Last name ______________________

First name ______________________

LARSON—OPER 731—CLASSROOM WORKSHEET 11
Basic Solutions, Extreme Points, Dimension

The ideas from 0.4 that I will try to illustrate with examples are:

1. A system of inequalities can be extended to a system of equations with (non-negative) *slack variables* and the feasible points in the first are in one-to-one correspondence with the solutions of the other.
2. Each *basic feasible solution* (algebra) in the simplex method corresponds to an *extreme point* (geometry).
3. The columns corresponding to the basic variables are linearly independent and the matrix formed from these columns is invertible (and a solution of the system of equations can be found by setting the non-basic variables to 0, and inverting this matrix).

1. Use slack variables and the Simplex Method to maximize the following LP. Keep track of the basic variables and resulting extreme points along the way.

$$z = x_1 + x_2$$

Subject to:

$$x_1 + x_2 \leq 1$$
$$x_i \geq 0.$$

2. Check that one each iteration of the Simplex Method that the columns corresponding to the basic variables are linearly independent.

3. Use slack variables and the Simplex Method to maximize the following LP. Keep track of the basic variables and resulting extreme points along the way.

$$z = x_1 + x_2$$

Subject to:

$$3x_1 + x_2 \leq 3$$
$$-x_1 + 3x_2 \leq 3$$
$$x_i \geq 0.$$

4. Check that one each iteration of the Simplex Method that the columns corresponding to the basic variables are linearly independent.

Dimension

5. What is the definition of the *dimension* of a polytope?

6. Argue that the dimension of a polytope $\mathcal{P} \subset \mathbb{R}^2$ is no more than 2.

7. Find the dimension of the polytopes defined by the linear inequalities in the above LPs.

8. Find the dimension of the polytope which is the convex hull of $X = \{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \}.$