Last name ________________________

First name ________________________

LARSON—MATH 511—HOMEWORK WORKSHEET 05
Concepts & Review

Explain your answers. An ideal answer will include appropriate definitions.

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \end{bmatrix}$.

1. A represents a function from $\mathbb{R}^3$ to $\mathbb{R}^2$. Let $\hat{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\hat{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

   $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ is the *standard basis* for $\mathbb{R}^3$.

   Find $A\hat{e}_1$, $A\hat{e}_2$, $A\hat{e}_3$.

2. Find the linearly independent columns of A.

3. Show that any other columns can be written as linear combinations of these.

4. Find a CR decomposition of A.

5. Show that the rows of A can be written as linear combinations of the rows of R.

6. Find the column space $C(A)$ of A.

7. Find a basis for $C(A)$.

8. What is the dimension of $C(A)$?

9. Find the row space $C(A^T)$ of A.

10. Find a basis for $C(A^T)$.

11. What is the dimension of $C(A^T)$?

12. Find the null space $N(A)$ of A.

13. Find a basis for $N(A)$.

14. What is the dimension of $N(A)$?

15. Is it true that $dim(C(A^T)) + dim(N(A)) = dim(\mathbb{R}^3)$?

16. Find the null space $N(A^T)$ of A^T.

17. Find a basis for $N(A^T)$.

18. What is the dimension of $N(A^T)$?

19. Is it true that $dim(C(A)) + dim(N(A^T)) = dim(\mathbb{R}^2)$?

20. Show that the basis vectors of $C(A^T)$ and of $N(A)$ are orthogonal.

21. Now explain why every vector in $C(A^T)$ and every vector in $N(A)$ are orthogonal.

22. Show that the basis vectors of $C(A)$ and of $N(A^T)$ are orthogonal.

23. Now explain why every vector in $C(A)$ and every vector in $N(A^T)$ are orthogonal.

More

All of Prof Strang's **Intro course** Linear Algebra lectures are here:

https://ocw.mit.edu/courses/18-06-linear-algebra-spring-2010/video_galleries/video-lectures/