Last name ______________________

First name ______________________

LARSON—MATH 556—CLASSROOM WORKSHEET 03
4 Fundamental Invariants.

Concepts & Notation

- *assignment problem*, *graph* G, *points* $V(G)$, *lines* $E(G)$, *adjacent*, *incident*. *line covering*, *line covering number* ρ, *matching*, *matching number* ν, point covering, *point covering number* τ, *independent set*, *independence number* α.

Review

A **bipartite graph** is a graph $G = (A, B)$ with bipartition $V(G) = A \cup B$ where every line has one endpoint in A and the other in B.

A **complete bipartite graph** is a graph $G = (A, B)$ with bipartition $V(G) = A \cup B$ where there is a line between every point in A and point in B. $K_{m,n}$ denotes the complete bipartite graph with $|A| = n$ and $|B| = m$.

Problems

Basic Definitions & the 4 fundamental invariants in Chp. 1

1. Draw the complete bipartite graph $K_{3,3}$.

2. Draw the complete bipartite graph $K_{3,4}$.

   A **star** is the complete bipartite graph $K_{1,n}$. ($K_{1,n}$ is the n-star, or star on $n + 1$ points).

3. Draw the star $K_{1,3}$.

4. Draw the 2-star and the 6-star.

A **matching** in a graph is a set of lines where no pair of lines is incident to the same point. A matching is **maximum** if there is no matching with larger cardinality. The cardinality of a maximum matching in graph G is the **matching number** $\nu(G)$.

5. Find a maximum matching for $K_{3,4}$. Then find ν. Can you find a matching which is maxim*al* (can't be extended) but is not maxim*um*?

A **line cover** in a graph is a set of lines which are incident to all the points in the graph. A line cover is *minimum* if there is no line cover with small cardinality. The cardinality of a minimum line cover in graph G is the *line covering number* $\rho(G)$.

6. Find a minimum line cover for $K_{3,4}$. Then find ρ. Can you find a line cover which is minim**al** (can't be reduced) but is not minim*um*?

An *independent set* in a graph is a set of points no pair of which are adjacent. An independent set is **maximum** if there is no independent set with larger cardinality. The cardinality of a maximum independent set in graph G is the *independence number* $\alpha(G)$.

7. Find a maximum independent set for $K_{3,4}$. Then find α. Can you find an independent set which is maxim**al** (can't be extended) but is not maxim*um*?

A **point cover** is a set of points which are incident to all the lines in the graph. A point cover is *minimum* if there is no point cover with smaller cardinality. The cardinality of a minimum matching in graph G is the **point covering number** $\tau(G)$.

8. Find a minimum point cover for $K_{3,4}$. Then find τ. Can you find a point cover which is minim*al* (can't be reduced) but is not minim*um*?

The Proof of Lemma 1.02

9. What relationship do you notice about ρ, ν and $|V(G)|$ in $K_{3,4}$? Draw another graph and see if this relationship holds. Can you draw a graph where it doesn't hold?

If G is a graph and H is also a graph the points and lines of which are also points and lines of G , then H is a **subgraph** of G. If H is a subgraph of G, and if every line joining two points of H which lies in G also lies in H then H is an **induced** subgraph of G. If X is a set of points in G then the **subgraph of G induced by X**, $G[X]$, is the induced subgraph of G having point set X.

10. Find a subgraph of $K_{3,4}$ which is not an induced subgraph of $K_{3,4}$. What can you say about any induced subgraph of $K_{3,4}$? An alternating sequence of points and lines,

beginning and ending with points, is called a **walk**. If all lines in a walk are distinct, the walk is called a **trail**. If, in addition, the points are also distinct, the trail is a **path**. A graph is **connected** if every two points are joined by a path. A maximal connected subgraph of a graph G is a **component** of G.

12. Draw a connected graph with a line so that removing the line makes the graph dis-connected.

13. Draw a connected graph where removing an line makes the graph dis-connected.

14. What is the fewest number of lines that need to be removed from $K_{3,4}$ in order to make it dis-connected?

Let C be any set of lines in a graph and let $V(C)$ be the set of points incident to those lines. $\langle C \rangle$ is the subgraph with point set $V(C)$ and lines C.

15. Draw $K_{3,4}$. Let C be any pair of non-incident lines. $\langle C \rangle$ and the induced subgraph $G[V(C)]$ are different. Why?

16. Let be C is a minimum line cover of a graph. What must $\langle C \rangle$ "look like"? Draw some examples. Will $\langle C \rangle$ be connected?