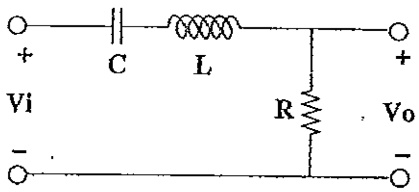
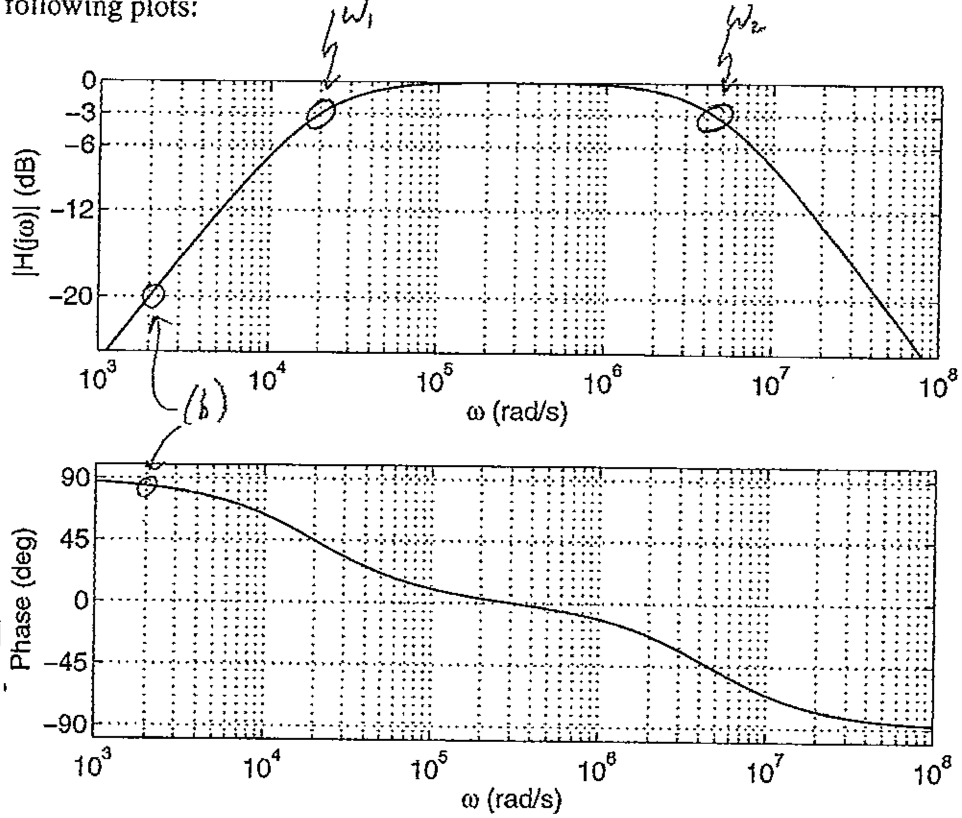
HW #3 BME 354 Spring 2015

~~BME153L.02 - Test #2~~ Name: Answer Key

## Problem #1 ~~[20 points]~~ (15/22)

You're given the following filter, and you measure the magnitude and phase of its transfer function ($H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$) in the lab, as shown in the following plots:




Code used to generate these plots attached.

(a) What type of filter is this (specify whether it is first- or second-order)? What is/are this filter's cutoff frequency/frequencies (reasonable estimates are okay)? [5 points]

BPF (2°)

$$\left.\begin{aligned} \omega_1 &\approx 2\times10^4 \text{ rad/s} \\ \omega_2 &\approx 4.5\times10^6 \text{ rad/s} \end{aligned}\right\} \text{-3dB points}$$

(b) If $V_i(t) = 0.5\cos(2000t)$ V, then what is $V_o(t)$ (again, reasonable estimates are okay)? [5 points]

$$-20\,\text{dB} = 20\log_{10}\left(\frac{|V_o|}{|V_i|}\right)\text{dB}$$

$$-1 = \log_{10}\left(\frac{|V_o|}{0.5}\right)$$

$$|V_o| = 0.05$$

$$\phi\Big|_{\omega=2000\,\text{rad/s}} \approx 85°$$

$$\boxed{V_o(t) = 0.05\cos(2000t + 85°)\ \text{V}}$$

Continue on next page...

