

Unit 5: Inference for categorical data

2. Comparing two proportions

Sta 104 - Summer 2015

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Slides posted at <http://bit.ly/sta104su15>

- ▶ Peer eval 2 opens today and closes Wednesday at midnight (note: different platform)

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CLT also describes the distribution of $\hat{p}_1 - \hat{p}_2$

$$(\hat{p}_1 - \hat{p}_2) \sim N \left(\text{mean} = (p_1 - p_2), SE = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} \right)$$

Conditions:

- ▶ Independence: Random sample/assignment + 10% rule
- ▶ Sample size / skew: At least 10 successes and failures

For HT where $H_0 : p_1 = p_2$, pool!

As with working with a single proportion,

- ▶ When doing a HT where $H_0 : p_1 = p_2$ (almost always for HT), use expected counts / proportions for S-F condition and calculation of the standard error.
- ▶ Otherwise use observed counts / proportions for S-F condition and calculation of the standard error.

Expected proportion of success for both groups when $H_0 : p_1 = p_2$ is defined as the *pooled proportion*:

$$\hat{p}_{\text{pool}} = \frac{\text{total successes}}{\text{total sample size}} = \frac{\text{suc}_1 + \text{suc}_2}{n_1 + n_2}$$

Clicker question

Suppose in group 1 30 out of 50 observations are successes, and in group 2 20 out of 60 observations are successes. What is the pooled proportion?

- (a) $\frac{30}{50}$
- (b) $\frac{20}{60}$
- (c) $\frac{30}{50} + \frac{20}{60}$
- (d) $\frac{30+20}{50+60}$
- (e) $\frac{\frac{30}{50} + \frac{20}{60}}{2}$

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“Healthy adults immunized with an experimental malaria vaccine, called PfSPZ may be completely protected from infection, according to government researchers.” reported Time magazine in Aug 2013. The vaccine contains weakened forms of the live parasite – *Plasmodium falciparum* – responsible for causing malaria. In a randomized trial, none of the six patients who received the vaccine developed malaria, while five of the six who were not vaccinated became infected. Do these data provide convincing evidence of a difference in rate of malaria infection?

Group	Outcome		
	Malaria	No malaria	
Vaccine	0	6	6
No vaccine	5	1	6
Total	5	7	12

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- If the S-F condition is met, can do theoretical inference: Z test, Z interval
- If the S-F condition is not met, must use simulation based methods: randomization test, bootstrap interval

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Group	Outcome		
	Malaria	No malaria	
Vaccine	0	6	6
No vaccine	5	1	6
Total	5	7	12

$$H_0 : p_T = p_C \quad H_A : p_T \neq p_C$$

Conditions:

1. Independence: Patients are randomly assigned to treatment groups
2. Success-failure: ?

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Clicker question

Assuming that the null hypothesis ($H_0 : p_T = p_C$) is true, which of the following is the pooled proportion of patients with malaria in the two groups?

- (a) $\frac{6}{12} = 0.5$
- (b) $\frac{5}{12} = 0.417$
- (c) $\frac{0}{5} = 0$
- (d) $\frac{6}{7} = 0.857$
- (e) $\frac{7}{12} = 0.583$

Group	Outcome		
	Malaria	No malaria	
Vaccine	0	6	6
No vaccine	5	1	6
Total	5	7	12

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Clicker question

Assuming that the null hypothesis ($H_0 : p_T = p_C$) is true, how many patients would we expect to get infected with malaria in the vaccine group?

- (a) $0.417 \times 12 = 5$
- (b) $0.417 \times 6 = 2.5$
- (c) $0.417 \times 5 = 2.085$
- (d) $0.5 \times 6 = 3$
- (e) $0.583 \times 12 = 7$

Group	Outcome		
	Malaria	No malaria	
Vaccine	0	6	6
No vaccine	5	1	6
Total	5	7	12

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Simulation scheme

- Use 12 index cards, where each card represents an experimental unit.
- Mark 5 of the cards as “malaria” and the remaining 7 as “no malaria”.
- Shuffle the cards and split into two groups of size 6, for vaccine and no vaccine.
- Calculate the difference between the proportions of “malaria” in the vaccine and no vaccine decks, and record this number.
- Repeat steps (3) and (4) many times to build a randomization distribution of differences in simulated proportions.

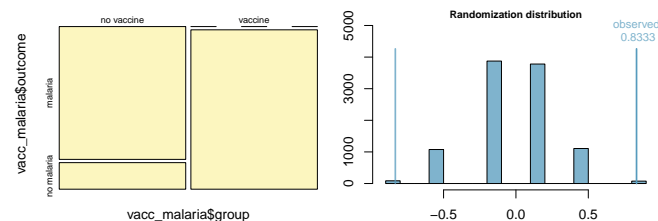
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Simulate in R

```
download("https://stat.duke.edu/~mc301/data/vacc_malaria.csv", destfile = "vacc_malaria.csv")
vacc_malaria = read.csv("vacc_malaria.csv")

inference(vacc_malaria$outcome, vacc_malaria$group, success = "malaria", est = "proportion",
  type = "ht", null = 0, alternative = "twosided", method = "simulation", seed = 1028)
```

```
Response variable: categorical, Explanatory variable: categorical
Difference between two proportions -- success: malaria
Summary statistics:
      x
y      no vaccine vaccine Sum
malaria      5      0      5
no malaria      1      6      7
Sum            6      6     12
Observed difference between proportions (no vaccine-vaccine) = 0.8333
H0: p_no vaccine - p_vaccine = 0
HA: p_no vaccine - p_vaccine != 0
p-value = 0.0152
```



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Application exercise: App Ex 5.2

See course website for details.

1. CLT also describes the distribution of $\hat{p}_1 - \hat{p}_2$
2. For HT where $H_0 : p_1 = p_2$, pool!
3. When S-F fails, simulate!