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First name _____

LARSON—MATH 310—CLASSROOM WORKSHEET 03
Mathematical Background.

Review

1. What is a *set*?
2. What is a *subset* of a set? (What does $S_1 \subseteq S_2$ mean?)
3. What is the *Cartesian product* of sets S_1 and S_2 ?
4. What is a *function*? (What does $f : D \rightarrow F$ mean?)

Chapter 1 of Klein's *Coding the Matrix* text

1. What are the real numbers \mathbb{R} ?
2. What is a *field*?
3. What are the *complex numbers*?
4. Why do non-zero complex numbers have a multiplicative inverse?
5. Why are the complex numbers a field?

6. How can we think of complex numbers *geometrically*?

7. What is the *absolute value* of a complex number $a + bi$?

8. How can we think of multiplication of a complex number by a scalar geometrically?

9. How can we think of multiplication of a complex number by i geometrically?

Chapter 2 of Klein's *Coding the Matrix* text

Definition. For a field \mathbb{F} and a positive integer n , a vector with n entries, each belonging to \mathbb{F} , is called an n -vector over \mathbb{F} . The set of n -vectors over \mathbb{F} is \mathbb{F}^n .

10. What are examples of vectors?

Definition 2.2.2: For a finite set D and a field \mathbb{F} , a D -vector over \mathbb{F} is a function from D to \mathbb{F} .

This is a computer scientist's definition; it lends itself to representation in a data structure. It differs in two important ways from a mathematician's definition.

- I require the domain D to be finite. This has important mathematical consequences: we will state theorems that would not be true if D were allowed to be infinite. There are important mathematical questions that are best modeled using functions with infinite domains, and you will encounter them if you continue in mathematics.
- The traditional, abstract approach to linear algebra does not directly define vectors at all. Just as a field is defined as a set of values with some operations ($+$, $-$, $*$, $/$) that satisfy certain algebraic laws, a vector space is defined as a set with some operations that satisfy certain algebraic laws; then vectors are the things in that set. This approach is more general but it is more abstract, hence harder for some people to grasp. If you continue in mathematics, you will become very familiar with the abstract approach.

Returning to the more concrete approach we take in this book, according to the notation from Section 0.3.3, we use \mathbb{F}^D to denote the set of functions with domain D and co-domain \mathbb{F} , i.e. the set of all D -vectors over \mathbb{F} .