

Last name \_\_\_\_\_

First name \_\_\_\_\_

**LARSON—MATH 550—CLASSROOM WORKSHEET 13**  
**Multiple Sums.**

**Concepts & Notation**

- Sec. 2.2. Two “tricks”.
- Sec. 2.3. Rules for sums. Perturbation method.
- Sec. 2.4. Multiple sums. General Distributive Law. Chebychev’s Monotonic Inequalities.

**Homework**

1. Find a single-sum formula for this double-sum:

$$S = \sum_{1 \leq j < k \leq n} (a_k - a_j)(b_k - b_j)$$

2. Use this to prove the following Chebyshev Monotonic Inequality:

$$\left(\sum_{k=1}^n a_k\right)\left(\sum_{k=1}^n b_k\right) \leq n\left(\sum_{k=1}^n a_k b_k\right) \text{ if } a_1 \leq \dots \leq a_n, b_1 \leq \dots \leq b_n.$$

3. Define  $\Delta f(x)$

4. Define  $x^m$ .

5. Define  $x^{\overline{m}}$ .

6. Find  $\Delta(x^m)$ .

7. Check:  $n! = n^{\underline{n}} = 1^{\underline{n}}$ .

8. How can we “investigate” the *General Law*?:

$$\sum_{0 \leq k \leq n} k^{\overline{m}} = \left. \frac{k^{\overline{m+1}}}{m+1} \right|_0^n$$