Last name ________________________

First name ________________________

LARSON—OPER 731—CLASSROOM WORKSHEET 11
Duality!

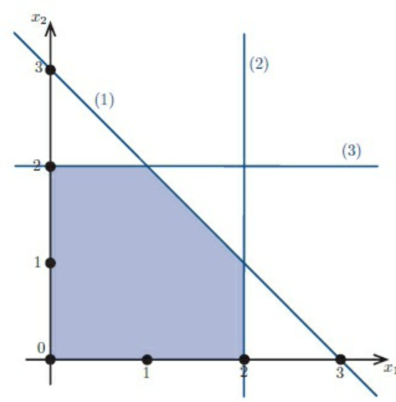
Concepts

- (Sec. 2.4) *basis*, *basic* variable, *nonbasic* variable, *basic solution*, *basic feasible solution*, *canonical form.*
- (Sec. 2.8) *hyperplane*, *halfspace*, *line*, *line segment*, *convex*, *polyhedron*, *tight* inequality, *extreme point.*
- (Sec. 3.1) *dual LP*, *Weak duality theorem.*

Review

1. **Claim**: Let A be a matrix with linearly independent rows and b be a vector. Let $P = \{x : Ax = b, x \geq \mathbb{O}\}$ and let $\bar{x} \in P$. Then $\bar{x}$ is an extreme point of P if and only if $\bar{x}$ is a basic feasible solution of $Ax = b$.

$$
\begin{array}{ll}
\max & (c_1,\ c_2)x \\
\text{s.t.} & \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} x \leq \begin{pmatrix} 3 \\ 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}. \qquad \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \\ (5) \end{matrix}
\end{array}
$$



2. Find all extreme points.

3. **Claim**: For a polyhedron $P = \{x \in \mathbb{R}^n : Ax \leq b\}$, $x \in \mathbb{R}^n$, and $A^{=}x \leq b^{=}$ tight for $\bar{x}$, $\bar{x}$ is an extreme point of P if and only if $rank(A^{=}) = n$.

4. Find all the basic feasible solutions for $Ax = b$ in the example above.

Duality

5. Consider the LP: $\max\{c^T x : Ax \leq b, x \geq \mathbb{O}\}$.

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ -1 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 20 \\ 18 \\ 8 \end{pmatrix} \quad c = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

Find the dual. Find feasible solutions for the primal and dual. Use these to estimate the optimal value of the primal objective function.

6. What is the *Weak Duality Theorem*?

7. We will consider a shortest-path LP and investigate how the dual can be interpreted.