**LARSON—MATH 255–CLASSROOM WORKSHEET 32**
**Graph Theory.**

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click "New", then "Worksheets", then call it **c32**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

2. (**Ramanujan**) 2, 9, 16, etc. can be written (uniquely) as the sum of 2 cubes ($1^3+1^3$, $1^3+2^3$, $2^3+2^3$, etc.). Find the smallest integer which can be written as the sum of 2 cubes in 2 different ways.

**Dictionaries**

A dictionary is a collection of key-value pairs (*keys* and *values*). Each pair is connected with a colon, and different pairs are separated by commas.

3. Here is a dictionary of name-age pairs: `my_dictionary={"Kyrie":30,"Cindy":23}`. Evaluate. Now evaluate `my_dictionary`.

4. To find Kyrie's age, evaluate `my_dictionary["Kyrie"]`. Now find Cindy's birthday.

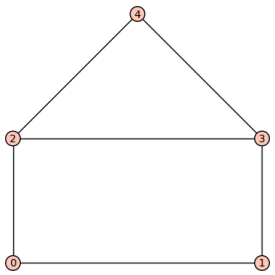
5. To get a list of all the *keys* use the `.keys()` method. Evaluate: `my_dictionary.keys()`.

6. And to get all the *values*, use the `.values()` method. Evaluate: `my_dictionary.values()`.

**Graphs & Graph Theory**

A **graph** is a mathematical object consisting of *dots* and *lines* (also called *vertices* and *edges*). The *order* of a graph is the number of vertices it has. The *size* of a graph is the number of edges it has. We can create our own graph using the `Graph()` constructor, and the `add_vertex()` and `add_edge()` methods.

7. Make the following graph, called "the house". Start by letting `house=Graph(5)`. When you are done you can view it with `house.show()`.



8. Try the following Sage Interact which shows some famous graphs and uses a *dictionary*:

```
@interact
def i_graph(graph=selector(["icosahedron", "dodecahedron",
    "tetrahedron", "octahedron"],
    label="Select a graph", default="tetrahedron")):

    dict={"icosahedron":graphs.IcosahedralGraph(),
    "dodecahedron":graphs.DodecahedralGraph(),
    "tetrahedron":graphs.TetrahedralGraph(),
    "octahedron":graphs.OctahedralGraph()}
    g=dict[graph]
    order = g.order()
    size=g.size()
    print("The {} has {} vertices and {} edges".format(graph,order,size))
    g.show()
```

9. Let's get acquainted with paths, cycles, stars, and complete graphs. Try:

```
@interact
def i_graph(graph=selector(["path", "cycle", "star", "complete"],
    label="Select a graph", default="path"),order=slider(3,20,1,3)):

    dict={"path":graphs.PathGraph(order),
    "cycle":graphs.CycleGraph(order),
    "star":graphs.StarGraph(order),
    "complete":graphs.CompleteGraph(order)}
    g=dict[graph]
    order = g.order()
    size=g.size()
    print("This graph has {} vertices and {} edges".format(graph,order,size))
    g.show()
```

Another way to represent a graph with order $n$ is with an $n \times n$ *adjacency matrix* $A$. If the vertices of the graph are $\{v_0, v_1, \ldots, v_{n-1}\}$ (or {1,2,...,n-1} for short) then the $A_{i,j}$ is 1 if there is an edge from vertex $i$ to vertex $j$, and 0 if there is not.

10. Try:

```
house.show()
house.adjacency_matrix()
```

Make sure you understand the pattern of 0's and 1's.

**Getting your classwork recorded**

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If CoCalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255 - c32 worksheet attached" (so that it will be properly recorded).