Last name ______________________

First name ______________________

LARSON—MATH 601—HOMEWORK WORKSHEET h11
Linear algebras & Polynomials

1. For $f, g \in \mathbb{F}^\infty$ with $f = (f_0, f_1, f_2, \ldots)$, and $g = (g_0, g_1, g_2, \ldots)$ we define a "vector multiplication" $fg \in \mathbb{F}^\infty$ where the value of the function fg for $n \in \mathbb{N}$ is $(fg)(n)$ or,

$$(fg)_n = \sum_{i=0}^{n} f_i g_{n-i},$$

which we identify with the n^{th} component of an infinite tuple.

   (a) Use the above definition to show that the "1" in $\mathbb{F}^\infty$ is $1 = (1, 0, 0, 0, \ldots)$, that is, for every $f \in \mathbb{F}^\infty$, $1f = f$.

   (b) For every $c \in \mathbb{F}$ let the "c function" in $\mathbb{F}^\infty$ be $c = (c, 0, 0, 0, \ldots)$. Use the above definition to show that $cf = (cf_0, cf_1, cf_2, \ldots)$.

   (c) Let $x = x^1 = (0, 1, 0, 0, \ldots)$ and $x^2 = (0, 0, 1, 0, 0, \ldots)$. Use the above definition to show that $x^2 = x^1 x^1$.

   (d) For $i \in \mathbb{N}$ let x^i be the all-zero-infinite tuple with a single 1 in the i^{th} coordinate. (Note that it follows that $x^0 = 1 \in \mathbb{F}^\infty$.) Use the above definition to show that $x^i x^1 = x^{i+1}$.

   (e) Use the above definition to show that $x^i x^j = x^{i+j}$.

2. Let $\mathbb{F}$ be a field.

   (a) Define $\mathbb{F}[x]$.

   (b) Explain why $\mathbb{F}[x]$ is a *vector space*.

   (c) Explain why $\mathbb{F}[x]$ is a *linear algebra*.

3. Explain the difference between polynomials and polynomial functions.

4. Give an example that illustrates how two different polynomials can correspond to the same function.

5. Let $p = (x - 2)(x - 3)(x - 1)$ be a polynomial over $\mathbb{R}$. Find $p(A)$ where:

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$