Math 107

Hypothesis Tests: p-values
(Sections 4.2 & 4.4)

Paul the Octopus

http://www.youtube.com/watch?v=3ESGpRUMj9E

Hypotheses

- In 2008, Paul the Octopus predicted 8 World Cup games, and predicted them all correctly.
- Is this evidence that Paul's chance of guessing correctly, *p*, is really greater than 50%?
- What are the appropriate hypotheses?

1. $\mathbf{H_0: \ p \neq 0.5}$ vs. $\mathbf{H_a: \ p = 0.5}$
2. $\mathbf{H_0: \ p = 0.5}$ vs. $\mathbf{H_a: \ p \neq 0.5}$
3. $\mathbf{H_0: \ p = 0.5}$ vs. $\mathbf{H_a: \ p > 0.5}$
4. $\mathbf{H_0: \ p > 0.5}$ vs. $\mathbf{H_a: \ p = 0.5}$

How do we measure how unusual a sample statistic is, if the null is true?

Measure Evidence Against the Null

To see if a statistic provides evidence against the null hypothesis, we need to understand what values of the sample statistic we would observe just by random chance, **assuming that the null hypothesis is true**.

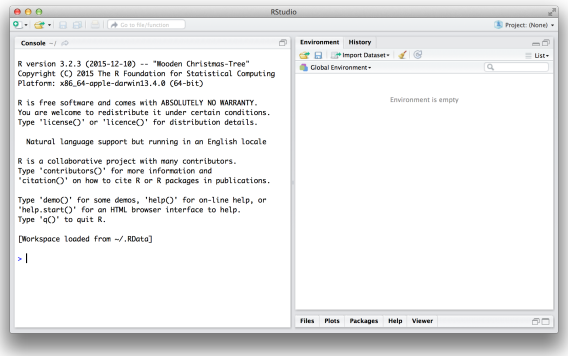
Paul the Octopus

- We need to know what values of the sample statistics we would observe just by random chance, if the null hypothesis were true.
- How could we figure this out?

Simulation

- We can simulate this choice with a fair coin.
- Heads = correct guess
- Tails = incorrect guess
- Chance of heads = 1/2 = probability Paul will guess correctly
- One repetition = one set of 8 coins flips

R Demo



Randomization Distribution

A **randomization distribution** is a collection of statistics from samples simulated by assuming that the null hypothesis is true.

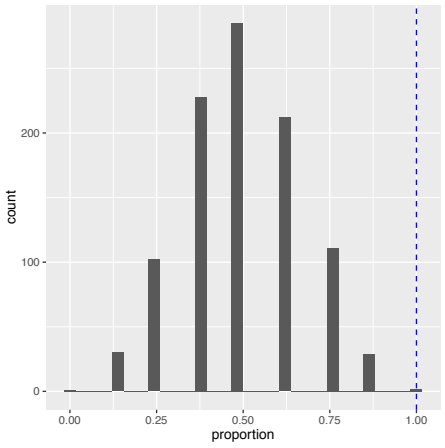
i.e. a distribution of what we would expect to observe by random chance, if the null hypothesis were true.

Quantifying Evidence

The **p-value** is the chance of obtaining a sample statistic as extreme as (or more extreme than) the observed sample statistic, if the null hypothesis is true

Calculation: the proportion of statistics in a randomization distribution that are as extreme as (or more extreme than) the observed sample statistic

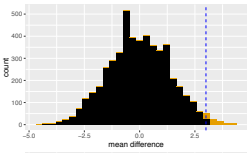
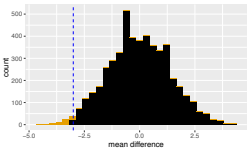
1000 Simulations



Alternative Hypothesis

- A **one-sided** alternative contains either > or <
- A **two-sided** alternative contains ≠
- The p-value is the proportion in the tail in the direction specified by H_a
- For a two-sided alternative, the p-value is twice the proportion in the smallest tail

p-value and H_a

Upper-tail (Right Tail)	H_0: $\mu_1 = \mu_2$ H_a: $\mu_1 > \mu_2$ Observe: 3	
Lower-tail (Left Tail)	H_0: $\mu_1 = \mu_2$ H_a: $\mu_1 < \mu_2$ Observe: -3	
Two-tailed	H_0: $\mu_1 = \mu_2$ H_a: $\mu_1 \neq \mu_2$ Observe: 3