

**LARSON—MATH 255—CLASSROOM WORKSHEET 15**  
**Scatter Plots & Recursion.**

1. (a) Start the Chrome browser.  
(b) Go to `http://cocalc.com`  
(c) Login using **your VCU email address** .  
(d) Click on our class Project.  
(e) Click “New”, then “Worksheets”, then call it **c15**.  
(f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be **#Problem 2**.

### Review

A **recursive** function is a function that calls itself. It must always have a *base case* so that the recursion eventually stops.

2. The *gcd* of 2 non-negative integers is their *greatest common divisor*. The following recursive function calculates the gcd of integers  $a$  and  $b$  using the fact (which can be proved) that, if  $a \geq b$  then  $\text{gcd}(a, b) = \text{gcd}(a - b, b)$ . It uses the fact that  $\text{gcd}(0, a) = \text{gcd}(a, 0) = a$ , for any non-negative integer  $a$ , as the base case.

```
def gcd(a,b):
    if a==0 or b==0:
        return max(a,b)
    else:
        return gcd(max(a,b)-min(a,b),min(a,b))
```

3. **Fibonacci!** The Fibonacci sequence  $F_n$  is defined as follows  $F_0 = 0$ ,  $F_1 = 1$  and  $F_n = F_{n-1} + F_{n-2}$  for  $n > 1$ . Write a recursive function `fib(n)` that computes the  $n^{\text{th}}$  Fibonacci number.
4. Try this for small values of  $n$  to make sure that it works, then try it for  $n = 10, 20, 30, 40, 50$ . Does it finish? If not, why not?!?!

### Random Values

5. `random()` returns a random number in  $[0, 1]$ . Execute it a few times to see what you get.
6. Define a function `my_mood()` which prints “I’m happy” or “I’m sad” randomly.

```
def my_mood():
    if random()<.5:
        print("I'm happy")
    else:
        print("I'm sad")
```

7. Use `random()` to define a function `coin_flip()` which randomly returns the string “H” (for heads) half the time and returns the string “T” (for tails) half the time. Try it a few times; your results will vary.
8. Run your coin flipping program 100 times and collect data. A random coin flipping program should come up heads about half the time. How many times do you get heads?
9. Now run your coin flipping program 1000 times and collect data. A random coin flipping program should come up heads about half the time. How many times do you get heads?

### **Investigate**

10. Start with any positive integer  $x$ . If  $x$  is even divide by 2. If  $x$  is odd, multiply by 3 and add 1. Repeat. Try this for several initial starting numbers  $x$ . What happens? (Do this all by hand—we’ll compute later).
11. Define a function `collatz(x)` that returns  $x$  if  $x$  is one, returns `collatz(3x+1)` if  $x$  is odd, and returns `collatz(x/2)` if  $x$  is even. This will be a recursive function (since it calls itself). What is the base case? Does it always terminate (return 1)?

### **Getting your classwork recorded**

When you are done, before you leave class...

- (a) Click the “Make pdf” (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then “Open”, then print or make a pdf using your browser).
- (b) Send me an email with an informative header like “Math 255 - c15 worksheet attached” (so that it will be properly recorded).
- (c) Remember to attach today’s classroom worksheet!