Last name ______________________

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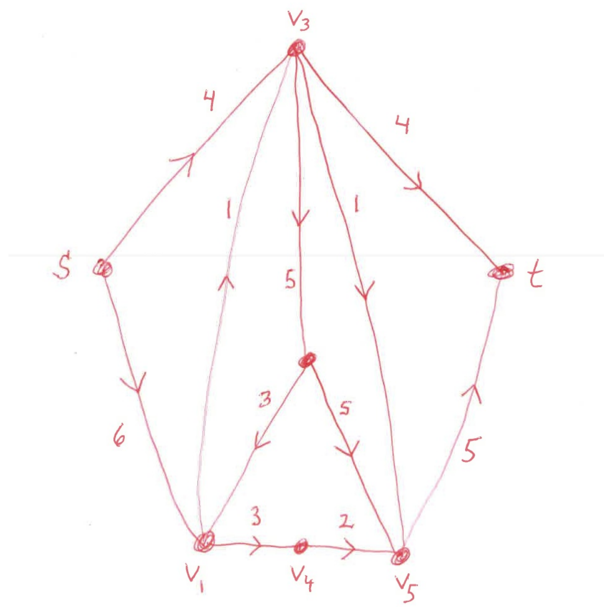
LARSON—MATH 356—CLASSROOM WORKSHEET 26
Network Flows & Min-Max Theorems



1. We found a flow with value 7 in this network. Find any flow with value 7 here.

2. If we were trying to improve this flow, find all scanned vertices before the algorithm terminates (re-do the last iteration of our scan-label algorithm). We'll need these ahead.

3. What is a *cut* in a network?

4. What is the *capacity* of a cut in a network?

5. What is the set of scanned vertices in the final iteration of our work finding a maximum flow?

6. What is the corresponding cut?

7. What is the capacity of this cut?

8. Check that the value of our final flow equals the capacity of this cut? (The Max Flow Min Cut Theorem says this is always the case).

9. The main tool we need in arguing that the value of a maximum flow in a network is the minimum capacity of a cut is the following lemma.

Lemma 3.4.1. *Let f be a flow of value Q in a network $\mathbf{X}$, and let $(W, \overline{W})$ be a cut in $\mathbf{X}$. Then*

$$Q = f(W, \overline{W}) - f(\overline{W}, W) \leq cap(W, \overline{W}). \tag{3.4.1}$$

Proof of lemma: The net flow out of s is Q. The net flow out of any other vertex $w \in W$ is 0. Hence, if $V(\mathbf{X})$ denotes the vertex set of the network $\mathbf{X}$, we obtain

$$\begin{aligned} Q &= \sum_{w \in W} \{f(w, V(\mathbf{X})) - f(V(\mathbf{X}), w)\} \\ &= f(W, V(\mathbf{X})) - f(V(\mathbf{X}), W) \\ &= f(W, W \cup \overline{W}) - f(W \cup \overline{W}, W) \\ &= f(W, W) + f(W, \overline{W}) - f(W, W) - f(\overline{W}, W) \\ &= f(W, \overline{W}) - f(\overline{W}, W). \end{aligned}$$

(Claim:) The value of a maximum flow in a network equals the capacity of a minimum cut.

10. What is a *min-max theorem*?

11. What is the significance or importance of min-max theorems?