Math 445 Name: _____
Midterm Exam – Part I
May 4, 2016

This exam is closed book and closed notes, except for one standard size sheets of paper (8.5" by 11") which can have notes on both sides. These sheets must be submitted with the exam. No copying, cheating, collaborations, computers, or cell phones are allowed. Show your work and write complete and coherent answers. Be sure to clearly label and justify your solutions to these problems. Illegible or unjustified solutions will not receive full credit. After you have completed the exam, please reaffirm the Lawrence University Honor Code.

1. Where do we expect a bootstrap distribution to be centered?

2. Where will a permutation distribution be centered?

3. Let $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_m$ be random samples drawn from two independent populations and assume that we are interested in the difference between the two population means.

   (a) Describe the algorithm for a two-sample bootstrap used to compare two sample means.

   (b) Once you have the bootstrap distribution, how would you create a 90% confidence interval for the difference in population means?

4. Let $X_1, \ldots, X_n$ be a random sample from a Gamma(α, θ), where $\theta > 0$ and α is a known constant. To answer this problem, use the following parameterization of the gamma distribution:

$$\frac{1}{\Gamma(\alpha)\theta^{\alpha}} x^{\alpha-1} e^{-x/\theta}, \ \mathrm{E}X = \alpha\theta, \ \mathrm{Var}X = \alpha\theta^2$$

(a) Find the method of moments estimator of θ.

(b) Is the method of moments estimator a MVUE?

(c) Is the method of moments estimator a consistent estimator of θ?

5. Let $Y_1, \ldots, Y_n$ be a random sample of discrete random variables with the common PDF

$$f(y|\theta) = \theta y^{\theta-1},\ 0 < x < 1$$

where $\theta > 0$.

(a) Find the maximum likelihood estimator of θ, $\widehat{\theta}_{ML}$.

The expected value and variance of the maximum likelihood estimator are given below:

$$E\left(\widehat{\theta}_{ML}\right) = \left(\frac{n}{n-1}\right)\theta \qquad Var\left(\widehat{\theta}_{ML}\right) = \left(\frac{n^2}{(n-1)^2(n-2)}\right)\theta^2$$

(b) Calculate the bias of $\widehat{\theta}_{ML}$. Is the estimator asymptotically unbiased?

(c) Calculate the mean square error of $\widehat{\theta}_{ML}$.

(d) Define a new estimator $\widehat{\theta}_U$ of θ to be:

$$\widehat{\theta}_U = \left(\frac{n-1}{n}\right)\widehat{\theta}_{ML}$$

Do you prefer this estimator to $\widehat{\theta}_{ML}$? Justify your answer.