Last name ______________________

First name ______________________

LARSON—MATH 550—CLASSROOM WORKSHEET 04
Mathematical Induction. Lines in the Plane.

Concepts & Notation

- Sec. 1.1 & Sec. 1.2 T_n, recurrence (recurrence relation), mathematical induction, basis, solving recurrences
- Sec. 2.1 $[m = n]$ notation, sum notations.
- Sec. 2.2 The "trick".

Induction

Let $P(n)$ be the (open) statement: "$2^0 + 2^1 + 2^2 + \ldots + 2^{n-1} = 2^n - 1$".

1. Check that $P(1)$, $P(2)$ and $P(3)$ are true (base cases).

2. Show: that $P(n)$ implies $P(n+1)$.
   That is, assume:
   $$2^0 + 2^1 + 2^2 + \ldots + 2^{n-1} = 2^n - 1$$
   and show:
   $$2^0 + 2^1 + 2^2 + \ldots + 2^{(n+1)-1} = 2^{n+1} - 1$$

3. What can you conclude?

Lines in the Plane

4. What is the maximum number of regions defined by n lines in the plane? Try the methodology developed in the Towers of Hanoi problem

   (a) *Name* the quantity you want to count/investigate.

   (b) Find some values of that quantity.

   (c) Find a recurrence relation for that quantity.

   (d) Use the recurrence to find more values of that quantity.

   (e) Use these values to *guess* a (non-recurrence closed-form) formula for that quantity.

   (f) *Prove* your formula.

5. (Sec. 2.2) Suppose $S_n = \sum_{k=0}^{n} a_k$ and $a_k = \alpha + \beta k$. Use our methodology to "solve" this recurrence.

(Sec. 2.2). Given a recurrence of the form $a_n T_n = b_n T_{n-1} + c_n$, you can get a "nicer" recurrence by multiplying through by:

$$s_n = \frac{a_{n-1} a_{n-2} \dots a_1}{b_n b_{n-1} \dots b_2}$$

6. What would this yield for $T_n = 2T_{n-1} + 1$?

7. What would this yield for $L_n = L_{n-1} + n$?