L3.1

Need formal lecture on mesh currents + Thevenin + Norton Equivalent ckts!

Post some practice problems ← KCL KVL

Lecture 3: mesh currents (briefly)

- Thevenin + Norton Equivalent ckts
- Source Transformations
- Possibly start with Phasors

NOTE:

due to MLK holiday:

1) office hours next week 8-9 am moved to Tues. morn.

2) homework 2 due → wed 1/21 @ 5

3) labs tomorrow be sure to do the prelab — lab safety training + bring your certificate to lab!

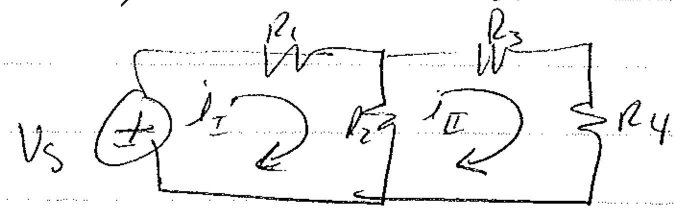
lab report: ← "short form"
→ answer all prelab ?'s, in lab ?'s + post lab ?'s
(write the ? and put answer...)
1/lab group.

review mesh currents:

meshes define closed loops
you will follow around ckt
to perform KVL

→ mesh currents will be your variables, and you will multiply them by each resistor (impedance) to determine voltage.

→ mesh current will define direction around the circuit



① identify meshes (closed loops)
② Keep ALL mesh currents CLOCKWISE!
③ KVL each mesh ← current direction defines polarity
④ Solve

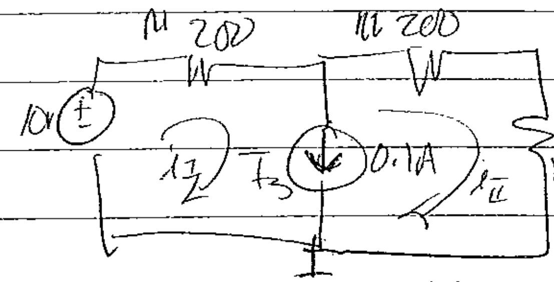
around I: $-V_s + i_I R_1 + R_2(i_I - i_{II}) = 0$

around II: $R_2(i_{II} - i_I) + R_3 i_{II} + R_4 i_{II} = 0$

i_{II} dictates direction

⑤ If ← you have a current source, don't know voltage for KVL
so, pick a different loop!
"supermesh"

mesh current example. from last class example.



current thru 100 i_3? (we had used V-divider + I-divider + superposition)

But, $i_3 = i_{II}$!

"Supermesh" (skip the current source)

$-10 + i_I 200 + i_{II} 200 + i_{II} 100 = \phi$

$I_S = i_I - i_{II}$ $\qquad i_I = I_S + i_{II}$ ← 2nd constraint

$-10 + (I_S + i_{II}) 200 + 300 i_{II} = 0$

$-10 + 200 I_S + 500 i_{II} = 0$

$i_{II} = \frac{10 - 200 I_S}{500} = \frac{10-20}{500} = \frac{-10}{500} = -.02$

What did we get last class?

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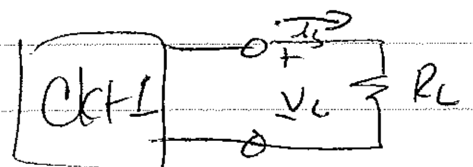
Another Tool:
Source Transformations:

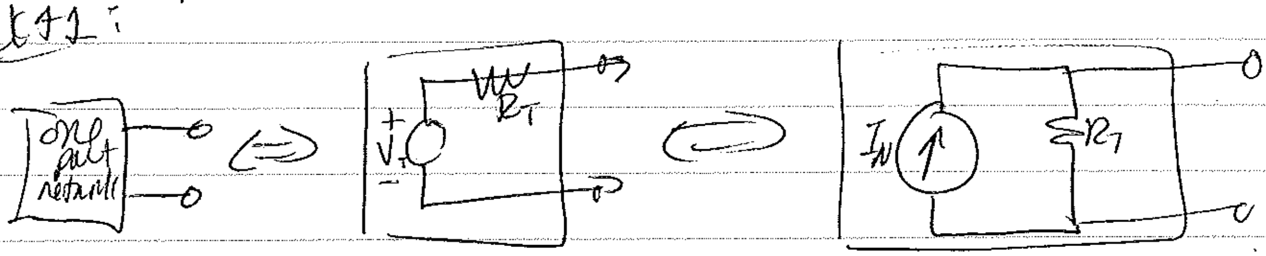
you might have noticed ⇒ sometimes ckts are easier to solve if only 1 type of source (either i or v) rather than 2 in one ckt

