Last name ________________________

First name ________________________

LARSON—MATH 610—CLASSROOM WORKSHEET 35
The Structure of Linear Operators.

Concepts & Notation

- (Sec. 5.3) *permutation*, $\det A$.
- (Sec. 6.2) *characteristic value*, *characteristic vector*, *characteristic polynomial*, *diagonalizable* linear operator.

Review

1. What is a *characteristic value* and a *characteristic vector* of a linear operator T from a vector space V over a field $\mathbb{F}$ to itself?

2. What is the *characteristic space* associated with a characteristic value c of a linear operator T from a vector space V over a field $\mathbb{F}$ to itself?

3. (**Claim:**) If matrices $A, B \in \mathbb{F}^{n\times n}$ are similar then $\det A = \det B$.

4. (**Claim:**) If T is a linear operator from a finite-dimensional vector space V over a field $\mathbb{F}$ to itself, and $\mathcal{B}$ and $\mathcal{B}'$ are bases for V then $\det([T]_{\mathcal{B}}) = \det([T]_{\mathcal{B}'})$.

5. What is $\det T$?

The Structure of a Linear Operator

6. What is a *diagonalizable* linear operator T?

7. What is the *characteristic polynomial* of a matrix A? What does it tell us?

8. Find the characteristic polynomial for $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. Find its roots (and thus the characteristic values for any linear transformation represented by A).

9. Find the corresponding characteristic vectors. (There is a small issue here. What is it?!?)

10. Does every matrix have characteristic values?

11. (**Claim:**) If T is a linear operator from a vector space V over a field $\mathbb{F}$ to itself then $T^2 = T \circ T$ is also in $\mathcal{L}(V)$. And $T^k \in \mathcal{L}(V)$.

12. (**Claim:**) If $T \in \mathcal{L}(V)$ then $cT \in \mathcal{L}(V)$ (for $c \in \mathbb{F}$).

13. (**Claim:**) If $T, T' \in \mathcal{L}(V)$ then $T + T' \in \mathcal{L}(V)$.

14. (**Claim:**) If $T \in \mathcal{L}(V)$ and $p \in \mathbb{F}[x]$ then $p(T) \in \mathcal{L}(V)$.

15. (**Claim:**) If $T \in \mathcal{L}(V)$, $T(\alpha) = c\alpha$ (for $c \in \mathbb{F}$, $\alpha \in V$), and $p \in \mathbb{F}[x]$ then $p(T)(\alpha) = p(c)\alpha$.