LARSON—MATH 255–HOMEWORK h13
Test 2 Review

1. Log in to your Sage/Cocalc account.

   (a) Start the Chrome browser.

   (b) Go to `http://cocalc.com` and sign in.

   (c) Click on the Project for our class.

   (d) Click "New", call it **h13**, then click "Sage Worksheet".

   (e) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

   (f) When you are finished with the worksheet, click "make pdf", email me the pdf (at `clarson@vcu.edu`, with a header that says **Math 255 h13 worksheet attached**). Send this **before** our Test 2.

   Here are the Test 2-type questions, together with a selection of **problems**. Go over your classroom worksheets as needed. You will be allowed to refer to your classroom worksheets on the Test.

2. Write a program to find a list of positive integers less than 100 that are multiples of 3 or 5 (with no repeated numbers).

3. Define a function `apples(a,b)` that takes positive integers a and b and prints "I picked a apples and b oranges."

4. Define a function `prime_sum(n)` which takes an integer n as input and returns the sum of the prime numbers up to n.

5. Define a function `constant_matrix(n,m,c)` which takes positive integers n, m and c as input and returns a n by m matrix whose entries are all c.

6. Define a function `sum_digits(n)` that takes an integer n and returns the sum of its digits.

7. Write a program to find all triples (a,b,c) with positive integers a,b,c ≤ 10 where $a^2 + b^2 = c^2$.

8. Define a function `a_count(s)` that takes a string s and counts the number of occurrences of the letter a in the string.

9. Write a program that produces 100 random numbers from the interval $[0, 100]$ and stores them in separate lines of a file called `rand_store.txt`.

10. Define a function `read_data(filename)` that opens a file called `filename` consisting of integers on separate lines, pulls out those numbers, puts them in a list, and returns that list of numbers. The function should not alter the contents of `filename`.

11. Use `list comprehension` to produce the cubes of the numbers in the list $L = [3, 7, 2, 5]$.

12. Consider the following sequence defined by a function on the positive integers. $T(1) = 3$, $T(2) = 4$, $T(3) = 5$, $T(n) = T(n-1) + T(n-2) + T(n-3)$. Define a recursive function `recurseT(n)` that takes a positive integer n as input and outputs $T(n)$. Find $T(10)$.

13. Consider the following sequence defined by a function on the positive integers. $T(1) = 3$, $T(2) = 4$, $T(3) = 5$, $T(n) = T(n-1) + T(n-2) + T(n-3)$. Define an *iterative* (non-recursive) function `iterateT(n)` that takes a positive integer n as input and outputs $T(n)$. Find $T(10)$.

14. Define a function `random_average(n)` to choose n random integers between 1 and 100 and find the average of these n numbers. Find `random_average(n)` for $n = 10$ to $n = 1000$. Use `scatter_plot` to display the results.

15. $3, 5$ and $5, 7$ and $11, 13$, etc are called *twin primes*. Find all of the twin primes less than 1000.

16. Define a function `prime_ratio(n)` that produces the ratio of primes up to n. Use your `prime_count(n)` in your definition. Test it (since there are 25 primes up to 100, the ratio is $\frac{25}{100}$).

17. 12 has 6 factors: $1, 2, 3, 4, 6, 12$. Find the positive integer no more than 100 with the most factors.

18. The Fibonacci sequence F_n is defined as follows $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n > 1$. What is the first term in the Fibonacci sequence to contain 2000 digits?

19. (Ramanujan revisited) We found that 1729 is the smallest number which is the sum of 2 cubes in 2 different ways ($1729 = 1^3 + 12^3 = 9^3 + 10^3$). Find the smallest integer which can be written as the sum of 2 *squares* in 2 different ways.

20. **Dictionary Problem**. Write a function `my_dictionary(n)` that takes a positive integer n as input and returns a dictionary where the keys are numbers between 1 and 15 (both included) and the values are the square of the keys.

    When $n = 15$ you should get:

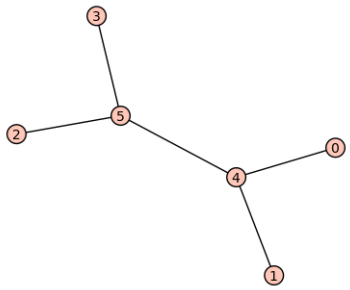
    {1: 1, 2: 4, 3: 9, 4: 16, 5: 25, 6: 36, 7: 49, 8: 64, 9: 81, 10: 100, 11: 121, 12: 144, 13: 169, 14: 196, 15: 225}

21. The sum of the reciprocals of the positive integers

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

    diverges (that is, the sum goes to infinity). Find the smallest integer m so that $\sum_{n=1}^{m} \frac{1}{n}$ is at least 6.

22. Make a Sage graph object with the name "milkbone":



23. Define a function `order_size(g)` which takes a graph g as input and returns the product of the number of vertices of the graph and the number of edges of the graph. Test it with the milkbone graph: `order_size(milkbone)` is 20.

24. If L is a list of integers, what command would you give to get a scatter plot that *visualizes* this data? What you write should work for *any* list L, but test it with `L=[2,3,5,7,11]`.

    (**Note:** a scatter plot that always produces the same straight line for any list of data is *not* a useful visualization of your data. You want something that has different pictures of different data sets).