Lecture 10: Probability

Probability basic

The probability $P(A)$ of an event is the ratio of possible outcomes of A and total outcomes. It is a number between 0 and 1.

Coin Flipping Examples:

- P("head" when flipping a coin)$= 1/2$ because head is one of the outcome of head and tail.
- P("HHH" when flipping 3 coins)$= 1/8$
- P(two heads when flipping 3 coins)$= 3/8$ because HHT, HTH, THH
- k-heads when flipping n coins

$$P(A) = \frac{\binom{n}{k}}{2^n}$$

Dice Rolling Examples:

- P("6" when rolling a dice)$= 1/6$.
- P(sum up to 12 when rolling 2 dices)$= 1/36$.
- P(sum up to 9 when rolling 2 dices)$= 4/36$ because $9 = 3 + 6 = 4 + 5 = 5 + 4 = 6 + 3$.
- P(sum up to 7 when rolling 2 dices)$= 6/36$.

Picking with(without) replacement:

- 3 green marbles and 4 red marbles in a bag.
- if picking with replacement, the probability of getting two green is $(3/7)^2 = 9/49$.
- if picking without replacement, the probability of getting two green is $3/7 * 2/6 = 6/42$.
- if picking without replacement, the probability of getting two red is $4/7 * 3/6 = 2/7$.

Card Drawing Examples:

- P(Diamond Ace when drawing from a deck of 52 cards)$= 1/52$.
- P(Diamond Ace then Heart Ace from a deck without replacement)$= 1/52 * 1/51$.
- P(Diamond Ace then Heart Ace from a deck with replacement)$= 1/52 * 1/52$.
- P(a black card and a red card from a deck without replacement)$= 1/2 * 26/51$.

More fun example:

- P(being an even number)$= 1/2$

- P(being a prime number)$= 0$
- P(raining chance)$= 40\%$
- Three boxes with one with one million dollars and two empty. You choose one, the winning chance is $1/3$. Someone knows the answer removing one empty box. Are you gonna switch?
  - what is the winning chance if you do not switch?
  - what is the winning chance if you switch?

More difficult examples:

- Permutations of "ALGEBRA" with "AA"
  - all permutations $7!/2$ because of the symmetry AA.
  - with AA together: $6!$
  - $P = \frac{6!}{7!/2} = 2/7$
- Rolling 3 dice exactly two 6s.
  - total outcome 6^3
  - possibility: $6a6$, $66a$, $a66$. So $3 * 5$.
  - $P = 15/216 = 5/72$.
- Selecting a committee of 5 out of 6 boys and 6 girls. What is chance there are exactly 3 girls in the committee?
  - total outcome? $\binom{12}{5}$.
  - the choice is $\binom{6}{3} * \binom{6}{2}$
- Rolling 2 dice with the second dice number larger than the first dice number.
  - total outcome 36
  - either $a > b$, $a = b$ or $a < b$, with the count 15, 6 and 15.
- Rolling 3 dice with the last dice number larger than the sum of first two numbers.
  - total outcome 216
  - Numerate all the cases.
- If raining chance is always $1/3$, what is the chance raining one day in next 4 days?
  - no rain chance: $2/3$
  - no rain at all $16/81$
  - complement: $65/81$