Last name _____

First name _____

LARSON—MATH 550—CLASSROOM WORKSHEET 11
Multiple Sums.

Concepts & Notation

- Sec. 2.2. Two "tricks".

- Sec. 2.3. Rules for sums. Perturbation method.

- Sec. 2.4. Multiple sums.

Homework

1. What does $\displaystyle\sum_{1\le i,j\le 3} a_i b_j$ *mean.*

2. Find a nice expression for $\displaystyle\sum_{1\le i,j\le 3} a_i b_j$ by rewriting as a double-sum.

3. Find a nice expression for $\displaystyle\sum_{1\le i,j\le n} a_i b_j$.

4. Find a nice expression for $\displaystyle\sum_{1\le i,j\le n} a_i a_j$.

5. Find the sum of the elements in the matrix:

$$
\begin{bmatrix}
a_1 a_1 & a_1 a_2 & \dots & a_1 a_{n-1} & a_1 a_n \\
a_2 a_1 & a_2 a_2 & \dots & a_2 a_{n-1} & a_2 a_n \\
\dots & \dots & & & \dots \\
a_{n-1} a_1 & a_{n-1} a_2 & \dots & a_{n-1} a_{n-1} & a_{n-1} a_n \\
a_n a_1 & a_n a_2 & \dots & a_n a_{n-1} & a_n a_n
\end{bmatrix}
$$

6. Find $T = \displaystyle\sum_{1 \leq i \leq j \leq n} a_i a_j$

   (the sum of the elements of the upper-triangle of an $n \times n$ matrix with entries $a_i a_j$).

7. Explain why this identity is true:

$$[1 \leq j < k \leq n] + [1 \leq k < j \leq n] = [1 \leq j, k \leq n] - [1 \leq j = k \leq n].$$

8. Expand and simplify:
$$\sum_{1 \leq i,j \leq 3} a_i b_i.$$

9. Find $S = \displaystyle\sum_{1 \leq j < kj \leq n} (a_k - a_j)(b_k - b_j)$