(c) Solve for the values of $R$ & $L$ needed to achieve this transfer function, assuming that $C$ = 5.55 nF and the quality factor for this circuit can be expressed as $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$. [10 points]

$$W_n = 3\times10^5 \text{ rad/s} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L\times(1\times10^{-9}F)}} \longrightarrow \boxed{L = 2mH}$$

$$Q = \frac{W_n}{B} = \frac{W_n}{W_2 - W_1} = \frac{3\times10^5}{(4.5\times10^6 - 2\times10^4)} = 0.067$$

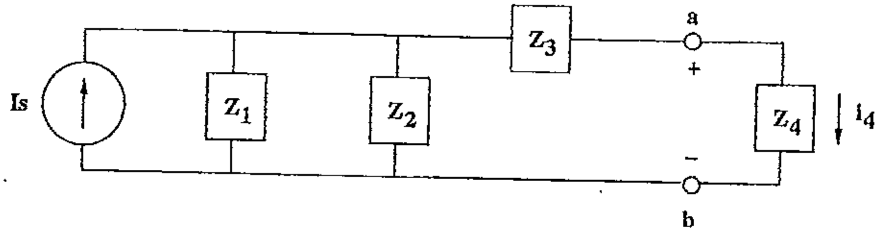
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 0.067 = \frac{1}{R}\sqrt{\frac{2\times10^{-3}}{5.55\times10^{-9}}}$$

$$\boxed{R \approx 9 k\Omega}$$

BME153L.02 - Test #2

Name: Answer Key

## Problem #~~3~~ [~~25~~ points] #2




$I_s(t) = 10\cos(1000t)$ mA
$Z_1 = 20\ \Omega$
$Z_2 = 30\ \Omega$
$Z_3 = ?$
$Z_4 =$ variable load

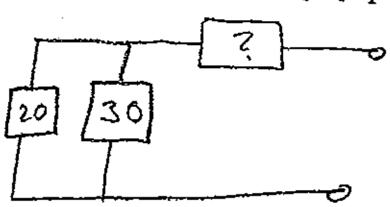
$Z_{1..4}$ represent discrete components in the circuit (i.e., resistors, capacitors, and inductors). When $Z_4 = \infty$ (open circuit across terminals $a$ & $b$), $v_{ab} = 120\cos(1000t)$ mV, and when $Z_4 = 0$ (short circuit across terminals $a$ to $b$), $i_4 = 9.86\cos(1000t + 9.46°)$ mA.

(a) What is the Norton impedance for the circuit as seen from $Z_4$ (i.e., $Z_4$ acts as the load)? [5 points]

$$Z_n = \frac{v_{oc}}{i_{sc}} = \frac{120\times10^{-3}}{9.86\times10^{-3}\angle 9.46°} = 12.17\angle{-9.46°}\ \Omega$$

$$= \boxed{12.0 - 2j\ \Omega}$$

(b) Given $Z_T$, what is $Z_3$? [5 points]




$$12 - 2j = Z_3 + 12$$

$$\boxed{Z_3 = -2j\ \Omega}$$

(c) Is $Z_3$ a resistor, a capacitor, or an inductor? What is the value of this component? [5 points]

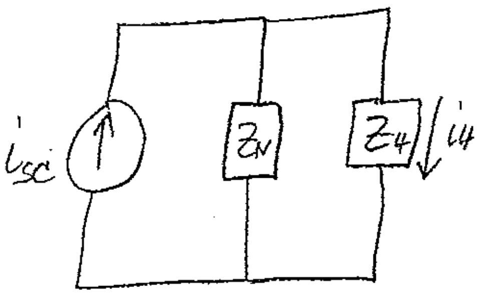
Capacitor ($-90°$ phase shift)

$$Z_3 = -2j = \frac{-j}{\omega C}\Big|_{\omega = 1000\ rad/s} \longrightarrow \boxed{C = 500\,\mu F}$$

Continue on the next page...

