Last name ______________________

First name ______________________

LARSON—MATH 310–CLASSROOM WORKSHEET 28
Eigenvalues

Review

1. How can we use Gaussian elimination to find the null space of a matrix?

Chapter 12 of Klein's *Coding the Matrix* text

If there is a non-zero vector $\vec{x}$ and scalar λ with $A\vec{x} = \lambda\vec{x}$ then λ is an *eigenvalue* of A and $\vec{x}$ is a corresponding *eigenvector*.

1. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. What does this matrix do?

2. Find the eigenvalues and corresponding eigenvectors.

3. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. A is *symmetric* (a theorem guarantees that it has real eigenvalues). Find the eigenvalues and corresponding eigenvectors.

4. The eigenvalues are different. A theorem guarantees that the corresponding eigenvectors are *orthogonal*. Check.

5. Check that these vectors are linearly independent. Why are orthogonal vectors guaranteed to be linearly independent?

6. Normalize these eigenvectors and make a matrix Q from them. Find Q^T. Then find QQ^T and Q^TQ.

7. A theorem guarantees that if D is the diagonal matrix with the eigenvalues on the diagonal then $A = Q^T DQ$. Check.

8. Not all matrices have real eigenvalues. Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. What does this matrix do?

9. Find the eigenvalues and corresponding eigenvectors.

10. *Why* doesn't this matrix have real eigenvectors?

11. The Fibonacci sequence $\{F_n\}$ is defined as $0, 1, 1, 2, 3, 5, \ldots$ where $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$. Check that this recursive definition gives the listed terms.

12. Let $F = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Find $F \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Let $F^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = F(F \begin{bmatrix} 1 \\ 0 \end{bmatrix})$. What do you see?

13. Let $F^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} = F^{n-1}(F \begin{bmatrix} 1 \\ 0 \end{bmatrix})$. How can we express what we're seeing in term of the Fibonacci recursion formula?

14. Write F as $Q^T DQ$ (for some diagonal matrix D and orthogonal matrix Q) to simplify the calculation of $F^{100} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$?