

Feel free to work with other students, but make sure you write up the homework and code on your own (no copying homework *or* code; no pair programming). Feel free to ask students or instructors for help debugging code or whatever else, though. The starter code for problem 2 part (d) can be found under the Resource tab on course website.

1 (Linear Transformation) Let $\mathbf{y} = A\mathbf{x} + \mathbf{b}$ be a random vector. show that expectation is linear:

$$\mathbb{E}[\mathbf{y}] = \mathbb{E}[A\mathbf{x} + \mathbf{b}] = A\mathbb{E}[\mathbf{x}] + \mathbf{b}.$$

Also show that

$$\text{cov}[\mathbf{y}] = \text{cov}[A\mathbf{x} + \mathbf{b}] = A\text{cov}[\mathbf{x}]A^\top = A\mathbf{\Sigma}A^\top.$$

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2 Given the dataset $\mathcal{D} = \{(x, y)\} = \{(0, 1), (2, 3), (3, 6), (4, 8)\}$

- (a) Find the least squares estimate $y = \boldsymbol{\theta}^\top \mathbf{x}$ by hand using Cramer's Rule.
- (b) Use the normal equations to find the same solution and verify it is the same as part (a).
- (c) Plot the data and the optimal linear fit you found.
- (d) Find randomly generate 100 points near the line with white Gaussian noise and then compute the least squares estimate (using a computer). Verify that this new line is close to the original and plot the new dataset, the old line, and the new line.

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