Last name _______________________

First name _______________________

LARSON—MATH 550—CLASSROOM WORKSHEET 12
Multiple Sums.

Concepts & Notation

- Sec. 2.2. Two "tricks".
- Sec. 2.3. Rules for sums. Perturbation method.
- Sec. 2.4. Multiple sums. General Distributive Law. Chebychev's Monotonic Inequalities.

Homework

1. Find a nice expression for $\displaystyle\sum_{1\le i,j\le n} a_i b_j$.

2. Find a nice expression for $\displaystyle\sum_{1\le i,j\le n} a_i a_j$.

3. Find the sum of the elements in the matrix:

$$\begin{bmatrix} a_1a_1 & a_1a_2 & \dots & a_1a_{n-1} & a_1a_n \\ a_2a_1 & a_2a_2 & \dots & a_2a_{n-1} & a_2a_n \\ \dots & \dots & & & \dots \\ a_{n-1}a_1 & a_{n-1}a_2 & \dots & a_{n-1}a_{n-1} & a_{n-1}a_n \\ a_na_1 & a_na_2 & \dots & a_na_{n-1} & a_na_n \end{bmatrix}$$

4. Find $T = \displaystyle\sum_{1\le i\le j\le n} a_i a_j$
   (the sum of the elements of the upper-triangle of an $n \times n$ matrix with entries $a_i a_j$).

5. Explain why this identity is true:

$$[1 \le j < k \le n] + [1 \le k < j \le n] = [1 \le j, k \le n] - [1 \le j = k \le n].$$

6. Expand and simplify:

$$\sum_{1 \le i, j \le 3} a_i b_i.$$

7. Find a single-sum formula for this double-sum:

$$S = \sum_{1 \le j < k \le n} (a_k - a_j)(b_k - b_j)$$

8. Use this to prove the following Chebyshev Monotonic Inequality:

$$(\sum_{k=1}^{n} a_k)(\sum_{k=1}^{n} b_k) \le n(\sum_{k=1}^{n} a_k b_k) \text{ if } a_1 \le \ldots a_n, b_1 \le \ldots \le b_n.$$