Lecture 2

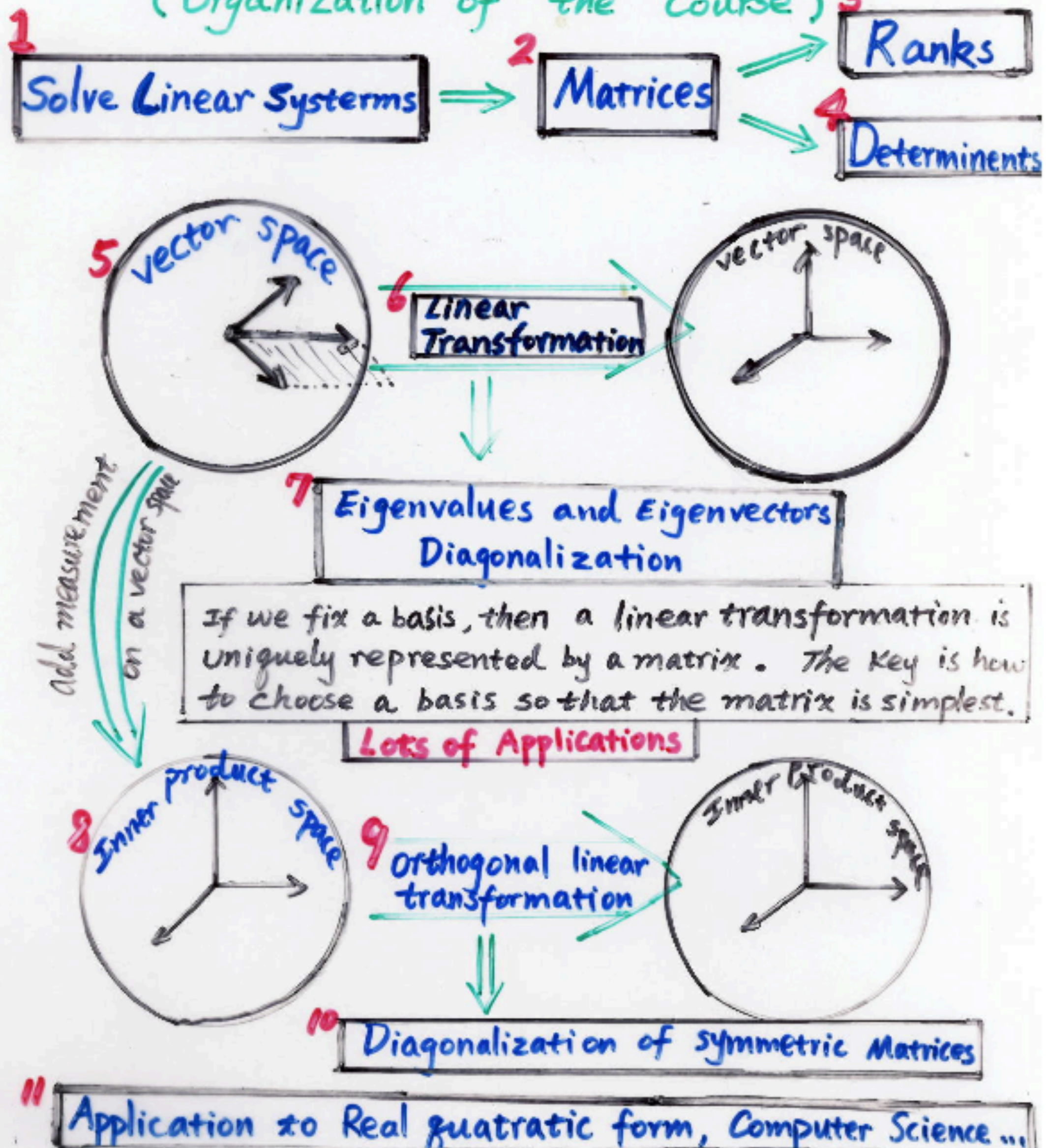
Review Linear Algebra
&
Ways of Geometric Thinking

