Last name ______________________

First name ______________________

LARSON—MATH 556–HOMEWORK WORKSHEET 13
Test 2 Review

You should know the following definitions, theorems, algorithms, and proofs for the test. Write out careful definitions, theorem statements, algorithms, proofs, and solutions. Turn these in at test time.

Definitions. Write each definition and give an example.

1. *spanning tree.*
2. *partially ordered set* (poset)
3. *Hasse diagram* of a poset.
4. *chain* in a poset.
5. *anti-chain* in a poset.
6. *permutation matrix.*
7. *doubly stochastic matrix.*
8. *convex combination* (of vectors, or matrices, etc).
9. (proper) *line coloring* of a graph.
10. *regular* graph.
11. *directed graph.*
12. *network.*
13. *flow* in a network.
14. *value* of a flow in a network.
15. *cut* $\nabla^+(A)$ in a network.
16. *capacity* of a cut $\nabla^+(A)$ (or a separator A) in a network.
17. f*-augmenting path* in a network.

Theorems. State and give an example, application or confirmation.

18. *Dilworth's Theorem.*
19. *Birkhoff von-Neumann Theorem.*
20. *Kőnig's Line Coloring Theorem.*

21. *Ford-Fulkerson Theorem.*

22. *Menger's Theorem.*

23. *Tutte's Theorem.*

Notation

24. Define $\chi_e(G)$, $c_o(G)$.

Algorithms

25. What is *Birkhoff's Algorithm*?

Proofs

26. **Show:** a regular bipartite graph has a perfect matching.

27. **Show:** A flow f in a network is maximum if and only if there are no f-augmenting paths.

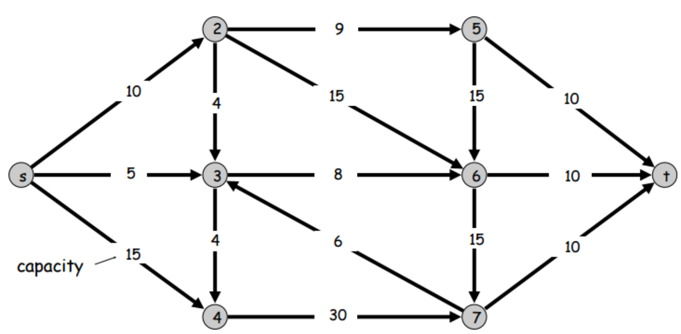
28. Explain how we used Kőnig's Minimax Theorem to prove Dilworth's Theorem. (What was the *main idea*? You don't need a full proof, but a *proof sketch* that discusses how we leveraged Kőnig's Theorem).

29. Explain how we used Hall's Theorem to prove the Birkhoff-von Neumann Theorem. (What was the *main idea*? You don't need a full proof, but a *proof sketch* that discusses how we leveraged Frobenius's Theorem).

Problems. Explain your answers.

30. Show that any connected graph has a spanning tree with the same matching number.

31. Show that set inclusion defines a partial order on any family of sets.



32. Find a maximum flow in this network (s is the source, t is the sink, capacities are indicated).

33. Use the Ford-Fulkerson Theorem to prove your flow is maximum.

34. Set all the capacities in the above network to 1. Find the maximum number of edge-disjoint directed paths from s to t. Use Menger's Theorem to prove it.