# Lecture 10: Probability

## Probability basic

The probability $P(A)$ of an event is the ratio of possible outcomes of $A$ and total outcomes. It is a number between $0$ and $1$.

**Example**:

- $P$("head" when flipping a coin)$= 1/2$ because head is one of the outcome of head and tail.
- $P$("HHH" when flipping 3 coins)$= 1/8$
- $P$(two heads when flipping 3 coins)$= 3/8$ because $HHT, HTH, THH$
- $k$-heads when flipping $n$ coins

$$P(A) = \frac{\binom{n}{k}}{2^n}$$

- $P$("6" when rolling a dice)$= 1/6$.
- $P$(sum up to $12$ when rolling $2$ dices)$= 1/36$.
- $P$(sum up to $9$ when rolling $2$ dices)$= 4/36$ because $9 = 3 + 6 = 4 + 5 = 5 + 4 = 6 + 3$.
- $P$(sum up to $7$ when rolling $2$ dices)$= 6/36$.
- Permutations of "ALGEBRA" with "AA"
  - all permutations $7!/2$ because of the symmetry AA.
  - with $AA$ together: $6!$
  - $P = \frac{6!}{7!/2} = 2/7$

**More fun example**:

- $P$(being an even number)$= 1/2$
- $P$(being a prime number)$= 0$
- $P$(raining chance)$= 40\%$
- Three boxes with one with one million dollars and two empty. You choose one, the winning chance is $1/3$. Someone knows the answer removing one empty box. Are you gonna switch?
  - what is the winning chance if you do not switch?
  - what is the winning chance if you switch?

## Picking with(without) replacement

- $3$ green marbles and $4$ red marbles in a bag.

- if picking with replacement, the probability of getting two green is $(3/7)^2 = 9/49$.
- if picking without replacement, the probability of getting two green is $3/7 * 2/6 = 6/42$.
- if picking without replacement, the probability of getting two red is $4/7 * 3/6 = 2/7$.

**More difficult examples**:

- Rolling $3$ dice exactly two $6$s.
  - total outcome $6^3$
  - possibility: $6a6$, $66a$, $a66$. So $3 * 5$.
  - $P = 15/216 = 5/72$.
- Selecting a committee of $5$ out of $6$ boys and $6$ girls. What is chance there are exactly $3$ girls in the committee?
  - total outcome? $\binom{12}{5}$.
  - the choice is $\binom{6}{3} * \binom{6}{2}$
- Rolling $2$ dice with the second dice number larger than the first dice number.
  - total outcome $36$
  - either $a > b$, $a = b$ or $a < b$, with the count $15$, $6$ and $15$.
- Rolling $3$ dice with the last dice number larger than the sum of first two numbers.
  - total outcome $216$
  - Numerate all the cases.
- If raining chance is always $1/3$, what is the chance raining one day in next $4$ days?
  - no rain chance: $2/3$
  - no rain at all $16/81$
  - complement: $65/81$