Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 34
The Structure of Linear Operators.

Concepts & Notation

- (Sec. 5.3) *permutation*, det A.
- (Sec. 6.2) *characteristic value*, *characteristic vector*, *characteristic polynomial*

Review

1. Suppose $A \in \mathbb{F}^{n\times n}$. There is a $n \times 1$ matrix α (not all-zero), and $c \in \mathbb{F}$ with $A\alpha = c\alpha$ iff $\det(A - cI) = 0$.

The Structure of a Linear Operator

2. What is a *characteristic value* and a *characteristic vector* of a linear operator T from a vector space V over a field $\mathbb{F}$ to itself?

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x_1, x_2) = (x_1, 2x_2)$. Find the characteristic values and corresponding characteristic vectors of T.

4. What is the *characteristic space* associated with a characteristic value c of a linear operator T from a vector space V over a field $\mathbb{F}$ to itself?

5. (**Claim:**) If matrices $A, B \in \mathbb{F}^{n\times n}$ are similar then $\det A = \det B$.

6. (**Claim:**) If T is a linear operator from a finite-dimensional vector space V over a field $\mathbb{F}$ to itself, and $\mathcal{B}$ and $\mathcal{B}'$ are bases for V then $\det([T]_{\mathcal{B}}) = \det([T]_{\mathcal{B}'})$.

7. What is $\det T$?

8. (**Claim:**) If T is a linear operator from a vector space V over a field $\mathbb{F}$ to itself then $T^2 = T \circ T$ is also in $\mathcal{L}(V)$. And $T^k \in \mathcal{L}(V)$.

9. (**Claim:**) If $T \in \mathcal{L}(V)$ then $cT \in \mathcal{L}(V)$ (for $c \in \mathbb{F}$).

10. (**Claim:**) If $T, T' \in \mathcal{L}(V)$ then $T + T' \in \mathcal{L}(V)$.

11. (**Claim:**) If $T \in \mathcal{L}(V)$ and $p \in \mathbb{F}[x]$ then $p(T) \in \mathcal{L}(V)$.

12. (**Claim:**) If $T \in \mathcal{L}(V)$, $T(\alpha) = c\alpha$ (for $c \in \mathbb{F}$, $\alpha \in V$), and $p \in \mathbb{F}[x]$ then $p(T)(\alpha) = p(c)\alpha$.