Last name _____

First name _____

LARSON—MATH 610–CLASSROOM WORKSHEET 16
Inner Products.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence.
(Chp. 2).pivot column decomposition, direct sum $\mathcal{U} \oplus \mathcal{V}$, *orthogonal* matrix, *unitary* matrix, *basis*, *dimension*, *linear transformation*.
(Chp. 3). *conformable* matrix addition and multiplication.
(Chp. 4). *nullity*, $A \oplus B$.
(Chp. 5). *inner product*, *inner product space*, $\langle\ \rangle$, *orthogonal* vectors, $\perp$, $||\cdot||$.

Review:

1. (**Notation**) How is the *direct sum* $A \oplus B$ defined?

2. What is the **Dimension Theorem** for linear transformations?

3. What is the **Rank-Nullity** Theorem (for matrices)?

Chp. 4 of Garcia & Horn, Matrix Mathematics

(**Theorem 4.2.1**). Let $A \in \mathbb{M}_{m\times n}(\mathbb{F})$. If $X \in \mathbb{M}_{p\times m}(\mathbb{F})$ has full column rank and $Y \in \mathbb{M}_{n\times q}(\mathbb{F})$ has full row rank then

$$rank(A) = rank(XAY).$$

1. What does this theorem say?

2. What is an example?

3. Why is it true?

Chp. 5 of Garcia & Horn, Matrix Mathematics

4. What is an *inner product* on a $\mathbb{C}$-vector space? What is the notation $\langle\ \rangle$?

5. What is an example?

6. What is an *inner product space*?

7. When are vectors *orthogonal*? What is the notation $\hat{u} \perp \hat{v}$?

8. What is a *norm* induced by an inner product? What is the notation $||\cdot||$?

9. What is a *unit* vector in an inner product space?

10. What properties does an inner product in a $\mathbb{C}$-vector space have?

11. What is the **Pythagorean Theorem** in an inner product space?