Last name _____

First name _____

LARSON—MATH 550—CLASSROOM WORKSHEET 40
Generating Functions & Fibonacci Numbers!.

Concepts & Notation

- Sec. 5.4: Convolutions, generating functions.
- Sec. 6.6 Fibonacci Numbers.

Fibonacci Numbers

We defined $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$.

Goals: We found the generating function $F(z) = \frac{z}{1-z-z^2}$ for $\langle F_n \rangle$ and now we'd use this to find a *formula* for the Fibonacci numbers F_n.

We will attempt a partial fraction decomposition $\frac{z}{1-z-z^2} = \frac{A}{1-\alpha z} + \frac{B}{1-\beta z}$.

1. First, find the sequence that the function $\frac{1}{1-\alpha z}$ generates.

2. Now find the sequence that the function $\frac{A}{1-\alpha z} + \frac{B}{1-\beta z}$ generates.

3. Now find A, B, α, β.

4. So what is our formula for F_n?

5. Let $\phi = \frac{1-\sqrt{5}}{2}$, $\hat{\phi} = \frac{1}{\phi}$, and re-write our formula for F_n.

6. Why is $F_n \sim \frac{1}{\sqrt{5}}\phi^n$ when n is large?

7. Use this formula to approximate F_{11}.

8. Show that $F_n = \lfloor \frac{1}{\sqrt{5}}\phi^n + \frac{1}{2} \rfloor$