Last name _____

First name _____

LARSON—OPER 731—CLASSROOM WORKSHEET 28
Matchings, Matching Matroid & Matching Polytope

1. What is a *matching*? in a graph? What is a *maximum matching*?
2. What is $\delta(v)$? If M is a matching, what vertices are *covered* by M? What vertices are *exposed* by M?
3. If M is a matching in a graph, what is an *alternating path*? What is an *augmenting path*?
4. What is *Berge's Theorem*? Why is it true?

(**Matching Matroid Theorem**) Let G be a graph and $W \subseteq V(G)$. $M = (E(M), \mathcal{I}(M))$ with $E(M) = W$ and

$$\mathcal{I}(M) = \{X \subseteq W : G \text{ has a matching that covers } X\}.$$

is a matroid.

5. Find the Matching Matroid M for the path on 3 vertices P_3 and $E(M) = V(P_3)$.
6. Why is the Matching Matroid Theorem true?

The **Matching Polytope** Let G be a graph with edges $E(G)$ and matchings $\mathcal{M}(G)$. The *matching polytope* of G, $\mathcal{P}_{\mathcal{M}(G)}$, is the convex hull of the characteristic vectors of the sets of edges which are matchings.

7. Find $\mathcal{P}_{\mathcal{M}(P_3)}$.

8. Find the polyhedron $\mathcal{P}$ defined by the following constraints: one non-negative variable x_i for each edge e_i of P_3 and one "vertex" constraint for each vertex of P_3 which models the requirement that the sum of the variables corresponding to the edges incident to any vertex is no more than 1.

9. Does $\mathcal{P} = \mathcal{P}_{\mathcal{M}(P_3)}$?

10. Find $\mathcal{P}_{\mathcal{M}(K_3)}$.

11. Find the polyhedron $\mathcal{P}$ defined by the following constraints: one non-negative variable x_i for each edge e_i of K_3 and one "vertex" constraint for each vertex of K_3 which models the requirement that the sum of the variables corresponding to the edges incident to any vertex is no more than 1.

12. Does $\mathcal{P} = \mathcal{P}_{\mathcal{M}(K_3)}$?

13. What is the **Matching Polytope Theorem**, why doesn't its use require checking exponentially many subsets, and what is it's significance?