Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 04
Review.

Concepts & Notation

- (Sec. 1.4) *row-reduced echelon matrix*, *zero matrix* $0^{m,n}$.
- matrix multiplication.
- (Sec. 1.5) *column matrix* B_j, *elementary matrix.*
- (Sec. 1.6) *left inverse*, *right inverse*, *invertible matrix*, *inverse* A^{-1}.

Problems

1. Let A and B be matrices. When is the product AB defined? If AB is defined, what are its entries?

2. Let
$$B = \begin{bmatrix} 5 & -1 & 2 \\ 15 & 4 & 8 \end{bmatrix}.$$
What are B_1, B_2, B_3? (`Notation`: $B = \begin{bmatrix} B_1 & B_2 & B_3 \end{bmatrix}$).

3. Let
$$A = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}.$$
Find AB_1, AB_2, AB_3. Check that $AB = \begin{bmatrix} AB_1 & AB_2 & AB_3 \end{bmatrix}$.

4. If AB is defined, and $B = \begin{bmatrix} B_1 \dots B_p \end{bmatrix}$, explain why $AB = \begin{bmatrix} AB_1 \dots AB_p \end{bmatrix}$.

5. Let $C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Check that: $A(BC) = (AB)C$.

6. Prove that matrix multiplication is associative: $A(BC) = (AB)C$ (assuming these products are defined).

7. What is a *left inverse* of a square matrix A? What is a *right inverse* of a square matrix A? When is A invertible?

8. Show that $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ is invertible. Find A^{-1}.

9. Show that $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is invertible. Find B^{-1}.

10. Suppose A is invertible. How can you show that its inverse is unique (and therefore our notation A^{-1} is unambiguous)?

11. Find AB, find $B^{-1}A^{-1}$ and check that $(AB)^{-1} = B^{-1}A^{-1}$.

12. Suppose A is invertible. Why does $(A^{-1})^{-1} = A$?