Math 142 Reading Week 5

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1 3.4.1 Fourier Transform Pairs

Let's take a look at some more common filter and signals:

- **Impulse:** The impulse response has a constant (all frequency) transform
- **Shifted Impulse:** The shifted impulse has unit magnitude and linear phase
- **Box filter** the box (moving average) filter

$$\text{box}(x) = \begin{cases} 1 & \text{if } |x| \le 1 \\ 0 & \text{else} \end{cases} \tag{1}$$

  has a sinc fourier transform

$$\text{sinc}(\omega) = \frac{\sin(\omega)}{\omega} \tag{2}$$

- **Tent:** The piecewise linear tent function

$$\text{tent}(x) = \max(0, 1 - |x|), \tag{3}$$

  has a sinc^2 Foruier transform.

2 3.4.2 Two-Dimensional Fourier Transforms

We can translate formulas and insights from one-dimensional images to two-dimensional images directly. Instead of just specifying a horizontal or vertical frequency ω_x or ω_y, we create an oriented sinusoid of frequency (ω_x, ω_y):

$$s(x, y) = sin(\omega_x x + \omega_y y) \tag{4}$$

The corresponding two-dimension Fourier transforms are then

$$H(\omega_x, \omega_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) \exp(-j(\omega_x x - \omega_y y)) dx\, dy, \tag{5}$$

and in the discrete domain,

$$H(k_x, k_y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} h(x, y) \exp(-j2\pi \frac{k_x x + k_y y}{MN}), \tag{6}$$

where M and N are the width and height of the image.

All of the Fourier transform properties mentioned in previous parts of the computer vision book hold for two-dimensional Fourier transforms as well, so long as we replace scalar variables with their two-dimensional vector counterparts.