Last name ______________________

First name ______________________

LARSON—MATH 610–CLASSROOM WORKSHEET 17
Inner Products.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence.
(Chp. 2).pivot column decomposition, direct sum $\mathcal{U}\oplus\mathcal{V}$, *orthogonal* matrix, *unitary* matrix, *basis*, *dimension*, *linear transformation*.
(Chp. 3). *conformable* matrix addition and multiplication.
(Chp. 4). *nullity*, $A\oplus B$.
(Chp. 5). *inner product*, *inner product space*, $\langle\ \rangle$, *orthogonal* vectors, $\perp$, $||\cdot||$.

Review:

1. (**Theorem 4.2.1**). Let $A\in\mathbb{M}_{m\times n}(\mathbb{F})$. If $X\in\mathbb{M}_{p\times m}(\mathbb{F})$ has full column rank and $Y\in\mathbb{M}_{n\times q}(\mathbb{F})$ has full row rank then

$$rank(A) = rank(XAY).$$

Chp. 5 of Garcia & Horn, Matrix Mathematics

1. What is an *inner product* on a $\mathbb{C}$-vector space? What is the notation $\langle\ \rangle$?

2. What is an example?

3. What is an *inner product space*?

4. When are vectors *orthogonal*? What is the notation $\hat{u}\perp\hat{v}$?

5. What is a *norm* induced by an inner product? What is the notation $||\cdot||$?

6. What is a *unit* vector in an inner product space?

7. What properties does an inner product in a $\mathbb{C}$-vector space have?

8. What is the **Pythagorean Theorem** in an inner product space?

The *projection* of $\hat{v}$ onto $\hat{u}$ is

$$\hat{x} = \langle \hat{v}, \frac{\hat{u}}{||\hat{u}||} \rangle \frac{\hat{u}}{||\hat{u}||}.$$

9. Find the projection of $\hat{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ on $\hat{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

10. Check that $\hat{v} - \hat{x}$ is orthogonal to $\hat{u}$ (and also to $\hat{x}$).

(**Theorem 5.4.14. Cauchy-Schwartz**). Let $\mathcal{V}$ be an inner product space with inner product $\langle\ ,\ \rangle$, and derived norm $||\ ||$. Then

$$|\langle \hat{u}, \hat{v} \rangle| \leq ||\hat{u}|| \cdot ||\hat{v}||.$$

11. Use Cauchy-Schwartz to show that, for $x_1, \ldots, x_n \in \mathbb{R}$ that $(\sum_{i=1}^{n} x_i)^2 \leq n \sum_{i=1}^{n} x_i^2$.