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LARSON—MATH 610—CLASSROOM WORKSHEET 18
Linear Transformations.

Concepts & Notation

- (Sec. 3.1) *linear transformation, rank, nullity.*

1. What is a *linear transformation* T from a vector space V into a vector space W ?
What is the *null space* of T ? What is the *range* of T ?

2. Consider the function $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(x_1, x_2) = (2x_1, 0, 0)$? Show it is a linear transformation.

3. Find the null space of T .

4. What is the *nullity* of T ?

5. Find the range of T .

6. What is the *rank* of T ?

7. **(Claim:)** If $\alpha_1, \dots, \alpha_k$ are linearly independent vectors in a vector space V and β_1, \dots, β_k are any vectors in a vector space W then there is a *unique* linear transformation T with $T(\alpha_1) = \beta_1, \dots, T(\alpha_k) = \beta_k$.

8. **(Rank-Nullity Theorem:)** If V is a finite-dimensional vector space then $\text{rank}(V) + \text{nullity}(V) = \dim(V)$.