Last name ______________________

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LARSON—MATH 550—CLASSROOM WORKSHEET 06
Vandermonde Matrices. The Trick.

Concepts & Notation

- Sec. 1.1 & Sec. 1.2 T_n, recurrence (recurrence relation), mathematical induction, basis, solving recurrences
- Sec. 2.1 $[m = n]$ notation, sum notations.
- Sec. 2.2 The "trick".

1. (Sec. 2.2) Suppose $S_n = \sum_{k=0}^{n} a_k$ and $a_k = k \cdot \alpha + \beta$. Use our methodology to "solve" this recurrence.
   (a) We let $R_0 = \beta$, and $R_n = R_{n-1} + a_n$ (then $S_n = R_n$).
   (b) We found $R_n - A(n)\alpha + B(n)\beta$ for some functions $A(n)$ and $B(n)$.
   (c) We found some terms for $A(n)$ and $B(n)$ and *guessed* that $A(n) = \frac{n(n+1)}{2}$ and $B(n) = n$.
   (d) Now we want to *prove* our guess is correct by checking that it agrees with the "facts".

Vandermonde Matrices.

2. Someone tells you that a sequence starts $2, 4, 6$ and asks you to guess the next term. You guess 8 but he claims that is 17. He claims that its not random. His sequence has a *rule*. In fact its a degree-3 polynomial!

   (a) Let $p(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$, with $p(1) = 2$, $p(2) = 4$, $p(3) - 6$ and $p(4) = 17$.

   (b) Write some equations we can use to solve for c_i in $p(x)$.

   (c) Re-write these equations in matrix form.

   (d) Argue that this matrix equation has a solution and thus there is a polynomial $p(x)$ that "works".

   (e) Conclude that, for any given sequence $a_1, a_2, \ldots, a_n$, you can make an argument that the next term is x (for any choice of x!)

(Sec. 2.2). Given a recurrence of the form $a_n T_n = b_n T_{n-1} + c_n$, you can get a "nicer" recurrence by multiplying through by (any constant multiple of):

$$s_n = \frac{a_{n-1} a_{n-2} \ldots a_1}{b_n b_{n-1} \ldots b_2}$$

3. What would this yield for $T_n = 2T_{n-1} + 1$?

4. What would this yield for $L_n = L_{n-1} + n$?