Last name ______________________

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LARSON—MATH 610—CLASSROOM WORKSHEET 23
Projections in Inner Product Spaces.

Concepts & Notation

- (Chp. 5) *eigenvalue*, *eigenvector*, *invariant subspace*, *minimal polynomial*,
- (Chp. 8) *generalized eigenvector*, *Cayley-Hamilton Theorem.*
- (Chp. 6) *dot product*, *inner product*, *inner product space*, *norm*, *orthogonal representation*, *Cauchy-Schwartz*, *orthonormal list*, *Gram-Schmidt*, *orthogonal projection.*

1. What is the *orthogonal complement* of a set U in an inner product space V?

2. For a subspace U of an inner product space V, what is the *orthogonal projection* operator P_U?

3.

6.55 Properties of the orthogonal projection P_U

Suppose U is a finite-dimensional subspace of V and $v \in V$. Then

(a) $P_U \in \mathcal{L}(V)$;

(b) $P_U u = u$ for every $u \in U$;

(c) $P_U w = 0$ for every $w \in U^\perp$;

(d) range $P_U = U$;

(e) null $P_U = U^\perp$;

(f) $v - P_U v \in U^\perp$;

(g) $P_U{}^2 = P_U$;

(h) $\|P_U v\| \le \|v\|$;

(i) for every orthonormal basis $e_1, \dots, e_m$ of U,

$$P_U v = \langle v, e_1\rangle e_1 + \cdots + \langle v, e_m\rangle e_m.$$

4. (**Minimizing the distance to a subspace**) Suppose U is a finite-dimensional subspace of V, $v \in V$, and $u \in U$. Then $||v = P_U v|| \leq ||v - u||$. Furthermore, the inequality above is an equality if and only if $u = P_U v$.

Linear Functionals and Riesz Representation Theorem

5. What is a *linear functional*?

6. What is the *Riesz Representation Theorem*?

Adjoint Operators

7. Let V, W be finite-dimensional inner product spaces and $T \in \mathcal{L}(V, W)$. What is the *adjoint* T^* of T?

8. Why does the adjoint exist?