Last name _____

First name _____

LARSON—MATH 310—CLASSROOM WORKSHEET 03
Dot Products

Review

- What is $\vec{v} \cdot \vec{w}$?
- What is $\|\vec{v}\|$?
- What is a *unit* vector?
- How can you find a unit vector in the direction of $\vec{v}$?
- Check that if $\vec{u}$ is a unit vector then $\vec{u} \cdot \vec{u} = 1$.
- Check that if $\vec{v}$ and $\vec{w}$ are vectors that point in the same direction then $\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\|$.

Let $\vec{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

1. Check that if the angle between $\vec{v}$ and $\vec{w}$ is θ then $\cos\theta = \dfrac{\vec{v}}{\|\vec{v}\|} \cdot \dfrac{\vec{w}}{\|\vec{w}\|}$.

2. Check that if $\vec{v}$ and $\vec{w}$ are perpendicular then $\vec{v} \cdot \vec{w} = 0$.

3. Let θ be the angle between $\vec{v}$ and $\vec{w}$. Find $\cos\theta$.

4. Find θ.

5. Find a (non-trivial) vector $\vec{u}$ which is perpendicular to $\vec{v}$.

Cauchy's Inequality. For any vectors $\vec{v}$, $\vec{w}$, $\vec{v} \cdot \vec{w} \leq \|\vec{v}\|\|\vec{w}\|$.

6. Check that Cauchy's Inequality holds for vectors $\vec{v}$, $\vec{w}$.

7. *Why* is Cauchy's Inequality true?

Triangle Inequality : $\|\vec{v} + \vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$.

8. Check that the Triangle Inequality holds for vectors $\vec{v}$, $\vec{w}$.

9. *Why* is the Triangle Inequality true?