LARSON—MATH 255–CLASSROOM WORKSHEET 20
Problems & Interacts

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) You should see an existing Project for our class. Click on that.
   (d) Click "New", then "Sage Worksheet", then call it **c20**.
   (e) For each problem number, label it in theSAGE cell where the work is. So for Problem 1, the first line of the cell should be `#Problem 1`.

Homework Problems

2. If $p = 120$ is the perimeter of a right triangle with integer length sides, $\{a, b, c\}$, there are exactly three solutions (three triples that are the sides of a right triangle): $\{20, 48, 52\}$, $\{24, 45, 51\}$, and $\{30, 40, 50\}$ (assuming $a \leq b \leq c$)..

   Write a function `solutions(p)` that finds the number of right triangles with integer length sides, $\{a, b, c\}$, and perimeter p.

3. For which value of $p \leq 1000$, is the number of solutions maximized (for which p has the most triples that work)?

Follow-up

4. Here's **a multi-step problem that builds on what we did**. Create a new file `two_hundred_numbers.txt` that consists of each line from `one_hundred_numbers.txt` written twice. (If you don't have `one_hundred_numbers.txt` In put a copy in your CoCalc Project Handouts fold).

Problems

5. The sum of the reciprocals of the positive integers

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

   diverges (that is, the sum goes to infinity).

   (a) Find the smallest integer m so that $\sum_{n=1}^{m} \frac{1}{n}$ is at least 2.
   (b) Find the smallest integer m so that $\sum_{n=1}^{m} \frac{1}{n}$ is at least 3.
   (c) Find the smallest integer m so that $\sum_{n=1}^{m} \frac{1}{n}$ is at least 4.

Interacts

There is a collection of examples of Sage INTERACTS at `http://wiki.sagemath.org/interact/`. Let's look at a few of these examples to see the kinds of things you can do with Sage.

6. Here is a simple Sage INTERACT with *default* values for a function f to be graphed, and the interval (a, b) that f will be graphed on. The Interact function is named "_", which is standard—as we will never refer to this function by name.

```
@interact
def _(f=x^2, a=-3, b=3):
    show(plot(f,(x,a,b)))
```

7. Now let's make this fancy with some *options*.

```
@interact
def _(f=input_box(x^2,width=20),
    axes=True,
    fill=True,
    zoom=range_slider(-3,3,default=(-3,3))):
    show(plot(f,(x,zoom[0], zoom[1]), axes=axes,fill=fill))
```

8. **Prime Numbers**. Try the following Sage Interact which visualizes the Prime Number Theorem (PNT).

```
@interact
def pnt(N=input_box(200)):
    show(plot(prime_pi,0,N,color="red")+plot(x/(log(x)-1),5,N,color="blue"))
```

   Try putting different numbers in the input box.

9. Let's do the same thing in a new way. Try:

```
@interact
def pnt2(N=(100,(2..1000000))):
    show(plot(prime_pi,0,N,color="red")+plot(x/(log(x)-1),5,N,color="blue"))
```

   What part of the code is producing the *slider*?

10. **Eigenvalues**. Try `M=identity_matrix(3)`. Evaluate M to see the entries. Change the upper right corner entry to 3. Use `M.` and TAB to find the eigenvalues and eigenvectors of M

11. Here's an INTERACT with an *update* button.

    ```
    @interact
    def _(m=("matrix", identity_matrix(2)), auto_update=False):
        print(m.eigenvalues())
    ```

    Try different values in the matrix and then and then click the update box. Now make a 3×3 matrix.

Getting your classwork recorded

When you are done, before you leave class...

1. Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If CoCalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

2. Send me an email with an informative header like "Math 255 - c20 worksheet attached" (so that it will be properly recorded).

3. Remember to attach today's classroom worksheet!