Last name ______________________

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LARSON—MATH 550—CLASSROOM WORKSHEET 26
Binomial Coefficients, the polynomial argument.

Concepts & Notation

- Sec. 5.1: Binomial coefficients, the Binomial Theorem, key formulas, the polynomial argument, Vandermonde's convolution.
- Sec. 5.2: the Sorting example.
- Sec. 5.3: Duplication formula, inversion, derangements.
- Sec. 5.4: Convolutions, generating functions.

Review

1. $\binom{n}{k}$ is the number of k-subsets of an n-set (for $n, k \in \mathbb{Z}^{\geq 0}$).
2. $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ $(0 \leq m \leq n,\ k, n \in \mathbb{Z})$.
3. We proved the *symmetry identity* $\binom{n}{k} = \binom{n}{n-k}$.
4. We proved the *addition formula*:
$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}.$$
5. The *binomial coefficients* $\binom{n}{k}$ $(n, k \in \mathbb{Z}^{\geq 0})$ can be generalized to $\binom{r}{k}$ $(r \in \mathbb{R}, k \in \mathbb{Z})$:
$$\binom{r}{k} = \frac{r^{\underline{k}}}{k!} \text{ (if } k \geq 0)$$
$$\binom{r}{k} = 0 \text{ (if } k < 0).$$
6. We proved the *absorbtion identity* $\binom{r}{k} = \frac{r}{k}\binom{r-1}{k-1}$ and the *absorbtion identity* variation $k\binom{r}{k} = r\binom{r-1}{k-1}$.
7. The (Newton's Generalized) *Binomial Theorem* says $(x+y)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^k y^{r-k}$. This agrees with our original formula when $r \in \mathbb{Z}$?

New

1. (**The Polynomial Argument**) Prove:

$$(r-k)\binom{r}{k} = r\binom{r-1}{k}.$$

2. (**Negating the Upper index)**. Prove:

$$\binom{r}{k} = (-1)^k\binom{k-r-1}{k}.$$

3. (**Vandermonde Convolution**) Prove ($n \in \mathbb{Z}$):

$$\sum_k \binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}.$$