Last name ______________________

First name ______________________

LARSON—OPER 731—CLASSROOM WORKSHEET 20
König's Theorem & Matroids

Totally Unimodular Matrices

1. What is a *totally unimodular matrix*?

2. Show that the vertex-edge incidence matrix of a directed graph is totally unimodular.

3. Show that the vertex-edge incidence matrix of a bipartite graph is totally unimodular.

4. **Total Unimodularity implies Integrality**. Show that any extreme point of the polyhedron $\mathcal{P}$ defined $Ax \leq b$ (and x_i nonnegative) has integer coordinates if A is totally unimodular and b is any vector with integer components.

König's Theorem

5. What is a *matching* in a graph, what is a *vertex cover*, what is a *bipartite graph*, and what is König's Theorem?

6. Represent finding a maximum matching in a bipartite graph as an integer programming problem and use total unimodularilty to show that the relaxation of this IP has integer solutions.

7. What does the dual of this IP model?

8. Use total unimodularilty to show that the relaxation of this dual IP has integer solutions. How does this prove König's Theorem?

Matroids

9. Why, if you have a set Y of 3 linearly independent vectors in $\mathbb{R}^3$ and a set X of 2 linearly independent vectors, must it be the case that there is a vector $v \in Y$ such that $X \cup \{v\}$ is linearly independent?

10. What is a *matroid*?

11. What is a *forest* in a graph?

12. Why, if the edges Y of a graph G induce a forest and the edges X of G induce a forest and $|Y| > |X|$, must it be the case that there is an edge $e \in Y$ such that $X \cup \{e\}$ induces a forest in G?