Last name _____

First name _____

LARSON—MATH 610–CLASSROOM WORKSHEET 14
Block Matrices.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence, pivot column decomposition, direct sum $\mathcal{U}\oplus\mathcal{V}$, *orthogonal* matrix, *unitary* matrix, *basis*, *dimension*, *linear transformation*, *conformable* matrix addition and multiplication.

Chp. 3 of Garcia & Horn, Matrix Mathematics

1. Check that:
$$AB = \begin{bmatrix} A_1 & A_2 \end{bmatrix} \begin{bmatrix} B_1^T \\ B_2^T \end{bmatrix} = A_1 B_1^T + A_2 B_2^T,$$
assuming all matrices are conformal.

2. What is the *inner product* of vectors $\hat{x}$ and $\hat{y}$?

3. What is the *outer product* of vectors $\hat{x}$ and $\hat{y}$?

4. Write a formula for the product AB in terms of an *outer product* of the columns of A and the rows of B.

5. Suppose A and B are invertible. Show $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$.

6. (**Notation**) How is the *direct sum* $A \oplus B$ defined?

Chp. 4 of Garcia & Horn, Matrix Mathematics

7. What is the **Dimension Theorem** for linear transformations?

8. What is the **Rank-Nullity** Theorem (for matrices)?