are positive integers and $a \neq 0$, then $a|b$ or $a|c$.

▷ $a = 4$, $b = 2$, $c = 2$, $b \cdot c = 4$

$a | bc$, but $a \nmid b$, $a \nmid c$.

▷ When a is a prime number, then the statement is true.

$b = p_1^{n_1} \cdots p_s^{n_s}$ $c = q_1^{m_1} \cdots q_t^{m_t}$

If $a | bc$, $a = p_i$ or q_j for some i, j.

▷ $a = 4$, $b = 2 \times 4567$ $c = 2 \times 4567$ $a \leq b$, $a \leq c$

$a | bc$ but $a \nmid b$, $a \nmid c$.

(4) Suppose that a and b are integers, $a \equiv 11 (mod 19)$, and $b \equiv 3 (mod 19)$. Find the integer c with $0 \leq c \leq 18$ such that

3min
- (a) $c \equiv 13a (mod 19)$
- (b) $c \equiv 8b (mod 19)$
- (c) $c \equiv a - b (mod 19)$

$$\begin{array}{r} 13 \\ \times\ 11 \\ \hline 13 \\ 13\ \ \\ \hline 143 \end{array} \qquad \begin{array}{r} 7 \\ 19\,\overline{)143} \\ 133 \\ \hline 10 \end{array}$$

$143 = 19 \times 7 + 10$

(a) $c \equiv 13a \equiv 13 \cdot 11 \equiv 143 \equiv 10 \mod 19$

$\equiv 19 \times 7 + 10 \equiv 0 \times 7 + 10 \equiv 10 \mod 19$

Therefore, $c = 10$.

(b) $c \equiv 8b \pmod{19}$ $\quad b \equiv 3 \pmod{19}$

$c \equiv 8b \equiv 8 \times 3 \equiv 24 \pmod{19}$

$\equiv 19 + 5 \equiv 5 \pmod{19}$

Therefore. $c = 5$.

(c) $c \equiv a-b \pmod{19}$ $\quad a \equiv 11,\ b \equiv 3 \pmod{19}$

$\equiv 11-3 \equiv 8 \pmod{19}$

Therefore, $c = 8$.

Discussion Activity:

Given that $5x \equiv 6 \pmod 8$, find x.

Key point: 5 is invertible mod 8

$5 \times 0 \equiv 0$

$5 \times 1 \equiv 5$

$5 \times 2 \equiv 2$

$5 \times 3 \equiv 7$

$5 \times 4 \equiv 4$

$5 \times 5 \equiv 25 \equiv 1 + 8 \times 3 \equiv 1$

$5 \times 5 \equiv 1 \pmod 8$

$5^{-1} \equiv 5 \pmod 8$

$5x \equiv 6 \pmod{8}$

$30 = 8 \times 3 + 6$

$5 \times 5 \times x \equiv 5 \times 6 \pmod{8}$

$\equiv 6 \pmod{8}$

$\equiv 1 \times x \equiv x$

$x \equiv 6 \pmod{8}$

$5x \equiv 6$

$x \equiv 6/5 \equiv 6 \cdot 5^{-1} \equiv 6 \times 5 \equiv 30 \equiv 6 \pmod{8}$

(10) Find:

(a) Find $\gcd(4567, 91837)$ and $\operatorname{lcm}(4567, 91837)$

(b) Find integers x, y such that $\gcd(4567, 91837) = 4567x + 91837y$

(c) Solve the congruence $91837x \equiv 1 (mod 4567)$

$497 = 91837 - 4567 \times 20$

$94 = 4567 - 497 \times 9$

$27 = 497 - 94 \times 5$

$13 = 94 - 27 \times 3$

$1 = 27 - 13 \times 2$

$$1 = 27 - 13 \times 2$$

$$= 27 - (94 - 27 \times 3) \times 2$$

$$= 27 \times 7 - 94 \times 2$$

$$= 497 \times 7 - 94 \times 37$$

$$= 497 \times 340 - 4567 \times 37$$

$$= 91837 \times 340 - 4567 \times 6837$$

2
4567 | 91837
9134
497

9
497 | 4567
4473
94

5
94 | 497
470
27