**Last name** ______________________

**First name** ______________________

# LARSON—MATH 601—CLASSROOM WORKSHEET 07
## Vector Spaces.

### Concepts & Notation

- (Sec. 1.5) *column matrix* $B_j$, *elementary matrix.*
- (Sec. 1.6) *left inverse*, *right inverse*, *invertible matrix*, *inverse* $A^{-1}$.
- (Sec. 2.1) *vector*, *vector space.*

1. (**Homework:**) Show that *any* subfield of the complex numbers $\mathbb{C}$ contains the rational numbers.

### Elementary Matrices

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. $A$ is a square $(n \times n)$ matrix. Argue that the following statement are equivalent:
   (a) That $A$ is invertible.
   (b) That the reduced matrix $R$ is the $(n \times n)$ identity matrix.
   (c) That $A$ is a product of elementary matrices.

### Vector Spaces

3. What is the prototypical example of a *vector space*?

4. What is the *formal* definition of a vector space?

5. What is a *vector*?

**Examples of vector spaces**

What needs to be checked in the following examples?

6. Any field $\mathbb{F}$ can be viewed as a vector space over itself.

7. Let $\mathbb{R}^n$ be the set of tuples $(a_1, a_2, \ldots, a_n)$ $(a_i \in \mathbb{R})$. Then $\mathbb{R}^n$ is a vector space over $\mathbb{R}$.

8. The space of a functions from a set $S$ to a field $\mathbb{F}$ is a vector space.

9. The complex numbers $\mathbb{C}$ over $\mathbb{R}$ (with scalar multiplication by real numbers specifically—and **not** by complex numbers generally).

10. What is a **linear combination** of vectors $\alpha_1, \alpha_2, \ldots, \alpha_n$ in a vector space $V$ over a field $\mathbb{F}$?