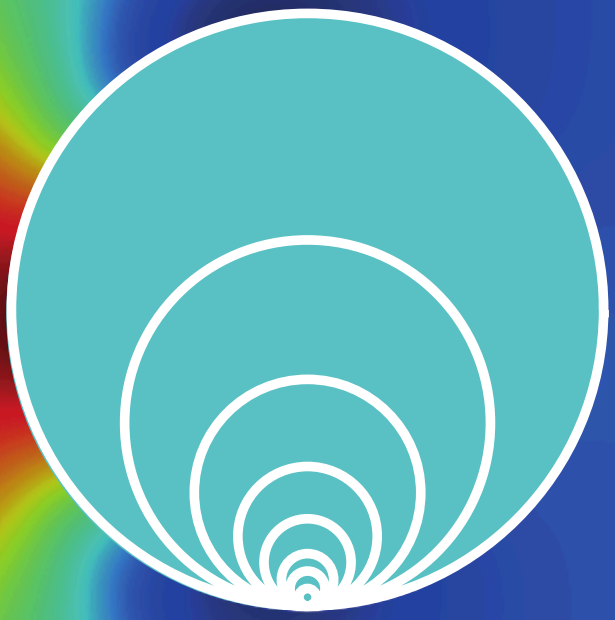
Program
and
Book of abstracts



YMMOR
2025

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Conference Program

Time Slot	Monday	Tuesday	Wednesday	Thursday	Friday
9.00am-10.30am	Introductory lectures	Max Beckermann	Michael Kartmann	Constantin Greif	Peter Oehme
		~~Sam Bender~~ Cancelled	Till Peters	Andreas Solheim	Alessandro Borghi
		Abbas Kabalan	Hongliang Mu	Jannis Marquardt	Niklas Reich
Coffee Break					
11.00am-12.30pm	Jakob Sudau	Jonas Nicodemus	*Hands-on* session	Alexandre Pasco	Moaad Khamlich
	Francesco Romor	Vishwas Kulkarni		Ivan Prusak	Closing remarks
	Sven Ullmann	Lucas Perrin		Zainab Farooq	
Lunch Break					
2.00pm-3.30pm	Isabella Carla Gonnella	Benedikt Klein	Excursion	Tobias Ehring	
	Anna Kovarnova	Francesco Carlo Mantegazza		Jakob Scheffels	
	Stefan Brunner	Tommaso Robbiani		Joshua Pughe-Sanford	
Coffee Break				Coffee Break	
4.00pm-5.00pm	Mattia Manucci	Robin Herkert		Leonidas Gkimisis	
	Zhuoyao Zeng	Yusuf Aydogdu		Lorenzo Tomada	
5.00pm-7.30pm	Ice-breaking				
7.30pm-11pm				Social dinner	

Table 1: YMMOR 2025 Conference Schedule.

Book of Abstracts

Reduced Models for Temperature Based Time of Death Estimation

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Estimating the time of death is crucial in forensic investigations. One of the most reliable and widely accepted method is estimating the cooling of the corpse. The prevailing approach is based on a purely phenomenological model. This leads to a lack of applicability to non-standard situations and difficulties in integrating multiple measurements. Newer models are mechanistic and based on the computational simulation of cooling of the corpse by solving the heat equation on CT-generated personalised anatomies so that different situations can be taken into account. Furthermore, this promises to provide higher accuracy as well as more reliable uncertainty quantification [3], but comes with its own challenges, such as a number of not precisely known parameters. These include the heat capacity values of different tissues, the exact point of measurement and the anatomy of the corpse [1, 2].

Solving the mechanistic model using finite elements is computationally expensive and as such not practical for real life application, so model order reduction can be used to reduce computational complexity and take advantage of an offline-online decomposition. The important quantity of interest that should be accurately reproduced in a reduced model is the temperature at the points of measurement. Due to practical limitations, this is usually only one point, a few at most. Consequentially a reduced model does not need to be accurate everywhere and we aim to adopt a goal-oriented approach based on proper orthogonal decomposition.

We discuss the general mechanistic model as well as relevant aspects of model order reduction.

References

[1] J. S. Subramaniam, M. Hubig, H. Muggenthaler, S. Schenkl, J. Ullrich, G. Pourtier, M. Weiser, and G. Mall. Sensitivity of temperature-based time since death estimation on measurement location. *International Journal of Legal Medicine*, 137:1815 – 1837, 2023. epub ahead of print.

[2] J. Ullrich, M. Weiser, J. Subramaniam, S. Schenkl, H. Muggenthaler, M. Hubig, and G. Mall. The impact of anatomy variation on temperature based time of death estimation. *International Journal of Legal Medicine*, 137:1615 – 1627, 2023.

