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LARSON—MATH 310—CLASSROOM WORKSHEET 22
Linear Dependence, Linear Independence, Basis, Dimension

Review: Chapter 4 of Klein's *Coding the Matrix* text

1. What is the *null space* of a matrix?

Matrix Inverses

1. What is the *inverse* of a matrix?
2. What are examples?
3. What is the importance of matrix inverses?

Chapter 5 of Klein's *Coding the Matrix* text

(**Question 5.2.5**) For a given vector space \mathcal{V} , what is the minimum number of vectors whose span equals \mathcal{V} ?

(**Definition 5.5.2**) Vectors $\hat{v}_1, \dots, \hat{v}_n$ are linearly dependent if the zero vector can be written as a nontrivial linear combination of the vectors:

$$0 = \alpha_1 \hat{v}_1 + \dots + \alpha_n \hat{v}_n$$

4. (**Example 5.5.3**) The vectors $[1, 0, 0]$, $[0, 2, 0]$, and $[2, 4, 0]$ are linearly dependent,

5. (**Example 5.5.4**) The vectors $[1, 0, 0]$, $[0, 2, 0]$, and $[0, 0, 4]$ are linearly independent.

(**Definition 5.6.1**) Let \mathcal{V} be a vector space. A *basis* for \mathcal{V} is a linearly independent set of generators for \mathcal{V} .

6. What are examples?

(**Lemma 5.7.1, Unique-Representation Lemma**) Let $\hat{a}_1, \dots, \hat{a}_n$ be a basis for a vector space \mathcal{V} . For any vector $\hat{v} \in \mathcal{V}$, there is exactly one representation of \hat{v} in terms of the basis vectors.

7. Why is Lemma 5.7.1 true?

(**Lemma 5.11.1, Exchange Lemma**) Suppose S is a set of vectors and A is a subset of S . Suppose \hat{z} is a vector in $\text{Span } S$ and not in A such that $A \cup \{\hat{z}\}$ is linearly independent. Then there is a vector $\hat{w} \in S - A$ such that $\text{Span } S = \text{Span } (\{z\} \cup S - \{\hat{w}\})$.

8. Why is Lemma 5.11.1 true?

(**Theorem 6.1.2, Basis Theorem**) Let \mathcal{V} be a vector space. All bases for \mathcal{V} have the same size.

9. Why is Theorem 6.1.2 true?