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LARSON—MATH 310—CLASSROOM WORKSHEET 20

Matrix-Matrix Multiplication

Review: Chapter 4 of Klein's *Coding the Matrix* text

1. What is the *column space* of a matrix?
2. What is the *row space* of a matrix?
3. (Sec. 4.5.4) What is a *matrix-vector equation*?

New

1. How can we view the following matrix-vector equation as a question about membership in the column space of the given matrix?

Example 4.6.8: Reformulating the triangular system of Example 2.11.1 (Page 130) as a matrix-vector equation, we obtain

$$\begin{bmatrix} 1 & 0.5 & -2 & 4 \\ 0 & 3 & 3 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix} * \mathbf{x} = [-8, 3, -4, 6]$$

2. What is the *null space* of a matrix?

3. Find:

Problem 4.7.3: For each of the given matrices, find a nonzero vector in the null space of the matrix.

$$1. \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

$$2. \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$3. \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Definition 4.6.1 (Dot-Product Definition of Matrix-Vector Multiplication): If M is an $R \times C$ matrix and \mathbf{u} is a C -vector then $M * \mathbf{u}$ is the R -vector \mathbf{v} such that $\mathbf{v}[r]$ is the dot-product of row r of M with \mathbf{u} .

- Find the following matrix-vector product using both the linear combination of columns definition and the dot product definition.

Example 4.6.2: Consider the matrix-vector product

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 10 & 0 \end{bmatrix} * [3, -1]$$

Definition 4.11.1 (Vector-matrix definition of matrix-matrix multiplication): For each row-label r of A ,

$$\text{row } r \text{ of } AB = (\text{row } r \text{ of } A) * B \quad (4.6)$$

- Use the “vector-matrix” definition of matrix-matrix multiplication to find the following products.

Problem 4.17.6: Compute:

$$1. \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

$$2. \begin{bmatrix} 2 & 4 & 1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 5 & 1 & 1 \\ 2 & 3 & 0 \end{bmatrix}$$

- Now use the “matrix-vector” definition of matrix-matrix multiplication to find those products.

Definition 4.11.3 (Matrix-vector definition of matrix-matrix multiplication): For each column-label s of B ,

$$\text{column } s \text{ of } AB = A * (\text{column } s \text{ of } B) \quad (4.7)$$

- Now use the “dot product” definition of matrix-matrix multiplication to find those products.

Definition 4.11.7 (Dot-product definition of matrix-matrix multiplication): Entry rc of AB is the dot-product of row r of A with column c of B .