Last name ______________________

First name ______________________

LARSON—MATH 350—HOMEWORK WORKSHEET 13
Test 2 Review

You should know the following definitions, theorems, algorithms, and proofs for the test. Write out careful definitions, theorem statements, algorithms, proofs, and solutions. Turn these in at test time.

Definitions, Notation & Tools. Write each definition and give an example.

1. What is the *Golden Ratio* ϕ?
2. What is a *sample space* S?
3. What is an *event*?
4. What is a *uniform sample space*?
5. What is the *probability* of an event E in a uniform sample space $P(E)$?
6. What is the *complement* $\bar{E}$ of an event E?
7. When are two events A and B are *independent*
8. What does it mean for integer a to *divide* integer b (that is, $a|b$)?
9. If a, b are integers and $b = aq + r$ (for integers q, r with $0 \leq r < a$), what are q and r called?
10. What is a *prime* number?
11. What does it mean for integers a and b to be *relatively prime*?
12. What is a *polygon*?
13. What is a *convex* polygon?

Theorems (1) (Write a clear, complete proof)

14. (**Claim:**) Every positive integer can be written as the product of primes, and this factorization is unique up to the order of the prime factors.
15. (**Claim:**) $\sqrt{2}$ is irrational.
16. (**Claim:**) There are infinitely many primes.
17. (**Claim:**) For every positive integer k, there exist k consecutive composite integers.

Theorems (2) (State, and give examples)

18. Fermat's Little Theorem.

19. Euler's Polyhedron Formula.

Problems

The terms of the Fibonacci sequence are given by the formula:

$$F_n = \frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n].$$

20. What happens to $(\frac{1-\sqrt{5}}{2})^n$ as $n \to \infty$?
21. Then find an approximation for $\frac{F_{n+1}}{F_n}$ (actually the limit as $n \to \infty$).
22. Why is $0 \leq P(E) \leq 1$?
23. Why does $P(\bar{E}) = 1 - P(E)$?
24. Consider the experiment of flipping a coin 5 times. Assume heads and tails are equally likely on each toss.
    - What is the total number of possible outcomes of this experiment?
    - What is the probability of getting exactly 3 heads?
    - What is the probability of getting exactly 5 heads?
    - Are the events of getting 3 heads and 5 heads independent?
25. Let a and b be positive integers, and p be a prime. Use the fact that each has a unique factorization into primes to **show** that, if $p|ab$ then $p|a$ or $p|b$.
26. We are given n natural numbers: $a_1, a_2, ..., a_n$. **Show** that we can choose a (nonempty) subset of these numbers whose sum is divisible by n.
27. We are given n numbers from the set $\{1, 2, ..., 2n-1\}$. **Show** that we can always find two numbers among these n numbers that are relatively prime to each other.
28. How many numbers are there up to 1200 that are relatively prime to 1200?
29. How many points of intersection do the diagonals of a convex n-gon have (inside the figure, assuming no 3 diagonals meet in the same point)?

New

30. **Show** that the prime factorization of a number n contains at most $\log_2 n$ prime factors.
31. How many integers are there that are not divisible by any prime larger than 20 and not divisible by the square of any prime?
32. Given $n \geq 3$ lines in general position (no two are parallel and no three go through a point), the lines will divide the plane into regions. **Show** that at least one of these regions will be a triangle.
33. The diagonals of a convex n-gon, divide the figure into regions (we assume no 3 diagonals go through the same point). How many regions are there?