Last name ______________________

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LARSON—MATH 610—CLASSROOM WORKSHEET 22
Linear Transformations.

Concepts & Notation

- (Sec. 3.1) *linear transformation*, *range*, *rank*, *null space*, *nullity.*
- (Sec. 3.2) $L(V, W)$, *linear operator*, *invertible* linear transformation, *non-singular* linear transformation.
- (Sec. 3.3) *isomorphism.*
- (Sec. 3.4) *matrix of* T *relative to (ordered) bases*, *similar matrices.*
- (Sec. 3.5) *linear functional.*

Review

1. (**Claim:**) If V and W are finite-dimensional vector spaces over a field $\mathbb{F}$ with $\dim V = \dim W$, and $T : V \to W$ is a linear transformation then the following are equivalent:

   (a) T is invertible,

   (b) T is non-singular,

   (c) T is onto (that is, the range of T is W).

New

2. If T an *isomorphism* of a vector space V onto a vector space W, why is T invertible and non-singular?

3. (**Claim:**) Every linear transformation T from an n-dimensional vector space V to an m-dimensional vector space W can be represented by a matrix A (with respect to specific bases for V and W; in particular, different bases yield different A's).

   We argued that if the bases are $\mathcal{B} = \{\alpha_1, \ldots, \alpha_n\}$ for V and $\mathcal{B}'$ for W, and $\alpha \in V$ and:

$$[T(\alpha)]_{\mathcal{B}'} = A[\alpha]_{\mathcal{B}},$$

   then:

$$A = [[T(\alpha_1)]_{\mathcal{B}'}[T(\alpha_2)]_{\mathcal{B}'} \ldots [T(\alpha_n)]_{\mathcal{B}'}].$$

   (**Notation:**) If V is a finite-dimensional vector space with basis $\mathcal{B} = \{\alpha_1, \ldots, \alpha_n\}$, and T is a linear transformation from V to itself, then $[T]_{\mathcal{B}}$ is the matrix A in:

$$[T(\alpha)]_{\mathcal{B}} = A[\alpha]_{\mathcal{B}}.$$

4. (**Example:**) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (x_1, 0)$. Let $\mathcal{B} = \{(1, 0), (0, 1)\}$ be the standard basis for $\mathbb{R}^2$. Find $[T]_{\mathcal{B}}$.

5. (**Example:**) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (x_1, 0)$. $\mathcal{B}' = \{(1, 1), (2, 1)\}$ is basis for $\mathbb{R}^2$. Find $[T]_{\mathcal{B}'}$.

6. What is the relationship between $[T]_{\mathcal{B}}$ and $[T]_{\mathcal{B}'}$?