Homework 4

Math 445, Spring 2017

Due Wednesday, May 3 by 4:30 p.m.

Instructions

- Please complete problems that require the use of R in R markdown. Theoretical problems may be handwritten. **Please knit your .Rmd file to either Word or PDF and print it for submission** with the remainder of the problems.
- All assignments should be stapled.
- Remember that the textbook data sets are contained in the `resampledata` R package.
- When completing a hypothesis test, be sure to outline all of the steps in your solution.

Assignment

Complete the following exercises from **Chapter 6** of the textbook.

Exercise 6

Exercise 11

Exercise 16

Notes on part b: If you use R to run the goodness-of-fit test, include the results (the value of the test statistic and associated p-value) and the R code used in your solution.

Notes on part c: To superimpose the Gamma(r, λ) PDF on a histogram (remember to change the scale of the y-axis to the density scale!), use the below layer:

```
stat_function(fun = dgamma, geom = "line", args = list(shape = r, rate = lambda))
```

where `r` and `lambda` are the estimates of r and λ, respectively.

To superimpose the Gamma(r, λ) CDF on a plot of the ECDF, use the below layer:

```
stat_function(fun = pgamma, geom = "line", args = list(shape = r, rate = lambda))
```

You can also experiment with the `color` and `linetype` arguments to make your plots easier to read.

Exercise 19

Exercise 23

In addition to the textbook problems, complete the following problem:

Problem: Text messages

The number of text messages sent per day by students at Lawrence is thought to follow a log-normal distribution. Let $X_1, X_2, \ldots, X_n$ be a random sample from LogNorm(μ, σ^2). The PDF of X_i is given by

$$f(x_i|\mu,\sigma^2) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left(\frac{-\left[\log(x)-\mu\right]^2}{2\sigma^2}\right), \quad x>0, \quad \mu\in\mathbb{R}, \quad \sigma>0$$

Note: $\exp x = e^x$.

(a) Let $Y \sim \mathcal{N}(\mu,\ \sigma^2)$. Define $X = e^Y$. Show that $X \sim \text{LogNorm}(\mu,\ \sigma^2)$.
(b) Calculate the mean of X. (*Hint:* this can be found directly using the PDF, or by exploiting it's relationship with the normal distribution via the MGF.)
(c) Calculate the variance of X. (*Hint:* this can be found directly using the PDF, or by exploiting it's relationship with the normal distribution via the MGF.)
(d) Find the method of moments estimators for μ and σ^2 by hand.
(e) What is the likelihood function for μ and σ^2?
(f) What is the log-likelihood function for μ and σ^2?
(g) Find the maximum likelihood estimators of μ and σ^2 by hand.
(h) Write a function in R called `lognorm_loglik` that computes the log-likelihood for a lognormal distribution. Include the code in your solution.
(i) The data set `stat111texts.csv` contains a variable, `texts`, which measures the typical number of text messages sent per day for a college student a certain prestigious university with a statistics department. In R, read in the data set, and calculate the mean, variance, and number of observations for the variable texts. Include both the code and output in your solution.
(j) Define a new variable called logtexts = log texts. Calculate the mean and variance of logtexts. Include both the code and output in your solution.
(k) Use your results from (i) and (j) to calculate the ML estimates for μ and σ^2.
(l) Use the `optim` function in R to calculate the maximum likelihood estimates of μ and σ^2. Do they agree with your previous calculations? If not, consider different starting values.