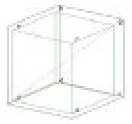

ch05 sec5.2 定积分

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取极限得定积分

```
md"""
##  取极限得定积分
"""
```

Definition

假定f在区间 $a\leq t\leq b$,定积分写为:

$$\int_{a}^{b}f(t)dt$$

当 $n \to +\infty, \Delta t \to 0$

$$\int_{a}^{b}f(t)dt=\lim_{n \to +\infty}(Left-hand-Sum)=\lim_{n \to +\infty}(\sum_{i=0}^{n-1} f(t_i)\Delta t)$$

$$\int_{a}^{b}f(t)dt=\lim_{n \to +\infty}(Right-hand-Sum)=\lim_{n \to +\infty}(\sum_{i=1}^{n} f(t_i)\Delta t)$$

左侧和与右侧和都成为黎曼和, f 称为被积函数, a,b 称为积分极限

```
md"""
!!! definition
    假定$f$在区间 $a\leq t\leq b$,定积分写为:

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    $$\int_{a}^{b}f(t)dt=\lim_{n \to +\infty}(Left-hand- Sum)=\lim_{n \to +\infty}
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(\sum_{i=1}^{n} f(t_i)\Delta t)$$

    左侧和与右侧和都成为黎曼和, $f$ 称为被积函数, $a,b$  称为积分极限
"""
```

Example

example1

计算$n=2$时, $\int_{a}^{b}\frac{1}{t}dt$ 的左和与右和

因为 $a=1,b=2,n=2$ 所以 $\Delta t=\frac{2-1}{2}=0.5$

```
md"""
!!! example
    example1

    计算$n=2$时, $\int_{a}^{b}\frac{1}{t}dt$ 的左和与右和



因为 $a=1,b=2,n=2$  所以 $\Delta t=\frac{2-1}{2}=0.5$
"""
```

```
0.5833
```

```julia
let
    a=1
    b=2
    n=2
    Δt=(b-a)/n
    tspan=a:Δt:b
    f(t)=1/t
    getnewarr(arr)=[f(t)*Δt for t in arr]    #计算每一个Δt 的值
    getsums(arr)=sum(arr)                    #求和
    get4digits(num)=round(num,digits=4)      #保留小数
    pipeline(arr)=arr|>getnewarr|> getsums|> get4digits  # 拼接管道操作

    @show leftsums=pipeline(tspan[1:2])
    @show rightsums=pipeline(tspan[2:3])

end
```

```
leftsums = pipeline(tspan[1:2]) = 0.8333
rightsums = pipeline(tspan[2:3]) = 0.5833
```

```
getRiemannSum (generic function with 1 method)
```

```julia
begin
    function getRiemannSum(a,b,n,func)
            a=a
            b=b
            n=n
            Δt=(b-a)/n
            tspan=a:Δt:b
            f=func
            len=size(tspan)[1]
            getnewarr(arr)=[f(t)*Δt for t in arr]    #计算每一个Δt 的值
            getsums(arr)=sum(arr)                    #求和
            get4digits(num)=round(num,digits=4)      #保留小数
            pipeline(arr)=arr|>getnewarr|> getsums|> get4digits  # 拼接管道操作
            res= Dict(
                "leftsum"=>pipeline(tspan[1:len-1]),
                "rightsums"=>pipeline(tspan[2:len]),

            )
            #@show res
            return res
     end


end
```

```
Dict("rightsums" ⇒ 0.6921, "leftsum" ⇒ 0.6941)
```

```julia
begin
    func1(t)=1/t
    n2= getRiemannSum(1,2,2,func1)
    n10=getRiemannSum(1,2,10,func1)
    n250=getRiemannSum(1,2,250,func1)
    @show n2 n10 n250

end
```

```
n2 = Dict("rightsums" => 0.5833, "leftsum" => 0.8333)
n10 = Dict("rightsums" => 0.6688, "leftsum" => 0.7188)
n250 = Dict("rightsums" => 0.6921, "leftsum" => 0.6941)
```

Note

当$n=250$时, 左侧和与右侧和之间的差距非常小,因此可以得出结论:

$$\int_a^b \frac{1}{t}dt \approx 0.69$$

Example

example2

计算$\int_{-1}^{1}\sqrt{1-x^2}dx$的积分

直接使用上面定义的求黎曼和公式:

```
Dict("rightsums" ⇒ 1.5704, "leftsum" ⇒ 1.5704)
```

```julia
begin
    func2(t)=sqrt(1-t^2)
    rn10= getRiemannSum(-1,1,10,func2)
    rn50=getRiemannSum(-1,1,50,func2)
    rn250=getRiemannSum(-1,1,250,func2)
    @show rn10 rn50 rn250

end
```

```
rn10 = Dict("rightsums" => 1.5185, "leftsum" => 1.5185)
rn50 = Dict("rightsums" => 1.5661, "leftsum" => 1.5661)
rn250 = Dict("rightsums" => 1.5704, "leftsum" => 1.5704)
```

```julia
@htl("""<script src="https://cdn.bootcdn.net/ajax/libs/mathjax/3.1.2/es5/tex-vg-full.js"></script>
    <script src="http://127.0.0.1:8080/tex-svg-full.min.js"></script>

""")
```