Last name ______________________

First name ______________________

LARSON—MATH 610—HOMEWORK WORKSHEET h16
Test 2 Review.

Write up a careful, complete test review and turn it in before our Test 2 on Thurs., May 11, 12:30-3:20. **Explain** everything.

1. What is a *linear transformation* T from a vector space V into a vector space W? What is the *null space* of T? What is the *range* of T?

2. Consider the function $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1, x_2) = (2x_1, 0, 0)$?

   (a) Show T is a linear transformation.

   (b) Find the null space of T.

   (c) What is the *nullity* of T?

   (d) Find the range of T.

   (e) What is the *rank* of T?

3. Let T be a linear transformation from a vector space V to a vector space W. **Show** that $T(0) = 0$.

4. **Show:** If $\alpha_1, \ldots, \alpha_n$ are a basis for a finite-dimensional vector space V and $\beta_1, \ldots, \beta_n$ are any vectors in a vector space W then there is a *unique* linear transformation T with $T(\alpha_1) = \beta_1, \ldots T(\alpha_n) = \beta_n$.

5. **Show:** Every n-dimensional vector space over a field $\mathbb{F}$ is isomorphic to $\mathbb{F}^n$.

6. Explain how to make the *collection* $L(V, W)$ of linear transformations from a vector space V to a vector space W into a *vector space*. (What needs to be shown?)

7. State and **prove** the Rank-Nullity Theorem.

8. Let T be an isomorphism from a vector space V to a vector space W. Let $\{\alpha_i : i \in \mathcal{I}\}$ be a basis for V. **Show** that $\{T(\alpha_i) : i \in \mathcal{I}\}$ is a **basis** for W.

9. What is an *invertible* linear transformation $T : V \to W$? Give an example.

10. What is a *singular* linear transformation $T : V \to W$? Give an example.

11. **Prove:** If V and W are finite-dimensional vector spaces over a field $\mathbb{F}$ with $\dim V = \dim W$, and $T : V \to W$ is a linear transformation then the following are equivalent:

    (a) T is invertible,

    (b) T is non-singular,

    (c) T is onto (that is, the range of T is W).

12. What is a *linear functional* on a vector space V over a field $\mathbb{F}$? Give an example.

13. What is $\mathbb{F}^\infty$?

14. Let $f, g \in \mathbb{F}^\infty$. How is fg defined? Give an example.

15. Let $\mathbb{F}$ be a field.

    (a) Define $\mathbb{F}[x]$.

    (b) Explain why $\mathbb{F}[x]$ is a *vector space.*

    (c) Explain why $\mathbb{F}[x]$ is a *linear algebra.*

16. What is a *monic* polynomial? Give an example.

17. What is an *ideal* in $\mathbb{F}[x]$?

18. What is a *principal* ideal in $\mathbb{F}[x]$? Give an example.

19. What is an n*-linear* function?

20. What is the "big" determinant formula?

21. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ Use this definition to find $\det A$.

22. **Show:** If A has a zero row then $\det A = 0$.

23. Suppose $A \in \mathbb{F}^{n\times n}$. **Show:** There is a $n \times 1$ matrix α (not all-zero), and $c \in \mathbb{F}$ with $A\alpha = c\alpha$ iff $\det(A - cI) = 0$.

24. What is a *characteristic value* and a *characteristic vector* of a linear operator T from a vector space V over a field $\mathbb{F}$ to itself?

25. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x_1, x_2) = (x_1, 2x_2)$. Find the characteristic values and corresponding characteristic vectors of T.

26. Consider $T : \mathbb{R}^2 \to \mathbb{R}^2$ from the previous example. Find $\det T$ by choosing a "nice" basis for $\mathbb{R}^2$. Explain.

27. What is the *characteristic polynomial* of a matrix A? What does it tell us?

28. Find the characteristic polynomial for $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. Find its roots. What do these roots tell us?

29. What is the *Cayley-Hamilton Theorem*?