

Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 10 Linear Transformations.

Concepts (Chp. 1): field, vector space, \mathcal{P} , \mathbb{F}^n , $\mathbb{M}_{m \times n}(\mathbb{F})$, subspace, null space, $\text{row}(A)$, $\text{col}(A)$, list of vectors, span of a list of vectors, linear independence, linear dependence, pivot column decomposition, direct sum $\mathcal{U} \oplus \mathcal{V}$, *orthogonal* matrix, *unitary* matrix, *basis*, *dimension*.

Review:

1. (Theorem 2.3.1) Let $A \in \mathbb{M}_{m \times n}(\mathbb{F})$. Then

$$\dim(\text{col}(A)) = \dim(\text{col}(A^T)) = \dim(\text{row}(A)) \leq \min\{m, n\}.$$

2. (Theorem 2.3.7. Full Rank Factorization). Let $A \in \mathbb{M}_{m \times n}(\mathbb{F})$ be non-zero, let $r = \text{rank}(A)$, and let the columns of $X \in \mathbb{M}_{m \times r}(\mathbb{F})$ be a basis for $\text{col}(A)$. Then there is a unique $Y \in \mathbb{M}_{r \times n}(\mathbb{F})$ such that $A = XY$. Moreover, $\text{rank}(Y) = r$, the rows of Y are a basis for $\text{row}(A)$ and $\text{null}(A) = \text{null}(Y)$.
3. What is the β -basis representation function? What are the coordinates of a vector with respect to a basis?
4. What is a linear transformation?

Chp. 2 of Garcia & Horn, Matrix Mathematics

1. What is $\mathfrak{L}(\mathcal{V}, \mathcal{W})$?
2. How does any matrix $A \in \mathbb{M}_{m \times n}$ define a linear transformation?
3. How does any linear transformation $T \in \mathcal{L}(\mathcal{V}, \mathcal{W})$ and bases $\beta = \hat{v}_1, \dots, \hat{v}_n$ of \mathcal{V} and $\gamma = \hat{w}_1, \dots, \hat{w}_m$ of \mathcal{W} define a matrix $A \in \mathbb{M}_{m \times n}$?

4. What is ${}_{\gamma}[T]_{\beta}$?
5. What is the β - γ *change-of-basis* matrix (notation: ${}_{\gamma}[I]_{\beta}$)?

6. What does it mean for matrices $A, B \in \mathbb{M}_n(\mathbb{F})$ to be *similar* (over \mathbb{F})?

(Corollary 2.6.5) Let $A, B \in \mathbb{M}(\mathbb{F})$. The following are equivalent:

- (a) A and B are similar.
- (b) There is an n -dimensional \mathbb{F} -vector space \mathcal{V} , bases β and γ for \mathcal{V} and a $T \in \mathfrak{L}(\mathcal{V})$ such that $A = {}_{\beta}[T]_{\beta}$ and $B = {}_{\gamma}[T]_{\gamma}$.

7. What does the Corollary mean?

8. Why is it true?