Last name ________________________

First name ________________________

LARSON—MATH 601—HOMEWORK WORKSHEET h14
The Determinant of a Linear Transformation

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x_1, x_2) = (3x_1 + x_2, x_2)$. $\mathcal{B} = \{\alpha_1, \alpha_2\} = (1, 1), (-1, 1)$ is a basis for $\mathbb{R}^2$. $\mathcal{B}' = \{\alpha'_1, \alpha'_2\} = (1, 2), (2, 1)$ is another basis.

1. Check that T is a linear transformation.

2. Find $[T]_{\mathcal{B}} = [[T(\alpha_1)]_{\mathcal{B}}\, T(\alpha_2)]_{\mathcal{B}}]$.

3. Find $[T]_{\mathcal{B}'} = [[T(\alpha'_1)]_{\mathcal{B}}\, T(\alpha'_2)]_{\mathcal{B}'}]$.

4. Find $P = [[\alpha_1]_{\mathcal{B}'}\ [\alpha_1]_{\mathcal{B}'}]$.

5. Find P^{-1}.

6. Check that $[T]_{\mathcal{B}}$ and $[T]_{\mathcal{B}'}$ are similar by showing that $[T]_{\mathcal{B}} = P^{-1}[T]_{\mathcal{B}'}P$.

7. Find $\det([T]_{\mathcal{B}})$.

8. Find $\det P$.

9. Find $\det P^{-1}$.

10. Find $\det([T]_{\mathcal{B}'})$.

11. Find $\det(P^{-1}[T]_{\mathcal{B}'}P)$.

12. Find $\det T$.

13. Find any characteristic values of T. For each characteristic value c, find a corresponding characteristic vector α.

14. Argue that $\mathbb{R}^2$ has a basis consisting of characteristic vectors of T.