Last name ________________________

First name ________________________

LARSON—MATH 310–CLASSROOM WORKSHEET 07
Bases and Dimension.

Review: $\mathbb{R}$, *field*, *complex numbers*, $\mathbb{R}^2$, $\mathbb{K}$, $\mathbb{K}^n$, *linear space* (or *vector space*), *subspace*, *linear map* (or *linear transformation*), *kernel*, *range*, *linear combination*, subspace *generated* by (or *spanned* by) a set of vectors, $\langle A \rangle$, *finite-dimensional vector space*, *linearly independent* set of vectors, *linearly dependent* set of vectors, *basis* of linear space.

From Chp. 3 of Tsukada, et al., Linear Algebra with Python

1. What are examples of linearly independent and linearly dependent sets of vectors? What is an example of a basis?

2. Why is every list of vectors containing $\vec{0}$ linearly dependent?

3. What is the *standard basis* of $\mathbb{K}_n$?

4. Let $\vec{v}$ be a vector in a linear space V with basis $X = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v_n}\}$. What is the *representation* of $\vec{v}$ with respect to basis X?

5. What is an example?

All bases have the same number of vectors.

6. Suppose V is a vector space with basis $X = \vec{v}_1, \vec{v}_2, \vec{v}_3$. Why is it impossible to add any other vector and still have a basis?

7. Suppose V is a vector space with basis $X = \vec{v}_1, \vec{v}_2, \vec{v}_3$ and $\vec{w}$ is a non-$\vec{0}$ vector. Why is it possible to replace one of the $\vec{v}_i$'s with $\vec{w}$ and still have a basis?

8. Suppose V is a vector space with basis $X = \vec{v}_1, \vec{v}_2, \vec{v}_3$, and $\vec{w}_1, \ldots, \vec{w}_j$ are linearly independent vectors. Why is $j \leq 3$?

9. Suppose V is a vector space with basis $X = \vec{v}_1, \vec{v}_2, \vec{v}_3$. Why can't V have a basis with less than 3 vectors?

10. Suppose V is a vector space with basis $X = \vec{v}_1, \vec{v}_2, \vec{v}_3$. Why does **every** basis for V have (exactly) three vectors?