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LARSON—MATH 610—CLASSROOM WORKSHEET 21
Inner product Spaces.

Concepts & Notation

- (Chp. 5) *eigenvalue, eigenvector, invariant subspace, minimal polynomial,*
- (Chp. 8) *generalized eigenvector, Cayley-Hamilton Theorem.*
- (Chp. 6) *dot product, inner product, inner product space, norm, orthogonal representation, Cauch-Schwartz, orthonormal list, orthogonal projection.*

Inner Product Spaces

1. What is an *orthonormal list* of vectors in an inner product space?
2. **(Claim)** An orthonormal list of vectors in an inner product space is linearly independent.
3. If (e_1, \dots, e_m) is an orthonormal list in an inner product space V (over \mathbb{F}) and $\alpha_1, \dots, \alpha_m \in \mathbb{F}$ then $\|\alpha_1 e_1 + \dots + \alpha_m e_m\|^2 = |\alpha_1|^2 + \dots + |\alpha_m|^2$.
4. What is an *orthonormal basis* in an inner product space?
5. If e_1, \dots, e_n is an orthonormal basis for an inner product space V , and $v \in V$, then

$$v = \langle v, e_1 \rangle e_1 + \dots + \langle v, e_n \rangle e_n,$$

and

$$\|v\|^2 = |\langle v, e_1 \rangle|^2 + \dots + |\langle v, e_n \rangle|^2.$$

6. What is the *Gram-Schmidt procedure*?
7. **(Existence of orthonormal basis)** Every finite-dimensional inner product space has an orthonormal basis.
8. **(Orthonormal list extends to orthonormal basis)** Suppose V is finite-dimensional. Then every orthonormal list of vectors in V can be extended to an orthonormal basis of V .

9. (**Upper-triangular matrix with respect to orthonormal basis**) Suppose $T \in \mathcal{L}(V)$ has an upper-triangular matrix with respect to some basis of V , then T has an upper-triangular matrix with respect to some orthonormal basis of V .
10. What is the *orthogonal complement* of a set U in an inner product space V ?
11. (**Claim:**) If U is a subspace of an inner product space V then $V = U \oplus U^\perp$.
12. For a subspace U of an inner product space V , what is the *orthogonal projection* operator P_U ?