Last name ______________________

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LARSON—MATH 610—CLASSROOM WORKSHEET 21
Linear Transformations.

Concepts & Notation

- (Sec. 3.1) *linear transformation*, *range*, *rank*, *null space*, *nullity.*
- (Sec. 3.2) $L(V, W)$, *linear operator*, *invertible* linear transformation, *non-singular* linear transformation.
- (Sec. 3.3) *isomorphism.*
- (Sec. 3.4) *matrix of* T *relative to (ordered) bases*

Review

1. (**Claim:**) If V and W are finite-dimensional vector spaces over a field $\mathbb{F}$ with $\dim V = \dim W$, and $T : V \to W$ is a linear transformation then the following are equivalent:
   (a) T is invertible,
   (b) T is non-singular,
   (c) T is onto (that is, the range of T is W).

New

2. If $T : V \to W$ is a linear transformation, when is T an *isomorphism* of V onto W? (If T is an isomorphism we say that vector spaces V and W are *isomorphic*).

3. If T an *isomorphism* of a vector space V onto a vector space W, why is T invertible and non-singular?

4. (**Claim:**) Every n-dimensional vector space over a field $\mathbb{F}$ is isomorphic to $\mathbb{F}^n$.

5. (**Claim:**) Every linear transformation T from an n-dimensional vector space V to an m-dimensional vector space W can be represented by a matrix A (with respect to specific bases for V and W; in particular, different bases yield different A's).

6. (**Example:**) Let T be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ defined by $T(x_1, x_2) = (x_1, x_2, x_3)$. $\mathcal{B} = \{(1,0),(0,1)\}$ is a basis for $\mathbb{R}^2$ and $\mathcal{B}' = \{(1,1,0),(0,1,0),(0,0,1)\}$ is a basis for $\mathbb{R}^3$. Find a 3×2 matrix A (with respect to these bases so that, for every $\alpha \in \mathbb{R}^2$, we have:
$$[T(\alpha)]_{\mathcal{B}'} = A[\alpha]_{\mathcal{B}}.$$