Last name _____

First name _____

LARSON—MATH 550—CLASSROOM WORKSHEET 18
Ceiling, floor, intervals, mod.

Concepts & Notation

- Sec. 3.2. interval notation.
- Sec. 3.4. mod notation.
- Sec. 4.4. Analyzing the size of $n!$
- Sec. 5.1. Binomial coefficients!

Interval Notation & Modulus

1. What are $[\alpha..\beta], [\alpha..\beta)$ (for $\alpha, \beta \in \mathbb{R}$)?

2. How many integers are in $[\alpha..\beta)$ (for $\alpha, \beta \in \mathbb{R}$)?

The *quotient* of positive integers n and m is $\lfloor n/m \rfloor$ and the *modulus* is the remainder of dividing n by m, denoted $n \mod m$.

3. Find $\lfloor 32/5 \rfloor$ and $32 \mod 5$ and check that $32 = \lfloor 32/5 \rfloor \cdot 5 + (32 \mod 5)$.

4. State a general law for positive integers n and m.

Estimating the size of $n!$

5. Check: $(n!)^2 = (1 \cdot 2 \ldots n)(1 \cdot 2 \ldots n) = \prod_{k=1}^{n} k(n+1-k)$

6. Check: $k(n+1-k) = \frac{1}{4}(n+1)^2 - (k - \frac{1}{2}(n+1))^2$

7. What is the smallest value of $k(n+1-k)$ (for $k, n \in \mathbb{Z}^+$, $k \leq n$)?

8. What is the largest value of $k(n+1-k)$ (for $k, n \in \mathbb{Z}^+$, $k \leq n$)?

9. What can we conclude?