Math107

Inference for a Single Mean
(Sections 6.4–6.6)

Central Limit Theorem

The sampling distribution of a sample mean is approximately normal and centered at μ, if n is sufficiently large.

$$\bar{\mathbf{x}} \sim \mathcal{N}\left(\mu,\ \frac{\sigma}{\mathbf{n}}\right)$$

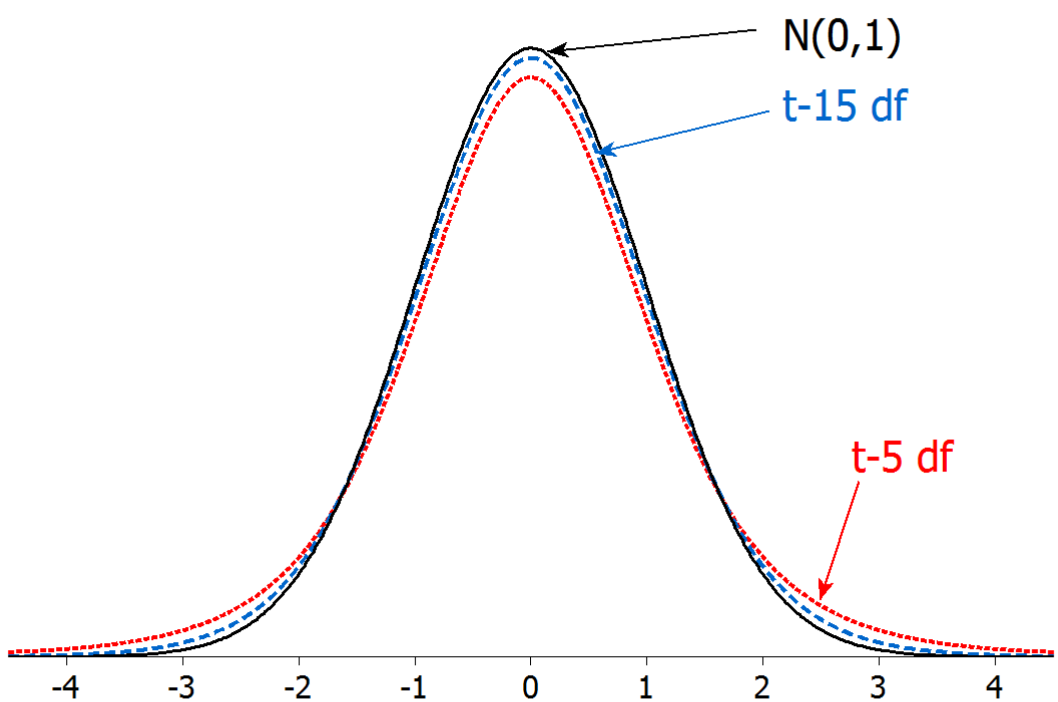
The normal approximation is usually good if n≥30.

t-distribution

t-distribution

- We don’t know the population standard deviation, σ, so we would like to plug-in the sample mean, s
- This changes the distribution of the z-statistic from normal to t
- The t-distribution is very similar to the normal distribution, but has fatter tails to account for the added uncertainty

t-distribution



- The *t*-distribution is characterized by its ***degrees of freedom (df)***
- **df = n - 1**
- The higher the degrees of freedom, the closer the *t*-distribution is to the standard normal

t-distribution

If a population with mean μ is approximately normal or if n is large (*n*≥30), the standardized statistic for a mean using the sample std. dev., *s,* follows a *t*-distribution with *n* – 1 degrees of freedom:

$$\frac{\mathbf{\overline{x}} - \mu}{\mathbf{s}/\sqrt{\mathbf{n}}} \sim \mathbf{t_{n-1}}$$

Small Samples

- If sample sizes are small, only use the *t*-distribution if the data look reasonably symmetric and do not have any extreme outliers.
- Even then, remember that it is just an approximation!
- In practice/life, if sample sizes are small, you should just use simulation methods (bootstrapping and randomization)

Aside: William Sealy Gosset

Confidence Intervals

Confidence Interval Formula

General formula:

