Math 142 Reading Week 7

Abhi Uppal

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1 4.1 Points and patches

Point features can be used to find a sparse set of corresponding locations in different images – which is the first step in a chain of prerequisites leading to stereo matching. These help us align images and perform object instance and category recognition.

There exist two main methods to finding feature points and their correspondences:

1. Find features that can be accurately tracked using a local search technique (correlation or least squares)
2. Independently detect features in all images and match features based on their appearance

Method 1 is better when you have a series of images taken in rapid succession, and method 2 is better for more spaced out photos.

This pipeline is split into four different stages:

1. **Feature Detection / Extraction:** each image is searched for locations that are likely to match in other images
2. **Feature Description:** Each region around keypoints are converted into a *descriptor* that can be matched against other descriptors
3. **Feature Matching:** Search for likely matching candidates in other images
4. **Feature Tracking:** Alternative that searches a small neighborhood around each detected feature (better for video processing)

1.1 4.1.1 Feature Detectors

The simplest possible matching criterion for two patches is the weighted summed squared difference:

$$E_{WSSD}(u) = \sum_i w(x_i)[I_1(x_i + u_i) - I_0(x_i)]^2, \tag{1}$$

where I_0 and I_1 are the two images being compared, u is the displacement vector, and $w(x)$ is a spatially-varying weighting function. If we compute this over a small variation Δu, we can compare an image patch against itself. This is known as an **autocorrelation function** or **surface**. Using a series expansion, we can approximate this as

$$E_{AC}(\Delta u) \approx \Delta u^\top A \Delta u, \tag{2}$$

where A is the autocorrelation matrix given by

$$A = w \times \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}. \tag{3}$$

Note that I_x denotes the partial derivative of the image with respect to x, and similarly for I_y. Some interesting relationships exist between this matrix A and its minimum (λ_0) and maximum (λ_1) eigenvalues. For some tolerance level α, we have:

$$\det(A) - \alpha\,\mathrm{trace}(A) = \lambda_0\lambda_1 - \alpha(\lambda_0 + \lambda_1)^2, \tag{4}$$

and

$$\frac{\det\ A}{\mathrm{tr}\ A} = \frac{\lambda_0\lambda_1}{\lambda_0 + \lambda_1} \tag{5}$$