Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 24
Linear Functionals.

Concepts & Notation

- (Sec. 3.5) *linear functional*, *trace*, *dual space*, V^*, *dual basis*, *annihilator*.
- (Sec. 4.1) *linear algebra*, $\mathbb{F}^\infty$, *algebra of formal power series*.

Review

1. What is the *dual space* V^*.

2. If $\mathcal{B} = \{\alpha_1, \ldots, \alpha_n\}$ is a basis for a vector space V, what is the *dual basis* $\mathcal{B}^*$?

New

3. (**Claim:**) If V is finite-dimensional then $\dim V = \dim V^*$.

Let V be a finite-dimensional vector space over a field $\mathbb{F}$ with basis $\mathcal{B} = \{\alpha_1, \ldots, \alpha_n\}$ and dual basis $\mathcal{B}^* = \{f_1, \ldots, f_n\}$.

4. (**Claim:**) For every linear functional f on V:

$$f = \sum_1^n f(\alpha_i) f_i.$$

5. (**Claim:**) For every vector $\alpha \in V$:

$$\alpha = \sum_1^n f_i(\alpha) \alpha_i.$$

6. If V is a vector space over a field $\mathbb{F}$ and $S \subseteq V$, what is the *annihilator* of S?

7. What is a *linear algebra*?

8. What are examples of linear algebras?

9. What is $\mathbb{F}^\infty$? Let $f, g \in \mathbb{F}^\infty$. How is fg defined?

10. What is $x \in \mathbb{F}^\infty$?