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First name _____

LARSON—MATH 556—HOMEWORK WORKSHEET 09

Posets!

1. (**Divisibility poset**). Let $X = \{2, 3, \dots, 20\}$ and define the divisibility relation " $|$ ": For $x, y \in X$, $x|y$ (that is, x divides y , or y is divisible by x) if there is an integer k such that $kx = y$. We showed that $P = (X, |)$ is a poset.
 - Find a largest antichain in P .
 - Find a minimal chain decomposition in P .
 - Prove that your antichain is indeed maximal and your chain decomposition is indeed minimal.
2. Let $X = \{x_1, x_2, \dots, x_{101}\}$ be a sequence of 101 positive integers. Argue that X contains either an 11-term increasing subsequence or an 11-term decreasing subsequence. (**Hint:** Define an appropriate poset).
3. Let

$$A = \begin{pmatrix} 7/12 & 0 & 5/12 \\ 1/6 & 1/2 & 1/3 \\ 1/4 & 1/2 & 1/4 \end{pmatrix}$$

Use Birkhoff's algorithm to show that A is a convex combination of permutation matrices. (So for each iteration, you'll need to define a bipartite graph G_i , find a perfect matching M_i , a corresponding permutation matrix P_i and number m_i . If you get a 0 matrix after k iterations, you will have $A = m_1P_1 + \dots + m_kP_k$ and $m_1 + \dots + m_k = 1$.)