LARSON—MATH 511—CLASSROOM WORKSHEET 08
Gilbert Strang Lectures 4 & 5.

More on Strang's Lectures

1. Find the eigenvalues of A= $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

2. Let λ_1 and λ_2 be the eigenvalues of A with corresponding (unit) eigenvectors $\hat{x}_1$ and $\hat{x}_2$. Let Q be the matrix whose columns are $\hat{x_1}$ and $\hat{x}_2$. What kind of matrix is Q?

3. Multiply out AQ and use this to get a "decomposition" of A in the form $Q\Lambda Q^T$.

4. What can we say about the relationship between A and Λ?

5. Why do symmetric matrices have real eigenvalues?

6. If S is symmetric why are eigenvectors corresponding to distinct eigenvalues orthogonal?

7. What is a *positive definite matrix*?

8. Is $S = \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}$ positive definite?

9. One equivalent condition is that a symmetric matrix S is positive definite if, for every vector $\hat{x}$, the *energy* $\hat{x}^T S\hat{x} > 0$. Show that S has positive energy for every vector $\hat{x}$.

10. We showed that A^TA is symmetric. If A^TA has linearly independent columns then A^TA is positive definite.

Sage/CoCalc

(a) Start the Chrome browser.

(b) Go to `http://cocalc.com`

(c) Login (likely using **your VCU email address**).

(d) You should see an existing Project for our class. Click on that.

(e) Click "New", then "Sage Worksheet", then call it **c08**.

11. Find the eigenvalues of S= $\begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}$.

12. Find the eigenvectors corresponding to the eigenvalues of S.

13. Let λ_1, λ_2 be the eigenvalues of A, with corresponding eigenvectors $\hat{x}_1$ and $\hat{x}_2$; and let Q be the matrix whose columns are $\hat{x_1}$ and $\hat{x}_2$.

14. How can we use SAGE to *check* that Q is orthogonal?

15. Let Λ be the diagonal matrix with λ_1 and λ_2 on the diagonal. Check that $S = Q\Lambda Q^T$.

16. Test random vectors $\hat{x}$ to see that $\hat{x}^T S\hat{x}$ is always positive.

Getting your classwork recorded

When you are done, before you leave class...

1. Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If CoCalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).
2. Send me an email with an informative header like "Math 511—c08 worksheet attached" (so that it will be properly recorded).
3. Remember to attach today's classroom worksheet!