LARSON—MATH 255–HOMEWORK h13
Test 2 Review

1. Log in to your Sage/Cocalc account.

   (a) Start the Chrome browser.

   (b) Go to `http://cocalc.com` and sign in.

   (c) Click on the Project for our class.

   (d) Click "New", call it **h13**, then click "Sage Worksheet".

   (e) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

   (f) When you are finished with the worksheet, click "make pdf", email me the pdf (at `clarson@vcu.edu`, with a header that says **Math 255 h13 worksheet attached**). Send this **before** our Test 2 (on Wed., Dec. 14 12:30).

   Here are the Test 2-type questions, together with a selection of **problems**. Go over your classroom worksheets as needed. You will be allowed to refer to your classroom worksheets on the Test.

2. Write a program to find the *set* of positive integers less than 100 that are multiples of 3 or 5.

3. Define a function `attitude()` which randomly prints "Don't look at me." or "Are you looking at me!?!?".

4. Let L be a list with 300 (random) entries. What would you write to get the *slice* of L consisting of the first ten elements of L?

5. Define a function `apples(a,b)` that takes positive integers a and b and prints "I picked a apples and b oranges."

6. Define a function `prime_sum(n)` which takes an integer n as input and returns the sum of the prime numbers up to n.

7. Define a function `constant_matrix(n,m,c)` which takes positive integers n, m and c as input and returns a n by m matrix whose entries are all c.

8. Let M be the 3×3 identity matrix. *Then* change the upper right corner entry to be 5.

9. Define a function `sum_digits(n)` that takes an integer n and returns the sum of its digits.

10. Write a program to find all triples (a,b,c) with positive integers a,b,c $\leq$ 10 where $a^2 + b^2 = c^2$.

11. Define a function `a_count(s)` that takes a string s and counts the number of occurrences of the letter a in the string.

12. Write a program that produces 100 random numbers from the interval $[0, 100]$ and stores them in separate lines of a file called `rand_store.txt`.

13. Define a function `read_data(filename)` that opens a file called `filename` consisting of integers on separate lines, pulls out those numbers, puts them in a list, and returns that list of numbers. The function should not alter the contents of `filename`.

14. Use `list comprehension` to produce the cubes of the numbers in the list $L = [3, 7, 2, 5]$.

15. Consider the following sequence defined by a function on the positive integers. $T(1) = 3$, $T(2) = 4$, $T(3) = 5$, $T(n) = T(n - 1) + T(n - 2) + T(n - 3)$. Define a recursive function `recurseT(n)` that takes a positive integer n as input and outputs $T(n)$. Find $T(10)$.

16. Consider the following sequence defined by a function on the positive integers. $T(1) = 3$, $T(2) = 4$, $T(3) = 5$, $T(n) = T(n - 1) + T(n - 2) + T(n - 3)$. Define an *iterative* (non-recursive) function `iterateT(n)` that takes a positive integer n as input and outputs $T(n)$. Find $T(10)$.

17. Write a Sage Interact that takes any function as input (so use an *input box*) and plots the graph of that function. Choose any function as the default input.

18. Define a function `random_average(n)` to choose n random integers between 1 and 100 and find the average of these n numbers. Find `random_average(n)` for $n = 10$ to $n = 1000$. Use `scatter_plot` to display the results.

19. Define a function `order_size(g)` which takes a graph g as input and returns the product of the number of vertices of the graph and the number of edges of the graph.

20. $3, 5$ and $5, 7$ and $11, 13$, etc are called *twin primes*. Find all of the twin primes less than 1000.

21. If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Write a program to find the sum of all the multiples of 3 or 5 below n.

22. 12 has 6 factors: $1, 2, 3, 4, 6, 12$. Find the positive integer no more than 100 with the most factors.

23. The Fibonacci sequence F_n is defined as follows $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n > 1$. What is the first term in the Fibonacci sequence to contain 1000 digits?

24. (Ramanujan revisited) We found that 1729 is the smallest number which is the sum of 2 cubes in 2 different ways ($1729 = 1^3 + 12^3 = 9^3 + 10^3$). Find the smallest integer which can be written as the sum of 2 *squares* in 2 different ways.

25. Find the sum of the digits in the number 100!

26. Write a program `digit_of_e(n)` that outputs the nth decimal digit of e.