[3] M. Weiser, B. Erdmann, S. Schenkl, H. Muggenthaler, M. Hubig, G. Mall, and S. Zachow. Uncertainty in temperature-based determination of time of death. *Heat and Mass Transfer*, 54(9):2815 – 2826, 2018.

Registration-based data assimilation from medical images

Francesco Romor[1], Felipe Galarce[2], Jia-Jie Zhu[3], and Alfonso Caiazzo[4]

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Image-based, patient-specific modeling of hemodynamics can improve diagnostic capabilities and provide complementary insights to better understand the after-effects of treatments. However, computational fluid dynamics simulations remain relatively costly in a clinical context. Moreover, projection-based reduced-order models and purely data-driven surrogate models struggle due to the high geometric variability of biomedical datasets. A possible solution is shape registration: a reference template geometry is designed from a cohort of available geometries, which can then be diffeomorphically mapped onto it. This provides a natural encoding that can be exploited by machine learning architectures and, at the same time, a reference computational domain in which efficient dimension-reduction strategies can be performed [2]. We compare state-of-the-art graph neural network models with recent data assimilation strategies [1] for the prediction of physical quantities and clinically relevant biomarkers in the context of aortic coarctation.

References

[1] F. Galarce, D. Lombardi, and O. Mula. State estimation with model reduction and shape variability. application to biomedical problems. *SIAM Journal on Scientific Computing*, 44(3):B805–B833, 2022.

[2] R. Guibert, K. Mcleod, A. Caiazzo, T. Mansi, M. A. Fernández, M. Sermesant, X. Pennec, I. E. Vignon-Clementel, Y. Boudjemline, and J.-F. Gerbeau. Group-wise construction of reduced models for understanding and characterization of pulmonary blood flows from medical images. *Medical image analysis*, 18(1):63–82, 2014.

Online adaptive surrogates for the value function of high-dimensional nonlinear optimal control problems

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We introduce a novel adaptive strategy for constructing a surrogate of the value function for high-dimensional, nonlinear optimal control problems of the form

$$v(x) = \min_{\mathbf{u}\in\mathcal{U}_\infty} \int_0^\infty h(\mathbf{x}(t)) + \mathbf{u}(t)^\top R\mathbf{u}(t)\,\mathrm{d}t \text{ subject to } \dot{\mathbf{x}}(t) = f(\mathbf{x}(t)) + g(\mathbf{x}(t))\mathbf{u}(t) \text{ and } \mathbf{x}(0) = x \in \mathcal{A}.$$

The proposed approach operates online and dynamically identifies the relevant regions of the state space where the surrogate value function must be sufficiently accurate. The quality of the surrogate value function $\tilde{v}(x)$ is quantified using the HJB residual, also referred to as the Bellman error (see [3]):

$$\mathrm{HJB}(x, \tilde{v}(x)) = \left| \min_{u\in\mathbb{R}^M} \left\{ (f(x) + g(x)u) \cdot \nabla_x \tilde{v}(x) + h(x) + u^\top R u \right\} \right|.$$

To construct the surrogate, we employ Hermite kernel regression (see [1]), a method particularly well-suited for function approximation using scattered data and derivative information. The training data for the surrogate originates from open-loop solutions of reduced-order optimal control problems, which are obtained through a Galerkin projection-based model order reduction. The Bellman error is computed on the full-order system to evaluate how well the Hermite kernel surrogate satisfies the HJB equation. If the error exceeds the prescribed tolerance threshold, the surrogate is adaptively refined by incorporating additional training data obtained online from the reduced-order optimal control solutions. Additionally, the reduced-order model itself can be improved with full-order data when the same HJB-based error indicator reveals that the current reduced system no longer provides a sufficiently accurate approximation of the full-order problem. The overall scheme, based on a hierarchical model structure involving full-order models, reduced-order models, and kernel-based surrogates, is inspired by the framework proposed in [2]. This adaptive approach ensures computational efficiency while maintaining accuracy in solving high-dimensional optimal control problems. Numerical experiments validate the effectiveness of the proposed scheme, demonstrating its capability to adaptively construct accurate value function surrogates while reducing computational costs.

References

[1] T. Ehring and B. Haasdonk. Hermite kernel surrogates for the value function of high-dimensional nonlinear optimal control problems. *Advances in Computational Mathematics*, 50(3):36, 2024.

[2] B. Haasdonk, H. Kleikamp, M. Ohlberger, F. Schindler, and T. Wenzel. A new certified hierarchical and adaptive RB-ML-ROM surrogate model for parametrized PDEs. *SIAM Journal on Scientific Computing*, 45(3):A1039–A1065, 2023.

