Last name ______________________

First name ______________________

LARSON—MATH 610–CLASSROOM WORKSHEET 13
Block Matrices.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence, pivot column decomposition, direct sum $\mathcal{U}\oplus\mathcal{V}$, *orthogonal* matrix, *unitary* matrix, *basis*, *dimension*, *linear transformation.*

Review:

1. $AB = A[B_1\ B_2] = [AB_1\ AB_2]$.
2. Find a formula for AB in terms of the rows of A.
3. Let $A \in \mathbb{M}_n$ be invertible. and R be a product of elementary matrices which code a sequence of row operations that reduces A to I. Then $RA = I$, and $R = A^{-1}$. Then,
$$R[A\ I] = [RA\ R] = [I\ A^{-1}].$$
If the block matrix $[A\ I]$ reduces to $[I\ X]$, then $X = A^{-1}$.

Chp. 3 of Garcia & Horn, Matrix Mathematics

1. What do we assume when we add matrices, $A + B$?

2. What do we assume when we multiply matrices, AB?

3. Check that $AB = \begin{bmatrix} X & Z \end{bmatrix}\begin{bmatrix} Y \\ W \end{bmatrix} = XY + ZW$, where
$$A = \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 3 & 4 \\ 0 & 5 & 0 \end{array}\right] = [X\ Z] \text{ and } B = \left[\begin{array}{cc} 3 & 0 \\ 1 & 4 \\ \hline 0 & 1 \end{array}\right] = \begin{bmatrix} Y \\ W \end{bmatrix}$$

4. Check that:

$$AB = \begin{bmatrix} A_1 & A_2 \end{bmatrix} \begin{bmatrix} B_1^T \\ B_2^T \end{bmatrix} = A_1 B_1^T + A_2 B_2^T,$$

   assuming all matrices are conformal.

5. What is the *inner product* of vectors $\hat{x}$ and $\hat{y}$?

6. What is the *outer product* of vectors $\hat{x}$ and $\hat{y}$?

7. Write a formula for the product AB in terms of an *outer product* of the columns of A and the rows of B.

8. Suppose A and B are invertible. Show $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$.

9. (**Notation**) How is the *direct sum* $A \oplus B$ defined?