Math 142

Professor Gu

A Big Picture of Linear Algebra

(Organization of the Course)



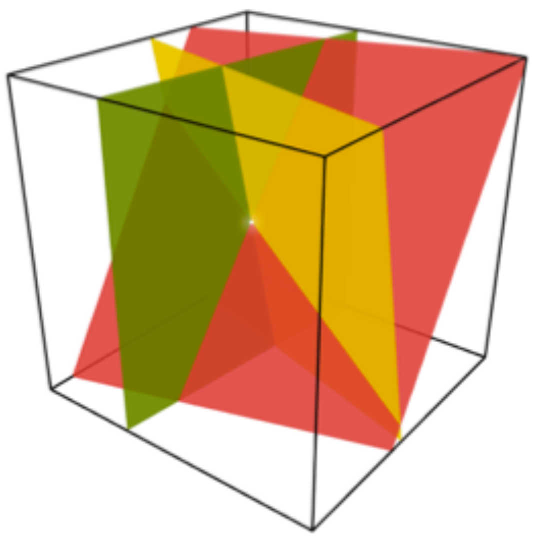
Please think each topic you learned in linear algebra ***geometrically***.

For examples:

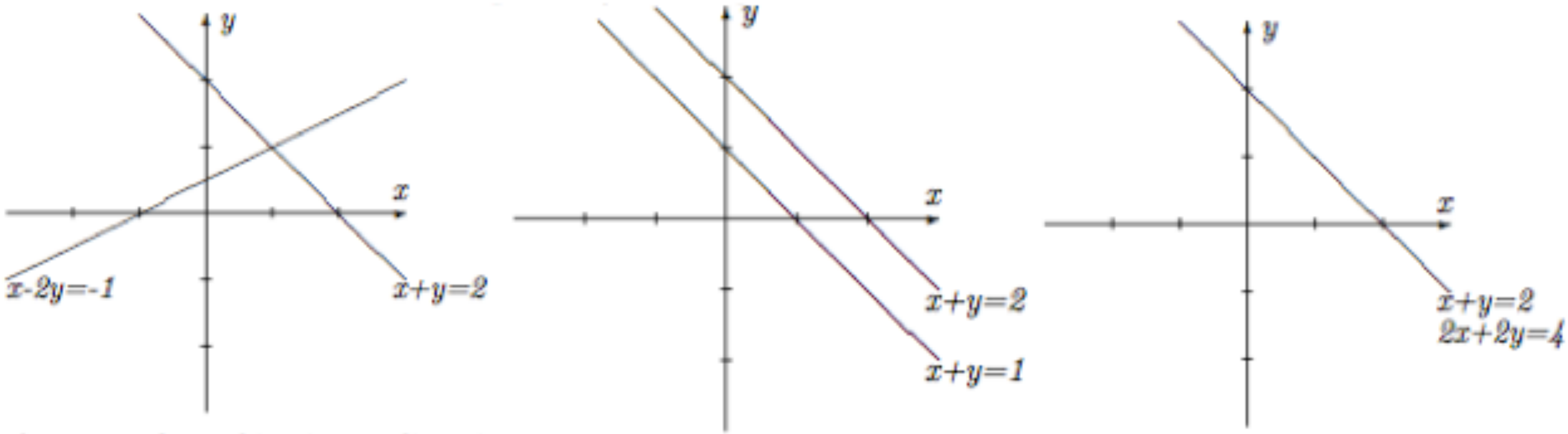
- Solving linear system
- Determinant
- Vector Space
- Linear transformation
- Eigenvalues and Eigenvectors
- Inner product

