Last name ______________________

First name ______________________

LARSON—MATH 310–CLASSROOM WORKSHEET 11
Linear Combinations, Span, Generating Set, Vector Space

Review: Chapter 3 of Klein's *Coding the Matrix* text

1. What is a *linear combination* of vectors $\hat{v_1}, \ldots, \hat{v_n}$?
2. What is the *span* of vectors $\hat{v_1}, \ldots, \hat{v_n}$?
3. Let $\mathcal{V}$ be a set of vectors. What is a *generating set* of vectors for $\mathcal{V}$?
4. What is $\mathbb{R}^n$?
5. What are the *standard* generators for $\mathbb{R}^n$?

1. Book problem:

> **Example 3.2.11:** I claim that $\{[3,0,0],[0,2,0],[0,0,1]\}$ is a generating set for $\mathbb{R}^3$. To prove that claim, I must show that the set of linear combinations of these three vectors is equal to $\mathbb{R}^3$. That means I must show two things:
>
> 1. Every linear combination is a vector in $\mathbb{R}^3$.
> 2. Every vector in $\mathbb{R}^3$ is a linear combination.
>
> The first statement is pretty obvious since $\mathbb{R}^3$ includes all 3-vectors over $\mathbb{R}$. To prove the second statement, let $[x,y,z]$ be any vector in $\mathbb{R}^3$. I must demonstrate that $[x,y,z]$ can be written as

2. Book problem:

> **Exercise 3.2.15:** For each of the subproblems, you are to investigate whether the given vectors span $\mathbb{R}^2$. If possible, write each of the standard generators for $\mathbb{R}^2$ as a linear combination of the given vectors. If doing this is impossible for one of the subproblems, you should first add one additional vector and then do it.
>
> 1. $[1,2],[3,4]$
> 2. $[1,1],[2,2],[3,3]$
> 3. $[1,1],[1,-1],[0,1]$

3. What are the 3 properties of a *vector space* $\mathcal{V}$?

4. Why is $\mathbb{R}^2$ a vector space?

5. For a field $\mathbb{F}$ and finite set D why is the collection of functions $\mathbb{F}^D$ a vector space?

6. If $\hat{v_1}, \hat{v_2}, \ldots, \hat{v_n}$ are vectors in a vector space $\mathcal{V}$, why is $\mathrm{Span}(\{\hat{v_1}, \hat{v_2}, \ldots, \hat{v_n}\})$ a vector space?

7. What is a *homogeneous linear system*?

8. Why are the solutions of a homogeneous linear system a vector space?

9. If $\mathcal{V}$ is a vector space, and $\mathcal{W} \subseteq \mathcal{V}$, when is $\mathcal{W}$ a *subspace* of $\mathcal{V}$?

10. What are examples of subspaces?