3. Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.

$f'(x) = x^3 \Rightarrow f(x) = \frac{x^4}{4} + C$.

$f(x_0)$ tangent to $x+y=0 \Rightarrow f'(x_0) =$ slope of "$(x+y)=0$" $= -1$

So $x_0^3 = -1 \Rightarrow x_0 = -1$.

Thus $(-1, f(-1))$ is on the line $x+y=0$

$\Downarrow$

$-1 + \frac{(-1)^4}{4} + C = 0 \Rightarrow C = \frac{3}{4}$.

Thus $f(x) = \frac{x^4}{4} + \frac{3}{4}$.

4. A stone was dropped off a cliff and hit the ground with a speed of 98 m/s. What is the height of the cliff? [Use the fact that acceleration due to gravity is -9.8 m/s^2.]

$S''(t) = -9.8\ m/s^2$

$S'(t) = (-9.8t + C)\ m/s$, since the initial velocity is 0, $C = 0$.

$= -9.8t\ m/s$. (The stone was "dropped")

$S(t) = (-4.9t^2 + D)\ m$.

Since when $S = 0$ m, $S'(t) = -98$ m/s, solving $-9.8t = -98$, we get $t = 10$.

Plug in $S(10) = 0$, we get $0 = -49 \times 10^2 + D$, thus $D = 490$

So $S(t) = (-4.9t^2 + 490)\ m$.

Thus the height of the cliff is $S(0) = 490$ m.