Problem 2:

2. (10%) What is compressive sensing? Give an example of a compressive sensing application and explain how it works.

Ans:

Compressive sensing in the classical sense is the ability to recover a signal with less samples than specified from the Nyquist Sampling Theorem. Central to compressive sensing is an incoherence between a sensing matrix and a representation matrix that allows for the above fore-mentioned recovery of the original signal. Incoherency between “matrix spaces” is best achieved when one of the spaces is sparse. Incoherency has been studied and shown to exist in a practical setting when the sensing matrix is random: Gaussian or Bernoulli. Remarkably, combining the representation matrix with the sensing matrix preserves the randomness of the original matrix. Finally, the computations required to solve the matrix inverse problem are usually of the form: (O) = k\*log(N), which is a very scalable solution for large data sets as N grows.

There are many applications, with MRI being one of the early raison d’etre for development. However, there is one, sparse Fourier that lends itself to a truly transformative technology that incorporates the spirit of compressive sensing.

Below are two uses of sparse Fourier transforms:

Chart

Description automatically generated with low confidence

While FFTs developed in the 1960s was a computational break thru for digital signal processing as it allowed discrete Fourier transforms to be computed in N\*Log N, this is still in polynomial time as opposed to N^2 for DFTs. With the advent of compressive sensing computations can now be reduced to k\*logK in best cases.

How do they work?

Diagram, timeline

Description automatically generated

First, does sparse sensing use the ideas of compressive sensing in its implementation? The two main ideas in compressive sensing, are sparsity and incoherency between representation and sensing spaces. The sparse Fourier meets our first criteria is that it is sparse or can be made sparse thru another basis function. The second incoherency is achieved by randomness of measurement.

The basic idea of the Sparse Fourier Transform is to perform a normal FFT at a lower sample rate, and in the process aliasing many of the signals of interest. However thru a co-prime sub-sampling scheme signals that are unique will collide and those that do not are really aliases that can be eliminated. With “random sampling” and applying the FFT a number of times the true signals will remain, with a large reduction in processing proportional to log K , where K is the number of signals.

Problem 3:

3. (30%) Based on the lecture demo on compressive imaging, use MATLAB to simulate the computations of a single pixel camera, which uses DMD to generate the random sensing matrix ( ∅𝑖,𝑗∈{0,1}).

Ans:

Here we used the code provided by Prof Liang and Romberg’s L1 optimization. We also use a coherence function that calculates the mutual incoherence of the matrix. While the RIP (Restricted Isometry Property) and the Null Space Property (NSP) provide guarantees for the recovery of a k-sparse signal, they are not easy to compute. The mutual coherence of a matrix, with a lower number meaning that the matrices, sensing, and representation matrices are more incoherent and gives a good recovery of k sparse vectors.

%---------------------------------------------------------------------

% file name : hmwk\_4\_prob\_3\_dmd\_simulation.m

% Student: Ray Duran

% Date: 10/16/21

% Class : EECS 590 Professor Liang, Fall Semester

% University of North Dakota

% Descr:

% Model a Digital Micro-Mirror Device(DMD) with a Bernouli Random Matrix

%

%

% Code Borrowed from :

% compressive\_sensing\_demo

% Use "L1 magic" by Justin Romberg.

% http://www.acm.caltech.edu/l1magic

% Download the L1 magic toolbox and add to the matlab dir

%---------------------------------------------------------------------

A = imread('cameraman.tif');

M = 50; %Downsample the image to 50x50

N = 50;

A = imresize(A,[M N]);

x = double(A(:));

n = length(x);

%\_\_\_MEASUREMENT MATRIX\_\_\_

m = 1000; % Number of samples

%Phi = randn(m,n); % Generate Gaussian distribute random numbers

p = 0.2;

Phi = binornd(1,p,m,n); % Generate Bernouli distribute random numbers

%\_\_\_COMPRESSION\_\_\_

y = Phi\*x;

%\_\_\_THETA\_\_\_

% NOTE: Avoid calculating Psi (nxn) directly to avoid memory issues.

Theta = zeros(m,n);

for ii = 1:n

ek = zeros(1,n);

ek(ii) = 1;

psi = idct(ek)';

Theta(:,ii) = Phi\*psi;

end

% Calculate coherence

u = mutual\_coherence(Theta);

lower\_bound\_on\_mutual\_coherence = ((N-M)/(M\*(N-1)))^0.5;

%\_\_\_l2 NORM SOLUTION\_\_\_ s2 = Theta\y; %s2 = pinv(Theta)\*y

s2 = pinv(Theta)\*y;

%\_\_\_BP SOLUTION\_\_\_

s1 = l1eq\_pd(s2,Theta,Theta',y,5e-3,20); % L1-magic toolbox

%\_\_\_IMAGE RECONSTRUCTIONS\_\_\_

x1 = zeros(n,1);

for ii = 1:n

ek = zeros(1,n);

ek(ii) = 1;

psi = idct(ek)';

x1 = x1+psi\*s1(ii);

end

%\_\_\_\_\_ Plots

coherence\_str = (['Coherence =' num2str(u)]);

figure('name','Compressive sensing image reconstructions')

subplot(1,2,1), imagesc(reshape(x,M,N)), xlabel('original'), axis image

subplot(1,2,2), imagesc(reshape(x1,M,N)), xlabel('basis pursuit'), axis image

str = sprintf( 'Digital Micro-Mirror Device Single Pixel Camera Simulation \n mututal coherence w/ Bernouli random matrix = %4.2f ',u)

sgtitle(str)

%subtitle(coherence\_str)

colormap gray

