Last name ______________________

First name ______________________

LARSON—MATH 310–CLASSROOM WORKSHEET 13
Vector Subspace, Homogeneous Linear System

Review: Chapter 3 of Klein's *Coding the Matrix* text

1. What are the 3 properties of a *vector space* $\mathcal{V}$?
2. Why is $\mathbb{R}^2$ a vector space?
3. If $\mathcal{V}$ is a vector space, and $\mathcal{W} \subseteq \mathcal{V}$, when is $\mathcal{W}$ a *subspace* of $\mathcal{V}$?
4. What are examples of subspaces?
5. Book problem:

Exercise 3.2.15: For each of the subproblems, you are to investigate whether the given vectors span $\mathbb{R}^2$. If possible, write each of the standard generators for $\mathbb{R}^2$ as a linear combination of the given vectors. If doing this is impossible for one of the subproblems, you should first add one additional vector and then do it.

1. $[1,2],[3,4]$
2. $[1,1],[2,2],[3,3]$
3. $[1,1],[1,-1],[0,1]$

New

1. For a field $\mathbb{F}$ and finite set D why is the collection of functions $\mathbb{F}^D$ a vector space?

2. If $\hat{v_1}, \hat{v_2}, \ldots, \hat{v_n}$ are vectors in a vector space $\mathcal{V}$, why is $\text{Span}(\{\hat{v_1}, \hat{v_2}, \ldots, \hat{v_n}\})$ a vector space?

3. What is a *homogeneous linear system*?

4. Why are the solutions of a homogeneous linear system a vector space?

5. Book problem:

Vector spaces

Problem 3.8.7: Prove or give a counterexample: "$\{[x, y, z] : x, y, z \in \mathbb{R}, x + y + z = 1\}$ is a vector space."

Problem 3.8.8: Prove or give a counterexample: "$\{[x, y, z] : x, y, z \in \mathbb{R}$ and $x + y + z = 0\}$ is a vector space."

Problem 3.8.9: Prove or give a counterexample: "$\{[x_1, x_2, x_3, x_4, x_5] : x_1, x_2, x_3, x_4, x_5 \in \mathbb{R}, x_2 = 0$ or $x_5 = 0\}$ is a vector space."