HOMEWORK ASSIGNMENT 1

1. Use the euclidean algorithm to compute $\gcd(252, 180)$ and to find integers u and v such that $252u + 180v = \gcd(252, 180)$.

2. Show that $\gcd(n! + 1, (n + 1)! + 1) = 1$.

3. Consider the following alternative characterization of the gcd :

"The gcd of a and b is the only positive integer d satisfying that:

(i) $d \mid a$ and $d \mid b$

(ii) If $d' \mid a$ and $d' \mid b$, then $d' \mid d$.

Using this, show that

(a) For all nonnegative integers a, b and c, we have

$$\gcd(ac, bc) = |c|.\gcd(a, b)$$

.

(b) For all a, b and c integers such that $a \mid bc$ and $\gcd(a, b) = 1$, we have that $a \mid c$.

4. (a) Given a and b relatively prime integers, show that $\gcd(a + b, a^2 - ab + b^2)$ is 1 or 3.

(b) If a, b, u, v are given integers such that $au + bv = 1$, show that a and b are relatively primes. Secondly, find a multiple of 19 and a multiple of 17 such that their difference is 5.

5. Solve the decryption code "A Piratical Treasure" from exercise 1.4 in Chapter 1 of the course textbook.