Last name _____

First name _____

LARSON—MATH 601—CLASSROOM WORKSHEET 05
Review.

Concepts & Notation

- (Sec. 1.4) *row-reduced echelon matrix*, *zero matrix* $0^{m,n}$.
- matrix multiplication.
- (Sec. 1.5) *column matrix* B_j, *elementary matrix*.
- (Sec. 1.6) *left inverse*, *right inverse*, *invertible matrix*, *inverse* A^{-1}.

Problems

1. If AB is defined, and $B = \begin{bmatrix} B_1 \dots B_p \end{bmatrix}$, explain why $AB = \begin{bmatrix} AB_1 \dots AB_p \end{bmatrix}$.

2. Show that $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ is invertible. Find A^{-1}.

3. Show that $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is invertible. Find B^{-1}.

4. Suppose A is invertible. How can you show that its inverse is unique (and therefore our notation A^{-1} is unambiguous)?

5. Find AB, find $B^{-1}A^{-1}$ and check that $(AB)^{-1} = B^{-1}A^{-1}$.

6. Suppose A is invertible. Why does $(A^{-1})^{-1} = A$?

Elementary Matrices

An elementary matrix E is one where left multiplication by E has the same effect as some row operation.

7. Suppose A is a 3×3 matrix. What elementary matrix E corresponds to scaling the first row by a (non-zero) constant c?

8. Why is E invertible? What is E^{-1}? Why is E^{-1} an elementary matrix?

9. Suppose A is a $n \times n$ matrix. What elementary matrix E corresponds to scaling the i^{th} row by a (non-zero) constant c?

10. Is E invertible?

11. Suppose A is a 3×3 matrix. What elementary matrix E corresponds to switching the 2^{nd} and 3^{rd} rows?

12. Why is E invertible? What is E^{-1}? Why is E^{-1} an elementary matrix?

13. Suppose A is a $n \times n$ matrix. What elementary matrix E corresponds to switching the i^{th} and j^{th} rows?

14. Is E invertible?

15. Suppose A is a 3×3 matrix. What elementary matrix E corresponds to adding a multiple c of the 2^{nd} row to the 3^{rd} row?

16. Why is E invertible? What is E^{-1}? Why is E^{-1} an elementary matrix?

17. Suppose A is a $n \times n$ matrix. What elementary matrix E corresponds to adding a multiple c of the i^{th} row to the j^{th} row?

18. Is E invertible?