LARSON—MATH 511—HOMEWORK WORKSHEET 18
Claims from Class

Be verbose. Write definitions, examples, etc, in order to be maximally clear.

1. Show that: if (non-zero) vectors $\hat{v}_1, \hat{v}_2, \ldots, \hat{v}_n$ are orthogonal then they are linearly independent.

2. Find examples Q_1, Q_2 of 3×3 orthogonal matrices. Explain. Show that $Q_1 Q_2$ is orthogonal.

3. Show that: if Q_1, Q_2 are square orthogonal matrices, then $Q_1 Q_2$ is orthogonal.

4. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$. Show that: the diagonal elements are eigenvalues of A.

5. Let A be any triangular matrix. Show that: the diagonal elements are eigenvalues of A.

6. What are *similar* matrices? Give an example. Show that: the matrices in your example have the same eigenvalues.

7. Show that: similar matrices have the same eigenvalues.

Gram-Schmidt

Idea: Given linearly independent vectors $\hat{a}_1$, $\hat{a}_2$,$\ldots$,$\hat{a}_n$, let $\hat{q}_1 = \frac{1}{||\hat{a}_1||}\hat{a}_1$, and at each step i ($i = 2, ..i = n$):

- Let $\hat{a}'_i$ be $\hat{a}_i$ minus the projection of $\hat{a}_i$ on each of the previously found $\hat{q}_1$,$\ldots$,$\hat{q}_{i-1}$.
- Let $\hat{q}_i = \frac{1}{||\hat{a}'_i||}\hat{a}'_i$.

8. Use Gram-Schmidt to find an orthogonal basis, $\hat{q}_1, \hat{q}_2, \hat{q}_3$, of the columns of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$

9. Let Q be the matrix whose columns are $\hat{q}_1$, $\hat{q}_2$, and $\hat{q}_3$. Write $A = QR$ (for some matrix R).

Improved Gram-Schmidt

Idea: Given linearly independent vectors $\hat{a}_1$, $\hat{a}_2$,...,$\hat{a}_n$, let $\hat{q}_j = \frac{1}{||\hat{a}_j||}\hat{a}_j$, where $||\hat{a}_i||$ is a maximum, and at each step i ($i = 2, ..i = n$):

- For remaining (not-yet-processed) $\hat{a}_i$'s, let new $\hat{a}_i$ be current $\hat{a}_i$ minus the projection of $\hat{a}_i$ on $\hat{q}_{i-1}$ (update $\hat{a}_i$'s on each step).
- Find the largest-norm remaining $\hat{a}_i$.
- Let $\hat{q}_i = \frac{1}{||\hat{a}_i||}\hat{a}_i$.

10. Use Gram-Schmidt to find an orthogonal basis, $\hat{q}_1, \hat{q}_2, \hat{q}_3$, of the columns of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$