Last name ______________________

First name ______________________

LARSON—MATH 550—CLASSROOM WORKSHEET 41
Generating Functions & Fibonacci Numbers!

Concepts & Notation

- Sec. 5.4: Convolutions, generating functions.
- Sec. 6.6 Fibonacci Numbers.

Fibonacci Numbers

We defined $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$.

Goals: We found the generating function $F(z) = \frac{z}{1-z-z^2}$ for $\langle F_n \rangle$ and now we'd use this to find a *formula* for the Fibonacci numbers F_n.

The partial fraction decomposition $\frac{z}{1-z-z^2} = \frac{A}{1-\alpha z} + \frac{B}{1-\beta z}$ gave us:

$$\langle F_n \rangle = \langle A\alpha^n + B\beta^n \rangle.$$

Since $\frac{A}{1-\alpha z} + \frac{B}{1-\beta z} = \frac{A(1-\beta z)+B(1-\alpha z)}{(1-\alpha z)(1-\beta z)}$, we get:

$$A + B = 0$$

$$A\beta + B\alpha = -1$$

$$\alpha + \beta = 1$$

$$\alpha\beta = -1$$

1. Now find A, B, α, β.

2. So what is our formula for F_n?

3. Let $\phi = \frac{1+\sqrt{5}}{2}$, $\hat{\phi} = \frac{1-\sqrt{5}}{2}$, and re-write our formula for F_n.

4. Why is $F_n \sim \frac{1}{\sqrt{5}}\phi^n$ when n is large?

5. Use this formula to approximate F_{11}.

6. Show that $F_n = \lfloor \frac{1}{\sqrt{5}}\phi^n + \frac{1}{2} \rfloor$