Bode Plots

Bode plots, named after Hendrick Bode, are log-log plots of magnitude vs. frequency and phase vs. frequency.

Specifically, a **magnitude** Bode plots presents decibels magnitude vs. log frequency:

$20\log_{10}|\frac{v_{out}}{v_{in}}(\omega)|$ vs. $\log_{10}(\omega/2\pi)$

A **phase** Bode plot plots the phase of $H(\omega)$ vs. log frequency.

The **logarithmic scale** allows presentation of a wide range of frequency, as well as transforming the curves into a more readable linear format. (Note that the frequency scale never reaches zero.)

Principles:

For a given transfer function $H(\omega)$:

- When $|H(\omega)|\ \alpha\ \omega$, the slope of the Bode plot $= +20$ dB/decade.
- When $|H(\omega)|\ \alpha\ 1/\omega$, the slope of the Bode plot $= -20$ dB/decade.
- Each ω or $j\omega$ in the numerator contributes $+20$ dB/decade.
- Each ω or $j\omega$ in the denominator contributes -20 dB/decade.
- Each term of form $1 + j\omega/\omega_c$ in the numerator is called a *zero*, and contributes $= +20$ dB/decade magnitude and a $+\pi/2$ phase shift above ω_c.
- Each term of form $1 + j\omega/\omega_c$ in the denominator is called a *pole*, and contributes $= -20$ dB/decade magnitude and a $-\pi/2$ phase shift above ω_c.

To sketch the Bode plots, evaluate the transfer function at the extremes of $\omega = 0,\ \omega = \infty$ and at ech cut-off frequency(s) ω_c.

Connect these points with line segments.

At ω_c, $|H(\omega)|$ will be down 3 dB relative to the pass-band, while the phase plot passes through ± 45 degrees relative to the pass-band.

For example, a low-pass filter has the transfer function:

$$\begin{aligned}
\frac{v_{out}}{v_{in}}(\omega) &= -\frac{Z_f}{Z_i} = -\frac{R_f||C}{R_i} \\
&= -\frac{\frac{(R_f/j\omega C)}{(1/j\omega C)+R_f}}{R_i} \\
&= -\frac{R_f}{(1+j\omega R_f C)R_i} = -\frac{R_f}{R_i}\frac{1}{1+j\omega R_f C}
\end{aligned} \tag{1}$$

At low frequencies, $|\frac{v_{out}}{v_{in}}(\omega)| \approx \frac{R_f}{R_i}$, while at high frequencies $|\frac{v_{out}}{v_{in}}(\omega)| \approx \frac{1}{\omega R_i C}$.

The corner frequency will be where $\frac{R_f}{R_i} = \frac{1}{\omega R_i C}$, or $\omega = 1/R_f C$.

Note that Bode plots are typically drawn showing ± 90 degrees. However, we consider phase to span 360 degrees, which is more correct and also indicates the 180 degree phase shift.

Generally, the order of the filter, indicated in simple circuits by the number of reactive components *active in a particular frequency band*, relates directly to the slope of the Bode plot in the cutoff region or regions.