Last name ______________________

First name ______________________

LARSON—MATH 610–CLASSROOM WORKSHEET 10
Block Matrices.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence, pivot column decomposition, direct sum $\mathcal{U} \oplus \mathcal{V}$, *orthogonal* matrix, *unitary* matrix, *basis*, *dimension*.

Review:

1. *Any* linearly independent set of vertices in a finite-dimensional vector space can be extended to a basis.
2. What is $\mathfrak{L}(\mathcal{V}, \mathcal{W})$?

Chp. 3 of Garcia & Horn, Matrix Mathematics

1. How does any matrix $A \in \mathbb{M}_{m\times n}$ define a linear transformation?

2. How does any linear transformation $T \in \mathcal{L}(\mathcal{V}, \mathcal{W})$ and bases $\beta = \hat{v}_1, \ldots, \hat{v}_n$ of $\mathcal{V}$ and $\gamma = \hat{w}_1, \ldots, \hat{w}_m$ of $\mathcal{W}$ define a matrix $A \in \mathbb{M}_{m\times n}$?

3. What is ${}_\gamma[T]_\beta$?

4. What is the β-γ *change-of-basis* matrix (notation: ${}_\gamma[I]_\beta$)?

5. What is an example?

Block Matrices

Let $A \in \mathbb{M}_{m\times r}$ and $B \in \mathbb{M}_{r\times n}$, and write $B = [B_1\ B_2]$, where B_1 is the first k columns of B and B_2 is the remaining $n-k$ columns of B. Then,

$$AB = A[B_1\ B_2] = [AB_1\ AB_2].$$

6. Check with $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 & 2 \\ 6 & 7 & 1 \end{bmatrix}$, where B_1 is the first two columns of B and B_2 is the remaining column.

Let $A \in \mathbb{M}_n$ be invertible. and R be a product of elementary matrices which code a sequence of row operations that reduces A to I. Then $RA = I$, and $R = A^{-1}$. Then,

$$R[A\ I] = [RA\ R] = [I\ A^{-1}].$$

If the block matrix $[A\ I]$ reduces to $[I\ X]$, then $X = A^{-1}$.