Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 25
Polynomials.

Concepts & Notation

- (Sec. 3.5) *linear functional*, *trace*, *dual space*, V^*, *dual basis*, *annihilator*.
- (Sec. 4.1) *linear algebra*, $\mathbb{F}^\infty$, *algebra of formal power series*.
- (Sec. 4.2) $\mathbb{F}[x]$, *degree*, *scalar* polynomial, *monic* polynomial..

1. If V is a vector space over a field $\mathbb{F}$ and $S \subseteq V$, what is the *annihilator* of S?

2. What is $\mathbb{F}^\infty$? Let $f, g \in \mathbb{F}^\infty$. How is fg defined?

3. What is $1 \in \mathbb{F}^\infty$?

4. What is $x \in \mathbb{F}^\infty$?

5. What is $x^2 \in \mathbb{F}^\infty$?

6. What is $\mathbb{F}[x]$?

7. What is the *degree* of $f \in \mathbb{F}[x]$?

8. What is a *scalar* polynomial? What is a *monic* polynomial?

9. (**Claim:**) If f and g are non-zero polynomials over a field $\mathbb{F}$ then:
   (a) fg is a non-zero polynomial;
   (b) $\deg(fg) = \deg(f) + \deg(g)$;
   (c) fg is a monic polynomial if and only if both f and g are monic polynomials;
   (d) fg is a scalar polynomial if and only if both f and g are scalar polynomials;
   (e) if $f + g \neq 0$ then $\deg(f + g) \leq \max\{\deg(f), \deg(g)\}$.