**BME153L.02 - Test #2** Name: Answer Key

(d) Solve for $i_4(t)$ if $Z_4$ is a 5 mH inductor. [5 points]




$$Z_4 = j\omega L = 5j\ \Omega$$

$$i_4 = 9.86\angle 9.46^\circ \quad \frac{12.17\angle -9.46^\circ}{12-2j+5j} \text{ mA}$$

$$i_4 = \frac{120}{12-2j+5j} = \frac{120}{12+3j} = 9.7\angle -14^\circ \text{ mA}$$

$$\boxed{i_4(t) = 9.7\cos(1000t - 14^\circ)\text{mA}}$$

(e) Solve for $v_{ab}(t)$ if $Z_4$ is a 5 mH inductor (same as (d)). [5 points]

$$v_{ab}(t) = v_4(t) = L\frac{di_4(t)}{dt} = (5\times10^{-3})\cancel{(-1000)}\cancel{(10^{-3})}(9.7)\sin(1000t-14^\circ)V$$

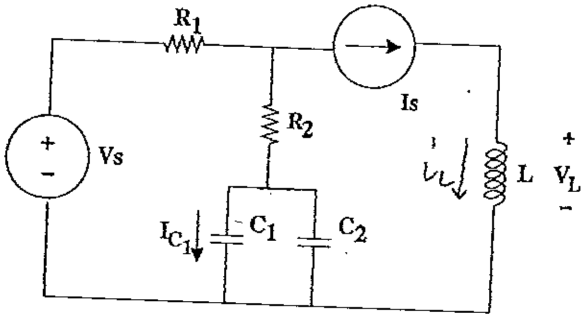
$$\boxed{v_{ab}(t) = -48.5\sin(1000t - 14^\circ)\text{mV}}$$

$$\boxed{v_{ab}(t) = 48.5\cos(1000t + 76^\circ)\text{mV}}$$

BME153L.02 - Test #2

Name: Answer Key

# Problem #~~1~~3 [~~15~~ 20 points] (20/18, 20)




$V_s(t) = 20\cos(100t)$ V
$I_s(t) = 10\cos(1000t + 20°)$ mA
$R_1 = 100\ \Omega$
$R_2 = 500\ \Omega$
$C_1 = 250\ \mu F$
$C_2 = 750\ \mu F$
$L = 5$ mH

Assume that all of the sources have been on for a long time (i.e., the circuit is in a steady-state condition).

(a) Solve for an expression for $V_L(t)$. [5 points]

$$v_L = i_L Z_L = (10\angle 20°)(j\,1000 \times 5 \times 10^{-3}) = 50\angle 110°$$

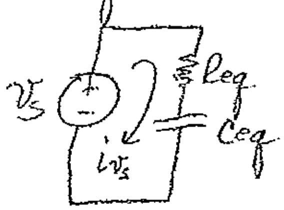
$$\boxed{v_L(t) = 50\cos(1000t + 110°)\ mV}$$

6

(b) Solve for an expression for $I_{C_1}(t)$. [15 points]

Using superposition...

$C_{eq} = C_1 + C_2 = 1000\,\mu F$




$R_{eq} = 600\ \Omega$

$Z_{C_{eq}} = \frac{-j}{\omega C} = -10j$

$$i_{v_S} = \frac{v_S}{Z_{eq} + Z_{C_{eq}}} = \frac{20\angle 0}{600 - 10j} = 0.0333\angle 1.0°$$

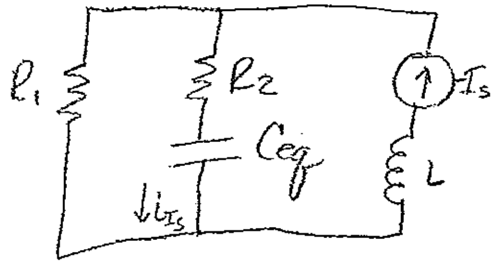
Current Division w/i $C_{eq}$: $i_{C_1 v_S} = i_{v_S}\frac{Z_{C_1}}{Z_{C_1} + Z_{C_2}} = 0.0333\angle 1.0° \underbrace{\frac{-13.33j}{-40j - 13.33j}}_{0.25}$

$$\boxed{i_{C_1 v_S}(t) = 0.0083\cos(100t + 1.0°)\ A}$$

Continue on the next page if necessary...

