Lecture 2 (Part2): Clever representations of elements of a manifold

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Example 1: Matrix representation of linear system

- Work out details with the students on the board.

Example 2: Matrix representation of a linear transformation

- Work out details with the students on the board.

Example 3: Matrix representation of a tensor

- Work out details with the students on the board.

Example 4: Matrix representation of inner product

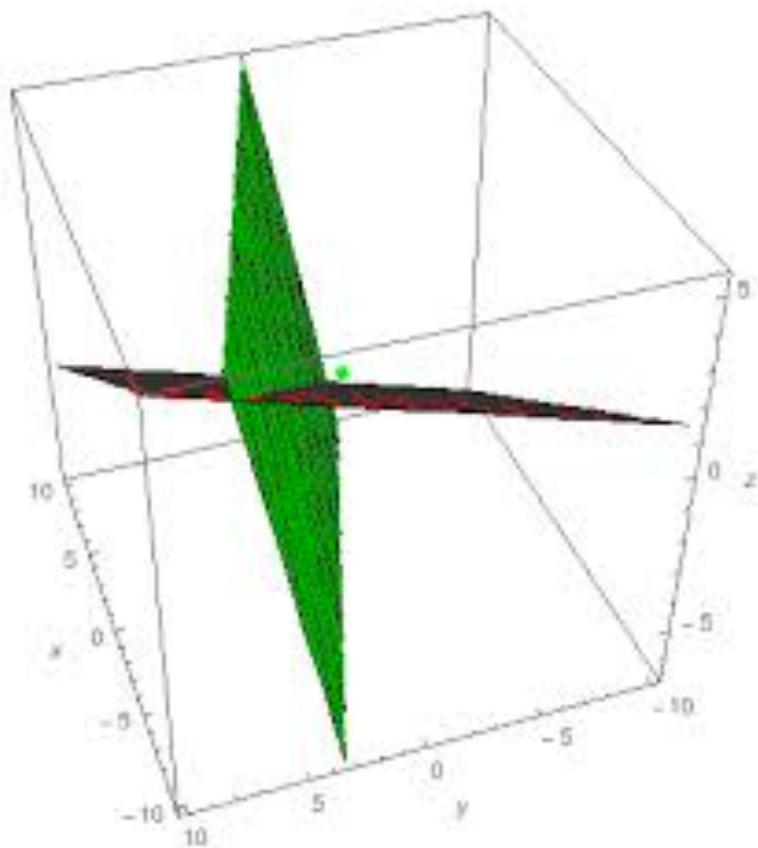
- Work out details with the students on the board.

Matrix representation of a plane
But subtle!

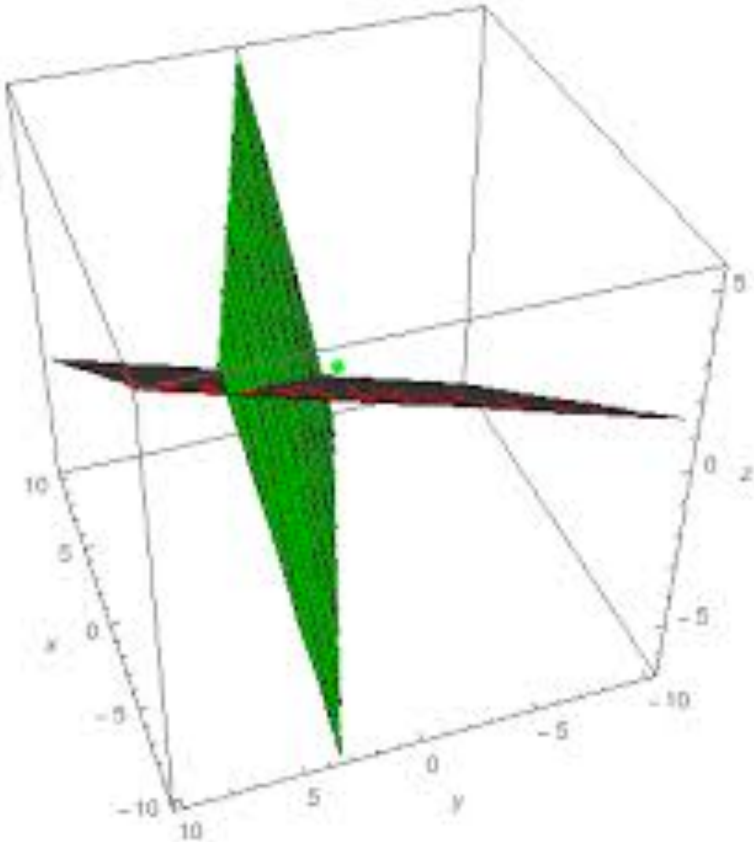
- Work out details with the students on the board.

