Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 05
Linear Maps.

Concepts & Notation

- (Chp. 1) *field* $\mathbb{F}$, *list*, *vector space*, $\mathbb{F}^n$, $\mathbb{F}^S$, $\mathbb{F}^\infty$, *subspace*, sums of subspaces, *direct sum*.
- (Chp. 2) *linear combination*, *span*, *finite-dimensional* vector space, *linear independence*, *basis*.
- (Chp. 3) *linear map*, *null space*, *range*, *injective*, *surjective*.

1. What is a *linear map*?

2. What is $\mathcal{L}(V, W)$?

3. What is $\mathcal{L}(V)$?

4. (**Linear map lemma**.) If $v_1, \ldots, v_n$ is a basis for vector space V and $w_1, \ldots, w_n$ is a basis for vector space W then there is a unique linear map $T : V \to W$ with $Tv_i = w_i$.

5. **Claim:** $\mathcal{L}(V, W)$ is a vector space.

6. Let $T \in \mathcal{L}(V, W)$. What is the *null space* of T? (**Notation:** *null* T).

7. Let $T \in \mathcal{L}(V, W)$. **Claim:** *null* T is a subspace of V.

8. Let $T \in \mathcal{L}(V, W)$. What does it mean for T to be *injective*.

9. Let $T \in \mathcal{L}(V, W)$. T is injective if and only if *null* $T = \{0\}$.

10. Let $T \in \mathcal{L}(V, W)$. What is the *range* of T? (**Notation:** *range* T).

11. What do we say if *range* $T = W$?