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LARSON—MATH 610—CLASSROOM WORKSHEET 37 The Structure of Linear Operators.

Concepts & Notation

- (Sec. 5.3) permutation, $\det A$.
- (Sec. 6.2) characteristic value, characteristic vector, characteristic polynomial, diagonalizable linear operator.

Review

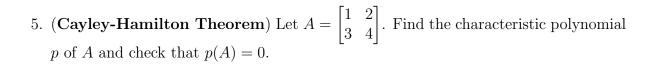
1. If $T \in \mathcal{L}(V)$ and $p \in \mathbb{F}[x]$ then $p(T) \in \mathcal{L}(V)$.

The Structure of a Linear Operator

2. (Claim:) If $T \in \mathcal{L}(V)$, $T(\alpha) = c\alpha$ (for $c \in \mathbb{F}$, $\alpha \in V$), and $p \in \mathbb{F}[x]$ then $p(T)(\alpha) = p(c)\alpha$.

3. What does it mean for a polynomial to annihilate a linear operator T?

4. What is the *minimal polymonial* of a linear operator T over a finite-dimensional vector space T? (Does it exist? What does it tell us?)



6. Use p(A) to find A^{-1} .

7. (Cayley-Hamilton Theorem) Let $A \in \mathbb{F}^{n \times n}$. Let $p \in \mathbb{F}[x]$ be the characteristic polynomial of A. Then p(A) = 0.

8. (Cayley-Hamilton Theorem) Let V be a finite-dimensional vector space over a field \mathbb{F} and $T \in \mathcal{L}$. Let $p \in \mathbb{F}[x]$ be the characteristic polynomial of T. Then p(T) = 0.