Geometric meaning of solving linear system

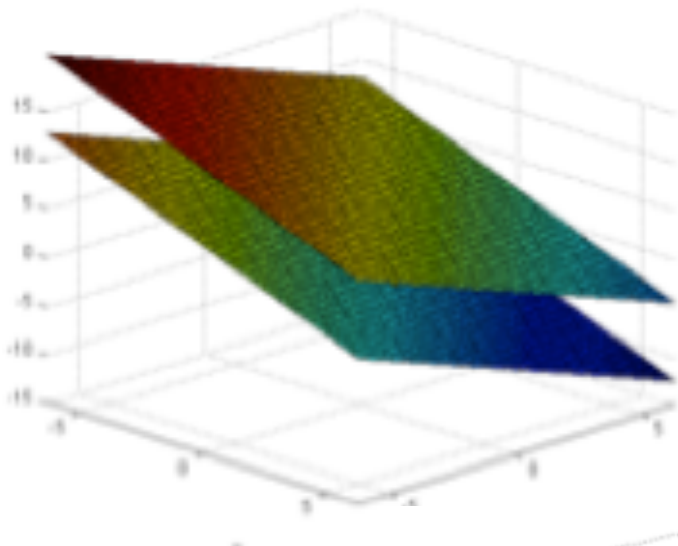
$$\begin{aligned} 3x + 2y - z &= 1 \\ 2x - 2y + 4z &= -2 \\ -x + \tfrac{1}{2}y - z &= 0 \end{aligned} \quad\Rightarrow\quad \begin{aligned} x &= 1 \\ y &= -2 \\ z &= -2 \end{aligned}$$

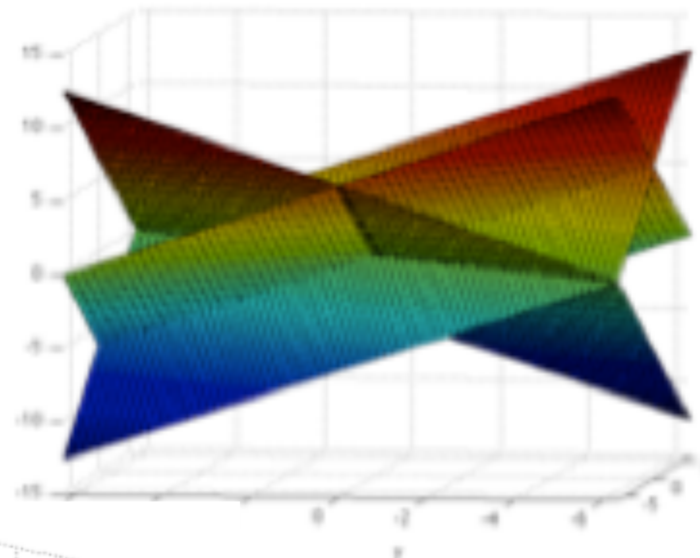


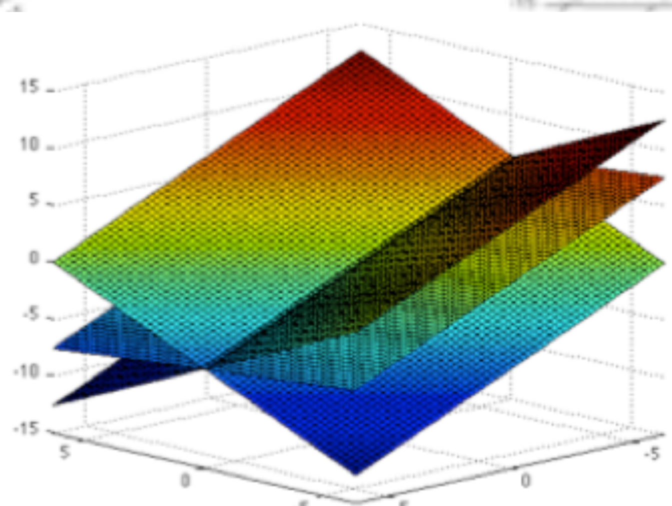
What kind of solutions that a linear system of two variables could have?



What kind of solutions that a linear system of three variables could have?

