Last name ________________________

First name ________________________

LARSON—MATH 550—CLASSROOM WORKSHEET 32
Inversion & Derangements.

Concepts & Notation

- Sec. 5.1: Binomial coefficients, the Binomial Theorem, key formulas, the polynomial argument, Vandermonde's convolution.
- Sec. 5.2: the Sorting example.
- Sec. 5.3: Duplication formula, inversion, derangements.
- Sec. 5.4: Convolutions, generating functions.

Inversion

Let $f(n)$ be a function defined on $\mathbb{Z}^{\geq 0}$, and define:

$$g(n) = \sum_k \binom{n}{k} (-1)^k f(k).$$

The **claim** is that:

$$f(n) = \sum_k \binom{n}{k} (-1)^k g(k).$$

1. Test this for $f(n) = 3 * n + 1$ and some values of $n \in \mathbb{Z}^{\geq 0}$

2. *Prove* that the inversion formula for $f(n)$ holds in general.

Derangements

A *derangement* of n objects is an ordering (listing, or permutation) of those objects so that none is in its original place; or, more formally, it is a bijection $\phi : [n] \to [n]$ such that $\phi(i) \neq i$, for every $i \in [n]$.

Let ¡n be the number of derangements of n objects.

Let $h(n, k)$ be the number of permutations of n objects where exactly k are fixed.

3. Find $h(4, 0)$, $h(4, 1)$, $h(4, 2)$, $h(4, 3)$, and $h(4, 4)$.

4. Why does

$$n! = \sum_k h(n, k)?$$

5. Why does

$$\sum_k h(n, k) = \sum_k \binom{n}{k} \text{¡}(n - k)$$