Recall

- Note tracking a plane changing with time is just tracking a curve on the unit sphere S^2.



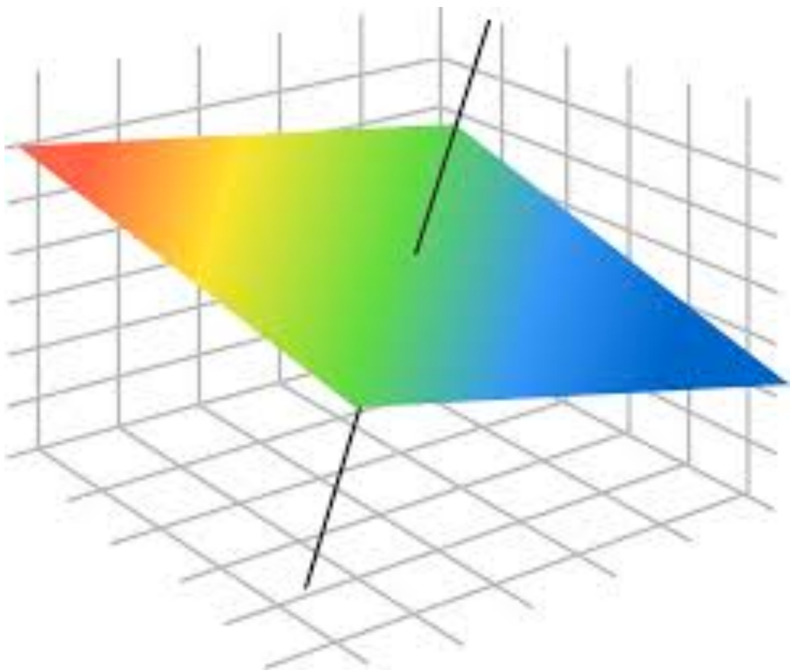
Question: What is the fastest way to change the green plane to the red plane?

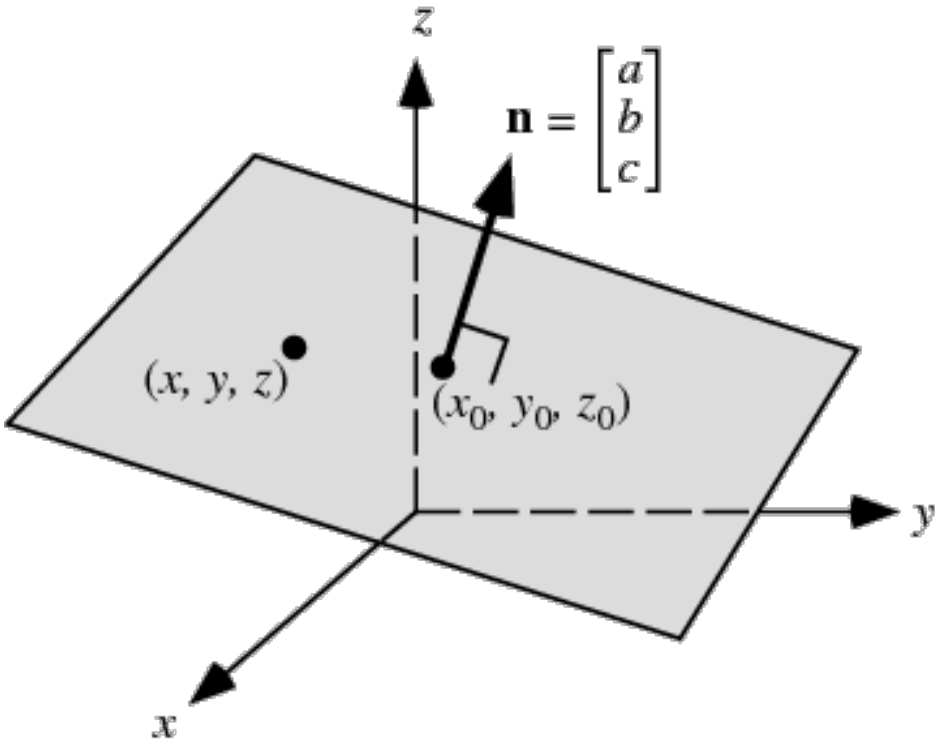


In general, what is a geodesic on the Grassmannian G_2R^3?

