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LARSON—MATH 310–CLASSROOM WORKSHEET 19
Column Space, Row Space, Null Space, Matrix-Matrix Multiplication

Review: Chapter 4 of Klein's *Coding the Matrix* text

1. How is matrix-vector multiplication defined?
2. How is vector-matrix multiplication defined?

New

1. What is the *column space* of a matrix?

2. What is the *row space* of a matrix?

3. (**Sec. 4.5.4**) What is a *matrix-vector equation*?

Computational Problem 4.5.13: *Solving a matrix-vector equation*

- *input:* an $R \times C$ matrix A and an R-vector $\boldsymbol{b}$
- *output:* the C-vector $\hat{\boldsymbol{x}}$ such that $A * \hat{\boldsymbol{x}} = \boldsymbol{b}$

4. How can we view the following matrix-vector equation as a question about membership in the column space of the given matrix?

Example 4.6.8: Reformulating the triangular system of Example 2.11.1 (Page 130) as a matrix-vector equation, we obtain

$$\begin{bmatrix} 1 & 0.5 & -2 & 4 \\ 0 & 3 & 3 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix} * \boldsymbol{x} = [-8, 3, -4, 6]$$

5. Solve this system.

Definition 4.6.1 (*Dot-Product* Definition of Matrix-Vector Multiplication): If M is an $R \times C$ matrix and $\boldsymbol{u}$ is a C-vector then $M * \boldsymbol{u}$ is the R-vector $\boldsymbol{v}$ such that $\boldsymbol{v}[r]$ is the dot-product of row r of M with $\boldsymbol{u}$.

6. Find the following matrix-vector product using both the linear combination of columns definition and the dot product definition.

Example 4.6.2: Consider the matrix-vector product

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 10 & 0 \end{bmatrix} * [3, -1]$$

7. (**Sec. 4.7.1**) What is the *null space* of a matrix?

8. What is *matrix-matrix* multiplication?

Definition 4.11.1 (*Vector-matrix* definition of matrix-matrix multiplication): For each row-label r of A,

$$\text{row } r \text{ of } AB = (\text{row } r \text{ of } A) * B \tag{4.6}$$

9. Use the "vector-matrix" definition of matrix-matrix multiplication to find the following products.

Problem 4.17.6: Compute:

1. $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$
2. $\begin{bmatrix} 2 & 4 & 1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 5 & 1 & 1 \\ 2 & 3 & 0 \end{bmatrix}$