Last name ______________________

First name ______________________

LARSON—OPER 731—CLASSROOM WORKSHEET 17
Totally Unimodular Matrices

Minkowski's Theorem

1. Argue that if $x \in \mathcal{P}$, a bounded polyhedron defined by a system of linear inequalities $Ax \leq b$, with extreme points X, x is a convex combination of extreme points of face $\mathcal{F}_1$ of $\mathcal{P}$, and x is a convex combination of extreme points of face $\mathcal{F}_2$ of $\mathcal{P}$, then x is a convex combination of points of X.

2. Given a bounded polyhedron $\mathcal{P}$ defined by a system of linear inequalities $Ax \leq b$, extreme points X, $x \in \mathcal{P}$, how can we show that $x \in conv(X)$?

Totally Unimodular Matrices

3. What is a *totally unimodular matrix*?

4. Check that the following matrix A is *totally unimodular*. $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$.

5. Let b be any vector in $\mathbb{R}^3$ with integer components. Use Cramer's rule to show that any extreme point of the polyhedron $\mathcal{P}$ defined $Ax \leq b$ (and x_i nonnegative) has integer coordinates.

6. Draw a directed graph with 4 vertices and 5 edges. Label the vertices.

7. Find its directed vertex-edge incidence matrix.

8. Show that this matrix is totally unimodular (List all its square submatrices and find the determinant of each, or make an *argument*.)

9. **Total Unimodularity implies Integrality**. Show that any extreme point of the polyhedron $\mathcal{P}$ defined $Ax \leq b$ (and x_i nonnegative) has integer coordinates if A is totally unimodular.