Lecture 7: Basic Counting

Counting patterns

- how many numbers in the series $1, 7, 13, 19, ..., 109$?
  - recall that this is an arithmetic series. So answer is

$$(109 - 1)/6 + 1 = 19$$

- how many two digit numbers are there?
  - two digit numbers starts with 10 and ends 99. So in total $99 - 10 + 1 = 90$.
- There is always a $+1$ term in many of those counting problems.

Counting outcomes

- How many possible outcomes when you flip
  - one coin? 2
  - the probability of getting the head is $1/2$.
  - two coins? $2 * 2 = 4$
  - the probability of getting two heads is $1/4$.
  - n coins? 2^n
- How many possible out comes when you roll
  - one dice? 6
  - n dice? 6^n
  - the probability of getting 6 is $1/6$.
- You have 3 choices of starters, 5 choices of entries, 3 choices of dessert. How many combinations?
  - $3 * 5 * 3 = 45$

Shaking hands and gift swap

- A family of 5 would like to send christmas gifts to everyone else. How many gifts they need to buy?
  - $5 * 4 = 20$.
  - A family of n would need $n * (n - 1)$.
- A group of 5 would like to shake hands with each other. How many handshakes in total?
  - $5 * 4/2 = 10$.
  - A family of n would need $n * (n - 1)/2$.

    - a second way: $4+3+2+1=10$
    - a third way: counting lines
- A group of 5 would like to pick 2 to form a Lego robot team. How many?
    - same as handshake problem.
    - how about a team of 3? how many?
- Pokemom deul?
    - Choose 3 out of 5 pokemoms for a fight with fight order $1, 2, 3$.
    - $5 * 4 * 3 = 60$

Venn Diagram

- In a class of 50, there are 25 students who has a cat, 20 who has a dog and 10 who does not own a pet at all. How many studnets who own both a cat and a dog?
    - $50 - 10 = 25 + 20 - x$
    - $x = 5$
- In a class of 50, there are 25 students who has a cat, 20 who has a dog and 5 who owns a dog and a cat. How many studnets who does not have neither a cat nor a dog?
    - total number of student who owns a dog or a cat $20 + 25 - 5 = 40$.
    - So 10.
- In a class, 20 own a cat, 25 own a dog, 15 own a fish, 7 own both a cat and a dog, 8 own both a dog and a fish, 5 owns both a cat and a fish, 3 owns all three. Everyone owns at least a dog, cat or fish. How many students in total?
    - own a cat and a dog only: $7 - 3 = 4$
    - own a cat and a fish only: $5 - 3 = 2$
    - own a fish and a dog only: $8 - 3 = 5$
    - own a cat only: $20 - 4 - 2 - 3 = 11$
    - own a dog only: $25 - 4 - 5 - 3 = 13$
    - own a fish only: $15 - 2 - 5 - 3 = 5$
    - total number $= 11 + 13 + 5 + 4 + 2 + 5 + 3 = 43$
    - total number alternative formula $= 20 + 25 + 15 - 7 - 8 - 5 + 3 = 43$
    - why? in terms of Venn diagram and overcounting.
- How many numbers less than 101 is divisible by 2, 3 and 5?
    - 50 divisible by 2, 33 divisible by 3, 20 divisible by 5
    - 16 divisible by $6 = 2 * 3$, 10 divisible by $10 = 2 * 5$, 6 divisible by 15
    - 3 divisible by 30
    - total $= 50 + 33 + 20 - 16 - 10 - 6 + 3 = 84$

Case works

- $4 * 4$ grid points contains how many squares as its vertices?
- AMC8 coat hanging problem