

This exam is closed book and closed notes. No copying, cheating, collaborations, computers, or cell phones are allowed. Show your work and write complete and coherent answers. Be sure to clearly label and justify your solutions to these problems. Illegible or unjustified solutions will not receive full credit. After you have completed the exam, please reaffirm the Lawrence University Honor Code.

Formulas:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$C = \sum_{\text{all cells}} \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

$$c = \sum_{i=1}^I \sum_{j=1}^J \frac{(n_{ij} - R_i C_j / n)^2}{R_i C_j / n}$$

$$df = (I - 1)(J - 1)$$

$$df = k - 1$$

$$df = k - \ell - 1$$

$$f_{\min}(x) = n(1 - F(x))^{n-1} f(x)$$

$$f_{\max}(x) = n(F(x))^{n-1} f(x)$$

Name	Param.	PMF or PDF	Mean	Variance	MGF
Bernoulli	$p$	$P(X = 1) = p$ $P(X = 0) = 1 - p$	$p$	$p(1 - p)$	$pe^t + (1 - p)$
Binomial	$n, p$	$\binom{n}{k} p^k (1 - p)^{n-k}$ $k \in \{0, 1, \dots, n\}$	$np$	$np(1 - p)$	$(pe^t + 1 - p)^n$
Geometric	$p$	$p(1 - p)^{k-1}$ $k \in \{1, 2, 3, \dots\}$	$\frac{1}{p}$	$\frac{(1 - p)}{p^2}$	$\frac{pe^t}{1 - (1 - p)e}$
Negative Binomial	$r, p$	$\binom{x-1}{r-1} p^r (1 - p)^{x-r}$ $x \in \{r, r + 1, r + 2, \dots\}$	$\frac{r}{p}$	$\frac{r(1 - p)}{p^2}$	$\left( \frac{pe^t}{1 - (1 - p)e} \right)^r$
Hypergeometric	$w, b, n$	$\frac{\binom{w}{k} \binom{b}{n-k}}{\binom{w+b}{n}}$ $k \in \{0, 1, \dots, n\}$	$\mu = \frac{nw}{w + b}$	$\left( \frac{w + b - n}{w + b - 1} \right) n \frac{\mu}{n} \left( 1 - \frac{\mu}{n} \right)$	
Poisson	$\lambda$	$\frac{e^{-\lambda} \lambda^k}{k!}$ $k \in \{0, 1, 2, \dots\}$	$\lambda$	$\lambda$	$e^{\lambda(e^t - 1)}$
Uniform	$a < b$	$\frac{1}{b - a}, x \in (a, b)$	$\frac{a + b}{2}$	$\frac{(b - a)^2}{12}$	$\frac{e^{tb} - e^{ta}}{t(b - a)}$
Normal	$\mu, \sigma^2$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$ $x \in \mathbb{R}$	$\mu$	$\sigma^2$	$e^{\mu t + (\sigma^2 t^2)/2}$
Exponential	$\lambda$	$\lambda e^{-\lambda x}, x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}$
Gamma	$r, \lambda$	$\frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$ $x > 0$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$	$\left( \frac{\lambda}{\lambda - t} \right)^a$
Beta	$a, b$	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}$ $0 < x < 1$	$\mu = \frac{a}{a + b}$	$\frac{\mu(1 - \mu)}{a + b + 1}$	$1 + \sum_{k=1}^{\infty} \left( \prod_{r=0}^{k-1} \frac{a + r}{a + b + r} \right) \frac{t^k}{k!}$
Chi-Square	$m$	$\frac{1}{2^{m/2} \Gamma(m/2)} x^{m/2-1} e^{-x/2}$ $x > 0$	$m$	$2m$	$\left( \frac{1}{1 - 2t} \right)^{m/2}$