

Last name \_\_\_\_\_

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**LARSON—MATH 310—CLASSROOM WORKSHEET 23**  
**Linear Dependence, Linear Independence, Basis, Dimension**

**Review**

1. What is the *inverse* of a matrix?
2. What is the importance of matrix inverses?  
(**Question 5.2.5**) For a given vector space  $\mathcal{V}$ , what is the minimum number of vectors whose span equals  $\mathcal{V}$ ?  
(**Definition 5.5.2**) Vectors  $\hat{v}_1, \dots, \hat{v}_n$  are *linearly dependent* if the zero vector can be written as a nontrivial linear combination of the vectors:  $0 = \alpha_1 \hat{v}_1 + \dots + \alpha_n \hat{v}_n$ .
3. What does it mean for vectors to be *linearly independent*?

**Chapter 5 of Klein's *Coding the Matrix* text**

1. What is an equivalent definition for a *linearly independent* set of vectors?

(**Definition 5.6.1**) Let  $\mathcal{V}$  be a vector space. A *basis* for  $\mathcal{V}$  is a linearly independent set of generators for  $\mathcal{V}$ .

2. What are examples?
3. Argue: the standard generating set for  $\mathbb{R}^2$  is a basis for  $\mathbb{R}^2$ .
4. Argue: the standard generating set for  $\mathbb{R}^3$  is a basis for  $\mathbb{R}^3$ .

5. Show: the vectors  $[1, 1, 1], [1, 1, 0], [0, 1, 1]$  are a basis for  $\mathbb{R}^3$ .

**(Lemma 5.7.1, Unique-Representation Lemma)** Let  $\hat{a}_1, \dots, \hat{a}_n$  be a basis for a vector space  $\mathcal{V}$ . For any vector  $\hat{v} \in \mathcal{V}$ , there is exactly one representation of  $\hat{v}$  in terms of the basis vectors.

6. Find the *coordinates* for  $[3, 3, 1]$  with respect to the standard basis for  $\mathbb{R}^3$ .
7. Find the *coordinates* for  $[3, 3, 1]$  with respect to the basis  $[1, 1, 1], [1, 1, 0], [0, 1, 1]$  for  $\mathbb{R}^3$ .

8. Why is Lemma 5.7.1 true?

**(Theorem 6.1.2, Basis Theorem)** Let  $\mathcal{V}$  be a vector space. All bases for  $\mathcal{V}$  have the same size.

9. What is the *dimension* of a vector space?
10. What are examples?
11. Given a generating set for a vector space  $\mathcal{V}$ , how can we find a basis for  $\mathcal{V}$ ?