Last name	
First name	

LARSON—OPER 731—CLASSROOM WORKSHEET 28 Matchings, Matching Matroid & Matching Polytope

- 1. What is a matching? in a graph? What is a maximum matching?
- 2. What is $\delta(v)$? If M is a matching, what vertices are *covered* by M? What vertices are *exposed* by M?
- 3. If M is a matching in a graph, what is an alternating path? What is an augmenting path?
- 4. What is *Berge's Theorem*? Why is it true?

(Matching Matroid Theorem) Let G be a graph and $W \subseteq V(G)$. $M = (E(M), \mathcal{I}(M))$ with E(M) = W and

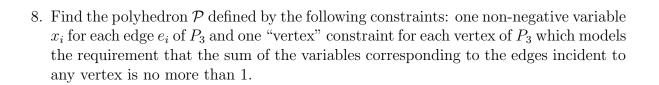
$$\mathcal{I}(M) = \{X \subseteq W : G \text{ has a matching that covers } X\}.$$

is a matroid.

- 5. Find the Matching Matroid M for the path on 3 vertices P_3 and $E(M) = V(P_3)$.
- 6. Why is the Matching Matroid Theorem true?

The Matching Polytope Let G be a graph with edges E(G) and matchings $\mathcal{M}(G)$. The matching polytope of G, $\mathcal{P}_{\mathcal{M}(G)}$, is the convex hull of the characteristic vectors of the sets of edges which are matchings.

7. Find $\mathcal{P}_{\mathcal{M}(P_3)}$.



9. Does
$$\mathcal{P} = \mathcal{P}_{\mathcal{M}(P_3)}$$
?

10. Find
$$\mathcal{P}_{\mathcal{M}(K_3)}$$
.

11. Find the polyhedron \mathcal{P} defined by the following constraints: one non-negative variable x_i for each edge e_i of K_3 and one "vertex" constraint for each vertex of K_3 which models the requirement that the sum of the variables corresponding to the edges incident to any vertex is no more than 1.

12. Does
$$\mathcal{P} = \mathcal{P}_{\mathcal{M}(K_3)}$$
?

13. What is the **Matching Polytope Theorem**, why doesn't its use require checking exponentially many subsets, and what is it's significance?