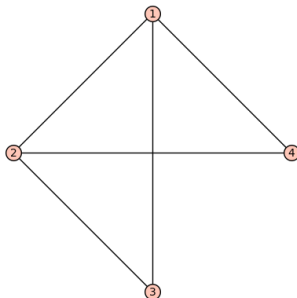**Last name** _____

First name _____

LARSON—OPER 731—HOMEWORK WORKSHEET 05
Test 1 Review

1. What is a *vertex packing*? Find a maximum vertex packing in the graph G above. Explain why it is maximum.

2. What is an *edge cover*? Find a minimum edge cover in the graph G above. Explain why it is minimum.

3. Write an Integer Program (IP) whose optimum is the size (cardinality) of a maximum vertex packing in the graph G above (that is, *model* this graph problem as an integer programming problem).

4. Use the Simplex Method to solve this LP:

   Maximize: $z = x_0 + x_1 + x_2 + x_3$

   Subject to:
   $$
   \begin{array}{rcrcrcrcl}
   x_0 & + & x_1 & & & & & \leq & 1 \\
   x_0 & & & + & x_2 & & & \leq & 1 \\
   & & x_1 & + & x_2 & & & \leq & 1 \\
   x_0 & + & & & & + & x_3 & \leq & 1 \\
   \end{array}
   $$

   with $x_i \in \mathbb{R}, x_i \geq 0$.

5. Find the *dual* of the LP in the last question.

6. State and prove the *Weak Duality Theorem.*

7. What is the *Strong Duality Theorem*? Give an example. Explain.

8. Use Fourier-Motzkin elimination to solve the following LP:

   Maximize:
   $$z = x_1 + x_2 + x_3$$

   Subject to:
   $$x_1 + x_2 \leq 1$$
   $$x_2 + x_3 \leq 1$$
   $$x_i \geq 0.$$

9. What is a *convex combination* of vectors? Give an example.

10. What are *affinely independent* vectors? Find an example of affinely independent vectors which are not linearly independent. Explain.

11. Argue that there can be no more than 3 affinely independent vectors in $\mathbb{R}^2$.

12. Define the *convex hull* of a finite point set X. Give an example.

13. What is *Weyl's theorem*?

14. What is a *polytope*?

15. Let $X = \{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}\}$. Use geometry to find inequalities whose feasible region is $conv(X)$.

16. Let $X = \{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}\}$. Use Fourier-Motzkin elimination to find inequalities whose feasible region is $conv(X)$.

17. What is the *Theorem of the Alternatives*?

18. Show that the following system $\mathcal{S}$ is *in*feasible:

$$x_1 + x_2 \leq 1$$
$$x_1 - x_2 \leq -2$$
$$x_1 \geq 0$$
$$x_2 \geq 0.$$

19. Use the Theorem of the Alternatives to find a system that *must* be feasible since $\mathcal{S}$ (from the last problem) is feasible. Explain.

20. Define complementary feasible solutions x and y of a primal LP $\mathcal{P}$ and its dual LP $\mathcal{D}$.

21. Give an example of an LP $\mathcal{P}$ and its dual LP $\mathcal{D}$ and complementary feasible solutions. Explain.

22. Give an example of an LP $\mathcal{P}$ and its dual LP $\mathcal{D}$, feasible solutions x and y, at least one non-optimal, and show that they are *not* complementary. Explain.

23. Let $C = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Show that the columns of C are linearly independent.

24. Let $C = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. What is the *rank* of C? Explain.

25. Let $C = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find a basis for the column space of C.

26. What is the *dimension* of a polytope?

27. What is the largest possible dimension of a polytope in $\mathbb{R}^n$? Explain.