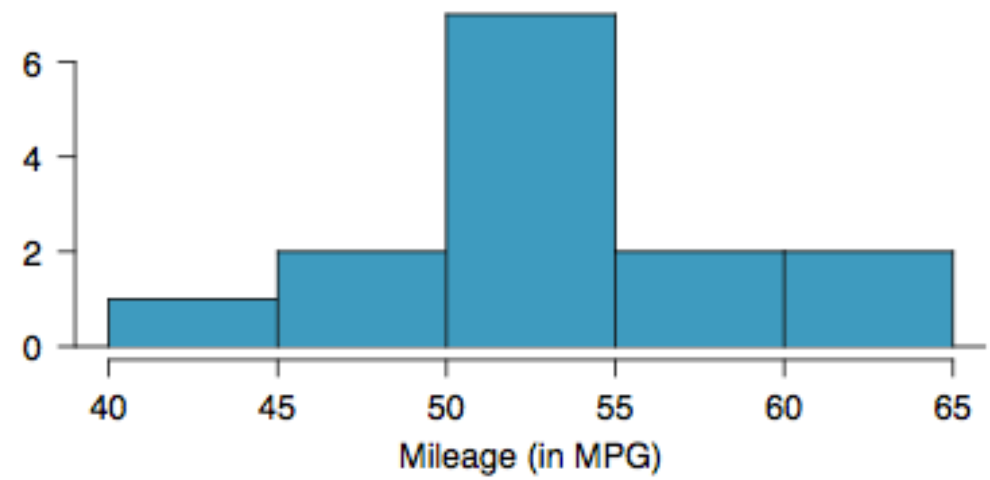
$$\text{sample statistic} \pm \mathbf{t}^* \times \text{SE}$$

Formula for a mean:

$$\overline{\mathbf{x}} \pm \mathbf{t}^* \cdot \frac{\mathbf{s}}{\sqrt{\mathbf{n}}}$$

Prius Fuel Economy

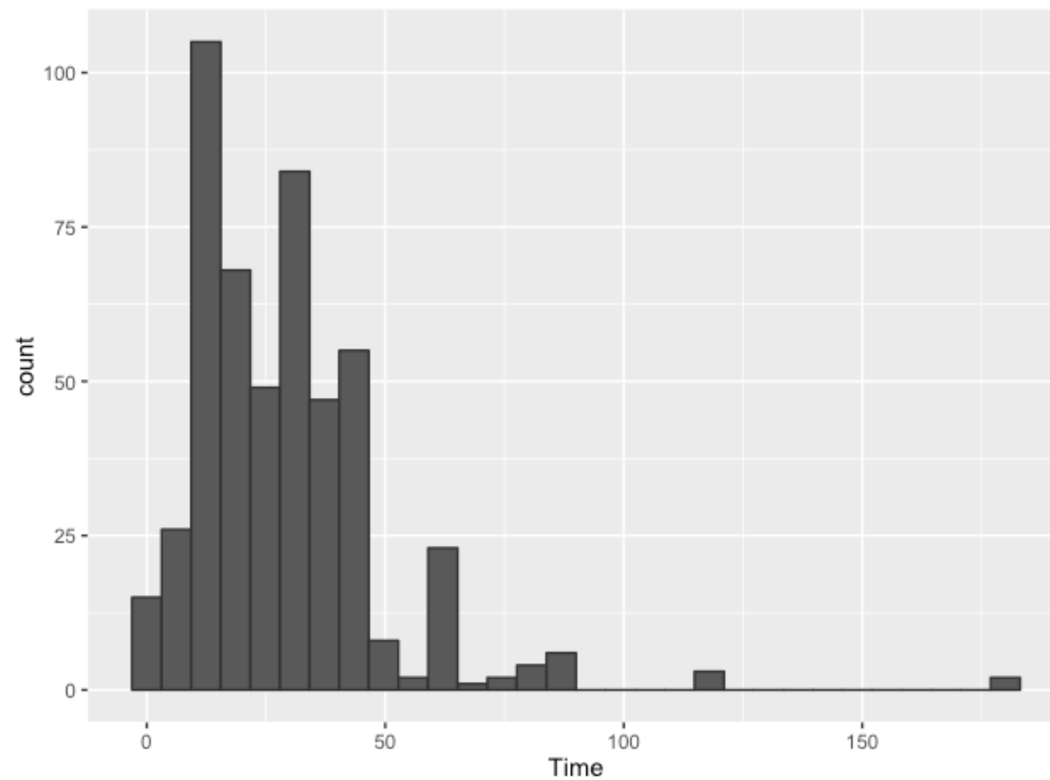
- Source: Fueleconomy.gov,
- Gas mileage (in MPG) from 14 users who drive a 2012 Toyota Prius
- Sample mean is 53.3 MPG
- Sample standard deviation is 5.2 MPG.



