Last name ______________________

First name ______________________

LARSON—MATH 601—HOMEWORK WORKSHEET h05
Vector Spaces.

The space of a functions from a set S to a field $\mathbb{F}$ is a vector space.

Let $V = \{f : S \to \mathbb{F} |$ f is a function$\}$,

with an "addition" $f +_V g$ (for $f, g \in V$) defined by the rule $(f +_V g)(s) = f(s) +_F g(s)$

and a "scalar multiplication" $c \cdot f$ (for $c \in \mathbb{F}$, $f \in V$) defined by $(c \cdot f)(s) = cf(s)$.

For the purpose of this first example only **only** indicate the field addition by '$+_F$' and the vector addition by '$+_V$'. They are **different**. I also indicated the scalar multiplication operator with a '$\cdot$' and the field multiplication by concatenation). Its always clear from the context which one is meant, but for one time only carefully distinguish between them.

1. Check all the vector space axioms for this example.
   (a) vector addition is commutative,
   (b) vector addition is associative,
   (c) there is an additive identity "0" (in V),
   (d) there are additive inverses "$-\alpha$" for each $\alpha \in V$,
   (e) $1 \cdot \alpha = \alpha$ (for $1 \in \mathbb{F}$ and $\alpha \in V$),
   (f) $(c_1 c_2) \cdot \alpha = c_1 \cdot (c_2 \cdot \alpha)$ (for $c_1, c_2 \in \mathbb{F}$, $\alpha \in V$),
   (g) $c \cdot (\alpha_1 +_V \alpha_2) = c \cdot \alpha_1 +_V c \cdot \alpha_2$ (for $c \in \mathbb{F}$, $\alpha_1, \alpha_2 \in V$), and
   (h) $(c_1 +_F c_2) \cdot \alpha = c_1 \cdot \alpha +_V c_2 \cdot \alpha$ (for $c_1, c_2 \in \mathbb{F}$, $\alpha \in V$).

2. Let $\mathbb{R}^n$ be the set of tuples $(a_1, a_2, \ldots, a_n)$ $(a_i \in \mathbb{R})$.
   Show: $\mathbb{R}^n$ is a vector space over $\mathbb{R}$.