Last name ______________________

First name ______________________

LARSON—MATH 310–HOMEWORK WORKSHEET 05

1. Write up a **neat** assignment on a **new sheet** of paper. (Do not cram your answers between the lines). Typed using LaTeXwould be even better.
2. **Number** your problems so that it is easy to see what work matches the assigned problems.
3. Be verbose. Remember that you do not understand a concept if you do not know an **examples**.

Problems

1. (**Maximal Linearly Independent Set Algorithm**). Let the columns of matrix A be $\vec{a}_1, \ldots \vec{a}_6$. Find a maximal set of linearly independent columns by greedily choosing the **last** (largest-index) non-zero column vector, adding the next smallest index column vector that is still linearly independent, and iterating (until no column remains). So, iterating from largest to smallest index.

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   What is the linearly independent set X produced at the end of this algorithm. **Write out** the 6 iterations, including your tests for linear independence/dependence.

2. Write each column vector that is **not** in X as a linear combination of the vectors in X.

3. What is the *column space* of A (what is the definition)?

4. Argue that $col(A)$, the column space of A, **is** the collection $\langle A \rangle$ of linear combinations of your A vectors (that is, show X is a basis for $col(A)$).

5. What is the definition for the *rank* of a collection of vectors?

6. What is the rank of $col(A)$?

7. Is it possible that, by considering the column vectors of A is some different order that you might get a collection X with a different number of vectors in it? Explain.