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LARSON—MATH 310–CLASSROOM WORKSHEET 24
Basis, Dimension, Column Space, Row Space

Review

1. What is an equivalent definition for a *linearly independent* set of vectors?
2. (**Definition 5.6.1**) Let $\mathcal{V}$ be a vector space. A *basis* for $\mathcal{V}$ is a linearly independent set of generators for $\mathcal{V}$.
3. Argue: the standard generating set for $\mathbb{R}^2$ is a basis for $\mathbb{R}^2$.
4. Argue: the standard generating set for $\mathbb{R}^3$ is a basis for $\mathbb{R}^3$.

Chapter 5 of Klein's *Coding the Matrix* text

1. Show: the vectors $[1,1,1],[1,1,0],[0,1,1]$ are a basis for $\mathbb{R}^3$.

(**Lemma 5.7.1, Unique-Representation Lemma)**) Let $\hat{a}_1,...,\hat{a}_n$ be a basis for a vector space $\mathcal{V}$. For any vector $\hat{v} \in \mathcal{V}$, there is exactly one representation of $\hat{v}$ in terms of the basis vectors.

2. Find the *coordinates* for $[3,3,1]$ with respect to the standard basis for $\mathbb{R}^3$.

3. Find the *coordinates* for $[3,3,1]$ with respect to the basis $[1,1,1],[1,1,0],[0,1,1]$ for $\mathbb{R}^3$.

4. Why is Lemma 5.7.1 true?

(**Theorem 6.1.2, Basis Theorem**) Let $\mathcal{V}$ be a vector space. All bases for $\mathcal{V}$ have the same size.

5. What is the *dimension* of a vector space?

6. What are examples?

7. Given a generating set for a vector space $\mathcal{V}$, how can we find a basis for $\mathcal{V}$?

8. How can we find a basis for the column space of a matrix? What is the *column rank* of a matrix?

Example 6.2.11: Consider the matrix

$$M = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 2 & 0 & 7 \\ 0 & 0 & 3 & 9 \end{bmatrix}$$

9. How can we find a basis for the row space of a matrix? What is the *row rank* of a matrix?

Fact: For any matrix M, the column rank of M equals its row rank (the clearest explanation of why this is true will follow after we have Gaussian Elimination as a tool).