Last name ______________________

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LARSON—MATH 610–CLASSROOM WORKSHEET 04
Vector Spaces and Subspaces.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence, pivot column decomposition, direct sum $\mathcal{U} \oplus \mathcal{V}$.

Review:

(**Theorem 1.3.3**). Let $\mathcal{V}$ be an $\mathbb{F}$-vector space and let $\mathcal{U}$ be a non-empty subset of $\mathcal{V}$. Then U is a subspace of $\mathcal{V}$ if and only if $cu + v \in \mathcal{U}$ whenever $u, v \in \mathcal{U}$ and $c \in \mathbb{F}$.

(**Theorem 1.4.10**). Let $Y = [\hat{y}_1 \hat{y}_2 \dots \hat{y}_p] \in \mathbb{M}_{m\times p}(\mathbb{F})$ and let $A \in \mathbb{M}_{m\times n}(\mathbb{F})$ then $col(Y) \subseteq col(A)$ if and only if $Y = AX$ for some $X \in \mathbb{M}_{n\times p}$.

1. What is an *orthogonal* matrix?

2. What is a *unitary* matrix?

3. Check that $\begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}$ is an example of a unitary matrix that is not orthogonal.

(**Theorem 1.5.9**). Let $\mathcal{U}$ and $\mathcal{W}$ be subspaces of an $\mathbb{F}$-vector space $\mathcal{V}$ and suppose that $\mathcal{U} \cap \mathcal{W} = \{0\}$. Then each vector in $\mathcal{U} \oplus \mathcal{W}$ is uniquely expressible as a sum of a vector in $\mathcal{U}$ and a vector in $\mathcal{W}$.

4. Why is this theorem true?

(**Theorem 1.7.1**) Let $\beta = \hat{v}_1, \ldots, \hat{v}_p$ be a nonzero list of vectors in an $\mathbb{F}$-vector space. There is an $s \in \{1, \ldots, p\}$ and unique indices $j_1, \ldots, j_s$ such that:

(a) $1 \leq j_1 < j_2 < j_s \leq p$.

(b) $\gamma = \hat{v}_{j_1}, \hat{v}_{j_2}, \ldots, \hat{v}_{j_s}$ is linearly independent.

(c) $span(\gamma) = span(\beta)$.

(d) If $j < j_1$ then $\hat{v}_j = \hat{0}$.

(e) If $s > 1$, $2 \leq k \leq s$ and $j_{k-1} < j < j_k$, then $\hat{v}_j \in span\{v_{j_1}, \hat{v}_{j_2}, \ldots, \hat{v}_{j_{k-1}}\}$.

5. What is this theorem about?

6. Why is this theorem true?