Last name _____

First name _____

LARSON—OPER 731—CLASSROOM WORKSHEET 15
Faces, Facets, Extreme Points and Minkowski's Theorem

Faces & Facets

1. What is a *face* of a polytope?

2. What is a *facet* of a polytope?

3. What is an *extreme point* of a polytope?

4. Find any faces in the polytope defined by the linear inequalities ($x \in \mathbb{R}$, $x_i \geq 0$), and find their dimensions. Identify facets and extreme points.

$$
\begin{array}{rcrcrcl}
x_0 & + & x_1 & & & \leq & 1 \\
x_0 & & & + & x_2 & \leq & 1 \\
& & x_1 & + & x_2 & \leq & 1 \\
\end{array}
$$

5. What is Weyl's Theorem?

6. What is *Minkowski's Theorem*?

7. Let $\mathcal{P}$ be the feasible region of the following system of inequalities ($x \in \mathbb{R}$, $x_i \geq 0$). Is it bounded?

$$
\begin{array}{rcrcrcl}
x_0 & + & x_1 & & & \leq & 1 \\
x_0 & & & + & x_2 & \leq & 1 \\
& & x_1 & + & x_2 & \leq & 1
\end{array}
$$

8. Find a convex hull description.