Last name _____

First name _____

LARSON—OPER 731—CLASSROOM WORKSHEET 06
Fourier-Motzkin Elimination—Weyl's Theorem

Fourier-Motzkin Elimination

1. Use Fourier-Motzkin elimination to solve the following LP:

   Maximize:

   $$z = x_1 + x_2 + x_3$$

   Subject to:

   $$x_1 + x_2 \leq 1$$
   $$x_2 + x_3 \leq 1$$
   $$x_i \geq 0.$$

   We found that this problem is equivalent to finding a maximum value for z in this system (I.):

   $$\begin{array}{rcrcrcl} -z & + & x_1 & + & x_2 & + & x_3 \leq 0 \\ z & - & x_1 & - & x_2 & - & x_3 \leq 0 \\ & & x_1 & + & x_2 & & \leq 1 \\ & & & & x_2 & + & x_3 \leq 1 \\ & - & x_1 & & & & \leq 0 \\ & & & - & x_2 & & \leq 0 \\ & & & & & - & x_3 \leq 0 \end{array}$$

   We found that this problem is equivalent to finding a maximum value for z in this system (II.):

   $$\begin{array}{rcrcrcl} -z & & + & x_2 & + & x_3 & \leq 0 \\ z & & & & - & x_3 & \leq 1 \\ & & & x_2 & & & \leq 1 \\ & & & x_2 & + & x_3 & \leq 1 \\ & & - & x_2 & & & \leq 0 \\ & & & & - & x_3 & \leq 0 \end{array}$$

   We found that this problem is equivalent to finding a maximum value for z in this system (III.):

   $$\begin{array}{rcrcl} -z & & + & x_3 & \leq 0 \\ & & & x_3 & \leq 1 \\ z & & - & x_3 & \leq 1 \\ & & - & x_3 & \leq 0 \end{array}$$

   We found that this problem is equivalent to finding a maximum value for z in this system (IV.). Find z and an optimal solution (x_1, x_2, x_3).

   $$\begin{array}{rl} -z & \leq 0 \\ z & \leq 2 \end{array}$$

Weyl's Theorem

2. Let $X = \{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}\}$. Find a system of linear inequalities so that the feasible region of that system is $conv(X)$.

3. Let $X = \{\begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}0\\2\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\}$. Find a system of linear inequalities so that the feasible region of that system is $conv(X)$.