

Last name \_\_\_\_\_

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**LARSON—MATH 310—CLASSROOM WORKSHEET 22**  
**Linear Dependence, Linear Independence, Basis, Dimension**

**Review:** Chapter 4 of Klein's *Coding the Matrix* text

1. What is the *null space* of a matrix?

**Matrix Inverses**

1. What is the *inverse* of a matrix?
2. What are examples?
3. What is the importance of matrix inverses?

**Chapter 5 of Klein's *Coding the Matrix* text**

**(Question 5.2.5)** For a given vector space  $\mathcal{V}$ , what is the minimum number of vectors whose span equals  $\mathcal{V}$ ?

**(Definition 5.5.2)** Vectors  $\hat{v}_1, \dots, \hat{v}_n$  are linearly dependent if the zero vector can be written as a nontrivial linear combination of the vectors:

$$0 = \alpha_1 \hat{v}_1 + \dots + \alpha_n \hat{v}_n$$

4. **(Example 5.5.3)** The vectors  $[1, 0, 0]$ ,  $[0, 2, 0]$ , and  $[2, 4, 0]$  are linearly dependent,

5. (**Example 5.5.4**) The vectors  $[1, 0, 0]$ ,  $[0, 2, 0]$ , and  $[0, 0, 4]$  are linearly independent.

(**Definition 5.6.1**) Let  $\mathcal{V}$  be a vector space. A *basis* for  $\mathcal{V}$  is a linearly independent set of generators for  $\mathcal{V}$ .

6. What are examples?

(**Lemma 5.7.1, Unique-Representation Lemma**) Let  $\hat{a}_1, \dots, \hat{a}_n$  be a basis for a vector space  $\mathcal{V}$ . For any vector  $\hat{v} \in \mathcal{V}$ , there is exactly one representation of  $\hat{v}$  in terms of the basis vectors.

7. Why is Lemma 5.7.1 true?

(**Lemma 5.11.1, Exchange Lemma**) Suppose  $S$  is a set of vectors and  $A$  is a subset of  $S$ . Suppose  $\hat{z}$  is a vector in  $\text{Span } S$  and not in  $A$  such that  $A \cup \{\hat{z}\}$  is linearly independent. Then there is a vector  $\hat{w} \in S - A$  such that  $\text{Span } S = \text{Span } (\{z\} \cup S - \{\hat{w}\})$ .

8. Why is Lemma 5.11.1 true?

(**Theorem 6.1.2, Basis Theorem**) Let  $\mathcal{V}$  be a vector space. All bases for  $\mathcal{V}$  have the same size.

9. Why is Theorem 6.1.2 true?