Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 37
The Structure of Linear Operators.

Concepts & Notation

- (Sec. 5.3) *permutation*, $\det A$.
- (Sec. 6.2) *characteristic value*, *characteristic vector*, *characteristic polynomial*, *diagonalizable* linear operator.

1. (**Claim:**) Let T be a linear operator on a finite-dimensional space V. Let $c_1, \ldots, c_k$ be the distinct characteristic values of T and let W_i be the null space of $(T - cI)$. The following are equivalent.

   (a) T is diagonalizable,

   (b) The characteristic polynomial for T is:

   $$p = (x - c_i)^{d_i} \ldots (x - c_k)^{d_k}$$

   where $\dim W_i = d_i$ (for $i = 1, \ldots, k$),

   (c) $\dim W_1 + \ldots + \dim W_k = \dim V$.

2. (**Claim:**) If T is a linear operator from a vector space V over a field $\mathbb{F}$ to itself then $T^2 = T \circ T$ is also in $\mathcal{L}(V)$. And $T^k \in \mathcal{L}(V)$.

3. (**Claim:**) If $T \in \mathcal{L}(V)$ then $cT \in \mathcal{L}(V)$ (for $c \in \mathbb{F}$).

4. (**Claim:**) If $T, T' \in \mathcal{L}(V)$ then $T + T' \in \mathcal{L}(V)$.

5. (**Claim:**) If $T \in \mathcal{L}(V)$ and $p \in \mathbb{F}[x]$ then $p(T) \in \mathcal{L}(V)$.

6. (**Claim:**) If $T \in \mathcal{L}(V)$, $T(\alpha) = c\alpha$ (for $c \in \mathbb{F}$, $\alpha \in V$), and $p \in \mathbb{F}[x]$ then $p(T)(\alpha) = p(c)\alpha$.

7. What does it mean for a polynomial to *annihilate* a linear operator T?

8. What is the *minimal polymonial* of a linear operator T over a finite-dimensional vector space T? (Does it exist? What does it tell us?)

9. (**Cayley-Hamilton Theorem**) Let $A \in \mathbb{F}^{n \times n}$. Let $p \in \mathbb{F}[x]$ be the characteristic polynomial of A. Then $p(A) = 0$.