Last name ________________________

First name ________________________

LARSON—MATH 610—CLASSROOM WORKSHEET 20
Linear Transformations.

Concepts & Notation

- (Sec. 3.1) *linear transformation*, *range*, *rank*, *null space*, *nullity*.
- (Sec. 3.2) $L(V,W)$, *linear operator*, *invertible* linear transformation, *non-singular* linear transformation.
- (Sec. 3.3) *isomorphism*.

Review

1. (**Claim:**) If $\alpha_1,\ldots,\alpha_n$ are a basis for a finite-dimensional vector space V and $\beta_1,\ldots,\beta_n$ are any vectors in a vector space W then there is a *unique* linear transformation T with $T(\alpha_1)=\beta_1,\ldots T(\alpha_n)=\beta_n$.
2. (**Claim:**) If A is an $m\times n$ matrix with entries in the field $\mathbb{F}$, then the row rank of A equals its column rank.

New

3. What is the space $L(V,W)$ of linear transformations from a vector space V to a vector space W?

4. What is an *invertible* linear transformation $T : V \to W$?

5. What is a *non-singular* linear transformation $T : V \to W$?

6. (**Claim:**) If V and W are finite-dimensional vector spaces over a field $\mathbb{F}$ with $\dim V = \dim W$, and $T : V \to W$ is a linear transformation then the following are equivalent:

   (a) T is invertible,

   (b) T is non-singular,

   (c) T is onto (that is, the range of T is W).

7. If $T : V \to W$ is a linear transformation, when is T an *isomorphism* of V onto W? (If T is an isomorphism we say that vector spaces V and W are *isomorphic*).

8. (**Claim:**) Every n-dimensional vector space over a field $\mathbb{F}$ is isomorphic to $\mathbb{F}^n$.