Last name ______________________

First name ______________________

LARSON—OPER 731—HOMEWORK h16
Test 2 Review

Concepts For each concept, give a **definition** and an **example**.

1. What is *complementary slackness*?
2. What is a *vertex packing* (*independent set*) in a graph?
3. How can we model finding a maximum vertex packing in a graph with an IP (What is the *vertex packing IP*?)
4. What is the *set cover* problem?
5. What is the relationship between the vertex cover problem and the vertex packing problem?
6. What is the *cone* of vectors $a^{(1)}, a^{(2)}, \ldots, a^{(k)}$ in $\mathbb{R}^n$.
7. What is a *directed graph*?
8. What is the vertex-arc incidence matrix of a directed graph?
9. What is a *totally unimodular matrix*?
10. Why is the vertex-edge incidence matrix of a directed graph totally unimodular?
11. What is an *s-t flow* in a directed graph with non-negative edge capacities?
12. What is the *value* of a flow?
13. What is a *matroid*?

Theorems. State and Prove the following theorems.

14. What is the *Complementary Slackness Theorem*?
15. What is *Farkas's Lemma*?

Problems Explain everything. As scientists it is never enough to write answers. They must be communicated—convincingly—to others.

16. (a) What is an example of a minimum cost perfect matching problem?
    (b) Model the problem as an IP.
    (c) Find its dual and interpret its *meaning*.
    (d) Given a dual feasible y, what is the *reduced cost* of an edge?
    (e) Given a minimum cost perfect matching IP and dual feasible y, explain why an optimal solution of the IP with reduced cost edges is an optimal solution of the original IP.

17. (a) Find the dual for following (primal) optimization problem:
max $(5, 3, 5)x$
subject to:
$$\begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 1 \end{pmatrix} x \leq \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$$
$x \geq \mathbb{O}$

(b) What are the complementary slackness conditions for an optimal solution to this primal-dual pair?

(c) Check that $\bar{x} = (1, -1, 1)^T$ is primal-feasible and $\bar{y} = (0, 2, 1)^T$ is dual-feasible.

(d) Check that $\bar{x}$ and $\bar{y}$ are optimal by verifying the complementary slackness conditions.

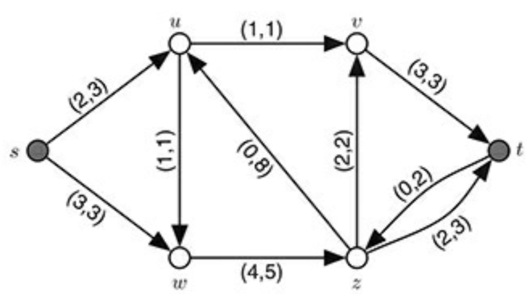
18. Find the cone of vectors $a^{(1)} = (2, -1)^T$, $a^{(2)} = (3, 1)^T$, $a^{(3)} = (2, 1)^T$ in $\mathbb{R}^2$.

19. (a) Check that $\bar{x} = (2, 1)^T$ is feasible for the linear program:
max $(\frac{3}{2}, \frac{1}{2})x$
subject to:
$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} x \leq \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$$
$x \geq \mathbb{O}$

(b) Identify which constraints are tight for $\bar{x}$. Let $J(\bar{x})$ be the corresponding row indices.

(c) What is the *cone of tight constraints* for $\bar{x}$ for polyhedron $(P) = \{x : Ax \leq b\}$ in this example?

20. Let $\bar{x}$ be a feasible solution to $max\{c^T x : Ax \leq b\}$. **Show**: $\bar{x}$ is optimal if and only if and only if c is in the cone of tight constraints for $\bar{x}$.



21. (a) The first numbers on each edge are flow values and the second numbers are edge capacities. Do the flow values indicate a valid flow? What is the value of this flow?

(b) Can you find a flow with a larger value in this network? If not, prove that this flow is maximum?

(c) What is an *s-t cut*? What is the *capacity* of an *s-t* cut?

(d) Find a minimum cut in this network. Prove that it is minimum.

22. (**Show:**). If a max-flow problem has integer capacities and an optimal solution, then there is an optimal *integer flow*.