Last name ______________________

First name ______________________

LARSON—MATH 310–CLASSROOM WORKSHEET 26
Gaussian Elimination!

Review

1. Given a generating set for a vector space $\mathcal{V}$, how can we find a basis for $\mathcal{V}$?
2. How can we find a basis for the column space of a matrix? What is the *column rank* of a matrix?
3. How can we find a basis for the row space of a matrix? What is the *row rank* of a matrix?
4. Can a set of vectors containing the 0-vector be linearly independent?
5. What is *echelon form*?

   Definition 7.1.1: An $m \times n$ matrix A is in *echelon form* if it satisfies the following condition: for any row, if that row's first nonzero entry is in position k then every previous row's first nonzero entry is in some position less than k.

$$\begin{bmatrix} 0 & 2 & 3 & 0 & 5 & 6 \\ 0 & 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 9 \end{bmatrix}$$

6. What things can we do to a system of equations that produces a new system with the same solutions?

Chapter 7 of Klein's *Coding the Matrix* text

1. What is a basis for the row space of a matrix in echelon form?

Gaussian elimination operations. Use these same three operations from solving systems of equations to produce a matrix *in echelon form* that represents a system of equations with the same solutions:

(a) Switch any two rows.

(b) Add a multiple of one row to another.

(c) Scale any row (by multiplying by a non-zero constant).

Gaussian elimination algorithm.

(a) Switch a not-yet-processed row with a left-most non-zero entry a to the top of the not-yet-processeded rows.

(b) Use this row, and *pivot* term a, to get 0's below a.

(c) Repeat on remaining unprocessed rows.

At the termination of this algorithm, the produced matrix is guaranteed to be (1) in echelon form, (2) with all 0's below the main diagonal, and (3) with all 0 rows at the bottom.

2. Use Gaussian elimination to produce an equivalent matrix in echelon form.

$$\begin{bmatrix} 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 6 & 7 \\ 0 & 0 & 0 & 9 & 8 \end{bmatrix}$$

3. Rewrite the following system as an augmented matrix and reduce solve using Gaussian elimination. Write the system of equations that the reduced matrix represents. Backsolve. Represent the solutions as a vector or vectors.

$$\begin{array}{rrcr} 2x & +3y & = & 13 \\ x & -y & = & -1 \end{array}$$

4. Let $F = \begin{bmatrix} 0 & 5 \\ -1 & 1 \end{bmatrix}$ Find F^{-1} if it exists.

5. How can we use Gaussian elimination to find the null space of a matrix?