LARSON—MATH 511—CLASSROOM WORKSHEET 07
Gilbert Strang Lecture 4.

More on Strang's Lectures

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \end{bmatrix}$.

1. Find AA^T and A^TA.

2. What do you notice? Is your observation *always* true?

3. What is an *eigenvalue* of a (square) matrix A?

4. What can you say about the eigenvalues of A= $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$?

5. Find the eigenvalues of A= $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

6. Let λ_1 and λ_2 be the eigenvalues of A with corresponding (unit) eigenvectors $\hat{x}_1$ and $\hat{x}_2$. Let Q be the matrix whose columns are $\hat{x_1}$ and $\hat{x}_2$. What kind of matrix is Q?

7. Multiply out AQ and use this to get a "decomposition" of A in the form $Q\Lambda Q^T$.

8. What can we say about the relationship between A and Λ?

Sage/CoCalc

(a) Start the Chrome browser.

(b) Go to `http://cocalc.com`

(c) Login (likely using **your VCU email address**).

(d) You should see an existing Project for our class. Click on that.

(e) Click "New", then "Sage Worksheet", then call it **c07**.

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \end{bmatrix}$.

9. Find AA^T and A^TA

10. Find the eigenvalues of A= $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

11. Find the eigenvectors corresponding to the eigenvalues of A.

12. Let λ_1, λ_2 be the eigenvalues of A, with corresponding eigenvectors $\hat{x}_1$ and $\hat{x}_2$; and let Q be the matrix whose columns are $\hat{x_1}$ and $\hat{x}_2$.

13. How can we use SAGE to *check* that a Q is orthogonal?

14. Let Λ be the diagonal matrix with λ_1 and λ_2 on the diagonal. Check that $A = Q\Lambda Q^T$.

Getting your classwork recorded

When you are done, before you leave class...

1. Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If CoCalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

2. Send me an email with an informative header like "Math 511—c07 worksheet attached" (so that it will be properly recorded).

3. Remember to attach today's classroom worksheet!