Last name ______________________

First name ______________________

LARSON—MATH 310–CLASSROOM WORKSHEET 08
Bases and Dimension.

Review: $\mathbb{R}$, *field*, *complex numbers*, $\mathbb{R}^2$, $\mathbb{K}$, $\mathbb{K}^n$, *linear space* (or *vector space*), *subspace*, *linear map* (or *linear transformation*), *kernel*, *range*, *linear combination*, subspace *generated* by (or *spanned* by) a set of vectors, $\langle A \rangle$, *finite-dimensional vector space*, *linearly independent* set of vectors, *linearly dependent* set of vectors, *basis* of linear space.

Review.

1. Every list of vectors containing is $\vec{0}$ linearly dependent.
2. What is the *standard basis* of $\mathbb{K}^n$?
3. Let $\vec{v}$ be a vector in a linear space V with basis $X = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v_n}\}$. What is the *representation* of $\vec{v}$ with respect to basis X?

From Chp. 3 of Tsukada, et al., Linear Algebra with Python

All bases have the same number of vectors.

1. Suppose V is a vector space with basis $X = \vec{v}_1, \vec{v}_2, \vec{v}_3$. Why is it impossible to add any other vector and still have a basis?
2. Suppose V is a vector space with basis $X = \vec{v}_1, \vec{v}_2, \vec{v}_3$ and $\vec{w}$ is a non-$\vec{0}$ vector. Why is it possible to replace one of the $\vec{v}_i$'s with $\vec{w}$ and still have a basis?
3. Suppose V is a vector space with basis $X = \vec{v}_1, \vec{v}_2, \vec{v}_3$, and $\vec{w}_1, \ldots, \vec{w}_j$ are linearly independent vectors. Why is $j \leq 3$?
4. Suppose V is a vector space with basis $X = \vec{v}_1, \vec{v}_2, \vec{v}_3$. Why can't V have a basis with less than 3 vectors?

5. Suppose V is a vector space with basis $X = \vec{v}_1, \vec{v}_2, \vec{v}_3$. Why does **every** basis for V have (exactly) three vectors?

6. What is the *dimension* of a linear/vector space?

7. What is the *rank* of a collection of vectors?

8. Let the columns of matrix A be $\vec{a}_1, \ldots \vec{a}_6$. Find a maximal set of linearly independent columns by greedily choosing the first non-zero column vector, adding the next available column vector, and iterating (until no column remain). Find the rank.

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

9. The *column space* of a matrix is the collection of linear combinations of the columns. Why is this collection a linear space?

10. Explain why the linearly independent column vectors of A we found are, in fact, a basis for the column space of A.