A ckt/port is defined by its i–v relations so



you can ~~replace~~ represent any one-port network by a voltage source in series with a RESISTOR; OR a current source in // w/a resistor
⇒ either one is equivalent to the original ckt as long as the v + i at the port are the same as the original.

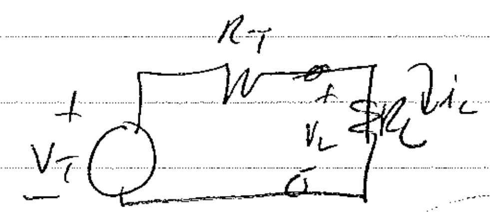
ckt 1:



if $\boxed{V_T = R_T I_N}$

why does this work? : (11)

~~Another example:~~

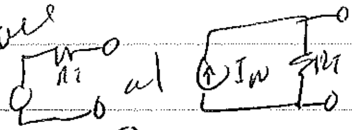


find $V_L + i_L$

V-divider

$$V_L = \frac{R_L}{R_T+R_L} V_T$$

$$i_L = V_L / R_L = \frac{V_T}{R_T+R_L}$$

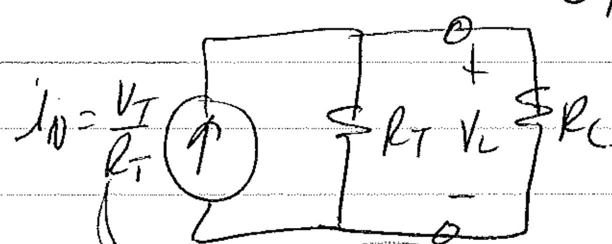


replace

Source xform:

using $V_T = i_N R_T$

$$i_N = \frac{V_T}{R_T}$$



$i_L \Rightarrow$ current divider:

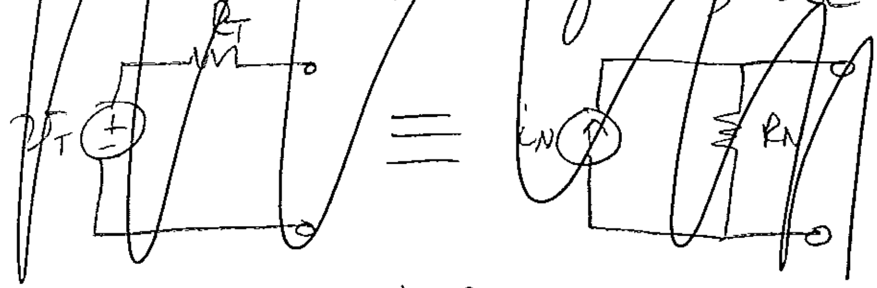
$$i_L = i_N \frac{\frac{1}{R_L}}{\frac{1}{R_T} + \frac{1}{R_L}} \left(\frac{R_T R_L}{R_T R_L}\right) = i_N \frac{R_T}{R_T+R_L} = \frac{V_T}{\cancel{R_T}} \frac{\cancel{R_T}}{R_T+R_L} = \frac{V_T}{R_T+R_L}$$

$$V_L = i_L R_L = V_T \frac{R_L}{R_T+R_L}$$

Same $i_L + V_L$ to load either way you represent it

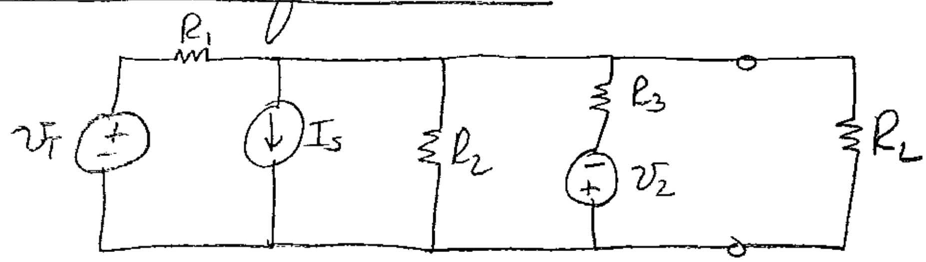
Norton Equiv.

