Last name ______________________

First name ______________________

LARSON—MATH 601—HOMEWORK WORKSHEET h10
Linear Transformations & Matrices.

1. Let T be a linear transformation from a vector space V to a vector space W. **Show** that $T(0) = 0$.

2. In class we proved: If T is an isomorphism from a vector space V to a vector space W then T is invertible and non-singular. This is true regardless of the dimensions of V and W (in particular do not assume that V and W are finite-dimensional). Write up a **nice proof**, including all necessary definitions.

3. Let T be an isomorphism from a vector space V to a vector space W. Let $\{\alpha_i : i \in \mathcal{I}\}$ (where $\mathcal{I}$ is a possibly infinite *index set*) be a basis for V. **Show** that $\{T(\alpha_i) : i \in \mathcal{I}\}$ is a **basis** for W.

4. Let T be an isomorphism from a vector space V to a vector space W with inverse T^{-1}. Let $\{\beta_j : j \in \mathcal{J}\}$ (where $\mathcal{J}$ is a possibly infinite *index set*) be a basis for W. **Show** that $\{T^{-1}(\beta_j) : j \in \mathcal{J}\}$ is a **basis** for V.

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (x_1, 0)$. Let $\mathcal{B} = \{\alpha_1, \alpha_2\} = \{(1,0),(0,1)\}$ be the standard basis in $\mathbb{R}^2$. Let $\mathcal{B}' = \{\alpha'_1, \alpha'_2\} = \{(1,1),(2,1)\}$ be another basis for $\mathbb{R}^2$.

   (a) Find $T(3,5)$.

   (b) Find $[T(3,5)]_{\mathcal{B}}$.

   (c) Find $[T]_{\mathcal{B}} = [[T(\alpha_1)]_{\mathcal{B}}[T(\alpha_2)]_{\mathcal{B}}]$.

   (d) Check that $[T(3,5)]_{\mathcal{B}} = [T]_{\mathcal{B}}[(3,5)]_{\mathcal{B}}$.

   (e) Show that $\mathcal{B}'$ is a basis for $\mathbb{R}^2$.

   (f) Find $[T(3,5)]_{\mathcal{B}'}$.

   (g) Find $[T]_{\mathcal{B}'} = [[T(\alpha'_1)]_{\mathcal{B}}[T(\alpha'_2)]_{\mathcal{B}'}]$.

   (h) Check that $[T(3,5)]_{\mathcal{B}'} = [T]_{\mathcal{B}'}[(3,5)]_{\mathcal{B}'}$.
   What is the relationship between $[T]_{\mathcal{B}}$ and $[T]_{\mathcal{B}'}$?

   (i) Find $P = [[\alpha'_1]_{\mathcal{B}}[\alpha'_2]_{\mathcal{B}}]$.

   (j) Find P^{-1}.

   (k) Check that $[T]_{\mathcal{B}'} = P^{-1}[T]_{\mathcal{B}}P$.

   (l) Find $Q = [[\alpha_1]_{\mathcal{B}'}[\alpha_2]_{\mathcal{B}'}]$.

   (m) Find Q^{-1}.

   (n) Check that $[T]_{\mathcal{B}} = Q^{-1}[T]_{\mathcal{B}'}Q$.