

Last name \_\_\_\_\_

First name \_\_\_\_\_

**LARSON—MATH 550—CLASSROOM WORKSHEET 15**  
**Ceiling, floor & more (notation).**

**Concepts & Notation**

- Sec. 2.6.  $\Delta$  operator. Rising and falling factorials.
- Sec. 3.1. floor and ceiling notation.
- Sec. 3.2. interval notation.
- Sec. 3.4. mod notation.
- Sec. 4.4. Analyzing the size of  $n!$
- Sec. 5.1. Binomial coefficients!

**Homework**

1. Define  $x^{\overline{m}}$ . Find  $5^{\overline{3}}$ .
  
  
  
  
  
  
  
  
  
  
2. Define  $x^{\overline{m}}$ . Find  $5^{\overline{3}}$ .
  
  
  
  
  
  
  
  
  
  
3. What are  $\lceil x \rceil$  and  $\lfloor x \rfloor$ ?
  
  
  
  
  
  
  
  
  
  
4. Claim:  $\lceil x \rceil = x \Leftrightarrow x$  is an integer  $\Leftrightarrow \lfloor x \rfloor = x$
  
  
  
  
  
  
  
  
  
  
5. Claim:  $\lceil x \rceil - \lfloor x \rfloor = [x \text{ is not an integer}]$
  
  
  
  
  
  
  
  
  
  
6. Claim:  $x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$ .

7. Claim:  $\lfloor x \rfloor = n \Leftrightarrow n \leq x < n + 1$ .

8. Claim:  $\lfloor x + n \rfloor = \lfloor x \rfloor + n$  (for  $n \in \mathbb{Z}, x \in \mathbb{R}$ ).

9. Does  $\lfloor nx \rfloor = nx$ ?

10. What are  $[\alpha..\beta], [\alpha..\beta)$  (for  $\alpha, \beta \in \mathbb{R}$ )?

11. How many integers are in  $[\alpha..\beta)$  (for  $\alpha, \beta \in \mathbb{R}$ )?

The *quotient* of positive integers  $n$  and  $m$  is  $\lfloor n/m \rfloor$  and the *modulus* is the remainder of dividing  $n$  by  $m$ , denoted  $n \bmod m$ .

12. Find  $\lfloor 32/5 \rfloor$  and  $32 \bmod 5$  and check that  $32 = \lfloor 32/5 \rfloor \cdot 5 + (32 \bmod 5)$ .

13. State a general law for positive integers  $n$  and  $m$ .