$R_T = R_N$ Instead of v_{OC}, $i_{SC} = i_N$

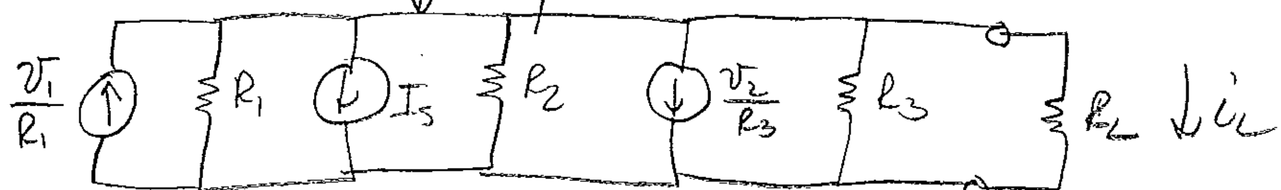


$v_T = i_N R_T$

Source Transformations

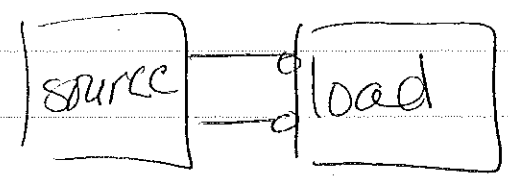


"simple" current divider



$$i_L = \left(\frac{v_1}{R_1} - I_S - \frac{v_2}{R_3}\right)\left[\frac{\frac{1}{R_L}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_L}}\right]$$

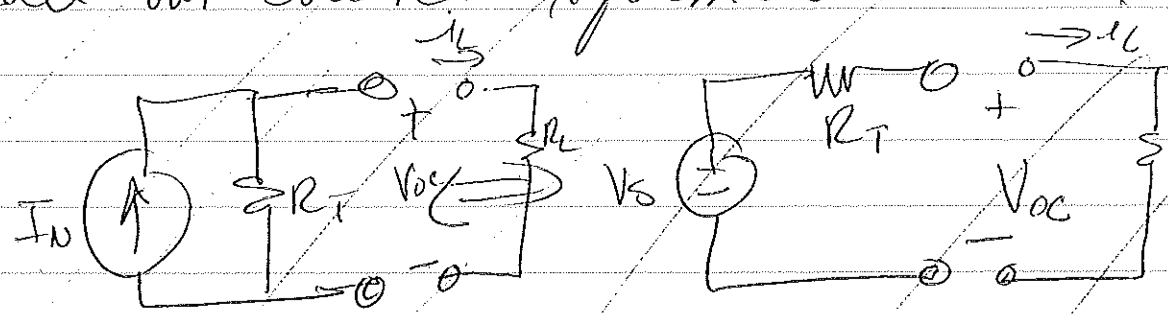
Today ⇒ divide ckt into "source" + "load" + simplify it ⇒ uses source transformations, ∥ + series combos, ~~[illegible]~~ "one port networks" + equivalent ckts ⇒



each box is 2 port device ⇒ describe by an i-v charac. (no matter what is in box)

Thevenin Thm: when viewed from the load, any network composed of ideal current + voltage sources + linear resistors may be represented by an equivalent ckt consisting of an ideal voltage source V_T w/ an equivalent Resistance in series R_T

Recall our source xforms:



where $V_S = I_N R_T$

in both cases V_{oc} + i_L are the same!

Last time we went from i-source to v-source & back w/ a resistor

This time ⇒ Combine entire subsets of elements into a single v-source + series R (or i-source & parallel R)

Why? If you have large parts of a ckt that never change ⇒ & say, a load resistance that does change, you simplify the non-changing parts so you can easily see i-v & determine impact of varying load w/ v-div. or i-div!

Really powerful! How do it?

