Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 03
Review.

Concepts & Notation

- (Sec. 1.1) *field* F, *subfield*.
- (Sec. 1.2) *homogenous* system of linear equations, *linear combination* of equations, *equivalent* systems of linear equations.
- (Sec. 1.3) *matrix of coefficients* of a system of linear equations, *matrix over the field* F, *elementary row operations* on a matrix, *row-equivalent* matrices, *row-reduced* matrix, *identity matrix* I, *Kronecker delta* δ.
- (Sec. 1.4) *row-reduced echelon matrix*, *zero matrix* $)^{m,n}$.
- matrix multiplication.

Problems

We showed that the matrix

$$A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 4 & 0 & -1 \\ 2 & 6 & -1 & 5 \end{bmatrix}$$

is row-equivalent to the row-reduced matrix

$$\begin{bmatrix} 0 & 0 & 1 & -\frac{11}{3} \\ 1 & 0 & 0 & \frac{17}{3} \\ 0 & 1 & 0 & -\frac{5}{3} \end{bmatrix}$$

1. Why is every $m \times n$ matrix A row-equivalent to a row-reduced (echelon) matrix?

2. We saw that the 3×4 matrix A corresponded to a homogeneous system of equations with infinitely many solutions. Why will *every* 3×4 matrix A corresponded to a system of equations with *some* non-trivial solution?

3. Why will *every* $m \times n$ with $m < n$ matrix A correspond to a system of equations with *some* non-trivial solution?

4. Show that the 2×2 matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is row-equivalent to the identity matrix I. Find the solutions of the corresponding homogeneous system $AX = 0$.

5. Let A be an $n \times n$ matrix. Explain why the homogeneous system $AX = 0$ has non-trivial solutions if and only if A is *not* row-equivalent to the identity matrix I.

6. Let A and B be matrices. When is the product AB defined? If AB is defined, what are its entries?

7. Find

$$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 5 & -1 & 2 \\ 15 & 4 & 8 \end{bmatrix}.$$