**Last name** _____

**First name** _____

**LARSON—MATH 550—HOMEWORK WORKSHEET h13**
**Fibonacci Numbers!**

**Fibonacci Numbers**

We defined $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$.

1. (**Kepler/Cassini**). Check that $F_{n+1}F_{n-1} - F_n^2 = (-1)^n$ holds for small values of $n$.

2. (**Kepler/Cassini**). Use induction to show that $F_{n+1}F_{n-1} - F_n^2 = (-1)^n$ $(n \in \mathbb{Z}^{\geq 0})$.

3. Check that $F_{n+k} = F_k F_{n+1} + F_{k-1}F_n$ holds for small values of $k$.

4. Use induction to show that $F_{n+k} = F_k F_{n+1} + F_{k-1}F_n$ $(k \in \mathbb{Z}^{\geq 0})$.

5. Set $k = n$ and argue that $F_{2n}$ is a multiple of $F_n$. (Is it really true? You should also check some concrete examples).

**Bonus**

6. Use induction to show that $F_{kn}$ is a multiple of $F_n$ for $k \in \mathbb{Z}^{\geq 0}$.