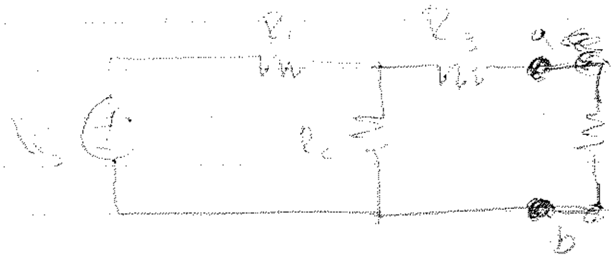
<u>Step 1</u>: Find equivalent resistance presented by ckt at its terminals (of port connecting ckt to load)

(3)

How? 1] Remove the load
2] zero all independent voltage & current sources
3] Compute total resistance btw load terminals after the load removed
⇒ This is the resistance that would be encountered by a current source connected to the circuit in place of the load.

ex:

circuit



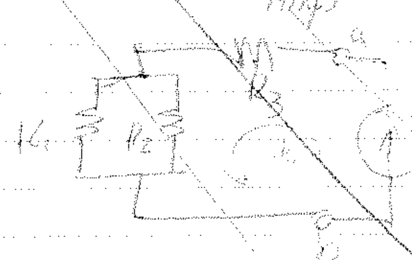
follow steps.

- start from the farthest point away from the load!

$R_T = R_1 \| R_2 + R_3$ a leaf or bud! follow path from a to b ...

You can also remove the circuit & connect a 1 Amp current source to terminals a-b

($V_{ab} = 1 R_T$) 1 Amp $V_{ab} = R_T$



$R_3 + R_1 \| R_2 = -V_{ab}$

R_T

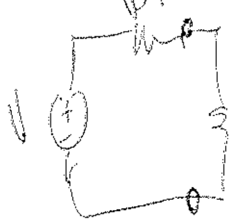
very little! confusing

(8)

1/2 way there!

Next need to compute the equivalent (Thevenin) source voltage. This is equal to the open circuit voltage present at the load terminals w/ the load removed.

Steps

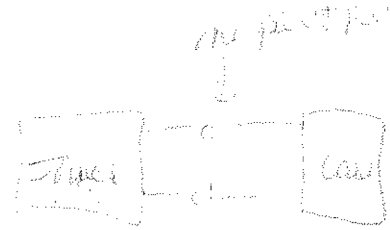


1) Remove load → load terminals are open circuit ← Keep sources!

2) Define V_{oc} = open circ. volt across open load terminals

3) apply any method (nodal analysis) to solve for V_{oc} — MESH OR NVA

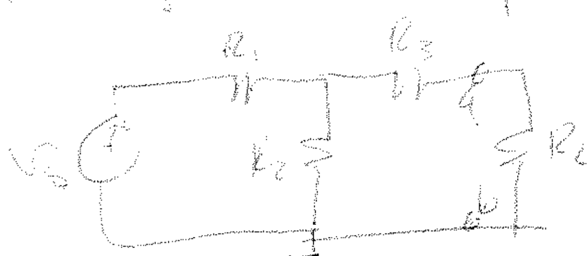
4) $V_T = V_{oc}$



$V_{oc} = V_T$

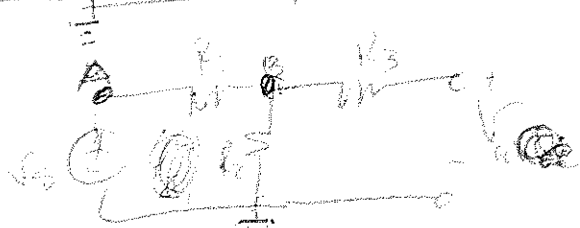
* No current flows through R_T when circuit is open, no voltage drop so
$V_T = V_{oc}$

Back to original example:



recall $R_T = R_1 \| R_2 + R_3$

follow steps!



note: no current flows through R_3 b/c open circuit ⇒ $V_B = V_{oc}$

Solve for $V_{TH} = V_B$ → nodal analysis

$V_A = V_S$

KCL @ B: $\frac{V_S - V_B}{R_1} = \frac{V_B}{R_2}$

$V_B\left(\frac{1}{R_2}+\frac{1}{R_1}\right) = \frac{V_S}{R_1}$

$V_B = V_S\left(\frac{1}{\cancel{R_1}}\right)\left(\frac{\cancel{R_1} R_2}{R_1+R_2}\right)$

