LARSON—MATH 255–CLASSROOM WORKSHEET 15
Recursion & Experiments.

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click "New", then "Worksheets", then call it **c16**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

Fibonacci!

The *Fibonacci sequence* F_n is defined as follows: $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n > 1$.

Here is a recursive function `fib(n)` that computes the nth Fibonacci number.

```
def fib(n):
    if n==0 or n==1:
        return n
    else:
        return fib(n-1)+fib(n-2)
```

2. Define a *non-recursive* (iterative) function `fib2(n)` that computes the nth Fibonacci number.

3. Try this for small values of n to make sure that it works, then try it for $n = 10, 20, 30, 40, 50$. Does it finish?

4. Solve the equation $\frac{a+b}{a} = \frac{a}{b}$, for a and b. Find $\frac{a}{b}$. Get a 10-digit approximation for this quantity (this is the Golden Ratio).

5. Define a function `fib_ratio(n)` which returns the ratio of the $(n+1)^{th}$ Fibonacci number to the n^{th}. find `fib_ratio(10)` and `fib_ratio(100)`. Compare this answer to your previous answer. What can you conjecture?

Random Values

6. `random()` returns a random number in $[0, 1]$. Execute it a few times to see what you get.

7. Use `random()` to define a function `coin_flip()` which randomly returns the string "H" (for heads) half the time and **returns** the string "T" (for tails) half the time. Try it a few times; your results will vary.

8. Run your coin flipping program 100 times and collect data. A random coin flipping program should come up heads about half the time. How many times do you get heads?

9. Now run your coin flipping program 1000 times and collect data. A random coin flipping program should come up heads about half the time. How many times do you get heads?

Investigate

10. Start with any positive integer x. If x is even divide by 2. If x is odd, multiply by 3 and add 1. Repeat. Try this for several initial starting numbers x. What happens? (Do this all by hand—we'll compute later).

11. Define a function `collatz(x)` that returns x if x is one, returns `collatz(3x+1)` if x is odd, and returns `collatz(x/2)` if x is even. This will be a recursive function (since it calls itself). What is the base case? Does it always terminate (return 1)?

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255 - c16 worksheet attached" (so that it will be properly recorded).

(c) Remember to attach today's classroom worksheet!