Claims from Class

Be verbose. Write definitions, examples, etc, in order to be maximally clear.

1. Let A be a square matrix with **columns** that are pair-wise orthonormal. Show that the **rows** are orthonormal.

2. Is this claim true if the condition on unit-length columns is dropped? Is it true that if the columns of a square matrix A are non-zero (but not necessarily unit length) and pair-wise orthogonal, that the rows are also pair-wise orthogonal? Either find an argument that this true—or find a counterexample (that shows that the claim is false).

Outer-Product Expansion

3. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$. Check that:

$$AB = \hat{a}_1 \hat{b}_1^T + \hat{a}_2 \hat{b}_2^T,$$

where $\hat{a}_i$'s are the columns of A and $\hat{b}_i^T$ are the rows of B.

4. **Why** is it true that, for an $m \times n$ matrix A with columns $\hat{a}_1, \ldots, \hat{a}_n$, and $n \times t$ matrix B, with rows $\hat{b}_1^T, \ldots, \hat{b}_n^T$, that:

$$AB = \hat{a}_1 \hat{b}_1^T + \hat{a}_2 \hat{b}_2^T + \ldots + \hat{a}_n \hat{b}_n^T.$$