Last name ________________________

First name ________________________

LARSON—MATH 310—CLASSROOM WORKSHEET 22
Determinants, Eigenvalues and Eigenvectors.

Review

The *determinant* of a square triangular matrix is the product of its diagonal entries.

Facts about a square matrix A. The following statements are all equivalent!

- RREF of A has a zero row.
- Any triangular matrix derived from A has a 0 on the diagonal.
- The rows of A are linearly dependent.
- A does not have an inverse.
- $\det(A) = 0$.

Determinant Computation Rules

- The determinant of a square matrix A equals the determinant of any matrix formed by a pivot operation.
- The determinant of a square matrix A equals *negative* the determinant of any matrix formed by switching 2 rows.

Let $A = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$.

If there is a non-zero vector $\vec{x}$ and scalar λ with $A\vec{x} = \lambda\vec{x}$ then λ is an *eigenvalue* of A and $\vec{x}$ is a corresponding *eigenvector*.

If $A\vec{x} = \lambda\vec{x}$, then $A\vec{x} - \lambda\vec{x} = 0$, and $(A - \lambda I)\vec{x} = 0$.

Since $\vec{x}$ is non-zero that means that $(A - \lambda I)$ is not invertible, that the RREF has a 0-row, and that $\det(A) = 0$.

1. Find $(A - \lambda I)$.

2. Use the 2×2 determinant formula to find $\det(A - \lambda I)$.
   (λ is a variable—so your answer will have λs in it).

3. Solve $\det(A - \lambda I) = 0$.

4. For each solution λ, write the equation $(A - \lambda I)\vec{x} = 0$, and solve for $\vec{x}$.

5. Check that your eigenvalue-eigenvector pairs work!