Math 142 Reading Week 11

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1 4.3 Lines

Edges and general curves are great at describing natural objects, but man-made objects tend to be full of straight lines. As such, we want to develop a framework for working with these.

2 4.3.1 Successive Approximation

Describing a curve as a series of 2D locations $x_i = x(s_i)$ provides a general representation suitable for matching and further processing. However, it's sometimes preferable to approximate such a curve with a simpler approximation like a piecewise-linear polyline.

The simplest method for doing so is to recursively subdivide the curve at the point furthest away from the line joining the two endpoints.

Once we have a line simplification, we can approximate the original curve using it.

3 4.3.2 Hough Transforms

Curve approximation of polynomials can often lead to successful line extraction, but lines in the real world are sometimes broken into disconnected components or are made up of many collinear line segments. We would like to group these collinear line segments together into extended lines.

The *Hough Transform* is a technique for having edges vote for plausible line locations. Each point votes for all possible lines passing through it, and lines corresponding to high accumulator or bin values are examined for potential line fits. However, a better approach is to use the local orientation at each edge to vote for a single accumulator cell.

Before voting for line hypotheses, we have to first choose a good representation. Let a line be parameterized by normal distance: $(\hat{n}, d)$. We adopt the convention that $\hat{n}$ points in the same direction as the image gradient $J(x)$. To get a two-parameter representation of the line, convert the normal vector into an angle:

$$\theta = \tan^{-}1\frac{n_y}{n_x}.$$

Given this parameterization, the Hough transform then fits lines to consituent edges.

4 4.3.3 Vanishing Points

Structurally important lines in many scenes have the same vanishing point since they are parallel in 3D (think: railway tracks). We can develop a simple Hough technique for detecting said vanishing points. The first step is to use a Hough transform to accumulate votes for likely vanishing point candidates.

Once the accumulator space is populated, peaks can be detected similarly as they were for line detection. Given a set of candidate line segments that voted for a vanishing point, you can fit a robust least squares model to estimate an accurate location for each vanishing point.