Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 11
Linear independence & Bases.

Concepts & Notation

- (Sec. 2.1) *vector*, *vector space*, *linear combination.*
- (Sec. 2.2) *subspace*, *subspace spanned by a set of vectors*, *span.*
- (Sec. 3.3) *linearly dependent/independent set of vectors*, *basis.*

Let $\mathbb{F}$ be a subfield of $\mathbb{C}$. Let V be the set of functions from $\mathbb{F}$ to $\mathbb{F}$ of the form

$$f(x) = c_0 + c_1x + c_2x^2 + \ldots + c_nx^n$$

(where $c_i \in \mathbb{F}$ and n is a non-negative integer).

1. What are some examples of elements of V?

2. **Claim:** V is a vector space with vector addition $f+g$ defined by $(f+g)(x) = f(x)+g(x)$ (for $f, g \in V$), and scalar multiplication cf defined by $(cf)(x) = c(f(x))$ (for $c \in \mathbb{F}$, $f \in V$).

3. Show that the vectors in V with degree at most 2 are a subspace of V.

If V is a vector space over a field $\mathbb{F}$ and $S \subset V$, the *subspace spanned by* S is the intersection of all subspaces of V containing S (S *spans* V); if $S = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ then *say* it is the *subspace spanned by* $\alpha_1, \alpha_2, \ldots, \alpha_n$.

Let $f_i(x) = x^i$ (for non-negative integers i). So $f_i \in V$.

4. **Show:** $S = \{f_i \mid i \in \mathbb{N}\}$ spans V.

5. **Claim:** S spans a vector space V if and only if V is the set of all (finite) linear combinations of vectors in S.

6. **Show:** $S = \{f_i \mid i \in \mathbb{N}\}$ spans V.

Vectors $\alpha_1, \ldots, \alpha_n$ in a vector space V over a field $\mathbb{F}$ are *linearly dependent* if $c_1\alpha_1 + \ldots c_n\alpha_n = 0$ where not all c_i's are 0 ($c_i \in \mathbb{F}$). If S is not linearly dependent then it is *linearly independent.*

7. **Show:** $S = \{f_i \mid i \in \mathbb{N}\}$ is linearly independent.

A *basis* for V is a set of linearly independent vectors which spans V. V is *finite-dimensional* if it has a finite basis.

8. **Show:** $S = \{f_i \mid i \in \mathbb{N}\}$ is a basis for V..

9. **Claim:** if V is a vector space spanned by vectors $\beta_1, \beta_2, \ldots, \beta_m$, then any set S of n vectors (with $n > m$) is linearly dependent.