Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 09
Vector Spaces.

Concepts & Notation

- (Sec. 2.1) *vector*, *vector space*, *linear combination*.
- (Sec. 2.2) *subspace*, *subspace spanned by a set of vectors*.
- (Sec. 3.3) *linearly independent set of vectors*, *basis*.

1. What is a *linear combination* of vectors $\alpha_1, \alpha_2, \ldots, \alpha_n$ in a vector space V over a field $\mathbb{F}$?

2. What is a *subspace* of a vector space V?

3. Show that $W = \{(x, 0) : x \in \mathbb{R}^2\}$ is a subspace of $\mathbb{R}^2$.

4. Let $W = \{(x, 0) : x \in \mathbb{R}^2\}$. Show that, for every $\alpha, \beta \in W$ and $c \in \mathbb{R}$, $c\alpha + \beta \in W$.

5. Let V is a vector space over a field $\mathbb{F}$, and $W \subseteq V$. **Show:** If, for every $\alpha, \beta \in W$ and $c \in \mathbb{F}$, $c\alpha + \beta \in W$, then W is a subspace of V.

If V is a vector space over a field $\mathbb{F}$ and $S \subset V$, the *subspace spanned by* S is the intersection of all subspaces of V containing S; if $S = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ then *say* it is the *subspace spanned by* $\alpha_1, \alpha_2, \ldots, \alpha_n$.

6. Find the subspace spanned by $\alpha = (1, 0)$ in $\mathbb{R}^2$.

7. Let $\alpha_1 = (1, 0), \alpha_2 = (0, 1) \in \mathbb{R}^2$. Show that, if $c_1\alpha_1 + c_2\alpha_2 = (0, 0)$ then $c_1 = c_2 = 0$.

Vectors $\alpha_1, \ldots, \alpha_n$ in a vector space V over a field $\mathbb{F}$ are *linearly independent* if $c_1\alpha_1 + \ldots c_n\alpha_n = 0$ $(c_i \in \mathbb{F})$ implies $c_1 = \ldots c_n$. If $\alpha_1, \ldots, \alpha_n$ are not linearly independent then they are *linearly dependent.*

8. Show $\alpha_1 = (1, 0), \alpha_2 = (0, 1) \in \mathbb{R}^2$ are linearly independent.