**BME153L.02 - Test #2** Name: Answer Key

Problem #4 continued...




$$i_{I_s} = -I_s \frac{100}{100+500-10j}$$

$$= -10\angle 20^\circ \frac{100}{600-j}$$

$$= \frac{-10\angle 20^\circ}{600\angle 0.1^\circ}\cdot(100) = -\frac{5}{3}\angle 19.9^\circ$$

$$Z_{eq} = \frac{-j}{\omega C} = -j$$

$$i_{C_1 I_s} = i_{I_s}(0.25) = -\frac{5}{12}\angle 19.9$$

$$i_{C_1 I_s}(t) = \left(-\frac{5}{12}\cdot 10^3\right)\cos(1000t+19.9^\circ)\,A$$

$$\boxed{i_{C_1}(t) = 8.3\cos(100t+1.0^\circ) - 0.417\cos(1000t+19.9^\circ)\,mA}$$

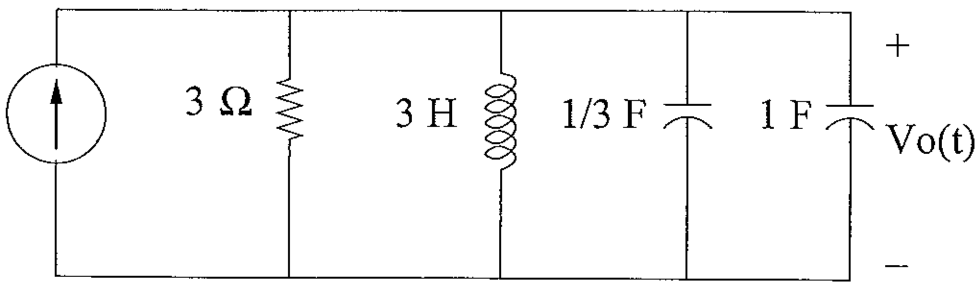
BME153L.001 - Test #2 Name:______________________

**Problem 4 [20 points]:** For the circuit below, use phasor techniques to solve for Vo(t):

Is = ~~10 cos 2t~~

$\times 2$ $10\angle 0$

$\omega = 2$




$4/3 \cdot 2 = 8/3$

combine

$= 1$ cap w/ $1\frac{1}{3}F = C_c$

all in || ⇒ use current divider,
get current through R, + solve for $V_o$.

$$i_R = i_s \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C_c} = i_s \frac{\frac{1}{3}}{\frac{1}{3} - j\frac{1}{6} + j\,\frac{8}{3}}$$

$$= i_s \frac{2}{2 - j + 16j} = i_s \frac{2}{2 + 15j} = i_s \frac{2}{15.13 \angle 82}$$

$$= 10\angle 0 \; \frac{2\angle 0}{15.13 \angle +82}$$

$$i_R = 1.32 \angle{-82} \qquad -82° = -1.438 \text{ rad.}$$

$$V_o = i_R R = 3.96 \angle{-82}$$ ← if correct but not final form,

$-2$

$$\boxed{V_o = 3.96 \cos(2t - 82°)}$$

If wrong eqn for || combo of 3 or more

$-3$

$$\frac{i_s}{\frac{1}{3} - \frac{j}{6} + \frac{8}{3}j}$$

BME153L.001 - Test #2 #5

Name:______________________________

**Problem 2 [30 points]:** Design a passive RC highpass filter that will attenuate signals with frequencies of 600 rad/sec by 20 dB.

4 (a) Draw your circuit.

4 (b) Derive its transfer function.

4 (c) What is your desired cutoff frequency?

3 (d) What component values will you use?

4 (e) Draw the Bode plot (both magnitude and phase) for your circuit.

4 (f) Now design a passive RL highpass filter with the same cutoff frequency and draw your circuit.

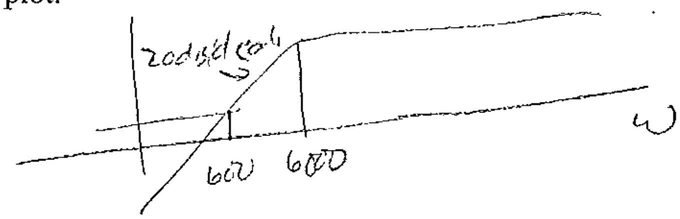
3 (g) What component values will you use?

