Last name _____

First name _____

LARSON—MATH 550—CLASSROOM WORKSHEET 02
Towers of Hanoi. Lines in the Plane.

Concepts & Notation

- (Chapter 1) T_n, recurrence (recurrence relation), mathematical induction, basis, solving recurrences

Towers of Hanoi

1. Let T_n be the minimum number of moves to solve the n disk Towers of Hanoi problem. Find T_1.

2. Explain why $T_n \leq 2T_{n-1} + 1$.

3. Explain why $T_n \geq 2T_{n-1} + 1$.

4. Explain why $T_n = 2T_{n-1} + 1$.

5. What is the *recurrence* for T_n?

6. Use the recurrence for T_n to find T_4, T_5 and T_6.

7. *Solve* the recurrence for T_n.

8. *Prove* the closed formula for T_n.

Lines in the Plane

9. What is the maximum number of regions defined by n lines in the plane? Try the methodology developed in the Towers of Hanoi problem

   (a) *Name* the quantity you want to count/investigate.

   (b) Find some values of that quantity.

   (c) Find a recurrence relation for that quantity.

   (d) Use the recurrence to find more values of that quantity.

   (e) Use these values to *guess* a (non-recurrence closed-form) formula for that quantity.

   (f) *Prove* your formula.