Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 19
Linear Transformations.

Concepts & Notation

- (Sec. 3.1) *linear transformation, rank, nullity.*

- (Sec. 3.2) $L(V, W)$, *linear operator*, *invertible* linear transformation, *non-singular* linear transformation.

Review

1. (**Rank-Nullity Theorem:**) If V is a finite-dimensional vector space and T is a linear transformation from V to a vector space W then $rank(T) + nullity(T) = dim(V)$.

New

2. (**Claim:**) If $\alpha_1, \ldots, \alpha_n$ are a basis for a finite-dimensional vector space V and $\beta_1, \ldots, \beta_n$ are any vectors in a vector space W then there is a *unique* linear transformation T with $T(\alpha_1) = \beta_1, \ldots T(\alpha_n) = \beta_n$.

3. (**Claim:**) If A is an $m \times n$ matrix with entries in the field $\mathbb{F}$, then the row rank of A equals its column rank.

4. What is the space $L(V, W)$ of linear transformations from a vector space V to a vector space W?

5. (**Claim:**) If V is an n-dimensional vector space and W is an m-dimensional vector space then $L(V, W)$ is finite-dimensional and has dimension mn.