Last name ______________________

First name ______________________

LARSON—MATH 550—CLASSROOM WORKSHEET 05
Mathematical Induction. The Trick.

Concepts & Notation

- Sec. 1.1 & Sec. 1.2 T_n, recurrence (recurrence relation), mathematical induction, basis, solving recurrences
- Sec. 2.1 $[m = n]$ notation, sum notations.
- Sec. 2.2 The "trick".

Induction

Let $P(n)$ be the (open) statement: "$a^0 + \ldots + a^{n-1} = \frac{a^n-1}{a-1}$ $(a \neq 1)$".

1. Check that $P(1)$, $P(2)$ and $P(3)$ are true (base cases).

2. Show: that $P(n)$ implies $P(n+1)$.
   That is, assume:
$$a^0 + \ldots + a^{n-1} = \frac{a^n - 1}{a - 1}$$
   and show:
$$a^0 + \ldots + a^{(n+1)-1} = \frac{a^{(n+1)} - 1}{a - 1}$$

3. What can you conclude?

4. Find
$$\sum_{k=0}^{5} \frac{1}{2^k}$$

5. (Sec. 2.2) Suppose $S_n = \sum_{k=0}^{n} a_k$ and $a_k = \alpha + \beta k$. Use our methodology to "solve" this recurrence.

(Sec. 2.2). Given a recurrence of the form $a_n T_n = b_n T_{n-1} + c_n$, you can get a "nicer" recurrence by multiplying through by (any constant multiple of):

$$s_n = \frac{a_{n-1} a_{n-2} \dots a_1}{b_n b_{n-1} \dots b_2}$$

6. What would this yield for $T_n = 2T_{n-1} + 1$?

7. What would this yield for $L_n = L_{n-1} + n$?