Last name ________________________

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LARSON—MATH 610—CLASSROOM WORKSHEET 24
Projections in Inner Product Spaces.

Concepts & Notation

- (Chp. 6) *dot product*, *inner product*, *inner product space*, *norm*, *orthogonal representation*, *Cauchy-Schwartz*, *orthonormal list*, *Gram-Schmidt*, *orthogonal complement*, *orthogonal projection*.
- (Chp. 7) *adjoint*.

1. For a subspace U of an inner product space V, what is the *orthogonal projection* operator P_U?

2. (**Minimizing the distance to a subspace**) Suppose U is a finite-dimensional subspace of V, $v \in V$, and $u \in U$. Then $||v - P_U v|| \leq ||v - u||$. Furthermore, the inequality above is an equality if and only if $u = P_U v$.

Linear Functionals and Riesz Representation Theorem

3. What is a *linear functional*?

4. What is the *Riesz Representation Theorem*?

Adjoint Operators

5. Let V, W be finite-dimensional inner product spaces and $T \in \mathcal{L}(V, W)$. What is the *adjoint* T^* of T?

6. Why does the adjoint exist?

7. Find the adjoint T^* of $T \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_3)$.

8. (**Claim**) The adjoint of a linear map on an inner product space is linear.