Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 28
Real Spectral Theorem.

Concepts & Notation

- (Chp. 6) *dot product*, *inner product*, *inner product space*, *norm*, *orthogonal representation*, *Cauchy-Schwartz*, *orthonormal list*, *Gram-Schmidt*, *orthogonal complement*, *orthogonal projection*.
- (Chp. 7) *adjoint*, *conjugate transpose*.

1. What is a *self-adjoint* linear operator (on an inner product space)?

2. Eigenvalues of self-adjoint operators are real.

3. What is the *real spectral theorem*?

4. Let $T \in \mathcal{L}(\mathbb{R}^3)$ be defined by $T(x_1, x_2, x_3) = (x_2, x_1 + x_3, x_2)$. Find the matrix for T with respect to the standard basis in $\mathbb{R}^3$.

5. What do you notice about this matrix?

6. Use the definition of the adjoint to check that $T = T^*$.

7. Find the eigenvalues of T and corresponding eigenvectors.

8. What do you notice about the eigenvalues of T?

9. What do you notice about the eigenvectors of T?

10. Normalize the eigenvectors of T and find the matrix of T with respect to this basis.

11. What is the connection to similar matrices?

12. What is the connection to graph theory?

13. What questions are there about the Test Review?