LARSON—MATH 511—HOMEWORK WORKSHEET 19
Randomized Matrix Multiplication

Sage/CoCalc

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login (likely using **your VCU email address**).
   (d) You should see an existing Project for our class. Click on that.
   (e) Click "New", then "Sage Worksheet", then call it **h19**.

   Annotate your Sage Worksheet verbosely. Answer any questions by writing *comments* in your worksheet.

2. Define small matrices A and B and find their (true) product AB.

```
A=matrix(RDF,2,3,[1..6])
B=matrix(RDF,3,2,[7..12])
A*B #and find their true product AB
```

3. Here's a function that does the randomized matrix multiplication Strang described using a choice of s columns of A (and the corresponding rows of B):

```
def randomized_matrix_multiplication(A,B,s):

    rows_columns_A = A.dimensions()
    rowsA = rows_columns_A[0]
    columnsA = rows_columns_A[1]
    rows_columns_B = B.dimensions()
    rowsB = rows_columns_B[0]
    columnsB = rows_columns_B[1]

    weights = []
    for i in range(columnsA):
        weight = (A.column(i).norm())*(B.row(i).norm())
        weights.append(weight)
    total_weight = sum(weights)
    probabilities = [weight/total_weight for weight in weights]

    Randomized_product = matrix(RDF, rowsA, columnsB, [0]*(rowsA*columnsB)) #

    X = GeneralDiscreteDistribution(probabilities)
    for i in range(s):
        index = X.get_random_element()
        summand = (1/(s*probabilities[index]))*matrix(RDF, rowsA, 1, A.column
        Randomized_product = Randomized_product + summand

    return Randomized_product
```

4. Try this `randomized_matrix_multiplication` function with our existing A, B and various choices of s ($s = 1, 2, 3$).

5. We can see what *average* output products look like by doing several experiments.

```
experiments = 100
s=3
total = randomized_matrix_multiplication(A,B,s) #initialization
for _ in [1..experiments-1]:
    summand = randomized_matrix_multiplication(A,B,s)
    total = total + randomized_matrix_multiplication(A,B,s)
(1.0/experiments)*total #this produces the average from our experiments
```

   How good is this average?

6. Let's try another example with something very simple and well-understood: diagonal matrices.

```
D1=diagonal_matrix([1,2,3])
D2=diagonal_matrix([11,12,-5])
```

   Evaluate $D1$, $D2$, and their (true) product $D1 * D2$.

7. Now we can try different numbers of experiments and different values for s.

```
experiments = 100
s=2
total = randomized_matrix_multiplication(D1,D2,s) #initialization
for _ in [1..experiments-1]:
    summand = randomized_matrix_multiplication(D1,D2,s)
    total = total + randomized_matrix_multiplication(D1,D2,s)
(1.0/experiments)*total
```

   How good is this average?

8. Let's try larger matrices now (say 100×100, and for convenience we'll just pick them randomly).

```
A = random_matrix(RDF, 100, 100)
B = random_matrix(RDF, 100, 100)
```

   These and their randomized products will be too big to inspect. How can we investigate the success of our randomized products? One idea is to subtract them from the true product and find the norm of this difference. If it were *exact* then the norm would be 0.

9. For our first run we'll repeat our experiment 100 times and randomly select $s = 5$ columns of A (and the corresponding rows of B).

```
experiments = 100
s=5
total = randomized_matrix_multiplication(A,B,s) #initialization
for _ in [1..experiments-1]:
    summand = randomized_matrix_multiplication(A,B,s)
    total = total + randomized_matrix_multiplication(A,B,s)
(A*B-(1.0/experiments)*total).norm()
```

How good is this average? Try larger values of s and larger numbers of experiments.

Getting your homework recorded

When you are done, ...

1. Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If CoCalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).
2. Send me an email with an informative header like "Math 511—h19 worksheet attached" (so that it will be properly recorded).
3. Remember to attach the homework worksheet!