

5. Let  $\mathbf{R}$  be the finite region bounded by  $y = 2 \ln x$ ,  $y = 6$  and  $x = 1$ . In the following manner, set up but do not evaluate definite integrals which represent the area of the region  $\mathbf{R}$ .

(a) Integrate with respect to  $x$ .

$$\int_1^{e^3} 6 - 2 \ln x \, dx$$

(b) Integrate with respect to  $y$ . (The integrands in parts (a) and (b) should be different.)

$$\int_0^6 e^{\frac{y}{2}} - 1 \, dy$$

6. Let  $\mathbf{R}$  be the finite region bounded by  $f(x) = x^2$  and  $g(x) = 3x$ . In the following manner, set up and evaluate definite integrals which represent the area of the region  $\mathbf{R}$ .

(a) Integrate with respect to  $x$ .

$$\int_0^3 3x - x^2 \, dx = \left. \frac{3x^2}{2} - \frac{x^3}{3} \right|_0^3 = \frac{9}{2}$$

(b) Integrate with respect to  $y$ . (The integrands in parts (a) and (b) should be different.)

$$\int_0^9 \sqrt{y} - \frac{y}{3} \, dy = \left. \frac{2}{3} y^{\frac{3}{2}} - \frac{y^2}{6} \right|_0^9 = \frac{9}{2}$$

7. Evaluate the following indefinite integrals.

(a)  $\int \cot x \, dx$

$$= \int \frac{\cos x}{\sin x} \, dx, \quad \text{let } u = \sin x.$$

(b)  $\int x\sqrt{2x+1}dx$

Let  $u = \sqrt{2x+1}$ .

(c)  $\int \cos^3(x)dx$

$= \int \cos x (\cos^2 x) dx = \int \cos x (1 - \sin^2 x) dx.$

Let  $u = \sin x$ .

(d)  $\int \frac{50x^9 \cos(x^{10})}{\sin(x^{10})} dx$

Let  $u = x^{10}$ , then use part a).

(e)  $\int \frac{6x^5}{x^3+1} dx$

$= \int \frac{6x^5 + 6x^2 - 6x^2}{x^3+1} dx = \int x^2 dx - \int \frac{6x^2}{x^3+1} dx.$

For second term, let  $u = x^3+1$ .