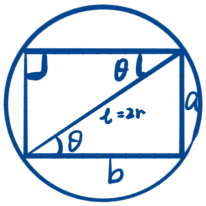
2. Find the dimensions of the rectangle of largest area that can be inscribe in a circle of radius r.



(Remark: If you have an inscribed right triangle of a circle, then the hypotenuse is the diameter of the circle.)

Sol1: $S = ab = \ell\cdot\sin\theta\cdot\ell\cdot\cos\theta = 4r^2\sin\theta\cos\theta = 2r^2\cos 2\theta.$

Solving for maximum on $\theta\in(0,\frac{\pi}{2}) \Rightarrow \theta=\frac{\pi}{4} \Rightarrow a=b=\sqrt{2}r$

Sol2: $S = ab = a\cdot\sqrt{\ell^2-a^2} = a\sqrt{4r^2-a^2}$

Solving for maximum on $a\in(0,2r) \Rightarrow a=\sqrt{2}r,\ b=\sqrt{2}r$

5. For what values of a and b is the following equation true?

$$\lim_{x\to0}\left(\frac{\sin 2x}{x^3}+\frac{b}{x^2}+a\right)=0$$

$$\lim_{x\to0}\frac{\sin2x}{x^3}+\frac{b}{x^2}+a = \lim_{x\to0}\frac{\overset{\to 0}{\sin2x+bx+ax^3}}{\underset{\to 0}{x^3}}$$

(By L'Hospital) $$= \lim_{x\to0}\frac{\overset{\to 2+b}{2\cos2x+b+3ax^2}}{\underset{\to 0}{3x^2}}$$

If $2+b\neq0$, then $\lim_{x\to0}\frac{\sin2x}{x^3}+\frac{b}{x^2}+a=\infty$ or $-\infty$. Thus $2+b=0$, i.e. $b=-2$.

So when $b=-2$, $$= \lim_{x\to0}\frac{\overset{\to 0}{-4\sin2x+6ax}}{\underset{\to 0}{6x}} = \lim_{x\to0}\frac{-8\cos2x+6a}{6}$$

Then $\lim_{x\to0}\frac{\sin2x}{x^3}+\frac{b}{x^2}+a=0 \Rightarrow \frac{-8\cos2x+6a}{6}\Big|_{x=0}=0 \Rightarrow -8\cos(0)+6a=0 \Rightarrow a=\frac{4}{3}$