1: For each of the sequences given, indicate whether it is increasing, non-increasing, decreasing, or non-decreasing. Any of these sequences may have multiple properties!

(a) $a_n = n^2 - 2n$ for $n \geq 1$.

(b) $a_n = n^2 - 3n$ for $n \geq 1$.

(c) $a_n = 2^n - n!$ for $n \geq 1$.

2: Under what conditions on r, the common ratio, and a, the initial value, is the geometric sequence $a_n = ar^n$ increasing? Decreasing?

3: Suppose someone takes out a home improvement loan for \$30,000. The annual interest on the loan is 6% and is compounded monthly. The monthly payment is \$600. Let a_n denote the amount owed at the end of the nth month. The payments start in the first month and are due the last day of every month.

(a) Give a recurrence relation for a_n.

(b) Suppose the borrower wants a lower monthly payment. How large does the monthly payment need to be in order to ensure the amount owed decreases each month?

4: Calculate the following:

(a) $\sum_{j=2}^{5}(2j-1)$

(b) $\sum_{j=0}^{2}(j+1)^2$

(c) $\sum_{k=0}^{100}(3+5k)$

5: **(a)** Rewrite the sum $\sum_{k=0}^{m+2}(k^2-4k+1)$ so that it ends on the mth term.

(b) Rewrite the sum $\sum_{k=0}^{q-1}(k^2+4k+3)$ so that it ends on the qth term.

(c) Rewrite the sum $\sum_{k=0}^{n+1}(j^2-2j+1)$ so that the index variable is $i = j-1$.

6: Prove by induction that for any positive integer n,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$$

1: For each of the sequences given, indicate whether it is increasing, non-increasing, decreasing, or non-decreasing. Any of these sequences may have multiple properties!

(a) ✓ $a_n = n^2 - 2n$ for $n \geq 1$.
$= n(n-2)$

5min
(b) $a_n = n^2 - 3n$ for $n \geq 1$.

(c) $a_n = 2^n - n!$ for $n \geq 1$.

(a) $a_n = n^2 - 2n$

1. $a_{n+1} - a_n$

$= [(n+1)^2 - 2(n+1)] - [n^2 - 2n]$

$= (n+1)^2 - n^2 - 2n - 2 + 2n$

$= 2n + 1 - 2 = 2n - 1 \geq 1 > 0$

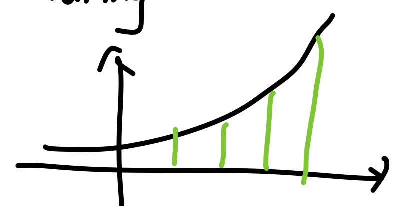
2. $\dfrac{a_{n+1}}{a_n} = \dfrac{(n+1)(n-1)}{n(n-2)} = \dfrac{n+1}{n} \cdot \dfrac{n-1}{n-2} > 1$

provided $n \geq 3$, $n = 1, 2$. compare term by terms

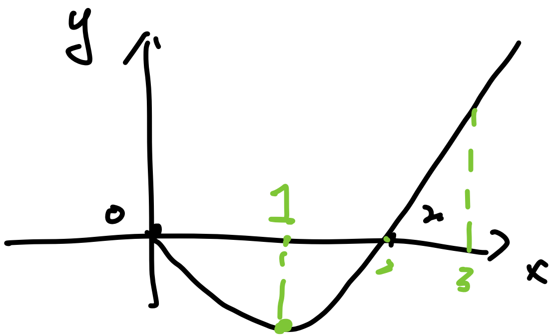
1. $a_{n+1} - a_n$ vs 0

2. If $a_n > 0$ or $a_n < 0$
$\dfrac{a_{n+1}}{a_n}$ vs 1

3. $a_n = f(n)$, where $f(x)$ function
Taking derivative



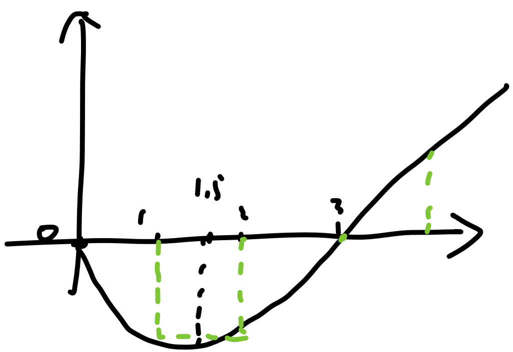
3. $f(x) = x^2 - 2x = x(x-2)$



$f(x)' = 2x - 2 \geq 0$ when $x \geq 1$.

> 0 when $x > 1$

(b) $a_n = n^2 - 3n \quad n \geq 1$



$f(x) = x^2 - 3x$

$= (x - \frac{3}{2})^2 - \frac{9}{4}$

non-decreasing.

