Last name ______________________

First name ______________________

LARSON—OPER 731—HOMEWORK WORKSHEET 07 (h07)
Polyhedra in Sage.

1. Log in to your Sage Cloud account.

   (a) Start Chrome browser.

   (b) Go to `http://cocalc.com`

   (c) Click "Sign In".

   (d) Click the project for our course.

   (e) Click "New", call it **h07**, then click "Sage Worksheet".

   Here's the Polyhedron we'd like to investigate: the feasible region of the following system of inequalities ($x \in \mathbb{R}$):

   $$x_1 + x_2 \leq 1$$
   $$x_1 \geq 0$$
   $$x_2 \geq 0.$$

   Polyhedron can be directly defined in Sage—but you have to enter the defining inequalities in an unusual form. Instead I prefer to define an LP first, where we can put the defining inequalities in in a standard human-readable way. And then we can easily produce the polyhedron of the feasible region of the defining inequalities. (The objective function we use for the LP is totally and completely irrelevant to the polyhedron, so I'll choose it to be the constant function 0 in the following example.

   `LP` be the name of our linear program. Of course, the name can be *anything*; it can be `Dantzig`, `D` or `Mathzilla`, anything that you are not already using for something else.

   We will also tell Sage that our objective is to find the maximum value of the objective function (the constant function 0—this is also irrelevant).

2. Enter and run:

```
LP = MixedIntegerLinearProgram(maximization=True)
x = LP.new_variable(nonnegative=True)
LP.set_objective(0)
LP.add_constraint(x[1] + x[2] <= 1)
```

3. Now we'll let P be the polyhedron defined by the feasible region of the defining inequalities.
   Run: `P=LP.polyhedron()`

4. Now we can find the vertices/*extreme points* of our polyhedron P:
   Run: `P.vertices()`. (Minkowski's Theorem says that the convex hull of these points *is* P).

5. You can also get a list of the vertices:
   Run: `P.vertices_list()`.

6. Imitate the last example to find the polyhedron P corresponding to the feasible region of the following system of inequalities. Find the vertices / extreme points.

$$3x_1 + x_2 \leq 3$$
$$x_1 + 3x_2 \leq 3$$
$$x_i \geq 0.$$

7. Imitate the last example to find the polyhedron P corresponding to the feasible region of the following system of inequalities. Find the vertices / extreme points.

$$\begin{array}{ccccccc} x_0 & + & x_1 & & & \leq & 1 \\ x_0 & & & + & x_2 & \leq & 1 \\ & & x_1 & + & x_2 & \leq & 1 \end{array}$$
$$x_i \geq 0.$$

8. Imitate the last example to find the polyhedron P corresponding to the feasible region of the following system of inequalities. Find the vertices/ extreme points.

$$\begin{array}{ccccccccc} x_0 & + & x_1 & & & & & \leq & 1 \\ x_0 & & & + & x_2 & & & \leq & 1 \\ & & x_1 & + & x_2 & & & \leq & 1 \\ x_0 & + & & & & + & x_3 & \leq & 1 \\ & & x_1 & & & + & x_3 & \leq & 1 \end{array}$$
$$x_i \geq 0.$$