LARSON—MATH 511—CLASSROOM WORKSHEET 18
Pseudo-inverses & Gram-Schmidt

Pseudo-inverses

1. Given a matrix A, how is the *pseudo-inverse* (Moore-Penrose inverse) A^+ defined?

2. Find the pseudo-inverse of $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

3. Does A^+ give the least-squares solution we previously got?

4. Check that $A^+ = A^{-1}$ when A is invertible and $\hat{x} = A^+\hat{b}$ is the solution of $A\hat{x} = \hat{b}$.

5. If $A\hat{x} = \hat{b}$ is not solvable, $\hat{x} = A^+\hat{b}$ is a "solution".

6. Solve $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Gram-Schmidt

7. Use Gram-Schmidt to find an orthogonal basis, $\hat{q}_1, \hat{q}_2, \hat{q}_3$, of the columns of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

8. Let Q be the matrix whose columns are $\hat{q}_1$, $\hat{q}_2$, and $\hat{q}_3$. Write $A = QR$ (for some matrix R).

9. What can we say about R? and how will this QR decomposition of A help us solve $A\hat{x} = \hat{b}$?

Sage/CoCalc

10. (a) Start the Chrome browser.
    (b) Go to `http://cocalc.com`
    (c) Login (likely using **your VCU email address**).
    (d) You should see an existing Project for our class. Click on that.
    (e) Click "New", then "Sage Worksheet", then call it **c18**.

11. Find the pseudo-inverse of $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

12. Use Sage to find a QR decomposition of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (these are not unique).

Getting your classwork recorded

When you are done, before you leave class...

1. Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If CoCalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).
2. Send me an email with an informative header like "Math 511—c18 worksheet attached" (so that it will be properly recorded).
3. Remember to attach today's classroom worksheet!