Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 08
Vector Spaces.

Concepts & Notation

- (Sec. 2.1) *vector*, *vector space*, *linear combination*.
- (Sec. 2.2) *subspace*

Vector Spaces

A *vector space* V over a field $\mathbb{F}$ consists of:

1. a field $\mathbb{F}$ of *scalars*,
2. a set V of *vectors*,
3. a *vector addition* "+" that is closed (so if $\alpha, \beta \in V$ then $\alpha + \beta \in V$), where:
   (a) addition is commutative,
   (b) addition is associative,
   (c) there is an additive identity "0",
   (d) and there are additive inverses "$-\alpha$" for each $\alpha \in V$.,
4. a *scalar multiplication* (so $c\alpha \in V$ for each $c \in \mathbb{F}$ and $\alpha \in V$), where:
   (a) $1\alpha = \alpha$,
   (b) $(c_1c_2)\alpha = c_1(c_2\alpha)$,
   (c) $c(\alpha_1 + \alpha_2) = c\alpha_1 + c\alpha_2$, and
   (d) $(c_1 + c_2)\alpha = c_1\alpha + c_2\alpha$.

1. What is the prototypical example of a *vector space*?

2. What is a *vector*?

Examples of vector spaces

3. Check all the vector space axions for the following example.

   The space of a functions from a set S to a field $\mathbb{F}$ is a vector space. Let $V = \{f : S \to \mathbb{F} |\text{ f is a function}\}$, with an "addition" $f + g$ (for $f, g \in V$) defined by the rule $(f + g)(s) = f(s) + g(s)$ and a "scalar multiplication" cf (for $c \in \mathbb{F}$, $f \in V$) defined by $(cf)(s) = cf(s)$.

   What needs to be checked in the following examples?

4. Any field $\mathbb{F}$ can be viewed as a vector space over itself.

5. Let $\mathbb{R}^n$ be the set of tuples $(a_1, a_2, \ldots, a_n)$ $(a_i \in \mathbb{R})$. Then $\mathbb{R}^n$ is a vector space over $\mathbb{R}$.

6. The complex numbers $\mathbb{C}$ over $\mathbb{R}$ (with scalar multiplication by real numbers specifically—and **not** by complex numbers generally).

7. What is a **linear combination** of vectors $\alpha_1, \alpha_2, \ldots, \alpha_n$ in a vector space V over a field $\mathbb{F}$?

8. If V is a vector space over a field $\mathbb{F}$ and $W \subseteq V$. When is W a *subspace* of V?