Last name _____

First name _____

LARSON—OPER 731—CLASSROOM WORKSHEET 18
Totally Unimodular Matrices

Totally Unimodular Matrices

1. What is a *totally unimodular matrix*?

2. Is it true that if you multiply a row of a totally unimodular matrix A by -1 to get matrix A' then A' is totally unimodular?

3. Let b be any vector in $\mathbb{R}^3$ with integer components. Use Cramer's rule to show that any extreme point of the polyhedron $\mathcal{P}$ defined $Ax \leq b$ (and x_i nonnegative) has integer coordinates.

4. Show that the vertex-edge incidence matrix of a directed graph is totally unimodular.

5. **Total Unimodularity implies Integrality**. Show that any extreme point of the polyhedron $\mathcal{P}$ defined $Ax \leq b$ (and x_i nonnegative) has integer coordinates if A is totally unimodular.

6. What is König's Theorem?

7. Represent finding a maximum matching in a bipartite graph as an integer programming problem and use total unimodularilty to show that the relaxation of this IP has integer solutions.

8. What does the dual of this IP model?

9. Use total unimodularilty to show that the relaxation of this dual IP has integer solutions. How does this prove König's Theorem?