LARSON—MATH 255–CLASSROOM WORKSHEET 24
Riemann Integrals.

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click "New", then "Worksheets", then call it **c24**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

Files

2. Now it is the case on any larger program that you will want to use functions you have previously defined. These are called *tools*. Instead of copying and pasting from your old code. You can save them as *files* and load them as needed.

   (a) Click "New". Type `heads_from_n_flips.sage` and then click "file". (You are making a **.sage** file *not* our usual Sage Worksheet file. These are regular text files that are loaded as Python files plus some *preprocessing*).
   (b) Define the function:

```
def heads_from_n_flips(n):
    heads=0
    for i in [1..n]:
        if random() < 0.5:
            heads=heads+1
    return heads
```

   (c) Click "Save" and then go back to your **c24** worksheet.
   (d) Type `load("heads_from_n_flips.sage")` and evaluate.
   (e) Now try `heads_from_n_flips(100)` a few times. You never need to write this function again. You have a tool!

3. Add a print statement to `heads_from_n_flips.sage` that indicates that the file has in fact been loaded. Test it.

Riemann Integration

Given a continuous function $f(x)$ on an interval $[a, b]$ we want to find the *area* between the curve, the x-axis and the lines $y = a$ and $y = b$. One way to do this is to use the Fundamental Theorem of Calculus and integrate. Unfortunately, it is difficult to find anti-derivatives for many (most) functions. So we need a different approach to get at least an approximate integral.

One way to do this is to slice up $[a,b]$ into n equal-sized intervals $[a_0,a_1]$, $[a_1,a_2]$,..., $[a_n,a_{n+1}]$ (where $a_1 = a$ and $a_{n+1} = b$), pick a point c_i from each interval $[a_i,a_{i+1}]$ and compute the area $f(c_i)\cdot\Delta$ of a rectangle, where Δ is the interval length $a_{i+1}-a_i$. There are different ways to pick the c_i's. You could pick the leftmost point of the interval, the midpoint, the rightmost point, or even a random point.

The *Riemann Integral* is defined to be the *limit* of these area approximations as n goes to infinity of this quantity.

Here is a function `leftpoint_riemann(f,a,b,n)` which computes the leftpoint Riemann sums for n equal intervals.

```
def leftpoint_riemann(f,a,b,n):
    area=0
    Delta=(b-a)/n
    for i in [0..(n-1)]:
        leftpoint=a+i*Delta
        area=area+f(leftpoint)*Delta
    return area*1.0
```

4. Find the integral of f(x)=x**2 on [0,3] (by hand).

5. Find the value of `leftpoint_riemann(f,a,b,n)` for f(x)=x**2 on [0,3] with $n = 2$, $n = 5$, $n = 10$ and $n = 100$. Here you are making the intervals smaller and smaller, giving a better and better approximation.

6. Given a continuous function f(x) on [a,b], define a function `rightpoint_riemann(f,a,b,n)` which computes the rightpoint Riemann sums for n equal intervals.

7. Find the values of `rightpoint_riemann(f,a,b,n)`for f(x)=x**2 on [0,3] with $n = 2$, $n = 5$, $n = 10$ and $n = 100$. Compare with your results for `leftpoint_riemann(f,a,b,n)`.

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255 - c24 worksheet attached" (so that it will be properly recorded).

(c) Remember to attach today's classroom worksheet!