$a_{n+1} - a_n = [(n+1)^2 - 3(n+1)] - [n^2 - 3n]$

$= (n+1)^2 - n^2 - 3n - 3 + 3n = 2n + 1 - 3$

$= 2n - 2 \geq 0$ non-decreasing

(C) $a_n = 2^n - n!$ $\quad n \geq 1$

$n=1$ $\quad 2-1=1$

$n=2$ $\quad 2\times2 - 1\times2 = 2$

$n=3$ $\quad 2\times2\times2 - 1\times2\times3 = 8-6=2$

$n=4$ $\quad 16-24=-8$

$n\geq4$ $\quad <0$

2^n

$$\underbrace{2\times2\times[2\times\cdots\times2]}_{n\ \ 2}$$

$n!$

$$1\times2\times\underbrace{[3\times4\times\cdots\times n]}$$

$$\frac{a_{n+1}}{a_n} = \frac{2^{n+1}-(n+1)!}{2^n - n!} = \frac{\frac{2^{n+1}}{n!} - n}{\frac{2^n}{n!} - 1} = \frac{n - \overset{<2}{\frac{2^n}{n!}\cdot 2}}{1 - \frac{2^n}{n!}^{<1}} > 1$$

Therefore is decreasing $(n\geq4)$

$1\leq n\leq3$ is non-increasing

$4\leq n$ is decreasing.

2: Under what conditions on r, the common ratio, and a, the initial value, is the geometric sequence $a_n = ar^n$ increasing? Decreasing?

$a < 0$ | $a = 0$ | $a > 0$

$r < 0$ | $r < 0$

$r = 0$ | $r = 0$

$0 < r < 1$ | $0 < r < 1$

$r = 1$ | $r = 1$

$1 < r$ | $1 < r$

- $a > 0$ $r < 0$ not monotonic.
- $a < 0$ $r > 1$ $a \cdot r^n$ $-1 \cdot 2^n = -2^n$

decreasing.

4: Calculate the following:

(a) $\sum_{j=2}^{5}(2j-1)$

5min

(b) $\sum_{j=0}^{2}(j+1)^2$

(c) $\sum_{k=0}^{100}(3+5k)$

(a) $\sum_{j=2}^{5}(2j-1) = (2\times 2-1) + (2\times 3-1) + (2\times 4-1) + (2\times 5-1)$

$= 3+5+7+9 = 24$

$\dfrac{2\cdot\sum_{j=2}^{5}(2j-1)}{2} = \Big[(2\times 2-1) + (2\times 3-1) + (2\times 4-1) + (2\times 5-1)$

$+ (2\times 5-1) + (2\times 4-1) + (2\times 3-1) + (2\times 2-1)\Big]/2$

$= \big[2\times(2+5)-2\big] + \big[2\times(3+4)-2\big] + \big[2\times(4+3)-2\big] + \big[2\times(5+2)-2\big] /2$

$= 4\times[2\times 7-2] = 4\times 12 /2 = 24$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$

(b) $$\sum_{j=0}^{2} (j+1)^2 = (0+1)^2 + (1+1)^2 + (2+1)^2$$

$$= 1 + 4 + 9$$

$$= 14$$

(c) $$\sum_{k=0}^{100} (3+5k) = \left(\sum_{k=0}^{100} 3\right) + \left(\sum_{k=0}^{100} 5k\right)$$

$$= \left(\sum_{k=0}^{100} 3\right) + 5\left(\sum_{k=0}^{100} k\right)$$

$$\sum_{k=0}^{100} 3 = \underbrace{3+3+\cdots+3}_{101} = 303$$

$$\sum_{k=0}^{100} k = \frac{100 \times 101}{2} = 5050$$

$$\sum_{k=0}^{100} (3+5k) = 303 + 5 \times 5050$$

$$= 303 + 25250$$

$$= 25553.$$

5: (a) Rewrite the sum $\sum_{k=0}^{m+2}(k^2 - 4k + 1)$ so that it ends on the mth term.

3min

(b) Rewrite the sum $\sum_{k=0}^{q-1}(k^2 + 4k + 3)$ so that it ends on the qth term.

(c) Rewrite the sum $\sum_{k=0}^{n+1}(j^2 - 2j + 1)$ so that the index variable is $i = j - 1$.

6: Prove by induction that for any positive integer n,

5min

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$$

$a_n = ar^n$

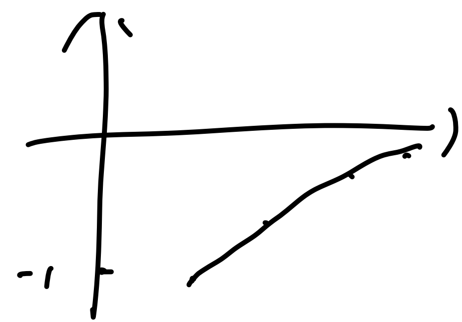
① $a=1,\ r=\frac{1}{2}$ $\quad a_n = (\frac{1}{2})^n$ $\searrow$

② $a=1,\ r=2$ $\quad a_n = 2^n$ $\nearrow$

③ $a=-1,\ r=\frac{1}{2}$ $\quad a_n = -\cdot(\frac{1}{2})^n$ $\nearrow$

④ $a=-1,\ r=2$ $\quad a_n = -2^n$ $\searrow$

⑤ $a=1 \quad r=-1 \quad \cdots$



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