Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 32
Determinants.

Concepts & Notation

- (Sec. 5.1) *n-linear* function, *alternating function.*
- (Sec. 5.2) *determinant function.*
- (Sec. 5.3) *permutation*, $\det A$.

Let A be an $n \times n$ matrix over a *commutative ring.* Let:

$$\det A = \sum_{\sigma \in S_n} [(sgn\ \sigma) \prod_{i=1}^{n} A_{i,\sigma(i)}]$$

$$= \sum_{\sigma \in S_n} (sgn\ \sigma) A_{1,\sigma(1)} A_{2,\sigma(2)} \dots A_{n,\sigma(n)}.$$

Review

1. $\det I = 1$.

2. If A is a diagonal matrix then $\det A = A_{11}A_{22}\dots A_{nn}$ (the product of the diagonal entries).

3. If A has a zero row then $\det A = 0$.

4. If A' is A with two rows switched then $\det A' = -\det A$.

5. If A has two rows which are the same then $\det A = 0$.

6. $\det(\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \dots, \alpha_{\sigma(n)}) = (sgn\ \sigma) \det(\alpha_1, \alpha_2, \dots, \alpha_n)$.

   Properties of $\det : K^{n \times n} \to K$

7. (**Claim:**) det is n-linear.

8. (**Claim:**) If A is row-reduced to (a diagonal matrix) R (using only the operation of adding a multiple of one row to another, and using k switches of pairs of rows) then $\det A = (-1)^k \det R$ (and $\det A$ is $\pm$ the product of the diagonal entries of R).

9. (**Claim:**) If $A \in \mathbb{F}^{n\times n}$, then A is invertible iff $\det A \neq 0$.

10. (**Claim:**) If $D, B \in \mathbb{F}^{n\times n}$ and D is diagonal then $\det DB = (\det D)(\det B)$.

11. (**Claim:**) If $A, B \in \mathbb{F}^{n\times n}$ then $\det AB = (\det A)(\det B)$.

The Structure of a Linear Operator

12. Suppose $A \in \mathbb{F}^{n\times n}$. There is a $n \times 1$ matrix α (not all-zero), and $c \in \mathbb{F}$ with $A\alpha = c\alpha$ iff $\det(A - cI) = 0$.