Due: Wed. Sept. 27

HMC Math 142 Fall 2017

Prof. Gu

Problem Set 4

Start this assignment before Sunday night!

Read:

- Baby Do Carmo, Differential Geometry of Curves and Surfaces: Sections 2-1, 2-2, Chapter 2
- Handout 5
- Lecture Notes

Do:

A: Problems on Reviewing of Rigid Motions in R^3.

- a) Show that the set of rigid motions $E(3)$ forms a group. (Later, we will see that $E(3)$ is in fact a Lie group.)

B: Other Problems

- a) Problem 2 on page 29, Section 1-6, Baby Do Carmo.
- b) Problem 3 on page 65, Section 2-2, Baby Do Carmo.
- c) Problem 5 on page 65, Section 2-2, Baby Do Carmo.
- d) Problem 10 on page 66, Section 2-2, Baby Do Carmo.
- e) Problem 16 on page 67, Section 2-2, Baby Do Carmo.

C: Exterior Wedge Product from Class

- a) Look at the definition in the link `https://en.wikipedia.org/wiki/Exterior_algebra` and prove that
  - $\mathbf{u} \wedge \mathbf{v} \wedge \mathbf{w} = (u_1v_2w_3 + u_2v_3w_1 + u_3v_1w_2 - u_1v_3w_2 - u_2v_1w_3 - u_3v_2w_1)(\mathbf{e}_1 \wedge \mathbf{e}_2 \wedge \mathbf{e}_3)$
  - $\dim \Lambda^k(V) = \binom{n}{k}$.
  - In characteristic 0, the 2-vector α has rank p if and only if $\underbrace{\alpha \wedge \cdots \wedge \alpha}_{p} \neq 0$ and $\underbrace{\alpha \wedge \cdots \wedge \alpha}_{p+1} = 0$