LARSON—MATH 255–CLASSROOM WORKSHEET 13
The Intermediate Value Theorem.

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click "New", then "Worksheets", then call it **c13**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

Challenges

2. **First Challenge**.

   2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder. What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

   We found the answer to this at the end of last class—but it still took a long time to compute—how can we leverage our *mathematical knowledge* to speed up our code?

Intermediate Value Theorem

If $f(x)$ is a continuous function, and $f(a) \leq c \leq f(b)$ then there is some real number x in the interval $[a, b]$ where $f(x) = c$ (that's the **Intermediate Value Theorem**). We will define a function that finds this x. We will do this in steps.

3. Given a continuous function $f(x)$, and numbers a, b and c, define a function `check_conditions(f,a,b,c)` that returns True if $f(a) \leq c \leq f(b)$ and False otherwise. Evaluate.

4. Let $f(x) = x^2$. Evaluate `check_conditions(f,1,2,3)` and `check_conditions(f,1,2,5)`. Is the output what you expected?

5. Given a continuous function $f(x)$, and numbers a, b and c, define a function `test_average(f,a,b,c)` that returns the tuple $(a, (a+b)/2)$ if $f((a+b)/2) \geq c$ and returns $((a+b)/2, b)$ if $f((a+b)/2) < c$.

6. What can you do now to find the intermediate value x where $f(x) = c$?

One idea is to successively find smaller and smaller intervals where the x must be (where $f(x) = c$). If we keep splitting our original interval in half, in n steps we will find that our x must be in an interval of length $\frac{1}{2^n}(b-a)$; in the limit this interval length is going to 0 and any x in the n^{th} interval will be within a very small error or tolerance.

7. Type and evaluate the following code. It first tests if a given function meets the conditions of the *Intermediate Value Theorem* (IVT). If so this means the desired x is in the interval $[a, b]$. If it does then it successively splits this interval to find out which half the x lives in. The version below does 10 splits (so the produced solution must be within $2^{-10}(b-a)$ of the correct x).

```
def IVT(f,a,b,c):
    if check_conditions(f,a,b,c)==False:
        print("The conditions of the IVT are not satisfied")
    else:
        for i in [1..10]:
            (a,b)=test_average(f,a,b,c)
    return a
```

$f(x)$ should still be the squaring function. Evaluate `f` to check. If not, let `f(x)=x**2`.

8. Find `IVT(f,1,2,2)`. This will give you an approximation of the square root of 2. Square your result to check how good the produced answer is.

9. Find `IVT(f,1,2,3)`. This will give you an approximation of the square root of 3. Square your result to check how good the produced answer is.

10. Modify the last program to do 20 iterations. Evaluate.

11. Find `IVT(f,1,2,2)` again. Square your result to check how good the produced answer is.

12. Find `IVT(f,1,2,3)` again. Square your result to check how good the produced answer is.

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255 - c13 worksheet attached" (so that it will be properly recorded).

(c) Remember to attach today's classroom worksheet!