Last name _____

First name _____

LARSON—OPER 731—CLASSROOM WORKSHEET 12
Dimension, Faces, Facets and Minkowski's Theorem

Dimension

1. What is the definition of the *dimension* of a polytope?

2. Argue that the dimension of a polytope $\mathcal{P} \subset \mathbb{R}^2$ is no more than 2.

3. Find the dimension of the polytope defined by the linear inequalities ($x \in \mathbb{R}$):

$$3x_1 + x_2 \leq 3$$

$$-x_1 + 3x_2 \leq 3$$

$$x_i \geq 0.$$

4. Find the dimension of the polytope which is the convex hull of $X = \{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \}$.

Faces & Facets

5. What is a *face* of a polytope?

6. Find any faces in the polytope defined by the linear inequalities ($x \in \mathbb{R}$)

$$\begin{aligned} 3x_1 + x_2 &\leq 3 \\ -x_1 + 3x_2 &\leq 3 \\ x_i &\geq 0. \end{aligned}$$

7. What is a *facet* of a polytope?

8. Find any facets in the polytope defined by the linear inequalities ($x \in \mathbb{R}$)

$$\begin{aligned} 3x_1 + x_2 &\leq 3 \\ -x_1 + 3x_2 &\leq 3 \\ x_i &\geq 0. \end{aligned}$$

9. What is *Minkowski's Theorem*?