Last name ______________________

First name ______________________

LARSON—MATH 601—HOMEWORK WORKSHEET h07
Test 1 Review.

Write up a careful, complete test review and turn it in before our Test 1 on Fri., Mar. 3. **Explain** everything.

1. What is a *field*? Give some examples.
2. Write this (homogeneous) system (over $\mathbb{Q}$) in matrix form, use the 3 row operations to find an equivalent system of equations in *row-reduced* form. Describe the solutions.

$$\begin{array}{ccccrcl} 2x_1 & -x_2 & +3x_3 & +2x_4 & = & 0 \\ x_1 & +4x_2 & & -x_4 & = & 0 \\ 2x_1 & +6x_2 & -x_3 & 5x_4 & = & 0 \end{array}$$

3. Why is every $m \times n$ matrix A row-equivalent to a row-reduced (echelon) matrix?
4. Why will *every* 3×4 matrix A corresponded to a system of equations with *some* non-trivial solution?
5. Let A be an $n \times n$ matrix. Explain why the homogeneous system $AX = 0$ has non-trivial solutions if and only if A is *not* row-equivalent to the identity matrix I.
6. Let A and B be matrices. When is the product AB defined? If AB is defined, what are its entries?
7. If AB is defined, and $B = \begin{bmatrix} B_1 \dots B_p \end{bmatrix}$, explain why $AB = \begin{bmatrix} AB_1 \dots AB_p \end{bmatrix}$.
8. Show that $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ is invertible. Find A^{-1}.
9. Suppose a matrix A is invertible. Show that its inverse is unique.
10. What is an *elementary matrix*? Give 3 importantly different examples (with explanation).
11. Suppose A is a 3×3 matrix. What elementary matrix E corresponds to adding a multiple c of the 2^{nd} row to the 3^{rd} row?
12. Why is E invertible? What is E^{-1}? Why is E^{-1} an elementary matrix?
13. $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is invertible. Show that A can be written as a product of elementary matrices.
14. Find A^{-1} as a product of elementary matrices—directly from your previous answer.
15. What is a *vector space*? What are prototypical examples.

16. Show that the set of $n \times 1$ column matrices with entries in a field $\mathbb{F}$ is a vector space over $\mathbb{F}$.

17. What is the Subspace Criterion Theorem?

18. Argue that $\alpha_1 = (1, 2, 3)$ and $\alpha_2 = (1, 1, 1)$ are linearly independent in $\mathbb{R}^3$. Find a vector α_3 so that $\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ is a basis for $\mathbb{R}^3$.

19. Let A be an $m \times n$ matrix. **Claim:** the $n \times 1$ column matrices X that are solutions to $AX = 0$ is a subspace of V.

20. Find the subspace spanned by $\alpha = (0, 1)$ in $\mathbb{R}^2$.

21. What does it mean for vectors $\alpha_1, \alpha_2, \ldots, \alpha_k$ to be linearly dependent? Give an example.

22. Show $\alpha_1 = (1, 0), \alpha_2 = (1, 1) \in \mathbb{R}^2$ are linearly independent.

23. Let $\mathbb{F}$ be a subfield of $\mathbb{C}$. Let V be the vector space of functions from $\mathbb{F}$ to $\mathbb{F}$ of the form
$$f(x) = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n$$
(where $c_i \in \mathbb{F}$ and n is a non-negative integer). Let W be the set of functions in V with maximum degree at most 3. Show W is a subspace of V.

24. Find a basis for W.

25. Show your basis spans W.

26. Show your basis is linearly independent.

27. What is the dimension of W?

28. **Prove:** Any linearly independent set in a finite-dimensional vector space V is part of (can be extended to) a (finite) basis for V.

29. Find the coordinate matrix $[\alpha]_{\mathcal{B}}$ of $\alpha = (2, 3)$ with respect to the basis $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(2, 0), (0, 1)\}$ for $\mathbb{R}^2$.

30. Find the coordinate matrix $[\alpha]_{\mathcal{B}'}$ of $\alpha = (2, 3)$ with respect to the basis $\mathcal{B}' = \{\alpha_1', \alpha_2'\} = \{(1, 0), (1, 1)\}$ for $\mathbb{R}^2$.

31. Find an invertible matrix P such that $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$.

32. Give an example of two non-row-reduced—but row-equivalent—matrices A and B. Show that they are row-equivalent.

33. What is the *row space* of a matrix?

34. Describe the row space of $A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

35. What is the *row rank* of a matrix?

36. Find the row rank of $A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.