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LARSON—MATH 610—CLASSROOM WORKSHEET 36 The Structure of Linear Operators.

Concepts & Notation

- (Sec. 5.3) permutation, $\det A$.
- (Sec. 6.2) characteristic value, characteristic vector, characteristic polynomial, diagonalizable linear operator.

Review

- 1. What is the *characteristic polynomial* of a matrix A? What does it tell us?
- 2. Does every matrix have characteristic values?
- 3. What is a diagonalizable linear operator T?

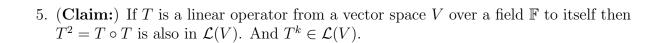
The Structure of a Linear Operator

- 4. (Claim:) Let T be a linear operator on a finite-dimensional space V. Let c_1, \ldots, c_k be the distinct characteristic values of T and let W_i be the null space of (T cI). The following are equivalent.
 - (a) T is diagonalizable,
 - (b) The characteristic polynomial for T is:

$$p = (x - c_i)^{d_i} \dots (x - c_k)^{d_k}$$

where dim $W_i = d_i$ (for i = 1, ..., k),

(c) $\dim W_1 + \ldots + \dim W_k = \dim V$.



6. (Claim:) If
$$T \in \mathcal{L}(V)$$
 then $cT \in \mathcal{L}(V)$ (for $c \in \mathbb{F}$).

7. (Claim:) If
$$T, T' \in \mathcal{L}(V)$$
 then $T + T' \in \mathcal{L}(V)$.

8. (Claim:) If
$$T \in \mathcal{L}(V)$$
 and $p \in \mathbb{F}[x]$ then $p(T) \in \mathcal{L}(V)$.

9. (Claim:) If
$$T \in \mathcal{L}(V)$$
, $T(\alpha) = c\alpha$ (for $c \in \mathbb{F}$, $\alpha \in V$), and $p \in \mathbb{F}[x]$ then $p(T)(\alpha) = p(c)\alpha$.

10. What does it mean for a polynomial to annihilate a linear operator T?

11. What is the $minimal\ polymonial$ of a linear operator T over a finite-dimensional vector space T?