**Last name** ______________________

**First name** ______________________

## LARSON—MATH 350—CLASSROOM WORKSHEET 28
## Geometry and Combinatorics: Euler's Polyhedron Formula!

### Review

Given a convex **polytope** in 3 dimensions, let $n$ be the number of its vertices, $e$ be the number of edges, and $f$ be the number of its faces.

- Draw a cube and calculate $n - e + f$.
- Draw a tetrahedron and calculate $n - e + f$.
- Draw an octahedron and calculate $n - e + f$.
- Can you make a conjecture?

### Planar Graphs

- What is a *graph*?
- What is a *planar graph*?
- What is a *connected* planar graph?
- Why can every convex polytope in 3-dimensional space ($\mathbb{R}^3$) be represented as a connected planar graph?

The *faces* of a polytope correspond to *regions* of a planar graph. We will prove our Euler's formula conjecture by proving the corresponding claim for connected planar graphs.

1. Argue that the conjecture is true for connected planar graphs with 2 faces/regions.

2. Argue that the conjecture is true for connected planar graphs with 3 edges.

3. Argue that the conjecture is true for all connected planar graphs with 3 vertices.

4. What inductive hypothesis should we make?

5. How will proof by induction work for graphs?

6. Prove our conjecture (for connected planar graphs).

**Fullerenes!**

A **fullerene** is a connected planar graph where all the regions/faces are pentagons or hexagons, and each vertex is incident to exactly three edges (the graph is *cubic* or *trivalent*).

7. Let $f_5$ be the number of pentagon and $f_6$ be the number of hexagons. What equation can we write?

8. Each vertex is incident to three edges and each edge is incident to two vertices. What equation can we write?

9. Write an equation relating the number of edges to the numbers of pentagons and hexagons.

10. Use these equations to solve for $f_5$.