Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 10
Linear independence & Bases.

Concepts & Notation

- (Sec. 2.1) *vector*, *vector space*, *linear combination*.
- (Sec. 2.2) *subspace*, *subspace spanned by a set of vectors*, *span*.
- (Sec. 3.3) *linearly independent set of vectors*, *basis*.

1. The set of $n \times 1$ column matrices with entries in a field $\mathbb{F}$ is a vector space over $\mathbb{F}$; call it V. Let A be an $m \times n$ matrix. **Claim:** the $n \times 1$ column matrices X that are solutions to $AX = 0$ is a subspace of V.

If V is a vector space over a field $\mathbb{F}$ and $S \subset V$, the *subspace spanned by* S is the intersection of all subspaces of V containing S (S *spans* V); if $S = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ then *say* it is the *subspace spanned by* $\alpha_1, \alpha_2, \ldots, \alpha_n$.

2. Find the subspace spanned by $\alpha = (1, 0)$ in $\mathbb{R}^2$.

3. Let $\alpha_1 = (1,0), \alpha_2 = (0,1) \in \mathbb{R}^2$. Show that, if $c_1\alpha_1 + c_2\alpha_2 = (0,0)$ then $c_1 = c_2 = 0$.

Vectors $\alpha_1, \ldots, \alpha_n$ in a vector space V over a field $\mathbb{F}$ are *linearly independent* if $c_1\alpha_1 + \ldots c_n\alpha_n = 0$ ($c_i \in \mathbb{F}$) implies $c_1 = \ldots c_n$. If $\alpha_1, \ldots, \alpha_n$ are not linearly independent then they are *linearly dependent.*

4. Show $\alpha_1 = (1,0), \alpha_2 = (0,1) \in \mathbb{R}^2$ are linearly independent.

A *basis* for V is a set of linearly independent vectors which spans V.

5. Show that $\alpha_1 = (1,0), \alpha_2 = (0,1) \in \mathbb{R}^2$ is a basis for $\mathbb{R}^2$.

6. Let $\mathbb{F}$ be a field, and $\mathbb{F}^n$ be the vector space of n-tuples with coordinates in $\mathbb{F}$, and ϵ_i be the 0-vector with a 1 in the i^{th}-coordinate. **Claim:** the set $S = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_n\}$ is a basis for $\mathbb{F}^n$.