Last name ________________________

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LARSON—MATH 550—CLASSROOM WORKSHEET 24
Binomial Coefficients, Theorems, Tools.

Concepts & Notation

- Sec. 5.1: Binomial coefficients, the Binomial Theorem, key formulas, the polynomial argument, Vandermonde's convolution.
- Sec. 5.2: the Sorting example.
- Sec. 5.3: Duplication formula, inversion, derangements.
- Sec. 5.4: Convolutions, generating functions.

Review

1. $\binom{n}{m}$ is the number of m-subsets of an n-set (for $n, m \in \mathbb{Z}^{\geq 0}$).
2. Find a formula for $\binom{n}{m}$ $(0 \leq m \leq n,\ m, n \in \mathbb{Z})$.
3. Argue the *symmetry identity* $\binom{n}{k} = \binom{n}{n-k}$.
4. prove the *addition formula*:
$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}.$$
5. Find the sum of $\binom{n}{0}, \binom{n}{1}, \ldots \binom{n}{n}$.
6. Find $(x+y)^n$.

Generalizing the Binomial Theorem

The *binomial coefficients* $\binom{n}{k}$ $(n, k \in \mathbb{Z}^{\geq 0})$ can be generalized to $\binom{r}{k}$ $(r \in \mathbb{R}, k \in \mathbb{Z})$:

$$\binom{r}{k} = \frac{r^{\underline{k}}}{k!} \text{ (if } k \geq 0)$$

$$\binom{r}{k} = 0 \text{ (if } k < 0).$$

We proved the *absorbtion identity* $\binom{r}{k} = \frac{r}{k}\binom{r-1}{k-1}$.

1. Argue the *absorbtion identity* variation $k\binom{r}{k} = r\binom{r-1}{k-1}$.

2. The (Newton's Generalized) *Binomial Theorem* says $(x+y)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^k y^{r-k}$. Does this agree with our formula when $r \in \mathbb{Z}$?

3. How can we prove the special case $(x+1)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^k$?

4. Find an expression for $\sqrt{x+1}$.

5. Check it for $x = 3$, $x = 1$.