Last name _____

First name _____

LARSON—MATH 550—CLASSROOM WORKSHEET 17
Ceiling, floor, intervals, mod.

Concepts & Notation

- Sec. 3.2. interval notation.
- Sec. 3.4. mod notation.
- Sec. 4.4. Analyzing the size of $n!$
- Sec. 5.1. Binomial coefficients!

Homework Hint

1. Show:
$$\sum_{1\leq j<k\leq n}(a_jb_k - a_kb_j)^2 = (\sum_{k=1}^{n} a_k^2)(\sum_{k=1}^{n} b_k^2) - (\sum_{k=1}^{n} a_kb_k)^2$$

2. (**Review**) Can you find an expression for $\lceil \lg n \rceil$ $(n \in \mathbb{Z})$?

Interval Notation & Modulus

3. What are $[\alpha..\beta], [\alpha..\beta)$ (for $\alpha, \beta \in \mathbb{R}$)?

4. How many integers are in $[\alpha..\beta)$ (for $\alpha, \beta \in \mathbb{R}$)?

The *quotient* of positive integers n and m is $\lfloor n/m \rfloor$ and the *modulus* is the remainder of dividing n by m, denoted $n \mod m$.

5. Find $\lfloor 32/5 \rfloor$ and $32 \mod 5$ and check that $32 = \lfloor 32/5 \rfloor \cdot 5 + (32 \mod 5)$.

6. State a general law for positive integers n and m.

Estimating the size of $n!$

7. Check: $(n!)^2 = (1 \cdot 2 \ldots n)(1 \cdot 2 \ldots n) = \prod_{k=1}^{n} k(n+1-k)$

8. Check: $k(n+1-k) = \frac{1}{4}(n+1)^2 - (k - \frac{1}{2}(n+1))^2$

9. What can we conclude?