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LARSON—MATH 310—CLASSROOM WORKSHEET 23
Linear Dependence, Linear Independence, Basis, Dimension

Review

1. What is the *inverse* of a matrix?
2. What is the importance of matrix inverses?

(Question 5.2.5) For a given vector space \mathcal{V} , what is the minimum number of vectors whose span equals \mathcal{V} ?

(Definition 5.5.2) Vectors $\hat{v}_1, \dots, \hat{v}_n$ are *linearly dependent* if the zero vector can be written as a nontrivial linear combination of the vectors: $0 = \alpha_1\hat{v}_1 + \dots + \alpha_n\hat{v}_n$.

3. What does it mean for vectors to be *linearly independent*?

Chapter 5 of Klein's *Coding the Matrix* text

1. What is an equivalent definition for a *linearly independent* set of vectors?

(Definition 5.6.1) Let \mathcal{V} be a vector space. A *basis* for \mathcal{V} is a linearly independent set of generators for \mathcal{V} .

2. What are examples?

3. Argue: the standard generating set for \mathbb{R}^2 is a basis for \mathbb{R}^2 .

4. Argue: the standard generating set for \mathbb{R}^3 is a basis for \mathbb{R}^3 .

5. Show: the vectors $[1, 1, 1], [1, 1, 0], [0, 1, 1]$ are a basis for \mathbb{R}^3 .

(Lemma 5.7.1, Unique-Representation Lemma) Let $\hat{a}_1, \dots, \hat{a}_n$ be a basis for a vector space \mathcal{V} . For any vector $\hat{v} \in \mathcal{V}$, there is exactly one representation of \hat{v} in terms of the basis vectors.

6. Find the *coordinates* for $[3, 3, 1]$ with respect to the standard basis for \mathbb{R}^3 .
7. Find the *coordinates* for $[3, 3, 1]$ with respect to the basis $[1, 1, 1], [1, 1, 0], [0, 1, 1]$ for \mathbb{R}^3 .
8. Why is Lemma 5.7.1 true?

(Theorem 6.1.2, Basis Theorem) Let \mathcal{V} be a vector space. All bases for \mathcal{V} have the same size.

9. What is the *dimension* of a vector space?
10. What are examples?
11. Given a generating set for a vector space \mathcal{V} , how can we find a basis for \mathcal{V} ?