Find and interpret a 90% confidence interval for average gas mileage of a 2012 Prius by drivers who participate on fueleconomy.gov.

Commuting in Atlanta

- Source: American housing survey by US Census Bureau
- Commute times (min) for random sample of 500 commuters
- Sample mean is 29.11 min
- Sample standard deviation is 20.7 min

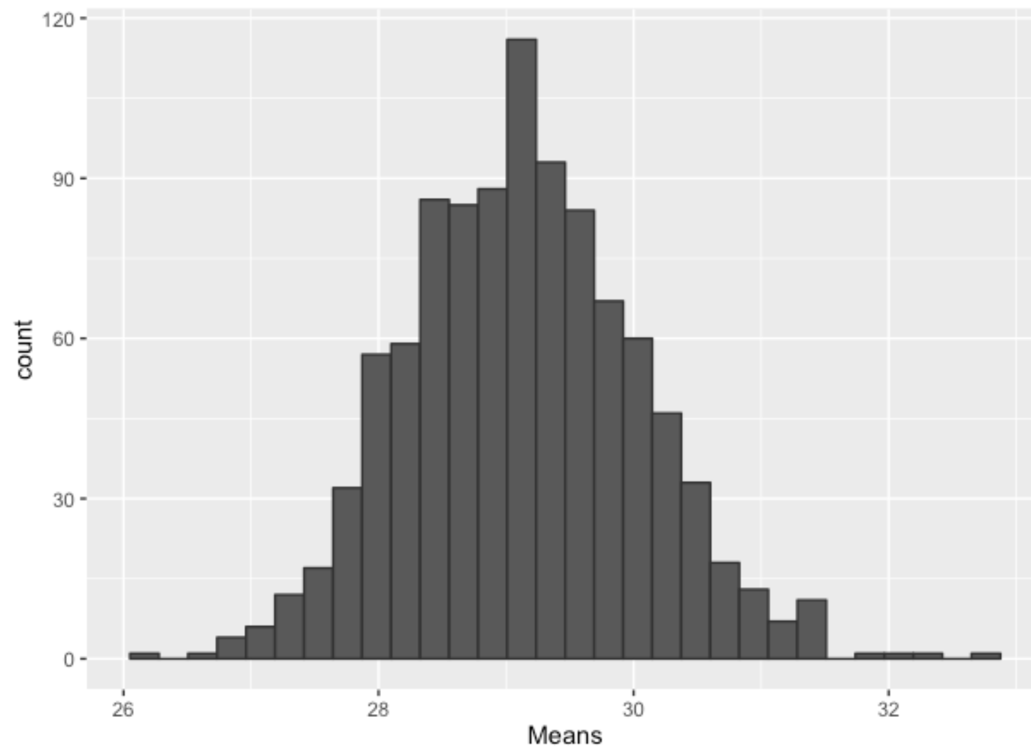


Find and interpret a 95% confidence interval for average commute time for someone living in Atlanta, GA

Commuting in Atlanta

Bootstrap results:

- mean = 29.1 min
- sd = 0.91



How does the 95% bootstrap confidence interval compare with the theory-based interval?

Using R

```
# Load the mosaic package
library(mosaic)

# Load the data
commute <- read.csv("data/CommuteAtlanta.csv")

# Constructing a 95% confidence interval
confint(t.test(~Time,
               data = commute,
               conf.level = 0.95))
```

Margin of Error

$$\mathbf{ME} = \mathbf{t}^* \cdot \frac{\mathbf{s}}{\sqrt{\mathbf{n}}}$$

You can choose your sample size in advance based on your desired margin of error!

$$\mathbf{n} = \left(\frac{\mathbf{t}^* \, \mathbf{s}}{\mathbf{ME}}\right)^{\mathbf{2}}$$

Margin of Error

$$\mathbf{n} = \left(\frac{\mathbf{t^* s}}{\mathbf{ME}}\right)^{\mathbf{2}}$$

- Problem 1: For t^*, need to know n.
  - Solution: Use z^* instead of t^* (they are usually close)
- Problem 2: For s, need data.
  - Solution: estimate s.
    1. Use data from a previous study or similar population
    2. Take a small pre-sample to estimate s
    3. Estimate the range (max – min) and use $s \approx$ range/4
    4. Make a reasonable guess.

