LARSON—MATH 310–HOMEWORK WORKSHEET 03

1. Write up a **neat** assignment on a **new sheet** of paper. (Do not cram your answers between the lines). Typed using LaTeXwould be even better.

2. **Number** your problems so that it is easy to see what work matches the assigned problems.

3. Be verbose. Remember that you do not understand a concept if you do not know an **examples**.

Problems

1. What is a *linear map* (or *linear transformation*) $f : V \rightarrow W$, from linear space V to linear space W?

2. Show that the map $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ where $f(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}) = v_1$, for every $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is a linear map.

3. Show that the map $f : \mathbb{R}^2 \rightarrow \mathbb{R}^{\not\vDash}$ where $f(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}) = (\begin{bmatrix} 0 \\ 3v_2 \\ 0 \end{bmatrix}$, for every $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is a linear map.

4. What is a *linear combination* of vectors $\vec{v}_1$, $\vec{v}_2$ in a linear space V (over field $\mathbb{K}$)?

5. Find the collection of all linear combinations of $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ in $\mathbb{R}^3$.

6. What is a *subspace* of a linear space?

7. Show that the collection of all linear combinations of $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ is a subspace of $\mathbb{R}^3$. (What needs to be shown?)

8. Show that the vectors $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$ are linearly independent.

9. Show that the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{e}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$ are linearly independent.

10. Show that the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{e}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ in $\mathbb{R}^3$ are linearly dependent.

11. What is a *basis* of a linear space?

12. Explain why the collection of vectors $\{\vec{e}_1, \vec{e}_3\}$ is not a basis for $\mathbb{R}^3$.