Answer: Let the unit normal of the green plane changing along a great circle of S^2 to the unit normal of the red plane!

A plane and a line is in 1-1 correspondence.





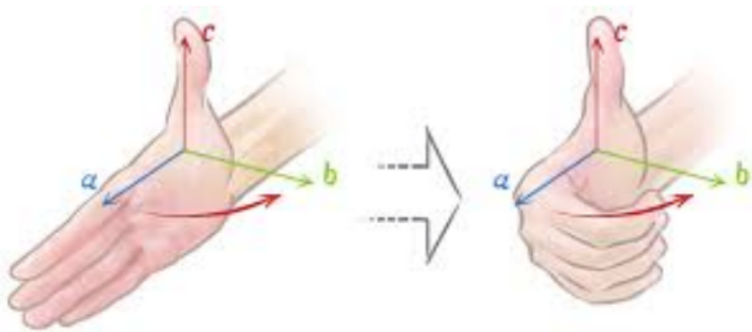
We can view n corresponding the face-up plane and (-n) corresponds to the face-down plane.
We call them oriented planes

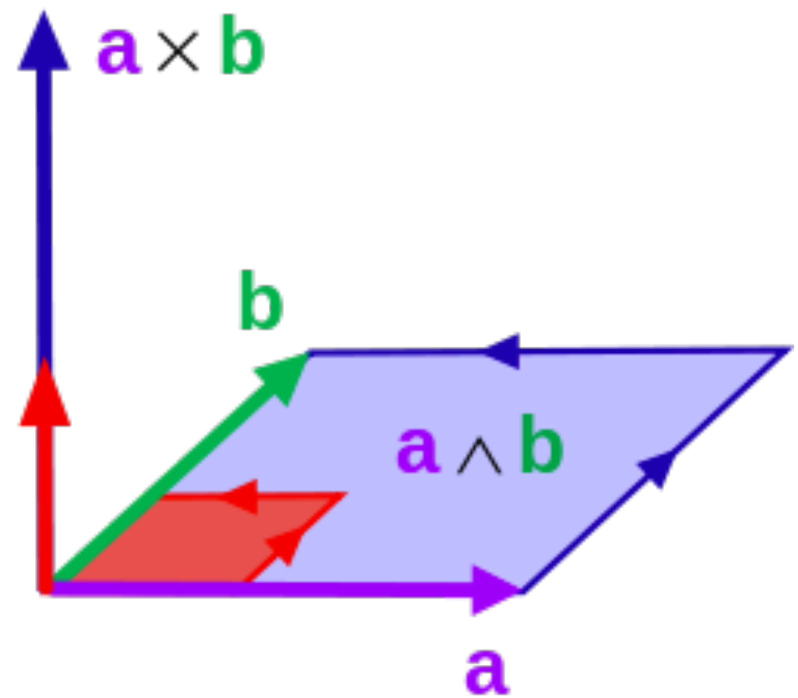
Note: Here we have used 1-1 corresponding between a 2-plane P in $\mathbf{R}^3$ and the normal vector n.

- This can be done so since R3 = P "+" n.
- What if we consider 2 planes in $\mathbf{R}^5$?
- We need to find different ways to represent 2-planes.

Another clever way to represent a 2-plane!

Oriented planes





Matrix representation:

- [a, b] represents the face-up plane.
- [b, a] represents the face-down plane.
- But if the frame {a, b} is rotated by an angle, then they still represent the same oriented plane!
- The notation $a \wedge b$ captures this change!

We mimic ideas in linear algebra

- Just like A is equivalent to EA, where E is an elementary matrix.

- Here a x b is equivalent to a’ x b’

- **We will try our best to represent elements on a manifold using matrices.** (For example, matrix representation of a linear map).

- For example, in theoretic physics, all the key quantities can be viewed as tensors.
- Each tensor is determined by its tensor components which are organized into a matrix (called a tensor matrix).
- For examples. Bilinear form (i.e. 2-tensor) and Inner project.
- Tensor flow is roughly talking about how one tensor matrix changes to next tensor matrix.

Mimic Linear Algebra --Matrix Representation of a rigid motion

- Recall linear system is represented by a matrix!