Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 14
Linear independence, Bases, Dimension.

Concepts & Notation

- (Sec. 2.1) *vector*, *vector space*, *linear combination*.
- (Sec. 2.2) *subspace*, *subspace spanned by a set of vectors*, *span*.
- (Sec. 2.3) *linearly dependent/independent set of vectors*, *basis*, *dimension*.
- (Sec. 2.4) *ordered basis*, *coordinates*, *coordinate matrix*, $[\alpha]_{\mathcal{B}}$.

A vector space V with a finite basis is **finite dimensional**.

We showed: If if V is a finite-dimensional vector space, then every basis has the same number of elements.

The **dimension** of a finite-dimensional vector space V is the number of elements in any basis of V and is denoted $\dim V$.

We showed: Let S be a linearly independent subset of a vector space V. If β is not in the subspace spanned by S then the set obtained by adding β to S is linearly independent.

1. **Claim:** Any linearly independent set in a finite-dimensional vector space V is part of (can be extended to) a (finite) basis for V.

Coordinates!

2. $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(1,0),(0,1)\}$ is a basis for $\mathbb{R}^2$. Let $\alpha = (2,3)$. Find x_1, x_2 such that $\alpha = x_1\alpha_1 + x_2\alpha_2$.

3. Argue that x_1 and x_2 are unique.

4. $\mathcal{B}' = \{\alpha_1', \alpha_2'\} = \{(1,0),(1,1)\}$ is a basis for $\mathbb{R}^2$. Let $\alpha = (2,3)$. Find x_1', x_2' such that $\alpha = x_1'\alpha_1' + x_2'\alpha_2'$.

5. What are the *coordinates* of a vector α in a vector space V with respect to an ordered basis $\mathcal{B} = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$? What is the *coordinate matrix* $[\alpha]_{\mathcal{B}}$?

6. $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(1,0),(0,1)\}$ is a basis for $\mathbb{R}^2$. Let $\alpha = (2,3)$. Find $[\alpha]_{\mathcal{B}}$.

7. $\mathcal{B}' = \{\alpha_1', \alpha_2'\} = \{(1,0),(1,1)\}$ is a basis for $\mathbb{R}^2$. Let $\alpha = (2,3)$. Find $[\alpha]_{\mathcal{B}'}$.

Claim: There is an invertible matrix P such that $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$. We will find P.

8. Find the coordinates for α_1' in terms of basis $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(1,0),(0,1)\}$, that is, find $[\alpha_1']_{\mathcal{B}}$.

9. Find the coordinates for α_2' in terms of basis $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(1,0),(0,1)\}$, that is, find $[\alpha_2']_{\mathcal{B}}$.

10. Let P be the matrix whose columns are $[\alpha_1']_{\mathcal{B}}$ and $[\alpha_2']_{\mathcal{B}}$. Check that $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$.

11. Why does that construction work?

12. Argue that P is invertible.