Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 27
Polynomials.

Concepts & Notation

- (Sec. 3.5) *linear functional*, *trace*, *dual space*, V^*, *dual basis*, *annihilator*.
- (Sec. 4.1) *linear algebra*, $\mathbb{F}^\infty$, *algebra of formal power series*.
- (Sec. 4.2) $\mathbb{F}[x]$, *degree*, *scalar* polynomial, *monic* polynomial..

Review

1. What is the *degree* of $f \in \mathbb{F}[x]$?

2. What is a *scalar* polynomial?

3. What is a *monic* polynomial?

New

4. (**Claim:**) If f and g are non-zero polynomials over a field $\mathbb{F}$ then:
   (a) fg is a non-zero polynomial;
   (b) $\deg(fg) = \deg(f) + \deg(g)$;
   (c) fg is a monic polynomial if and only if both f and g are monic polynomials;
   (d) fg is a scalar polynomial if and only if both f and g are scalar polynomials;
   (e) if $f + g \neq 0$ then $\deg(f + g) \leq \max\{\deg(f), \deg(g)\}$.

5. What is the difference between a *polynomial* and a *polynomial function*?

6. If $p \in \mathbb{F}[x]$ and α is an element of a linear algebra, what is $p(\alpha)$?

7. Let p $= x - 5 \in \mathbb{R}[x]$ and $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$. Find $p(A)$.

8. What is a *root* of a polynomial?

9. What is an *ideal* in $\mathbb{F}[x]$?

10. (**Claim:**) Every ideal in $\mathbb{F}[x]$ is *principle*.

11. What is a *determinant* function?