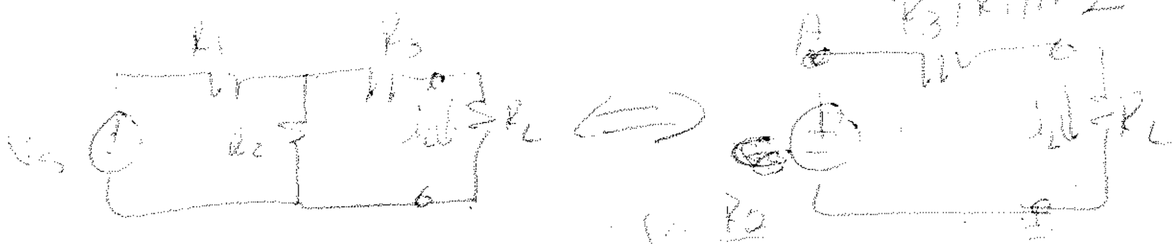
– circuit analysis is mostly algebra!

$$\boxed{V_{TH} = V_B = V_S\frac{R_2}{R_1+R_2}}$$

$R_1||R_2 = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = \frac{1}{\frac{R_2+R_1}{R_1R_2}}$

So, our equivalent circuits are:

$R_3 + R_1||R_2 = R_3 + \frac{R_1R_2}{R_1+R_2}$



$V_{TH} = V_S\frac{R_2}{R_1+R_2}$

The current drawn by the load is the same in both circuits!

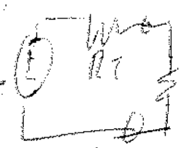
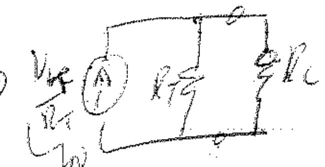
$R_1||R_2 = \frac{R_1R_2}{R_1+R_2}$

$$i_L = \frac{V_S\left(\frac{R_2}{R_1+R_2}\right)}{R_3 + \frac{R_1R_2}{R_1+R_2} + R_L} = \frac{V_{TH}}{R_{TH}+R_L}$$

Section 3.6

Norton's theorem, Thevenin's theorem are duals.

The Norton theorem: when viewed from the load, any network composed of ideal voltage & current sources & linear resistors may be represented by an equivalent ckt consisting of an ideal current source i_N in parallel with an equivalent resistance R_T;

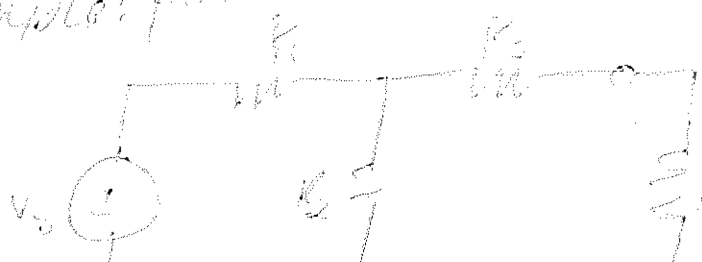
⇒ could compute Thevenin +  ⇒ 

How compute the Norton current directly?

equal to short circuit current that would flow if the load were replaced by a short circuit.

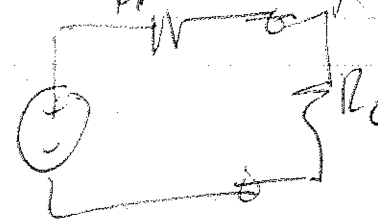
1] replace load w/ short circuit
2] define i_{sc} as Norton equivalent current
3] solve for i_{sc} using any method

Example: from last time:



⇒ $V_S \frac{R_2}{R_1+R_2} = V_T$

from last time $R_T = R_3 + \frac{R_1R_2}{R_1+R_2}$



recall: equivalent resistance: remove load, zero all sources & get R_T

$R_T = R_1 \| R_2 + R_3$

(10)

Next: solve for $i_{sc} = i_N$:

$V_A = V_S$

nodal analysis

@ B

$$\frac{V_S - V_B}{R_1} = \frac{V_B}{R_2} + \frac{V_B}{R_3}$$

$$\frac{V_S}{R_1} = V_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = V_B\left(\frac{R_2R_3 + R_1R_3 + R_1R_2}{R_1R_2R_3}\right)$$

