Last name ______________________

First name ______________________

LARSON—MATH 310–CLASSROOM WORKSHEET 09
Matrices.

Review: $\mathbb{R}$, *field*, *complex numbers*, $\mathbb{R}^2$, $\mathbb{K}$, $\mathbb{K}^n$, *linear space* (or *vector space*), *subspace*, *linear map* (or *linear transformation*), *kernel*, *range*, *linear combination*, subspace *generated* by (or *spanned* by) a set of vectors, $\langle A \rangle$, *finite-dimensional vector space*, *linearly independent* set of vectors, *linearly dependent* set of vectors, *basis* of linear space, *dimension*, *rank* of a collection of vectors.

Review.

1. What is our *algorithm* for finding a maximal set of linearly independent vectors from any list of vectors?

From Chp. 4 of Tsukada, et al., Linear Algebra with Python

1. Let the columns of matrix A be $\vec{a}_1, \ldots \vec{a}_6$. Find a maximal set of linearly independent columns by greedily choosing the first non-zero column vector, adding the next available column vector, and iterating (until no column remain). Find the rank.

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. The *column space* of a matrix is the collection of linear combinations of the columns. Why is this collection a linear space?

3. Explain why the linearly independent column vectors of A we found are, in fact, a basis for the column space of A.

4. What is $M_{\mathbb{K}}(m,n)$?

5. How can we add two matrices?

6. How can we multiply a matrix by a scalar?

7. Why is $M_{\mathbb{K}}(m,n)$ a linear space?

8. Let A be a matrix with n columns. $A = \begin{bmatrix} \vec{a}_1 & \dots & \vec{a}_n \end{bmatrix}$ and $\hat{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$. What is $A\hat{x}$?

9. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $\hat{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. What is $A\hat{x}$?

10. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $\hat{x} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$. What is $A\hat{x}$?