Last name _____

First name _____

LARSON—MATH 550—CLASSROOM WORKSHEET 13
Multiple Sums.

Concepts & Notation

- Sec. 2.3. Rules for sums. Perturbation method.
- Sec. 2.4. Multiple sums. General Distributive Law. Chebychev's Monotonic Inequalities.
- Sec. 2.6. Δ operator. Rising and falling factorials.

Homework

1. Find a single-sum formula for this double-sum:

$$S = \sum_{1 \le j < k \le n} (a_k - a_j)(b_k - b_j)$$

2. Use this to prove the following Chebyshev Monotonic Inequality:

$$(\sum_{k=1}^{n} a_k)(\sum_{k=1}^{n} b_k) \le n(\sum_{k=1}^{n} a_k b_k) \text{ if } a_1 \le \ldots a_n, b_1 \le \ldots \le b_n.$$

3. Define $\Delta f(x)$

4. Define $x^{\underline{m}}$.

5. Define $x^{\overline{m}}$.

6. Find $\Delta(x^{\underline{m}})$.

7. Check: $n! = n^{\underline{n}} = 1^{\underline{n}}$.

8. How can we "investigate" the *General Law*?:

$$\sum_{0 \leq k < n} k^{\underline{m}} = \left. \frac{k^{\underline{m+1}}}{m+1} \right|_0^n$$