Last name ______________________

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LARSON—MATH 610–CLASSROOM WORKSHEET 09
Linear Transformations.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence, pivot column decomposition, direct sum $\mathcal{U} \oplus \mathcal{V}$, *orthogonal* matrix, *unitary* matrix.

Review:

(**Corollary 2.1.11, All Bases have Same Cardinality**). If r and n are positive integers and $\hat{v}_1, \ldots, \hat{v}_r$ and $\hat{w}_1, \ldots, \hat{w}_n$ are bases of an $\mathbb{F}$-vector space $\mathcal{V}$ then $r = n$.

Chp. 2 of Garcia & Horn, Matrix Mathematics

1. What is the *dimension* of a vector space?

2. What are examples?

(**Theorem 2.3.1**) Let $A \in \mathbb{M}_{m\times n}(\mathbb{F})$. Then

$$dim(col(A)) = dim(col(A^T)) = dim(row(A)) \leq \min\{m, n\}.$$

3. What is this theorem about?

4. Why is this true?

(**Theorem 2.3.7. Full Rank Factorization**). Let $A \in \mathbb{M}_{m\times n}(\mathbb{F})$ be non-zero, let $r = rank(A)$, and let the columns of $X \in \mathbb{M}_{m\times r}(\mathbb{F})$ be a basis for $col(A)$. Then there is a unique $Y \in \mathbb{M}_{r\times n}(\mathbb{F})$ such that $A = XY$. Moreover, $rank(Y) = r$, the rows of Y are a basis for $row(A)$ and $null(A) = null(Y)$.

5. What is this theorem about?

6. Why is this true?

7. What is the β-*basis representation function*? What are the *coordinates* of a vector with respect to a basis?

8. What is a *linear transformation*? What is $\mathcal{L}(\mathcal{V}, \mathcal{W})$?

9. How does any matrix $A \in \mathbb{M}_{n\times n}$ define a linear transformation?

10. How does any linear transformation $T \in \mathcal{L}(\mathcal{V}, \mathcal{W})$ and bases $\beta = \hat{v}_1, \ldots, \hat{v}_n$ of $\mathcal{V}$ and $\gamma = \hat{w}_1, \ldots, \hat{w}_m$ of $\mathcal{W}$ define a matrix $A \in \mathbb{M}_{m\times n}$?