Math 445 Midterm – Part I

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1. The following need no justification. Simply state whether each statement is true or false.
   (a) If $\widehat{\theta}_1$ is an unbiased estimator for θ and $\widehat{\theta}_2$ is biased for θ, then $\mathrm{MSE}(\widehat{\theta}_1) < \mathrm{MSE}(\widehat{\theta}_2)$.
   (b) Any MVUE will be unbiased for θ.
   (c) Any unbiased estimator will be consistent for θ.
   (d) It is possible for the maximum likelihood estimator for θ to be a biased estimator of θ.
   (e) The p-value is the probability of exceeding the observed test statistic, given that the null hypothesis is true.
2. Let $X_1, X_2, \ldots, X_n$ be iid random variables with common PDF $f(x|\theta)$, $\theta > 0$, where

$$f(x|\theta) = \begin{cases} \theta 2^{-\theta} x^{\theta-1} & \text{if } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

   In answering the following, state any standard results that you are using.
   (a) Find $\widehat{\theta}$, the maximum likelihood estimator of θ based on $X_1, X_2, \ldots, X_n$, and carefully argue that it does indeed maximize the likelihood.
   (b) Find $\widetilde{\theta}$, the method of moments estimator of θ based on $X_1, X_2, \ldots, X_n$.
   (c) Show that $\widetilde{\theta}$ is a consistent estimator of θ.
3. Suppose that $X_1, X_2, \ldots, X_n$ form a random sample from a Poisson distribution with unknown mean λ ($\lambda > 0$). Show that $\overline{X}_n$ is a MVUE.

Note: There was a fourth problem on this exam, but we are covering material in a slightly difference order.

Table of Distributions

Name	Param.	PMF or PDF	Mean	Variance	MGF
Bernoulli	p	$P(X=1)=p$ $P(X=0)=q$	p	pq	pe^t+q
Binomial	n,p	$\binom{n}{k}p^k q^{n-k}$ $k\in\{0,1,\dots,n\}$	np	npq	$(pe^t+1-p)^n$
Geometric	p	pq^k $k\in\{0,1,2,\dots\}$	$\frac{q}{p}$	$\frac{q}{p^2}$	$\frac{p}{1-qe^t}$
Negative Binomial	r,p	$\binom{r+n-1}{r-1}p^r q^n$ $k\in\{0,1,2,\dots\}$	$\frac{rq}{p}$	$\frac{rq}{p^2}$	$\left(\frac{p}{1-qe^t}\right)^r$
Hypergeometric	w,b,n	$\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$ $k\in\{0,1,\dots,n\}$	$\mu=\frac{nw}{w+b}$	$\left(\frac{w+b-n}{w+b-1}\right)n\frac{\mu}{n}\left(1-\frac{\mu}{n}\right)$	
Poisson	λ	$\frac{e^{-\lambda}\lambda^k}{k!}$ $k\in\{0,1,2,\dots\}$	λ	λ	$e^{\lambda(e^t-1)}$
Uniform	$a<b$	$\frac{1}{b-a},\ x\in(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Normal	μ,σ^2	$\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/\left(2\sigma^2\right)}$ $x\in\mathbb{R}$	μ	σ^2	$e^{\mu t+(\sigma^2t^2)/2}$
Exponential	λ	$\lambda e^{-\lambda x},\ x>0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}$
Gamma	a,λ	$\frac{\lambda^a}{\Gamma(a)}x^{a-1}e^{-\lambda x}$ $x>0$	$\frac{a}{\lambda}$	$\frac{a}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-t}\right)^a$
Beta	a,b	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$ $0<x<1$	$\mu=\frac{a}{a+b}$	$\frac{\mu(1-\mu)}{a+b+1}$	$1+\sum_{k=1}^{\infty}\left(\prod_{r=0}^{k-1}\frac{a+r}{a+b+r}\right)\frac{t^k}{k!}$
Chi-Square	n	$\frac{1}{2^{n/2}\Gamma(n/2)}x^{n/2-1}e^{-x/2}$ $x>0$	n	$2n$	$\left(\frac{1}{1-2t}\right)^{n/2}$