Last name _____

First name _____

LARSON—OPER 731—CLASSROOM WORKSHEET 21
König's Theorem & Matroids

König's Theorem

1. What is a *matching* in a graph, what is a *vertex cover*, what is a *bipartite graph*, and what is König's Theorem?

2. Represent finding a maximum matching in a bipartite graph as an integer programming problem and use total unimodularilty to show that the relaxation of this IP has integer solutions.

3. What does the dual of this IP model?

4. Use total unimodularilty to show that the relaxation of this dual IP has integer solutions. How does this prove König's Theorem?

Matroids

5. Why, if you have a set Y of 3 linearly independent vectors in $\mathbb{R}^3$ and a set X of 2 linearly independent vectors, must it be the case that there is a vector $v \in Y$ such that $X \cup \{v\}$ is linearly independent?

6. What is a *matroid*?

7. What is a *tree*? What is a *spanning tree* in a graph?

8. What can you say about the number of edges in a tree?

9. What is a *forest* in a graph? What is a *component* of a graph? What is κ? What can you say about the number of edges in a forest?

10. Why, if the edges Y of a graph G induce a forest and the edges X of G induce a forest and $|Y| > |X|$, must it be the case that there is an edge $e \in Y$ such that $X \cup \{e\}$ induces a forest in G?