LARSON—OPER 731—CLASSROOM WORKSHEET 21
Matroids

Matroids

1. *Why*, if you have a set Y of 3 linearly independent vectors in $\mathbb{R}^3$ and a set X of 2 linearly independent vectors, must it be the case that there is a vector $v \in Y$ such that $X \cup \{v\}$ is linearly independent?

2. What is a *matroid*?

3. What is a *tree*? What is a *spanning tree* in a graph?

4. What can you say about the number of edges in a tree?

5. What is a *forest* in a graph? What is a *component* of a graph? What is κ? What can you say about the number of edges in a forest?

6. Why, if the edges Y of a graph G induce a forest and the edges X of G induce a forest and $|Y| > |X|$, must it be the case that there is an edge $e \in Y$ such that $X \cup \{e\}$ induces a forest in G?

Checking the Matroid Axioms

7. What is a *linear matroid*?

8. What is a *graphic matroid*?

9. What is a *uniform matroid*?