Last name _____

First name _____

LARSON—MATH 310–HOMEWORK WORKSHEET 13
Test 2 Review.

General Instructions

1. **You should know the following definitions, and corresponding examples, for the test. Write out careful definitions and problem solutions. Turn these in at test time**.
2. Write up a **neat** assignment on a **new sheet** of paper. (Do not cram your answers between the lines).
3. **Number** your problems so that it is easy to see what work matches the assigned problems.
4. Remember to **give examples** (you do not understand a concept unless you can provide an example of it).

Definitions. Write each definition **and give an example**.

1. *vector space*.
2. *homogeneous linear system*
3. *linear-combination definition of matrix-vector multiplication*.
4. *linear-combinations definition of vector-matrix multiplication*.
5. *upper-triangular matrix*.
6. *diagonal matrix*.
7. *inverse* of a matrix.
8. *transpose* of a matrix.
9. *linearly dependent* vectors.
10. *linearly independent* vectors.
11. *basis* of a vector space.
12. *column space* of a matrix.
13. *null space* of a matrix.
14. *rank* of a matrix.
15. *dimension* of a vector space.
16. *eigenvalue* of a square matrix.

17. *eigenvector* of a square matrix.

Algorithms. State each algorithm carefully.

18. *Greedy algorithm for finding a basis for the column space of a matrix.*

Theorems. State each theorem carefully.

19. *Basis Theorem.*

Problems. Show your work, and **explain** your answers.

20. Show: $\{[x, y, z] : x, y, z \in \mathbb{R}^3, x + y + z = 1\}$ is not a vector space (so, it's not a subspace of $\mathbb{R}^3$.

21. Show: $\{[x, y, z] : x, y, z \in \mathbb{R}^3, x + y + z = 0\}$ is a vector space (so, it is a subspace of $\mathbb{R}^3$.

22. If $\hat{v_1}, \hat{v_2}, \ldots, \hat{v_n}$ are vectors in a vector space $\mathcal{V}$, why is Span$(\{\hat{v_1}, \hat{v_2}, \ldots, \hat{v_n}\})$ a vector space?

23. Are the vectors $[1, 0, 0]$, $[0, 2, 0]$, and $[2, 4, 0]$ are linearly dependent or linearly independent?

24. Are the vectors $[1, 0, 0]$, $[0, 2, 0]$, and $[0, 0, 4]$ are linearly dependent or linearly independent?

25. Why can't a set of vectors containing the 0-vector be linearly independent?

26. Argue: the standard generating set for $\mathbb{R}^2$ is a basis for $\mathbb{R}^2$.

27. Use our greedy algorithm to find a basis for the column space of $A = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 2 & 0 & 7 \\ 0 & 0 & 3 & 9 \end{bmatrix}$.

28. Let $A = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 2 & 0 & 7 \\ 0 & 0 & 3 & 9 \end{bmatrix}$. Let $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \\ 3 & 5 \end{bmatrix}$. Find AB.

29. Rewrite the following system as an augmented matrix and reduce solve using Gaussian elimination. Write the system of equations that the reduced matrix represents. Backsolve. Represent the solutions as a vector or vectors.

$$\begin{array}{rrcr} 2x & +3y & = & 13 \\ x & -y & = & -1 \end{array}$$

30. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find the eigenvalues and corresponding eigenvectors.