Last name ________________________

First name ________________________

LARSON—MATH 610–CLASSROOM WORKSHEET 15
Block Matrices.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence.
(Chp. 2).pivot column decomposition, direct sum $\mathcal{U} \oplus \mathcal{V}$, *orthogonal* matrix, *unitary* matrix, *basis*, *dimension*, *linear transformation*.
(Chp. 3). *conformable* matrix addition and multiplication.
(Chp. 4). *nullity*, $A \oplus B$.

Review:

1. $AB = \begin{bmatrix} A_1 & A_2 \end{bmatrix} \begin{bmatrix} B_1^T \\ B_2^T \end{bmatrix} = A_1 B_1^T + A_2 B_2^T$.
2. What is the *inner product* of vectors $\hat{x}$ and $\hat{y}$?
3. What is the *outer product* of vectors $\hat{x}$ and $\hat{y}$?
4. Write a formula for the product AB in terms of an *outer product* of the columns of A and the rows of B.
5. If A and B are invertible then $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$.

Chp. 4 of Garcia & Horn, Matrix Mathematics

1. (**Notation**) How is the *direct sum* $A \oplus B$ defined?
2. What is the **Dimension Theorem** for linear transformations?

3. What is the **Rank-Nullity** Theorem (for matrices)?

4. What is an example?

(**Theorem 4.2.1**). Let $A \in \mathbb{M}_{m\times n}(\mathbb{F})$. If $X \in \mathbb{M}_{p\times m}(\mathbb{F})$ has full column rank and $Y \in \mathbb{M}_{n\times q}(\mathbb{F})$ has full row rank then

$$rank(A) = rank(XAY).$$

5. What does this theorem say?

6. What is an example?

7. Why is it true?