LARSON—MATH 255–CLASSROOM WORKSHEET 25
Riemann Integrals.

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click "New", then "Worksheets", then call it **c25**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

Riemann Integration

Given a continuous function $f(x)$ on an interval $[a, b]$ we want to find the *area* between the curve, the x-axis and the lines $y = a$ and $y = b$. One way to do this is to use the Fundamental Theorem of Calculus and integrate. Unfortunately, it is difficult to find anti-derivatives for many (most) functions. So we need a different approach to get at least an approximate integral.

One way to do this is to slice up $[a, b]$ into n equal-sized intervals $[a_0, a_1]$, $[a_1, a_2]$,..., $[a_n, a_{n+1}]$ (where $a_1 = a$ and $a_{n+1} = b$), pick a point c_i from each interval $[a_i, a_{i+1}]$ and compute the area $f(c_i) \cdot \Delta$ of a rectangle, where Δ is the interval length $a_{i+1} - a_i$. There are different ways to pick the c_i's. You could pick the leftmost point of the interval, the midpoint, the rightmost point, or even a random point.

The *Riemann Integral* is defined to be the *limit* of these area approximations as n goes to infinity of this quantity.

Here is a function `leftpoint_riemann(f,a,b,n)` which computes the leftpoint Riemann sums for n equal intervals.

```
def leftpoint_riemann(f,a,b,n):
    area=0
    Delta=(b-a)/n
    for i in [0..(n-1)]:
        leftpoint=a+i*Delta
        area=area+f(leftpoint)*Delta
    return area*1.0
```

2. Find the integral of f(x)=x**3 on [-3,3] (by hand), and then do some experiments with your `leftpoint_riemann` function.

3. Given a continuous function f(x) on [a,b], define a function `rightpoint_riemann(f,a,b,n)` which computes the rightpoint Riemann sums for n equal intervals.

4. Find the values of `rightpoint_riemann(f,a,b,n)` for f(x)=x**3 on [-3,3] with $n = 2$, $n = 5$, $n = 10$ and $n = 100$. Compare with your results for `leftpoint_riemann(f,a,b,n)`.

Working with Files

Reading in, and working with, data files is an important ability. First we will create a data file. Then we will read it in line-by-line, and then we will work with the data.

An important thing to know/note is that a file is actually a big *string*. You can read the lines of a file with `readline()`. Those lines are also strings (and not numbers - despite how they look). If you want numbers they must be converted to numbers.

5. (a) Go to: `http://projecteuler.net/problem=13`. Copy the one hundred 50-digit numbers there.
   (b) Click "New", type in `one_hundred_numbers.txt` as the name of your file, then click "File".
   (c) Paste in your numbers and "Save".
   (d) Now go back to your **c25** worksheet.
   (e) Type in:

```
data=open("one_hundred_numbers.txt")
numbers=[]
number_string=data.readline()
while(number_string!=""):
    number=Integer(number_string)
    numbers.append(number)
    number_string=data.readline()
```

   You have a *list* of numbers. You can use built-in Sage functions to find out statistics about this list.

6. How many numbers are there?
7. What is the biggest number?
8. What is the sum of these numbers?
9. What is the average of these numbers?
10. What is their median?
11. Now we will create a file `testio.txt` in *write* mode (hence the "w"), and write something to it. The *close* command forces the writing to happen and flushes the *buffer*. Now you can't write anything else to the file without reopening it.

```
datafile=open("testio.txt","w")
datafile.write("hello world!")
datafile.close()
```

12. Go to Files, find `testio.txt` and click on it to see what's in there.

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255 - c25 worksheet attached" (so that it will be properly recorded).