Project 1

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1 Exercise 1

Exercise 1.1 Set up a system of linear equations based on this problem from the *Nine Chapters on the Mathematical Art*.

The text then shows three columns set up on a counting board (a tool for mathematical calculation) in the following manner:

The given augmented matrix is:

$$\begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 1 & 1 \\ 26 & 34 & 39 \end{matrix}$$

Solution:
Setting the system of linear equations:

$$\begin{aligned} 3x_1 + 2x_2 + x_3 = 39 \\ 2x_1 + 3x_2 + x_3 = 34 \\ x_1 + 2x_2 + 3x_3 = 26 \end{aligned} \tag{1}$$

Exercise 1.2 Write the equations you found in Exercise 1.1 as an augmented matrix. How does your matrix compare with the numbers on the counting board?

Solution:
Setting the augmented matrix found in the **Exercise 1.1**:

$$\begin{vmatrix} 3 & 2 & 1 & 39 \\ 2 & 3 & 1 & 34 \\ 1 & 2 & 3 & 26 \end{vmatrix}$$

The major difference between the matrices is the order how the numbers are listed. According to the book *Nine Chapters on the Mathematical Art*, they vertically listed up the variables and created a matrix in this same fashion. The way how seprating the coefficient variables is also following their writing order; from right to left and top to bottom. As this way of writing order used during Han Dynasty, creating matrix also followed the same method that they used for writing chiense letters.

Exercise 1.3 Use row operations to get your augmented matrix from Exercise 1.2 into row-echelon form and solve the system of equations.

Work to reduced row-echelon form,
first with j = 1 ,

$$\begin{bmatrix} 3 & 2 & 1 & 39 \\ 2 & 3 & 1 & 34 \\ 1 & 2 & 3 & 26 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & 26 \\ 2 & 3 & 1 & 34 \\ 3 & 2 & 1 & 39 \end{bmatrix}$$

$$\xrightarrow{-2R_1 + R_2} \begin{bmatrix} 1 & 2 & 3 & 26 \\ 0 & -1 & -5 & -18 \\ 3 & 2 & 1 & 39 \end{bmatrix} \xrightarrow{-3R_1 + R_3} \begin{bmatrix} 1 & 2 & 3 & 26 \\ 0 & -1 & -5 & -18 \\ 0 & -4 & -8 & -39 \end{bmatrix}$$

Now, with j = 2,

$$\xrightarrow{-4R_2 + R_3} \begin{bmatrix} 1 & 2 & 3 & 26 \\ 0 & -1 & -5 & -18 \\ 0 & 0 & 12 & 33 \end{bmatrix} \xrightarrow{2R_2 + R_1} \begin{bmatrix} 1 & 0 & -7 & -10 \\ 0 & -1 & -5 & -18 \\ 0 & 0 & 12 & 33 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 0 & -7 & -10 \\ 0 & 1 & 5 & 18 \\ 0 & 0 & 12 & 33 \end{bmatrix}$$

And finally, with j = 3,

$$\xrightarrow{\frac{1}{12}R_3} \begin{bmatrix} 1 & 0 & -7 & -10 \\ 0 & 1 & 5 & 18 \\ 0 & 0 & 1 & \frac{11}{4} \end{bmatrix} \xrightarrow{-5R_3 + R_2} \begin{bmatrix} 1 & 0 & -7 & -10 \\ 0 & 1 & 0 & \frac{17}{4} \\ 0 & 0 & 1 & \frac{11}{4} \end{bmatrix} \xrightarrow{7R_3 + R_1} \begin{bmatrix} \boxed{1} & 0 & 0 & \frac{37}{4} \\ 0 & \boxed{1} & 0 & \frac{17}{4} \\ 0 & 0 & \boxed{1} & \frac{11}{4} \end{bmatrix}$$

Thus,

$$\begin{bmatrix} 1 & 0 & 0 & \frac{37}{4} \\ 0 & 1 & 0 & \frac{17}{4} \\ 0 & 0 & 1 & \frac{11}{4} \end{bmatrix}$$

Therefore, this system of equation is:

$$x_1 = \frac{37}{4} = 9.25$$

$$x_2 = \frac{17}{4} = 4.25$$

$$x_3 = \frac{11}{4} = 2.75$$

Exercise 1.4 Notice that the methods used in the *Nine Chapters* are very similar to our modern approach to solving a system of equations. Verify that the solution that you obtained in Exercise 1.3 is the same as the solution obtained when solving the equations given by the columns on the counting board shown below Exercise 1.3.

Solution:
The given first counting board found in the Exercise 1.1 is as follows,

$$\begin{array}{ccc} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 1 & 1 \\ 26 & 34 & 39 \end{array}$$

and we will call it as **A** which is written in an aumented matrix

$$\mathbf{A} = \left| \begin{array}{cccc} 3 & 2 & 1 & 39 \\ 2 & 3 & 1 & 34 \\ 1 & 2 & 3 & 26 \end{array} \right|$$

The given second counting board found in the Exercise 1.3 is as follows,

$$\begin{array}{ccc} 0 & 0 & 3 \\ 0 & 5 & 2 \\ 36 & 1 & 1 \\ 99 & 24 & 39 \end{array}$$

and we will call it as **B** which is written in an aumented matrix

$$\mathbf{B} = \left| \begin{array}{cccc} 3 & 2 & 1 & 39 \\ 0 & 5 & 1 & 24 \\ 0 & 0 & 36 & 99 \end{array} \right|$$

According to the definition of Row-Equivalent Matrices, we can transform the augmented matrices **A** and **B** into Reduced Row-Echelon Form

$$
\mathbf{A} = \left|\begin{array}{cccc} 3 & 2 & 1 & 39 \\ 2 & 3 & 1 & 34 \\ 1 & 2 & 3 & 26 \end{array}\right| \xrightarrow{\text{RREF}} \left|\begin{array}{cccc} \boxed{1} & 0 & 0 & \frac{37}{4} \\ 0 & \boxed{1} & 0 & \frac{17}{4} \\ 0 & 0 & \boxed{1} & \frac{11}{4} \end{array}\right|
$$

$$
\mathbf{B} = \left|\begin{array}{cccc} 3 & 2 & 1 & 39 \\ 0 & 5 & 1 & 24 \\ 0 & 0 & 36 & 99 \end{array}\right| \xrightarrow{\text{RREF}} \left|\begin{array}{cccc} \boxed{1} & 0 & 0 & \frac{37}{4} \\ 0 & \boxed{1} & 0 & \frac{17}{4} \\ 0 & 0 & \boxed{1} & \frac{11}{4} \end{array}\right|
$$

Thus, two matrices **A** and **B** are row-equivalent, and these two augmented matrices which derived from the counting board are having the same solutions.

Exercise 1.5 Translate Borrel's language into a system of simultaneous linear equations using the variables A, B and C.

Solution:

3A	1B	1C	[42	1ST
1A	4B	1C	[32	2ND
1A	1B	5C	[40	3RD

Since the