Example

- Suppose we want to estimate the average GPA at Lawrence with a margin of error of 0.1 with 95% confidence.
- Determine how large a sample we would need to meet our requirements.

Hypothesis Tests

Hypothesis Test for μ

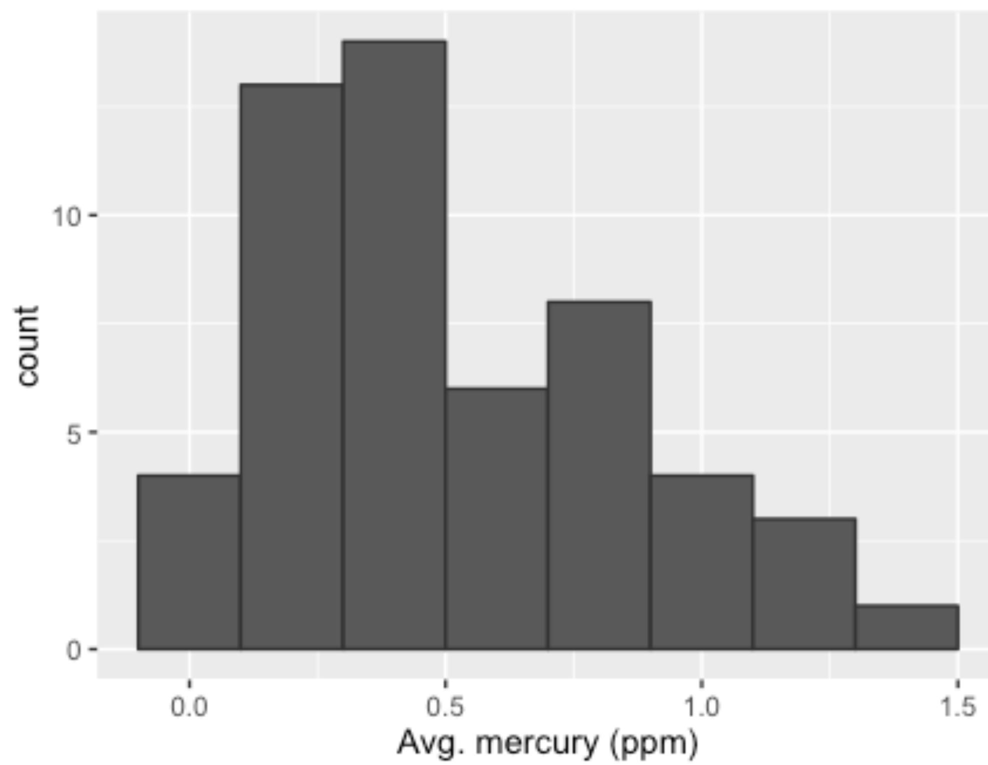
1. State the hypotheses
2. Check the conditions necessary to use the normal distribution
3. Compute the test statistic
4. Compute the p-value
5. Make a decision and state its implications in the context of the problem

Mercury Content in Fish

- Data: Average mercury levels for a sample of large mouth bass from 53 randomly selected lakes in FL
- Sample mean: 0.527
- Sample standard deviation: 0.341

Mercury Content in Fish

- Data: Average mercury levels (ppm) for a sample of large mouth bass from 53 randomly selected lakes in FL
- Sample mean: 0.527 ppm
- Sample standard deviation: 0.341 ppm



Mercury Content in Fish

a) The Food and Drug Administration (FDA) has a limit on the mercury concentration in fish of 1.0 ppm. Is there evidence that the average mercury concentrations in FL lakes is greater than this cutoff?

b) In Canada, the limit is 0.5 ppm. Is there evidence that the average mercury concentrations in FL lakes is greater than this cutoff?

Using R

```
# Load the data
lakes <- read.csv("data/FloridaLakes.csv")

# Running test for H_0: mu = 1 vs. H_a: mu < 1
result1 <- t.test(~ AvgMercury, data = lakes,
                  mu = 1, alternative = "less")

# Running test for H_0: mu = 0.5 vs. H_a: mu > 0.5
result2 <- t.test(~ AvgMercury, data = lakes,
                  mu = 1, alternative = "greater")
```