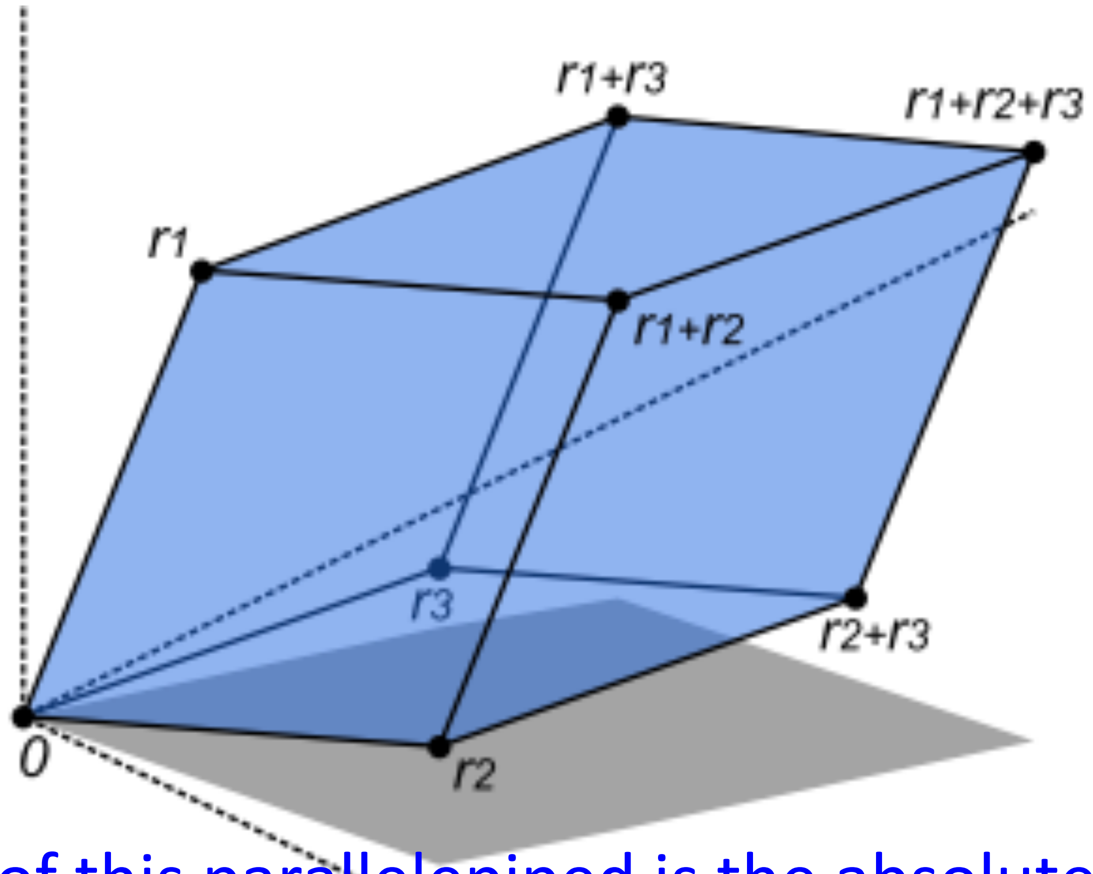






Key: Understand the solution space geometrically.

Understand the geometry of determinant of a matrix



The volume of this parallelepiped is the absolute value of the determinant of the matrix formed by the rows constructed from vectors r1, r2, and r3.

True or False

- A linear system could have 3 solutions.
- n+1 vectors in R^n must be linear dependent.
- Determinant of n linearly dependent vectors must be equal to zero.

What will happen if the following matrix applying to a vector in R^2?

$$\begin{matrix} 1 & 1 \\ -1 & 1 \end{matrix}$$

Does this matrix has real eigenvalues? Is it diagonalizable?

- Answer:

- How about the following matrix:

1 a

0 1

Matrix representation

- Given a matrix A.
- Go to the board and list all possible ways that this matrix A can represent.
- *E.g. The matrix A could represent a quadratic form as we saw from last class.*

Can the following matrix be used to define an inner product?

3 1

1 3

Or

1   2

2   1