Last name ______________________

First name ______________________

LARSON—MATH 310–CLASSROOM WORKSHEET 25
Column Space, Row Space, Echelon form

Review

1. (**Lemma 5.7.1, Unique-Representation Lemma)**) Let $\hat{a}_1, ..., \hat{a}_n$ be a basis for a vector space $\mathcal{V}$. For any vector $\hat{v} \in \mathcal{V}$, there is exactly one representation of $\hat{v}$ in terms of the basis vectors.

2. Find the *coordinates* for $[3, 3, 1]$ with respect to the basis $[1, 1, 1], [1, 1, 0], [0, 1, 1]$ for $\mathbb{R}^3$.

3. (**Theorem 6.1.2, Basis Theorem**) Let $\mathcal{V}$ be a vector space. All bases for $\mathcal{V}$ have the same size.

4. What is the *dimension* of a vector space?

5. Given a generating set for a vector space $\mathcal{V}$, how can we find a basis for $\mathcal{V}$?

Chapter 6 of Klein's *Coding the Matrix* text

1. How can we find a basis for the column space of a matrix? What is the *column rank* of a matrix?

Example 6.2.11: Consider the matrix

$$M = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 2 & 0 & 7 \\ 0 & 0 & 3 & 9 \end{bmatrix}$$

2. How can we find a basis for the row space of a matrix? What is the *row rank* of a matrix?

3. Can a set of vectors containing the 0-vector be linearly independent?

Chapter 7 of Klein's *Coding the Matrix* text

Definition 7.1.1: An $m \times n$ matrix A is in *echelon form* if it satisfies the following condition: for any row, if that row's first nonzero entry is in position k then every previous row's first nonzero entry is in some position less than k.

$$\begin{bmatrix} 0 & 2 & 3 & 0 & 5 & 6 \\ 0 & 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 9 \end{bmatrix}$$

4. What is *echelon form*?

5. What are examples?

6. What is a basis for the row space of a matrix in echelon form?

7. What things can we do to a system of equations that produces a new system with the same solutions?

(**Gaussian elimination**). Use these same thee operations to produce a matrix *in echelon form* that represents a system of equations with the same solutions.

8. Use Gaussian elimination to produce an equivalent matrix in echelon form.

$$\begin{bmatrix} 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 6 & 7 \\ 0 & 0 & 0 & 9 & 8 \end{bmatrix}$$