Last name ______________________

First name ______________________

LARSON—MATH 550—CLASSROOM WORKSHEET 31
The Sorting Example, Inversion & Derangements.

Concepts & Notation

- Sec. 5.1: Binomial coefficients, the Binomial Theorem, key formulas, the polynomial argument, Vandermonde's convolution.
- Sec. 5.2: the Sorting example.
- Sec. 5.3: Duplication formula, inversion, derangements.
- Sec. 5.4: Convolutions, generating functions.

The Duplication Formula

We showed:

$$r^{\underline{k}}(r-\frac{1}{2})^{\underline{k}} = (2r)^{\underline{2k}}/2^{2k} \text{ for } k \in \mathbb{Z}^{\geq 0}.$$

1. Try it with some values for r and k.

Derangements

A *derangement* of n objects is an ordering (listing, or permutation) of those objects so that none is in its original place; or, more formally, it is a bijection $\phi : [n] \to [n]$ such that $\phi(i) \neq i$, for every $i \in [n]$.

Let ¡n be the number of derangements of n objects.

2. Find ¡2.

3. Find ¡3.

4. Find ¡4.

Inversion

Let $f(n)$ be a function defined on $\mathbb{Z}^{\geq 0}$, and define:

$$g(n) = \sum_k \binom{n}{k} (-1)^k f(k).$$

The **claim** is that:

$$f(n) = \sum_k \binom{n}{k} (-1)^k g(k).$$

5. Test this for $f(n) = n^2$ and some values of $n \in \mathbb{Z}^{\geq 0}$

6. *Prove* that the inversion formula for $f(n)$ holds in general.