Last name ________________________

First name ________________________

LARSON—MATH 556–HOMEWORK WORKSHEET 06
Trees and Forests

An **acyclic** graph is a graph that does not have any cycles.

A **forest** is another name for an acyclic graph. Note that, according to the definition a forest may have more than one component.

A **tree** is a connected acyclic graph.

The **degree** of a point v in a graph, denoted $d(v)$ is the number of points v is adjacent to. A **pendant** (or **leaf**) is a point with degree 1. A **branch point** (or **branching point**) in a forest is a point with degree at least three.

1. (**Claim from class**) Prove that every tree with at least two points has at least two pendants.

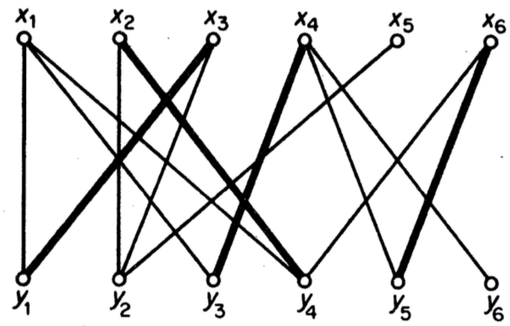
   Induction in graph theory usually works as follows. Assume that all graphs (trees, etc) with less than p points have some property. Then we will consider an arbitrary graph on p points and shows that it also has this property. We conclude that all graphs have this property.

   Let G be a graph with p points. Let v be a point of G (often with some specific property, for instance, v may be chosen to have minimum degree). Let $G' = G - v$.

2. Prove that every tree is bipartite.

3. Prove that, for any tree with more than one point, the number of pendants of the tree is more than the number of its branch points.

4. A *spanning tree* in a connected graph is a subgraph that is a tree. Prove that every connected graph has a spanning tree.



5. Let $A = \{x_1, x_2, \ldots, x_6\}$, $B = \{y_1, y_2, \ldots, y_6\}$, and the shaded lines be the initial matching M. Use the Hungarian Method to find a maximum matching in this graph. For each iteration define A_1, B_1, F, X and Y.

   In the final iteration, $Y = V(F) \cap B$ and $X = A \setminus V(F)$ must be a point cover. Check that the cardinality of this point cover and the cardinality of your matching are the same.