Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 37
The Structure of Linear Operators.

Concepts & Notation

- (Sec. 5.3) *permutation*, det A.
- (Sec. 6.2) *characteristic value*, *characteristic vector*, *characteristic polynomial*, *diagonalizable* linear operator.

Review

1. If $T \in \mathcal{L}(V)$ and $p \in \mathbb{F}[x]$ then $p(T) \in \mathcal{L}(V)$.

The Structure of a Linear Operator

2. (**Claim:**) If $T \in \mathcal{L}(V)$, $T(\alpha) = c\alpha$ (for $c \in \mathbb{F}$, $\alpha \in V$), and $p \in \mathbb{F}[x]$ then $p(T)(\alpha) = p(c)\alpha$.

3. What does it mean for a polynomial to *annihilate* a linear operator T?

4. What is the *minimal polymonial* of a linear operator T over a finite-dimensional vector space T? (Does it exist? What does it tell us?)

5. (**Cayley-Hamilton Theorem**) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find the characteristic polynomial p of A and check that $p(A) = 0$.

6. Use $p(A)$ to find A^{-1}.

7. (**Cayley-Hamilton Theorem**) Let $A \in \mathbb{F}^{n \times n}$. Let $p \in \mathbb{F}[x]$ be the characteristic polynomial of A. Then $p(A) = 0$.

8. (**Cayley-Hamilton Theorem**) Let V be a finite-dimensional vector space over a field $\mathbb{F}$ and $T \in \mathcal{L}$. Let $p \in \mathbb{F}[x]$ be the characteristic polynomial of T. Then $p(T) = 0$.