Last name _____

First name _____

LARSON—MATH 550—CLASSROOM WORKSHEET 39
Generating Functions & Fibonacci Numbers!.

Concepts & Notation

- Sec. 5.4: Convolutions, generating functions.
- Sec. 6.6 Fibonacci Numbers.

Fibonacci Numbers

We defined $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$.

1. (**Bee Trees**). A male bee has a single female parent. A female bee has one male parent and one female parent. Draw a tree representing the "ancestors" of a male bee.

2. (**Bee Trees**). Let $B_1 = 1$, representing a male bee. Then that bee has one (female) ancestor one generation back; and represent this by $B_2 = 1$. This bee has two parents, so our original bee has 2 ancestors two generations ago; represent this by $B_3 = 2$. How many ancestors B_n does our original bee have after $n-1$ generations? Explain.

3. (**Kepler/Cassini**). Check that $F_{n+1}F_{n-1} - F_n^2 = (-1)^n$ holds for small values of n.

Goals: We'd like to find the generating function $F(z)$ for $\langle F_n \rangle$ and use this to find a *formula* for the Fibonacci numbers F_n.

4. Find a relationship between $F(z)$, $zF(z)$ and $z^2F(z)$, and solve to get a formula for $F(z)$.

5. Now we'd like to use this formula for $F(z)$ to find a new representation of the sequence it is the generating function for. We will attempt a partial fraction decomposition. First, find the sequence that the function $\frac{1}{1-\alpha z}$ generates.

6. Now find the sequence that the function $\frac{A}{1-\alpha z} + \frac{B}{1-\beta z}$ generates.