LARSON—MATH 601—CLASSROOM WORKSHEET 05
Review.

Concepts & Notation

- (Sec. 1.5) *column matrix B_j, elementary matrix.*

- (Sec. 1.6) *left inverse, right inverse, invertible matrix, inverse A^{-1}.*

- (Sec. 2.1) *vector, vector space.*

Elementary Matrices

An elementary matrix E is one where left multiplication by E has the same effect as some row operation.

1. Suppose a matrix A can be row-reduced to a matrix R (in row-reduced echelon form) by row operations $\epsilon_1, \ldots, \epsilon_k$, corresponding to elementary matrices $E_1, \ldots, E_k$. Write R in terms of these matrices.

2. We argued that elementary matrices E_i are invertible (and thus have inverses E_i^{-1}). Write A in terms of these matrices.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

3. Show

   (a) That the reduced matrix R is the (3×3) identity matrix.

   (b) That A is a product of elementary matrices.

   (c) That A is invertible. Find A^{-1}.

$$B = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

4. Show

    (a) That the reduced matrix R is *not* the (3×3) identity matrix.

    (b) That B is *not* a product of elementary matrices.

    (c) That B is *not* invertible.

5. A is a square $(n \times n)$ matrix. Argue that the following statement are equivalent:

    (a) That A is invertible.

    (b) That the reduced matrix R is the $(n \times n)$ identity matrix.

    (c) That A is a product of elementary matrices.

Vector Spaces

6. What is the prototypical example of a *vector space*?

7. What is the *formal* definition of a vector space?

8. What is a *vector*?

9. What are some examples of vector spaces?