Last name ______________________

First name ______________________

LARSON—MATH 610–CLASSROOM WORKSHEET 02
Vector Spaces and Subspaces.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence, pivot column decomposition, direct sum $\mathcal{U} \oplus \mathcal{V}$.

Review: *invertible* matrix, *conjugate* of a matrix, $\bar{A}$, *conjugate transpose* of a matrix, A^*, *symmetric* matrix, *Hermitian* matrix, *normal* matrix.

Think: symmetric $\subseteq$ Hermitian $\subseteq$ normal.

1. What is a *vector space*?

2. What is a *subspace* of a vector space?

(**Theorem 1.3.3**). Let $\mathcal{V}$ be an $\mathbb{F}$-vector space and let $\mathcal{U}$ be a non-empty subset of $\mathcal{V}$. Then U is a subspace of $\mathcal{V}$ if and only if $cu + v \in \mathcal{U}$ whenever $u, v \in \mathcal{U}$ and $c \in \mathbb{F}$.

3. Why is this theorem true?

(**Theorem 1.4.10**). Let $Y = [\hat{y}_1 \hat{y}_2 \ldots \hat{y}_p] \in \mathbb{M}_{m\times p}(\mathbb{F})$ and let $A \in \mathbb{M}_{m\times n}(\mathbb{F})$ then $col(Y) \subseteq col(A)$ if and only if $Y = AX$ for some $X \in \mathbb{M}_{n\times p}$.

4. Why is this theorem true?

5. What is an *orthogonal* matrix?

6. What is a *unitary* matrix?

7. (**Theorem 1.5.9**). Let $\mathcal{U}$ and $\mathcal{W}$ be subspaces of an $\mathbb{F}$-vector space $\mathcal{V}$ and suppose that $\mathcal{U} \cap \mathcal{W} = \{0\}$. Then each vector in $\mathcal{U} \oplus \mathcal{W}$ is uniquely expressible as a sum of a vector in $\mathcal{U}$ and a vector in $\mathcal{W}$.

8. Why is this theorem true?