Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 15
Coordinates, Row Equivalence.

Concepts & Notation

- (Sec. 2.4) *ordered basis*, *coordinates*, *coordinate matrix*, $[\alpha]_{\mathcal{B}}$.
- (Sec. 2.5) *row rank*.

Basis Extension Theorem: Any linearly independent set in a finite-dimensional vector space V is part of (can be extended to) a (finite) basis for V.

1. $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(1,0),(0,1)\}$ is a basis for $\mathbb{R}^2$. Let $\alpha = (2,3)$. Find x_1, x_2 such that $\alpha = x_1\alpha_1 + x_2\alpha_2$. Find the coordinate matrix $[\alpha]_{\mathcal{B}}$.

2. Let $\mathcal{B}' = \{\alpha_1', \alpha_2'\} = \{(\cos\theta, \sin\theta), (-\sin\theta, \cos\theta)\}$ (θ is a fixed real number). Show $\mathcal{B}'$ is a basis for $\mathbb{R}^2$.

3. Let $\alpha = (2,3)$. Find x_1', x_2' such that $\alpha = x_1'\alpha_1' + x_2'\alpha_2'$. Find the coordinate matrix $[\alpha]_{\mathcal{B}'}$.

4. Find the coordinates for α_1' in terms of basis $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(1,0),(0,1)\}$, that is, find $[\alpha_1']_{\mathcal{B}}$.

5. Find the coordinates for α_2' in terms of basis $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(1,0),(0,1)\}$, that is, find $[\alpha_2']_{\mathcal{B}}$.

6. Let P be the matrix whose columns are $[\alpha_1']_{\mathcal{B}}$ and $[\alpha_2']_{\mathcal{B}}$. Check that $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$.

7. Why does that construction work?

8. Does P depend on α?

9. Argue that P is invertible.

10. **Explain:** For a vector space V with bases $\mathcal{B}$ and $\mathcal{B}'$, and vector $\alpha \in V$, there is an invertible matrix P such that $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$.

11. Explain how you can find a matrix P' so that $[\alpha]_{\mathcal{B}'} = P'[\alpha]_{\mathcal{B}}$. What is P'?

12. What is the *row space* of a matrix? Why is it a vector space? Describe the row space of $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

13. What is the *row rank* of a matrix? Find the row rank of $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.