Last name ______________________

First name ______________________

LARSON—MATH 550—CLASSROOM WORKSHEET 22
Pascal's Triangle & Binomial Coefficients.

Concepts & Notation

- Sec. 5.1. Binomial coefficients, Pascal's triangle.

We let $\binom{n}{m}$ be the number of m-subsets of an n-set.

1. Find a formula for $\binom{n}{m}$ $(0 \le m \le n,\ m, n \in \mathbb{Z})$.

2. Argue the *symmetry identity* $\binom{n}{k} = \binom{n}{n-k}$.

3. We discovered that, in order to *prove* that this triangle is the same as Pascal's triangle, we'd need to prove the *addition formula*:

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}.$$

4. We drew a Pascal-style triangle where the 0^{th} (top) level is the single number $\binom{0}{0}$, and where the n^{th} level is the $(n+1)$ numbers $\binom{n}{0}, \binom{n}{1}, \ldots \binom{n}{n}$. Find the sums of the rows. How can we interpret these? How can we prove them?

5. Find $(x+y)^2$.

6. Find $(x+y)^3$ by multiplying $(x+y)(x+y)^2$.

7. Find $(x+y)^4$ by multiplying $(x+y)(x+y)^3$.

8. Find $(x+y)^n$. How can you reason about this?

The *binomial coefficients* $\binom{n}{k}$ $(n, k \in \mathbb{Z}^{\geq 0})$ can be generalized to $\binom{r}{k}$ $(r \in \mathbb{R}, k \in \mathbb{Z})$:

$$\binom{r}{k} = \frac{r^{\underline{k}}}{k!} \text{ (if } k \geq 0)$$

$$\binom{r}{k} = 0 \text{ (if } k < 0).$$

9. Find $\binom{-1}{2}$.

10. Find $\binom{-1}{-1}$.

11. Find $\binom{-1}{0}$.

12. Find $\binom{\pi}{2}$.

13. Argue the *absorbtion identity* $\binom{r}{k} = \frac{r}{k}\binom{r-1}{k-1}$.

14. The (Newton's Generalized) *Binomial Theorem* says $(x+y)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^k y^{r-k}$. Does this agree with our formula when $r \in \mathbb{Z}$?

15. Find $(x+1)^e$.