Last name ______________________

First name ______________________

LARSON—MATH 310–CLASSROOM WORKSHEET 27
Gaussian Elimination, Eigenvalues

Review

1. What is a basis for the row space of a matrix in echelon form?
2. **Gaussian elimination operations**. Use these same three operations from solving systems of equations to produce a matrix *in echelon form* that represents a system of equations with the same solutions:
   (a) Switch any two rows.
   (b) Add a multiple of one row to another.
   (c) Scale any row (by multiplying by a non-zero constant).
3. **Gaussian elimination algorithm**.
   (a) Switch a not-yet-processed row with a left-most non-zero entry a to the top of the not-yet-processeded rows.
   (b) Use this row, and *pivot* term a, to get 0's below a.
   (c) Repeat on remaining unprocessed rows.

   At the termination of this algorithm, the produced matrix is guaranteed to be (1) in echelon form, (2) with all 0's below the main diagonal, and (3) with all 0 rows at the bottom.

Chapter 7 of Klein's *Coding the Matrix* text

1. Use Gaussian elimination to produce an equivalent matrix in echelon form.

   Let
$$A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 2 & 6 \end{bmatrix}.$$

2. How can we use Gaussian elimination to find the null space of a matrix?

Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ -2 & -3 \end{bmatrix} \text{ and } \hat{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

3. (a) Form the augmented matrix corresponding to the system $A\hat{x} = \hat{b}$.

   (b) Get this matrix to row-reduced echelon form.

   (c) Find conditions on b_1, b_2, b_3 for the system to have a solution.

Chapter 12 of Klein's *Coding the Matrix* text

If there is a non-zero vector $\vec{x}$ and scalar λ with $A\vec{x} = \lambda\vec{x}$ then λ is an *eigenvalue* of A and $\vec{x}$ is a corresponding *eigenvector*.

Note: If $A\vec{x} = \lambda\vec{x}$, then $A\vec{x} - \lambda\vec{x} = 0$, and $(A - \lambda I)\vec{x} = 0$.

4. What are examples?