Last name _____

First name _____

LARSON—MATH 350—CLASSROOM WORKSHEET 27
Geometry and Combinatorics

Review

- What does it mean for a figure to be *convex*?
- A collection of lines are in **standard position** if all lines intersect with all others (no parallel lines) and no more than 2 intersect at any point.
- How many regions would be formed by n lines in standard position?
- Show that for any 5 points in the plane, there are 4 which form a convex quadrilateral (4-gon).
- What is the *Happy End Problem*

Euler's Formula

Given a convex **polytope** in 3 dimensions, let n be the number of its vertices, e be the number of edges, and f be the number of its faces.

1. Draw a cube and calculate $n - e + f$.

2. Draw a tetrahedron and calculate $n - e + f$.

3. Draw an octahedron and calculate $n - e + f$.

4. Can you make a conjecture?

Planar Graphs

5. What is a *graph*?

6. What is a *planar graph*?

7. What is a *connected* planar graph?

8. Why can every convex polytope in 3-dimensional space ($\mathbb{R}^3$) be represented as a connected planar graph?

   The *faces* of a polytope correspond to *regions* of a planar graph. We will prove our Euler's formula conjecture by proving the corresponding claim for connected planar graphs.

9. Argue that the conjecture is true for connected planar graphs with 2 faces/regions.

10. Argue that the conjecture is true for connected planar graphs with 3 edges.

11. What inductive hypothesis should we make?

12. How will proof by induction work for graphs?