Last name _	
First name	

LARSON—MATH 550—CLASSROOM WORKSHEET 24 Binomial Coefficients, Theorems, Tools.

Concepts & Notation

- Sec. 5.1: Binomial coefficients, the Binomial Theorem, key formulas, the polynomial argument, Vandermonde's convolution.
- Sec. 5.2: the Sorting example.
- Sec. 5.3: Duplication formula, inversion, derangements.
- Sec. 5.4: Convolutions, generating functions.

Review

- 1. $\binom{n}{m}$ is the number of *m*-subsets of an *n*-set (for $n, m \in \mathbb{Z}^{\geq 0}$).
- 2. Find a formula for $\binom{n}{m}$ $(0 \le m \le n, m, n \in \mathbb{Z})$.
- 3. Argue the symmetry identity $\binom{n}{k} = \binom{n}{n-k}$.
- 4. prove the addition formula:

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}.$$

- 5. Find the sum of $\binom{n}{0}$, $\binom{n}{1}$, ... $\binom{n}{n}$.
- 6. Find $(x+y)^n$.

Generalizing the Binomial Theorem

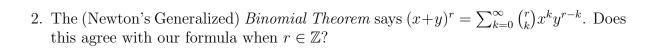
The binomial coefficients $\binom{n}{k}$ $(n, k \in \mathbb{Z}^{\geq 0})$ can be generalized to $\binom{r}{k}$ $(r \in \mathbb{R}, k \in \mathbb{Z})$:

$$\binom{r}{k} = \frac{r^{\underline{k}}}{k!} \text{ (if } k \ge 0)$$

$$\binom{r}{k} = 0 \text{ (if } k < 0).$$

We proved the absorbtion identity $\binom{r}{k} = \frac{r}{k} \binom{r-1}{k-1}$.

1. Argue the absorbtion identity variation $k\binom{r}{k} = r\binom{r-1}{k-1}$.



3. How can we prove the special case
$$(x+1)^r = \sum_{k=0}^{\infty} {r \choose k} x^k$$
?

4. Find an expression for
$$\sqrt{x+1}$$
.

5. Check it for
$$x = 3$$
, $x = 1$.