Last name _____

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LARSON—MATH 610—CLASSROOM WORKSHEET 31
Determinants.

Concepts & Notation

- (Sec. 5.1) *n-linear* function, *alternating function.*

- (Sec. 5.2) *determinant function.*

- (Sec. 5.3) *permutation*, det A.

Let A be an $n \times n$ matrix over a *commutative ring.* Let:

$$\det A = \sum_{\sigma \in S_n} \left[(sgn\ \sigma) \prod_{i=1}^{n} A_{i,\sigma(i)} \right]$$

$$= \sum_{\sigma \in S_n} (sgn\ \sigma) A_{1,\sigma(1)} A_{2,\sigma(2)} \dots A_{n,\sigma(n)}.$$

Review

1. What is a *determinant* function?

2. What is an example of a determinant function?

   Properties of $\det : K^{n \times n} \to K$

3. (**Claim:**) $\det I = 1$.

4. (**Claim:**) If A is a diagonal matrix then $\det A = A_{11} A_{22} \dots A_{nn}$ (the product of the diagonal entries).

5. (**Claim:**) If A has a zero row then $\det A = 0$.

6. (**Claim:**) If A' is A with two rows switched then $\det A' = -\det A$.

7. (**Claim:**) If A has two rows which are the same then $\det A = 0$.

8. (**Claim:**) $\det(\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \ldots, \alpha_{\sigma(n)}) = (sgn\ \sigma) \det(\alpha_1, \alpha_2, \ldots, \alpha_n)$.

9. (**Claim:**) $\det$ is n-linear.

10. (**Claim:**) If A is row-reduced to (a diagonal matrix) R (using only the operation of adding a multiple of one row to another, and using k switches of pairs of rows) then $\det A = (-1)^k \det R$ (and $\det A$ is $\pm$ the product of the diagonal entries of R).

11. (**Claim:**) If $A \in \mathbb{F}^{n \times n}$, then A is invertible iff $\det A \neq 0$.