

Last name \_\_\_\_\_

First name \_\_\_\_\_

**LARSON—MATH 550—CLASSROOM WORKSHEET 08**  
**Quicksort—Homework—Sum Rules.**

**Concepts & Notation**

- Sec. 1.1 & Sec. 1.2  $T_n$ , recurrence (recurrence relation), mathematical induction, basis, solving recurrences
- Sec. 2.1  $[m = n]$  notation, sum notations.
- Sec. 2.2 Two “tricks”.
- Rules for sums

**The Quicksort Recurrence**

The famous quicksort algorithm for sorting a list (of items from a linearly ordered collection) takes  $C_n$  steps on average where:

$$C_0 = C_1 = 0$$

$$C_n = (n + 1) + \frac{2}{n} \sum_{k=0}^{n-1} C_k \text{ (for } n > 1)$$

1. We found

$$nC_n = (n + 1)C_{n-1} + 2n,$$

let

$$s_n = \frac{2}{n(n + 1)},$$

multiplied through to get

$$\frac{2}{n + 1}C_n = \frac{2}{n}C_{n-1} + \frac{4}{n + 1}, (n > 1).$$

Now let

$$U_n = \frac{2}{n + 1}C_n, (n > 1).$$

Solve the recurrence for  $U_n$  to get a formula for  $C_n$ .

## Homework

3. Find a recurrence for the Towers of Hanoi variation where no direct move is allowed from the initial tower to the terminal tower.
4. Find and solve a recurrence for the maximum number of bounded regions formed by  $n$  lines.