Last name ______________________

First name ______________________

LARSON—MATH 610–CLASSROOM WORKSHEET 12
Block Matrices.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence, pivot column decomposition, direct sum $\mathcal{U}\oplus\mathcal{V}$, *orthogonal* matrix, *unitary* matrix, *basis*, *dimension*, *linear transformation.*

Review:

1. How does any matrix $A\in\mathbb{M}_{m\times n}$ define a linear transformation?
2. How does any linear transformation $T\in\mathcal{L}(\mathcal{V},\mathcal{W})$ and bases $\beta=\hat{v}_1,\ldots,\hat{v}_n$ of $\mathcal{V}$ and $\gamma=\hat{w}_1,\ldots,\hat{w}_m$ of $\mathcal{W}$ define a matrix $A\in\mathbb{M}_{m\times n}$?
3. What is ${}_\gamma[T]_\beta$?
4. What is the β-γ *change-of-basis* matrix (notation: ${}_\gamma[I]_\beta$)?

Chp. 4 of Garcia & Horn, Matrix Mathematics

1. Find $A\hat{x}$ where $A=\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}$ and $\hat{x}=\begin{bmatrix}x\\y\\z\end{bmatrix}$.

2. Let $A\in\mathbb{M}_{m\times r}$ and $B\in\mathbb{M}_{r\times n}$, and write $B=[B_1\ B_2]$, where B_1 is the first k columns of B and B_2 is the remaining $n-k$ columns of B. Then,
$$AB=A[B_1\ B_2]=[AB_1\ AB_2].$$

3. Check with $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}$, $B=\begin{bmatrix}4&5&2\\6&7&1\end{bmatrix}$, where B_1 is the first two columns of B and B_2 is the remaining column.

4. Find $\hat{x}^TB$ where $B=\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}$ and $\hat{x}=\begin{bmatrix}x\\y\\z\end{bmatrix}$.

5. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 & 2 \\ 6 & 7 & 1 \end{bmatrix}$. Check:

$$AB = \begin{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} B \\ \\ \begin{bmatrix} 3 & 4 \end{bmatrix} B \end{bmatrix}$$

6. Find $\begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} B$ where $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, where $A \in \mathbb{M}_{2\times 3}$.

7. Find a formula for AB in terms of the rows of A.

8. Let $A \in \mathbb{M}_n$ be invertible. and R be a product of elementary matrices which code a sequence of row operations that reduces A to I. Then $RA = I$, and $R = A^{-1}$. Then,

$$R[A\ I] = [RA\ R] = [I\ A^{-1}].$$

If the block matrix $[A\ I]$ reduces to $[I\ X]$, then $X = A^{-1}$.

9. Check that:

$$AB = \begin{bmatrix} A_1 & A_2 \end{bmatrix} \begin{bmatrix} B_1^T \\ B_2^T \end{bmatrix} = A_1 B_1^T + A_2 B_2^T.$$

10. Write a formula for the product AB in terms of an *outer product* of the columns of A and the rows of B.