${f Last\ name}$		
First name		

LARSON—MATH 601—CLASSROOM WORKSHEET 15 Coordinates, Row Equivalence.

Concepts & Notation

- (Sec. 2.4) ordered basis, coordinates, coordinate matrix, $[\alpha]_{\mathcal{B}}$.
- (Sec. 2.5) row rank.

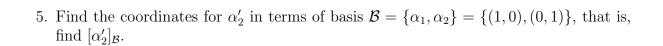
Basis Extension Theorem: Any linearly independent set in a finite-dimensional vector space V is part of (can be extended to) a (finite) basis for V.

1. $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(1, 0), (0, 1)\}$ is a basis for \mathbb{R}^2 . Let $\alpha = (2, 3)$. Find x_1, x_2 such that $\alpha = x_1\alpha_1 + x_2\alpha_2$. Find the coordinate matrix $[\alpha]_{\mathcal{B}}$.

2. Let $\mathcal{B}' = \{\alpha'_1, \alpha'_2\} = \{(\cos \theta, \sin \theta), (-\sin \theta, \cos \theta)\}$ (θ is a fixed real number). Show \mathcal{B}' is a basis for \mathbb{R}^2 .

3. Let $\alpha = (2,3)$. Find x_1', x_2' such that $\alpha = x_1'\alpha_1' + x_2'\alpha_2'$. Find the coordinate matrix $[\alpha]_{\mathcal{B}'}$.

4. Find the coordinates for α'_1 in terms of basis $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(1, 0), (0, 1)\}$, that is, find $[\alpha'_1]_{\mathcal{B}}$.



- 6. Let P be the matrix whose columns are $[\alpha'_1]_{\mathcal{B}}$ and $[\alpha'_2]_{\mathcal{B}}$. Check that $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$.
- 7. Why does that construction work?
- 8. Does P depend on α ?
- 9. Argue that P is invertible.
- 10. **Explain:** For a vector space V with bases \mathcal{B} and \mathcal{B}' , and vector $\alpha \in V$, there is an invertible matrix P such that $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$.
- 11. Explain how you can find a matrix P' so that $[\alpha]_{\mathcal{B}'} = P'[\alpha]_{\mathcal{B}}$. What is P'?

- 12. What is the row space of a matrix? Why is it a vector space? Describe the row space of $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.
- 13. What is the *row rank* of a matrix? Find the row rank of $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.