

LARSON—MATH 255—CLASSROOM WORKSHEET 21
Problems & Interacts

1. (a) Start the Chrome browser.
(b) Go to `http://cocalc.com`
(c) You should see an existing Project for our class. Click on that.
(d) Click “New”, then “Sage Worksheet”, then call it **c21**.
(e) For each problem number, label it in the SAGE cell where the work is. So for Problem 1, the first line of the cell should be **#Problem 1**.

Interacts

Here's one more example of a CoCalc INTERACT.

2. Try to figure out what the following Sage Interact will do. Then type it in and try it.

```
x0 = 0
f = sin(x)*e^(-x)
p = plot(f,-1,5)
dot = point((x0,f(x=x0)),pointsize=80,color="red")
@interact
def i_taylor(order=(5,(1..12))):
    ft = f.taylor(x,x0,order)
    print("The function is {}".format(f))
    print("The order-{} Taylor series at x={} is {}".format(order,x0,ft))
    pt = plot(ft,-1, 5, color="green")
    show(dot + p + pt, ymin = -.5, ymax = 1)
```

Problems

3. (**Euclid**) Let p_1, p_2, \dots, p_k be a list of any k primes. The product p of these primes plus one is

$$p = p_1 \cdot p_2 \cdot \dots \cdot p_k + 1$$

p is either a prime (different from each of these k primes) or it has a prime factor q also different from each of p_1, p_2, \dots, p_k (If p is prime then q is just p). This is the main idea in Euclid's proof that there are infinitely many primes. Write a program to find this prime number q given any list, p_1, p_2, \dots, p_k , of primes.

4. $n!$ means $n \times (n-1) \times \dots \times 3 \times 2 \times 1$. For example, $10! = 10 \times 9 \times \dots \times 3 \times 2 \times 1 = 3628800$, and the sum of the digits in the number $10!$ is $3 + 6 + 2 + 8 + 8 + 0 + 0 = 27$. Find the sum of the digits in the number $100!$ (100-factorial).

5. (**Goldbach**). Goldbach conjectured that every even number larger than 2 is the sum of two primes. So $4 = 2 + 2$, $6 = 3 + 3$, $8 = 3 + 5$, etc. Despite much work (with real progress in the last 100 years) the conjecture remains unresolved (open). It is known to be true up to some ginormous n .

Write a program `goldbach(n)` that takes an even integer greater than 2 as input and returns two primes p_1 and p_2 so that $n = p_1 + p_2$.

Getting your classwork recorded

When you are done, before you leave class...

1. Click the “Make pdf” (Adobe symbol) icon and make a pdf of this worksheet. (If CoCalc hangs, click the printer icon, then “Open”, then print or make a pdf using your browser).
2. Send me an email with an informative header like “Math 255 - c21 worksheet attached” (so that it will be properly recorded).
3. Remember to attach today’s classroom worksheet!