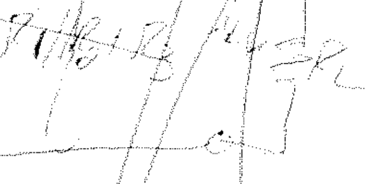
more to left & invert

$$\frac{V_S}{\cancel{R_1}} \cdot \frac{\cancel{R_1}R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2} = V_B$$

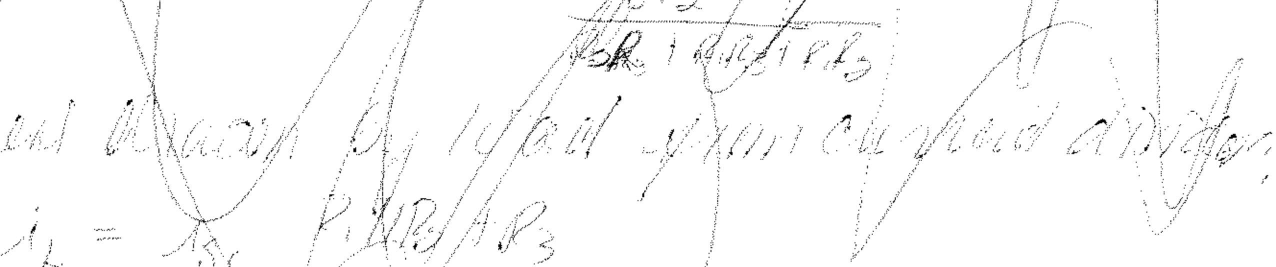
$$\boxed{i_{sc} = \frac{V_B}{R_3} = \frac{V_S R_2}{R_2R_3 + R_1R_3 + R_1R_2}} = i_N ;$$

$R_1 \| R_2 + R_3$

$\Rightarrow$



~~so equivalent circuits:~~

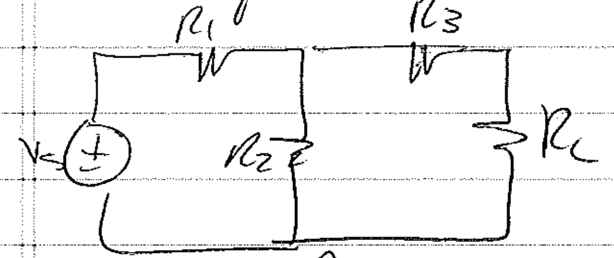


$\frac{V_S R_2}{R_2R_3 + R_1R_3 + R_1R_2}$

current division by ideal current divider:

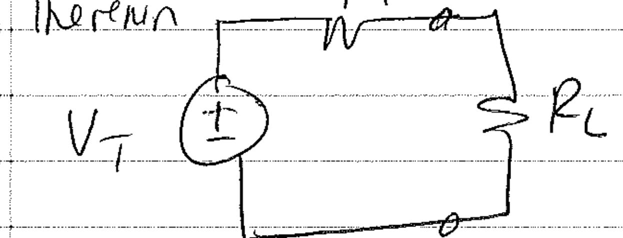
$$i_L = i_{sc}\,\frac{R_1\|R_2 + R_3}{R_1\|R_2 + R_3 + R_L}$$

so ⇒ equivalent ckts:



⇕

Thevenin

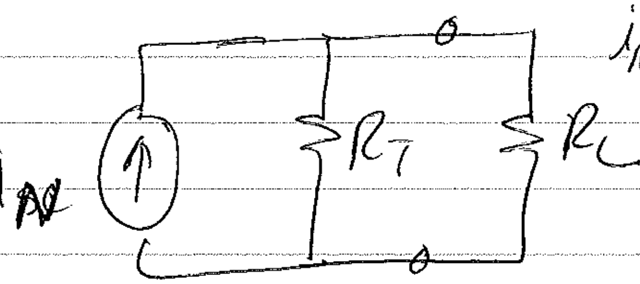


$V_T = V_s \frac{R_2}{R_1+R_2}$

$R_T = R_1 \| R_2 + R_3 = \frac{R_1R_2}{R_1+R_2} + R_3$

⇕

Norton:



$i_N = i_{sc} = V_s \frac{R_2}{R_2R_3 + R_1R_3 + R_1R_2}$

$R_T = R_1 \| R_2 + R_3$
$= \frac{R_1R_2}{R_1+R_2} + R_3$

Recall source xforms:

$V_T = i_N R_T$

is this true for our example?

prove to yourself!