

Last name _____

First name _____

LARSON—MATH 601—CLASSROOM WORKSHEET 17
Linear Transformations.

Concepts & Notation

- (Sec. 2.4) *ordered basis, coordinates, coordinate matrix, $[\alpha]_{\mathcal{B}}$.*
- (Sec. 2.5) *row rank.*
- (Sec. 3.1) *linear transformation, rank, nullity.*

1. What is the *row space* of a matrix? Why is it a vector space? Describe the row space

of
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

2. What is the *row rank* of a matrix? Find the row rank of
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

3. **Claim:** Row-equivalent matrices have the same row-space.

4. **Claim:** If matrix A is equivalent to row-reduced echelon R and to $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then
- $$R = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

5. What is a *linear transformation* T from a vector space V into a vector space W ? What is the *null space* of T ? What is the *range* of T ?
6. What is the zero transformation 0 from a vector space V to a vector space W ? Show it is a linear transformation. Find the null space of 0 . Find the range of 0 .
7. What are the *rank* and *nullity* of a linear transformation T ? Assume V and W are finite dimensional vector spaces. What is the rank and nullity of the zero transformation 0 ?
8. What is the identity transformation I from a vector space V to itself? Show it is a linear transformation. Find the kernel, range, rank and nullity of I .