Last name _____

First name _____

LARSON—MATH 550—CLASSROOM WORKSHEET 03
Mathematical Induction. Lines in the Plane.

Concepts & Notation

- (Chapter 1) T_n, recurrence (recurrence relation), mathematical induction, basis, solving recurrences

Induction

Let $P(n)$ be the (open) statement: "$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$".

1. Check that $P(1)$ and $P(2)$ are true (base case).

2. Show: that $P(n)$ implies $P(n+1)$.
   That is, assume:

$$1 + 2 + 3 + \ldots + (n-1) = \frac{(n-1)((n-1)+1)}{2},$$

   and show:

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

3. What can you conclude?

Lines in the Plane

4. What is the maximum number of regions defined by n lines in the plane? Try the methodology developed in the Towers of Hanoi problem

   (a) *Name* the quantity you want to count/investigate.

   (b) Find some values of that quantity.

   (c) Find a recurrence relation for that quantity.

   (d) Use the recurrence to find more values of that quantity.

   (e) Use these values to *guess* a (non-recurrence closed-form) formula for that quantity.

   (f) *Prove* your formula.