

LARSON—MATH 255—HOMEWORK WORKSHEET h06
Visualizing data, Simulations, Collatz

1. Create a Cocalc/Sage Cloud account.
 - (a) Start the Chrome browser.
 - (b) Go to `http://cocalc.com`
 - (c) You should see an existing Project for our class. Click on that.
 - (d) Click “New”, then “Sage Worksheet”, then call it **h06**.
 - (e) For each problem number, label it in the SAGE cell where the work is. So for Problem 1, the first line of the cell should be `#Problem 1`.

Primes

2. Define a function `prime_count(n)` that counts the primes up to n . Test it (there are 25 primes for instance in the integers from 1 to 100).
3. Define a function `prime_ratio(n)` that produces the ratio of primes up to n . Use your `prime_count(n)` in your definition. Test it (since there are 25 primes up to 100, the ratio is $\frac{25}{100}$).
4. Make a scatter plot to visualize values of `prime_ratio(n)` for n from 100 to 1000.

Coin Flip Simulations

5. Use `random()` to define a function `coin_flip()` which randomly returns the string “H” (for heads) half the time and **returns** the string “T” (for tails) half the time. Try it a few times; your results will vary.
6. Define a function `heads(n)` that simulates flipping a coin n times and counting the number of “heads” that were flipped. Use your `coin_flip()` in your definition. Test it (you would expect roughly $\frac{n}{2}$ heads).
7. Define a function `heads_ratio(n)` that simulates flipping a coin n times and returns the ratio of heads you flipped. Use your `heads(n)` in your definition. Test it (you would expect values near 0.5).

More Collatz

In class we defined a function `collatz(x)` that returns x if x is one, and returns `collatz(3x+1)` if x is odd, and returns `collatz(x/2)` if x is even. It is conjectured that this function always returns 1 (so starting at any positive integer and repeating this process, you will eventually produce 1).

8. Now let's see how many steps it takes until we get to 1 (so if you input 4, you then get 2, and then 1; that's 2 steps). Now maybe our recursive collatz function is less useful. Define a function `collatz_one_step(n)` that inputs n , returns 1 if n is 1, returns $\frac{n}{2}$ if n is even, and returns $3n + 1$ if n is odd.
9. Now define a function `collatz_count(n)` that takes a positive integer n as input and returns the number of steps it takes to get to 1. Use your `collatz_one_step(n)` in your definition. Test it (you should get 2 if your input n is 4 and 0 if your input n is 1).

Getting your homework recorded

When you are done, before you leave class...

- (a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If CoCalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).
- (b) Send me an email with an informative header like "Math 255 - h06 worksheet attached" (so that it will be properly recorded).
- (c) Remember to attach today's classroom worksheet!