Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 13
Linear independence, Bases, Dimension.

Concepts & Notation

- (Sec. 2.1) *vector*, *vector space*, *linear combination.*
- (Sec. 2.2) *subspace*, *subspace spanned by a set of vectors*, *span.*
- (Sec. 3.3) *linearly dependent/independent set of vectors*, *basis*, *dimension.*

Let $\mathbb{F}$ be a subfield of $\mathbb{C}$. Let V be the set of functions from $\mathbb{F}$ to $\mathbb{F}$ of the form

$$f(x) = c_0 + c_1x + c_2x^2 + \ldots + c_nx^n$$

(where $c_i \in \mathbb{F}$ and n is a non-negative integer).

We **proved:** A set of vectors S spans a vector space V if and only if V is the set of all (finite) linear combinations of vectors in S.

Let $f_i(x) = x^i$ (for non-negative integers i). So $f_i \in V$.

We showed: $S = \{f_i \mid i \in \mathbb{N}\}$ spans V.

Vectors $\alpha_1, \ldots, \alpha_n$ in a vector space V over a field $\mathbb{F}$ are *linearly dependent* if $c_1\alpha_1 + \ldots c_n\alpha_n = 0$ where not all c_i's are 0 ($c_i \in \mathbb{F}$). If S is not linearly dependent then it is *linearly independent.*

We showed: $S = \{f_i \mid i \in \mathbb{N}\}$ is linearly independent.

A *basis* for a vector space V is a set of linearly independent vectors which spans V. V is *finite-dimensional* if it has a finite basis.

We showed: $S = \{f_i \mid i \in \mathbb{N}\}$ is a basis for V.

We showed: if V is a vector space spanned by vectors $\beta_1, \beta_2, \ldots, \beta_m$, then any set S of n vectors (with $n > m$) is linearly dependent.

1. **Claim:** if V is a vector space spanned by vectors $\beta_1, \beta_2, \ldots, \beta_m$, then any independent set of vectors is finite and has no more than m elements.

A vector space V with a finite basis is **finite dimensional**.

2. **Claim:** If if V is a finite-dimensional vector space, then every basis has the same number of elements.

The **dimension** of a finite-dimensional vector space V is the number of elements in any basis of V and is denoted $\dim V$.

3. Find $\dim \mathbb{R}^2$.

4. Find $\dim \mathbb{F}^n$.

5. Find the dimension of the vector space of polynomials over a field $\mathbb{F}$ of degree at most 2.

6. **Claim:** Let S be a linearly independent subset of a vector space V. If β is not in the subspace spanned by S then the set obtained by adding β to S is linearly independent.

7. **Claim:** Any linearly independent set in a finite-dimensional vector space V is part of (can be extended to) a (finite) basis for V.