Last name	
First name	

## LARSON—MATH 601—CLASSROOM WORKSHEET 03 Review.

## Concepts & Notation

- (Sec. 1.1) field F, subfield.
- (Sec. 1.2) homogenous system of linear equations, linear combination of equations, equivalent systems of linear equations.
- (Sec. 1.3) matrix of coefficients of a system of linear equations, matrix over the field F, elementary row operations on a matrix, row-equivalent matrices, row-reduced matrix, identity matrix I, Kronecker delta  $\delta$ .
- (Sec. 1.4) row-reduced echelon matrix, zero matrix  $)^{m,n}$ .
- matrix multiplication.

## **Problems**

We showed that the matrix

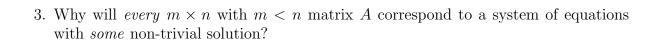
$$A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 4 & 0 & -1 \\ 2 & 6 & -1 & 5 \end{bmatrix}$$

is row-equivalent to the row-reduced matrix

$$\begin{bmatrix} 0 & 0 & 1 & -\frac{11}{3} \\ 1 & 0 & 0 & \frac{17}{3} \\ 0 & 1 & 0 & -\frac{5}{3} \end{bmatrix}$$

1. Why is every  $m \times n$  matrix A row-equivalent to a row-reduced (echelon) matrix?

2. We saw that the  $3 \times 4$  matrix A corresponded to a homogeneous system of equations with infinitely many solutions. Why will every  $3 \times 4$  matrix A corresponded to a system of equations with some non-trivial solution?



4. Show that the  $2 \times 2$  matrix  $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$  is row-equivalent to the identity matrix I. Find the solutions of the corresponding homogeneous system AX = 0.

5. Let A be an  $n \times n$  matrix. Explain why the homogeneous system AX = 0 has non-trivial solutions if and only if A is *not* row-equivalent to the identity matrix I.

6. Let A and B be matrices. When is the product AB defined? If AB is defined, what are its entries?

7. Find

$$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 5 & -1 & 2 \\ 15 & 4 & 8 \end{bmatrix}.$$