Last name ______________________

First name ______________________

LARSON—MATH 610–CLASSROOM WORKSHEET 02
Linear Algebra Background.

Concepts (Chp. 1): field, vector space, $\mathcal{P}$, $\mathbb{F}^n$, $\mathbb{M}_{m\times n}(\mathbb{F})$, subspace, null space, row(A), col(A), list of vectors, span of a list of vectors, linear independence, linear dependence, pivot column decomposition, direct sum $\mathcal{U} \oplus \mathcal{V}$.

1. What is an *invertible* matrix?

2. What is the *conjugate* of a matrix $A \in \mathbb{M}_{m\times n}$?

3. What is the *conjugate transpose* (or *adjoint*) A^* of a matrix $A \in \mathbb{M}_{m\times n}$?

4. What is a *symmetric* matrix?

5. What is a *Hermitian* matrix? Check that $\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ is Hermitian but not symmetric.

Think: symmetric $\subseteq$ Hermitian $\subseteq$ normal.

6. What is a *normal* matrix? Check that $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ is normal but not Hermitian.

7. What is a *vector space*?

8. What is a *subspace* of a vector space?

(**Theorem 1.3.3**). Let $\mathcal{V}$ be an $\mathbb{F}$-vector space and let $\mathcal{U}$ be a non-empty subset of $\mathcal{V}$. Then U is a subspace of $\mathcal{V}$ if and only if $cu + v \in \mathcal{U}$ whenever $u, v \in \mathcal{U}$ and $c \in \mathbb{F}$.

9. Why is this theorem true?

(**Theorem 1.4.10**). Let $Y = [\hat{y}_1 \hat{y}_2 \dots \hat{y}_p] \in \mathbb{M}_{m\times p}(\mathbb{F})$ and let $A \in \mathbb{M}_{m\times n}(\mathbb{F})$ then $col(Y) \subseteq col(A)$ if and only if $Y = AX$ for some $X \in \mathbb{M}_{n\times p}$.

10. Why is this theorem true?