Last name ______________________

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LARSON—MATH 610—CLASSROOM WORKSHEET 23
Linear Functionals.

Concepts & Notation

- (Sec. 3.1) *linear transformation*, *range*, *rank*, *null space*, *nullity.*
- (Sec. 3.2) $L(V, W)$, *linear operator*, *invertible* linear transformation, *non-singular* linear transformation.
- (Sec. 3.3) *isomorphism.*
- (Sec. 3.4) *matrix of T relative to (ordered) bases*, *similar matrices.*
- (Sec. 3.5) *linear functional*, *trace*, *dual space*, V^*, *dual basis*, *annihilator.*

1. What is a *linear functional* on a vector space V over a field $\mathbb{F}$?

2. (**Example**) What is the *trace* $tr(A)$ of an $n \times n$ matrix A? **tr** is a linear functional on the vector space of $n \times n$ matrices over a field $\mathbb{F}$.

3. (**Example**) Let V be the vector space of polynomial functions from a vector space V (over a field $\mathbb{F}$) to $\mathbb{F}$. Let $t \in \mathbb{F}$. $L_t(p) = p(t)$ is a linear functional on V $(p \in V)$.

4. (**Example**) Let $C([0,1])$ be the vector space of continuous real-valued functions on the interval $[0,1]$. Then $L(g) = \int_0^1 g(t)dt$ is a linear functional on $C([0,1])$ $(g \in C([0,1])$.

5. What is the *dual space* V^*.

6. (**Claim:**) If V is finite-dimensional then $\dim V = \dim V^*$.

7. If $\mathcal{B} = \{\alpha_1, \ldots, \alpha_n\}$ is a basis for a vector space V, what is the *dual basis* $\mathcal{B}^*$?

Let V be a finite-dimensional vector space over a field $\mathbb{F}$ with basis $\mathcal{B} = \{\alpha_1, \ldots, \alpha_n\}$ and dual basis $\mathcal{B}^* = \{f_1, \ldots, f_n\}$.

8. (**Claim:**) For every linear functional f on V:

$$f = \sum_1^n f(\alpha_i) f_i.$$

9. (**Claim:**) For every vector $\alpha \in V$:

$$\alpha = \sum_1^n f_i(\alpha) \alpha_i.$$

10. If V is a vector space over a field $\mathbb{F}$ and $S \subseteq V$, what is the *annihilator* of S?