**1:** For each of the sequences given, indicate whether it is increasing, non-increasing, decreasing, or non-decreasing. Any of these sequences may have multiple properties!
**(a)** $a_n = n^2 - 2n$ for $n \geq 1$.

**(b)** $a_n = n^2 - 3n$ for $n \geq 1$.

**(c)** $a_n = 2^n - n!$ for $n \geq 1$.

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**2:** Under what conditions on $r$, the common ratio, and $a$, the initial value, is the geometric sequence $a_n = ar^n$ increasing? Decreasing?

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**3:** Suppose someone takes out a home improvement loan for \$30,000. The annual interest on the loan is 6% and is compounded monthly. The monthly payment is \$600. Let $a_n$ denote the amount owed at the end of the $n$th month. The payments start in the first month and are due the last day of every month.
**(a)** Give a recurrence relation for $a_n$.

**(b)** Suppose the borrower wants a lower monthly payment. How large does the monthly payment need to be in order to ensure the amount owed decreases each month?

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**4:** Calculate the following:
**(a)** $\sum_{j=2}^{5}(2j-1)$

**(b)** $\sum_{j=0}^{2}(j+1)^2$

**(c)** $\sum_{k=0}^{100}(3+5k)$

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**5:** **(a)** Rewrite the sum $\sum_{k=0}^{m+2}(k^2-4k+1)$ so that it ends on the $m$th term.

**(b)** Rewrite the sum $\sum_{k=0}^{q-1}(k^2+4k+3)$ so that it ends on the $q$th term.

**(c)** Rewrite the sum $\sum_{k=0}^{n+1}(j^2-2j+1)$ so that the index variable is $i = j - 1$.

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**6:** Prove by induction that for any positive integer $n$,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$$