

Last name \_\_\_\_\_

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**LARSON—MATH 310—CLASSROOM WORKSHEET 24**  
**Basis, Dimension, Column Space, Row Space**

**Review**

1. What is an equivalent definition for a *linearly independent* set of vectors?
2. (**Definition 5.6.1**) Let  $\mathcal{V}$  be a vector space. A *basis* for  $\mathcal{V}$  is a linearly independent set of generators for  $\mathcal{V}$ .
3. Argue: the standard generating set for  $\mathbb{R}^2$  is a basis for  $\mathbb{R}^2$ .
4. Argue: the standard generating set for  $\mathbb{R}^3$  is a basis for  $\mathbb{R}^3$ .

**Chapter 6 of Klein's *Coding the Matrix* text**

1. Show: the vectors  $[1, 1, 1]$ ,  $[1, 1, 0]$ ,  $[0, 1, 1]$  are a basis for  $\mathbb{R}^3$ .

(**Lemma 5.7.1, Unique-Representation Lemma**) Let  $\hat{a}_1, \dots, \hat{a}_n$  be a basis for a vector space  $\mathcal{V}$ . For any vector  $\hat{v} \in \mathcal{V}$ , there is exactly one representation of  $\hat{v}$  in terms of the basis vectors.

2. Find the *coordinates* for  $[3, 3, 1]$  with respect to the standard basis for  $\mathbb{R}^3$ .
3. Find the *coordinates* for  $[3, 3, 1]$  with respect to the basis  $[1, 1, 1]$ ,  $[1, 1, 0]$ ,  $[0, 1, 1]$  for  $\mathbb{R}^3$ .
4. Why is Lemma 5.7.1 true?

**(Theorem 6.1.2, Basis Theorem)** Let  $\mathcal{V}$  be a vector space. All bases for  $\mathcal{V}$  have the same size.

5. What is the *dimension* of a vector space?
6. What are examples?
7. Given a generating set for a vector space  $\mathcal{V}$ , how can we find a basis for  $\mathcal{V}$ ?
8. How can we find a basis for the column space of a matrix? What is the *column rank* of a matrix?

**Example 6.2.11:** Consider the matrix

$$M = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 2 & 0 & 7 \\ 0 & 0 & 3 & 9 \end{bmatrix}$$

9. How can we find a basis for the row space of a matrix? What is the *row rank* of a matrix?

**Fact:** For any matrix  $M$ , the column rank of  $M$  equals its row rank (the clearest explanation of why this is true will follow after we have Gaussian Elimination as a tool).