

Last name \_\_\_\_\_

First name \_\_\_\_\_

**LARSON—MATH 310—HOMEWORK WORKSHEET 02**  
**First Concepts from Linear Algebra.**

1. What are the real numbers  $\mathbb{R}$ ?
2. What is a *field*?
3. What are the *complex numbers*?
4. Find the multiplicative inverse of  $3 + 4i$ ? Explain.
5. Why are the complex numbers a field?
6. What is  $\mathbb{R}^2$ ?
7. How can we interpret the *vectors* in  $\mathbb{R}^2$  geometrically?
8. Explain addition in  $\mathbb{R}^2$  geometrically. Consider  $\vec{v} + \vec{w}$ , where  $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$  and  $\vec{w} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$
9. Explain scalar multiplication in  $\mathbb{R}^2$  geometrically. Consider  $c\vec{v}$  where  $c = 4$  and  $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ .
10. What is  $\mathbb{K}$ ?
11. What is a *linear space* (or *vector space*) over a field  $\mathbb{K}$ ?
12. Argue that  $\mathbb{R}^2$  is a vector space.
13. What is  $\mathbb{K}^n$ ?
14. What is a *subspace* of a vector space? Give an example.
15. What is a *linear map* (or *linear transformation*)  $f : V \rightarrow W$ , from linear space  $V$  to linear space  $W$ ?
16. Consider the linear map  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  where  $f\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = v_1$ , for every  $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ . Find the *kernel* of  $f$ .
17. Consider the linear map  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  where  $f\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = v_1$ , for every  $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ . Find the *range* of  $f$ .