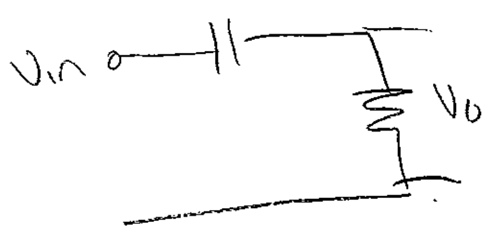
NMI.

4 (h) Describe the differences (if any) you would expect in the Bode plots between your RL and RC filters. NOTE: You do not have to draw this Bode plot.

HPF 600 by 20 dB ⇒

$\omega_0 = 6000$ rad/sec



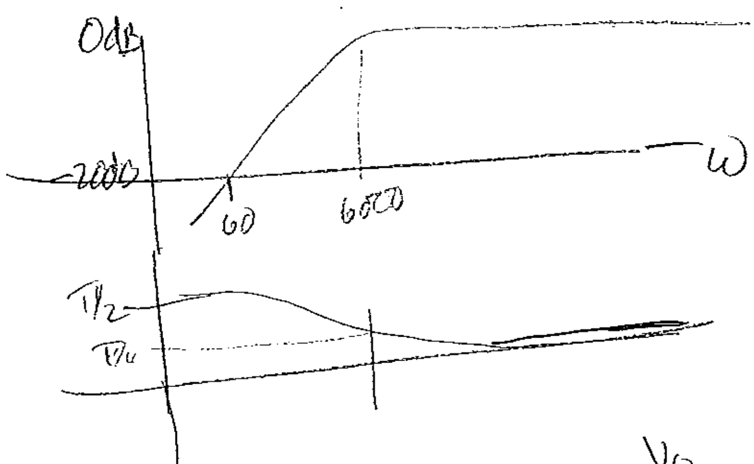



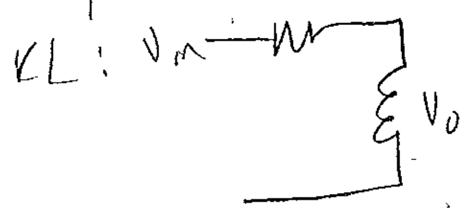

$$\frac{V_o}{V_{in}} = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega RC}{1+j\omega RC}$$

Let R = 1K ⇒ $\omega_0 = \frac{1}{RC}$

$6000(1K) = \frac{1}{C}$ $C = 166.67nF$

$$\left|\frac{V_o}{V_{in}}\right| = \pi/2 - \tan^{-1}\omega RC$$




$$\frac{V_o}{V_{in}} = \frac{j\omega L}{R+j\omega L} \Rightarrow \frac{j\omega L/R}{1+j\omega L/R}$$

R = 1K ⇒ $6000 = \frac{R}{L}$

$L = \frac{1000}{6000} = .1667H$

(6)

Key points from the battery question:

- Batteries utilize oxidation-reduction (redox) reactions in half cells (cathode and anode) to generate a voltage difference across two terminals.
- Rechargeable batteries can have charge in these half cells replenished by having a reversed voltage difference applied across their terminals.
- The individual cells of different types of batteries utilize different redox reactions that are responsible for different typical single cell voltages.

| Battery Type | Half Cell Reagents | Single Cell Voltage (V) | Notes |
|---|---|---|---|
| Alkaline | Zn, Mn | 1.5 | not rechargeable |
| Lead Acid | Pb, $HSO_4$ | 2.1 | heavy |
| NiCad | Ni, Cd | 1.2 | charging "memory" (bad) |
| NiMH | Ni, metal alloy | 1.2 | not as heavy; need smart charger |
| LiIon | Li, C | 3.7 | very high energy density; fire hazard |

- BME design considerations:
  - Safety
  - Weight
  - Durability
  - Biocompatibility
  - Energy density / life expectancy

Lithium "whiskers" can form in the cells, leading to internal shorts, high currents → heat → fire

Safeguards include: Regulating recharge ckts to prevent "trickle charges" that causes plating of lithium; also prevents overcharging.

(5)