**Last name** _____

**First name** _____

## LARSON—MATH 310–CLASSROOM WORKSHEET 12
## Matrices.

**Review**: $\mathbb{R}$, *field*, *complex numbers*, $\mathbb{R}^2$, $\mathbb{K}$, $\mathbb{K}^n$, *linear space* (or *vector space*), *subspace*, *linear map* (or *linear transformation*), *kernel*, *range*, *linear combination*, subspace *generated* by (or *spanned* by) a set of vectors, $\langle A \rangle$, *finite-dimensional vector space*, *linearly independent* set of vectors, *linearly dependent* set of vectors, *basis* of linear space, *dimension*, *rank* of a collection of vectors, $M_{\mathbb{K}}(m,n)$.

**Review**.

1. If $A \in M_{\mathbb{K}}(m,n)$ and $B \in M_{\mathbb{K}}(n,p)$, how is $AB$ defined?

2. What is the *dot product* $\hat{x} \cdot \hat{y}$ of vectors $\hat{x}, \hat{y} \in \mathbb{K}^n$?

3. What is the $(i,j)$ element of $AB$? (What is a formula?)

4. Is matrix multiplication *commutative*?

### Properties of Matrix Multiplication

1. Why is matrix multiplication *associative*?

2. Why does matrix multiplication *distribute* over addition?

3. What does it mean for a matrix $A$ to have an inverse?

4. What is an example?

5. What is the notation for "A-inverse" (in the case $A$ is *invertible*)?

6. (**Lemma**) If $X = \vec{v}_1, \ldots, \vec{v}_n$ is a basis for a vector space $V$, and $\vec{v} \in V$, then there are unique scalars $c_1, \ldots, c_n$ so that $\vec{v} = c_1 \vec{v}_1 + \ldots + c_n \vec{v}_n$.

7. Let $\vec{v}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ is a basis for $\mathbb{R}^2$. Let $\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.
   Find scalars $c_1, c_2$ so that $\vec{v} = c_1 \vec{v}_1 + c_2 \vec{v}_2$.

8. Find the inverse of $A = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$.

9. **Argue:** For a square matrix $A$ of order $n$ (so $A \in \mathbb{M}_{\mathbb{K}}(n, n)$), $A$ is invertible if and only if the columns of $A$ are linearly independent.