Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 18
Linear Transformations.

Concepts & Notation

- (Sec. 3.1) *linear transformation*, *rank*, *nullity*.

1. What is a *linear transformation* T from a vector space V into a vector space W? What is the *null space* of T? What is the *range* of T?

2. Consider the function $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1, x_2) = (2x_1, 0, 0)$? Show it is a linear transformation.

3. Find the null space of T.

4. What is the *nullity* of T?

5. Find the range of T.

6. What is the *rank* of T?

7. (**Claim:**) If $\alpha_1, \ldots, \alpha_n$ are a basis for a finite-dimensional vector space V and $\beta_1, \ldots, \beta_n$ are any vectors in a vector space W then there is a *unique* linear transformation T with $T(\alpha_1) = \beta_1, \ldots T(\alpha_n) = \beta_n$.

8. (**Rank-Nullity Theorem:**) If V is a finite-dimensional vector space and T is a linear transformation from V to a vector space W then $rank(T) + nullity(T) = dim(V)$.