Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 02
Review.

Concepts & Notation

- (Sec. 1.1) *field* F, *subfield*.
- (Sec. 1.2) *homogenous* system of linear equations, *linear combination* of equations, *equivalent* systems of linear equations.
- (Sec. 1.3) *matrix of coefficients* of a system of linear equations, *matrix over the field* F, *elementary row operations* on a matrix, *row-equivalent* matrices, *row-reduced* matrix, *identity matrix* I, *Kronecker delta* δ.

Problems

1. Give an example of a linear combination of the following system of equations:

$$\begin{array}{ccccccc} 2x_1 & -x_2 & +3x_3 & +2x_4 & = & 0 \\ x_1 & +4x_2 & & -x_4 & = & 0 \\ 2x_1 & +6x_2 & -x_3 & 5x_4 & = & 0 \end{array}$$

2. Explain why any solution of the original system must be a solution to your linear combination.

3. What are the three elementary row operations? Why will each of them produce a system of equations that is equivalent to the original system?

4. Write this (homogeneous) system (over $\mathbb{Q}$) in matrix form, use the 3 row operations to find an equivalent system of equations in *row-reduced* form. Describe the solutions.

$$\begin{array}{ccccccc} 2x_1 & -x_2 & +3x_3 & +2x_4 & = & 0 \\ x_1 & +4x_2 & & -x_4 & = & 0 \\ 2x_1 & +6x_2 & -x_3 & 5x_4 & = & 0 \end{array}$$

5. Write this (non-homogeneous) system in matrix form, use the 3 row operations to find an equivalent system of equations in *row-reduced* form, and find any condition on the y's that are required for any solution.

$$\begin{array}{ccccc} x_1 & -2x_2 & +x_3 & = & y_1 \\ 2x_1 & +x_2 & +x_3 & = & y_2 \\ & +5x_2 & -x_3 & = & y_3 \end{array}$$