

Last name _____

First name _____

LARSON—MATH 511—HOMEWORK WORKSHEET 23

Test 2 Review

Write up a careful, complete test review and turn it in **before** our Test 2 on Thursday, Dec. 15 @ 8:00. **Explain** everything.

1. Suppose A is a matrix and \hat{v} is an eigenvector of $A^T A$ with $\lambda \neq 0$. Find an eigenvector of AA^T .
2. What is a *positive semi-definite* matrix. Give an example.
3. Explain why, for *any* matrix A that $A^T A$ is positive semi-definite.
4. What is the *rank* of a matrix?
5. Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 1 \end{bmatrix}$. What is the rank of A ?
6. Follow all the steps to find the SVD of A . (We'll need $A = U\Sigma V^T$ for the next questions about A).
7. What is a *singular value* of a matrix?
8. What are the singular values of A ?
9. We used Gram-Schmidt to find an orthogonal basis for the column space of A . Is there an orthogonal basis for the column space of A that we can “read off” from $A = U\Sigma V^T$?
10. Let \hat{x} be a vector. We can use the SVD of A to describe $A\hat{x}$ as a rotation, followed by a scaling, followed by a rotation. Explain.
11. What is the (Moore-Penrose) *pseudo-inverse* of a matrix?
12. Find the pseudo-inverse A^+ of A .
13. Find AA^+ and A^+A .

14. Explain why no matrix (including A^+) can be the inverse of A .
15. Do A and A^+ have the same rank? Is it true in general that a matrix B and its pseudo-inverse B^+ have the same rank?
16. Write A as a sum of rank-1 matrices.
17. What is the Eckart-Young Theorem?
18. Find the “best” rank-1 approximation of A (explain what you mean by “best”).
19. What is the norm $\|A - \sigma_1 u_1 v_1^T\|$ when the largest rank-1 piece of A is removed?
20. What are the singular values of $A - \sigma_1 u_1 v_1^T$ and its rank?