Last name _	
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LARSON—MATH 610—CLASSROOM WORKSHEET 32 Determinants.

Concepts & Notation

- (Sec. 5.1) *n-linear* function, alternating function.
- (Sec. 5.2) determinant function.
- (Sec. 5.3) permutation, $\det A$.

Let A be an $n \times n$ matrix over a commutative ring. Let:

$$\det A = \sum_{\sigma \in S_n} [(sgn \ \sigma) \prod_{i=1}^n A_{i,\sigma(i)}]$$

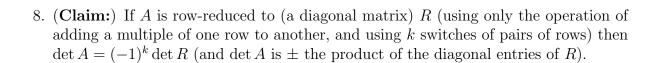
$$= \sum_{\sigma \in S_n} (sgn \ \sigma) A_{1,\sigma(1)} A_{2,\sigma(2)} \dots A_{n,\sigma(n)}.$$

Review

- 1. $\det I = 1$.
- 2. If A is a diagonal matrix then det $A = A_{11}A_{22}...A_{nn}$ (the product of the diagonal entries).
- 3. If A has a zero row then $\det A = 0$.
- 4. If A' is A with two rows switched then $\det A' = -\det A$.
- 5. If A has two rows which are the same then $\det A = 0$.
- 6. $\det(\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \dots, \alpha_{\sigma(n)}) = (sgn \ \sigma) \det(\alpha_1, \alpha_2, \dots, \alpha_n).$

Properties of det: $K^{n \times n} \to K$

7. (Claim:) det is n-linear.



9. (Claim:) If
$$A \in \mathbb{F}^{n \times n}$$
, then A is invertible iff det $A \neq 0$.

10. (Claim:) If
$$D, B \in \mathbb{F}^{n \times n}$$
 and D is diagonal then $\det DB = (\det D)(\det B)$.

11. (Claim:) If
$$A, B \in \mathbb{F}^{n \times n}$$
 then $\det AB = (\det A)(\det B)$.

The Structure of a Linear Operator

12. Suppose $A \in \mathbb{F}^{n \times n}$. There is a $n \times 1$ matrix α (not all-zero), and $c \in \mathbb{F}$ with $A\alpha = c\alpha$ iff $\det(A - cI) = 0$.