Last name ______________________

First name ______________________

LARSON—MATH 601—CLASSROOM WORKSHEET 06
Review.

Concepts & Notation

- (Sec. 1.5) *column matrix* B_j, *elementary matrix.*
- (Sec. 1.6) *left inverse*, *right inverse*, *invertible matrix*, *inverse* A^{-1}.
- (Sec. 2.1) *vector*, *vector space.*

Elementary Matrices

An elementary matrix E is one where left multiplication by E has the same effect as some row operation.

1. Suppose a matrix A can be row-reduced to a matrix R (in row-reduced echelon form) by row operations $\epsilon_1, \ldots, \epsilon_k$, corresponding to elementary matrices $E_1, \ldots, E_k$. Write R in terms of these matrices.

2. We argued that elementary matrices E_i are invertible (and thus have inverses E_i^{-1}). Write A in terms of these matrices.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

3. Show
   (a) That the reduced matrix R is the (3×3) identity matrix.
   (b) That A is a product of elementary matrices.
   (c) That A is invertible. Find A^{-1}.

$$B = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

4. Show

   (a) That the reduced matrix R is *not* the (3×3) identity matrix.

   (b) That B is *not* a product of elementary matrices.

   (c) That B is *not* invertible.

5. A is a square $(n \times n)$ matrix. Argue that the following statement are equivalent:

   (a) That A is invertible.

   (b) That the reduced matrix R is the $(n \times n)$ identity matrix.

   (c) That A is a product of elementary matrices.

Vector Spaces

6. What is the prototypical example of a *vector space*?

7. What is the *formal* definition of a vector space?

8. What is a *vector*?

9. What are some examples of vector spaces?