Last name _____
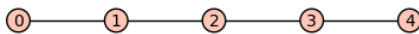
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LARSON—OPER 731—HOMEWORK WORKSHEET 09
Matchings, Integer Programs and Totally Unimodular Matrices

1. This *path graph* P_5 is *bipartite*. Explain.

2. What is a *matching* in a graph?

3. Find a maximum matching in P_5 and argue that it is maximum.

4. *Model* the problem of finding a maximum matching (and finding the *number* of edges in a maximum matching) as an integer program (IP). Explain how your IP in fact models this combinatorial problem: what do your variables represent, what do your constraints model, what will a solution corresponding to the IP optimum represent.

5. What is the constraint matrix A from your IP model? Explain why it is totally unimodular.

6. Relax your IP to an LP. How do you know that this LP is guaranteed to have an integer optimum?

7. Solve the LP.

8. Write the dual LP (you do not need to derive it. You can write it directly from the primal LP). Find a dual feasible solution that *proves* that your primal solution is optimal. Explain.