[3] R. Kamalapurkar, P. Walters, J. Rosenfeld, and W. Dixon. *Reinforcement Learning for Optimal Feedback Control*. Springer International Publishing, 2018.

A metric perspective on Reduced Order Models: Wasserstein Gradient Flows for Advection-Diffusion dynamics

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Reduced Order Models (ROMs) are simplified representations of large systems designed to reduce the computational cost while retaining the essential features. Their key challenges are the identification of such important features and the a-priori quantification of the error committed in the reduced representation. This talk introduces ROMs in metric spaces, highlighting the novelties of this perspective and addressing the advantages it brings to tackle the above-mentioned points.
Metric ROMs [2, 1] are built to accurately capture what is known about the geometric structure of the problem. By identifying significant topological features of the potentially nonlinear data manifold at disposal, this approach provides geometry-aware and computationally efficient models, addressing some relevant limitations of traditional ROM techniques. Indeed, the metric space framework not only enhances the construction of nonlinear ROMs but also offers hints for error quantification. Additionally, it establishes an interesting connection between ROMs and learning-based models, as, in the metric context, the reduced model is often derived by minimizing some distance between a chosen quantity of interest and its reduced representation, mirroring the loss minimization processes in machine learning algorithms.
This talk will deepen some recent advances in the construction of ROMs within the Wasserstein metric space for gradient flow-based dynamics [4]. Such a setting is particularly well-suited for handling both advective and diffusive behaviors, whose coexistence is notoriously problematic for the identification of accurate ROMs [3]. This builds upon ongoing work in collaboration with Dr. Federico Pichi, Prof. Olga Mula, Dr. Rafael Bailo, and Prof. Gianluigi Rozza.

References

[1] B. Battisti, T. Blickhan, G. Enchery, V. Ehrlacher, D. Lombardi, and O. Mula. Wasserstein model reduction approach for parametrized flow problems in porous media. *ESAIM: Proceedings and Surveys*, 73:28–47, 2023.

[2] V. Ehrlacher, D. Lombardi, O. Mula, and F.-X. Vialard. Nonlinear model reduction on metric spaces. application to one-dimensional conservative pdes in wasserstein spaces. *ESAIM: Mathematical Modelling and Numerical Analysis*, 54(6):2159–2197, 2020.

[3] I. C. Gonnella, F. Pichi, and G. Rozza. Nonlinear reduction strategies for data compression: a comprehensive comparison from diffusion to advection problems. *arXiv:2501.12816*, 2025.

[4] F. Santambrogio. Euclidean, metric, and Wasserstein gradient flows: an overview. *Bull. Math. Sci.*, 7(1):87–154, Mar. 2017.

Megaframe – a scalable variant of shifted proper orthogonal decomposition for model order reduction of flows with particles

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Flows with several moving particles that describe, e.g., microfluidic reactors or pneumatic conveying, are representative of transport-dominated systems. As such, they cannot be well expressed as a superposition of a few basis functions, and common mode-based approaches for model order reduction utilizing for example the proper orthogonal decomposition (POD), fail. The shifted proper orthogonal decomposition (sPOD) [2], approximates the field by a sum of co-moving frames, in which the data are stationary and can be described with a small number of basis functions.

Shifted POD works with known transports and optimizes which data are sorted into which co-moving frame of reference [2, 1]. The approximation is then realized as

$$Q \approx \sum_{k=1}^{K} \mathcal{T}^{\Delta_k}\left(Q_k\right), \quad \operatorname{rank}\left(Q_k\right) = \ell_k, \tag{1}$$

where $Q = [\boldsymbol{q}(t_1), \dots \boldsymbol{q}(t_{\mathrm{N_t}})]$ is a matrix of snapshots, $\{Q_k\}_{k=1,\dots K}$ matrices comprising the co-moving frames and $\mathcal{T}^{\Delta_k}$ discrete versions of transport operators, $\mathcal{T}^{\Delta_k}\left(q(x,t)\right) := q(x - \Delta_k(t), t)$.

The overall rank of the sPOD reconstruction is equal to the sum of ranks of all the co-moving frames. When applying the method to flows with particles, the intuitive choice is to assign each particle its own frame. The rank of the sPOD reconstruction then grows with a growing number of particles.

However, for multiple particles of the same shape and size, one can assume that the flow fields around them would be similar and try to approximate them with the same basis functions. In our work, we approximate the field by a sum of co-moving frames $\hat{Q}_k$ analogously to sPOD; however, we then stack the frames into a single 'megaframe' and fix its rank to $\hat{\ell}$ as

$$Q \approx \sum_{k=1}^{K} \mathcal{T}^{\Delta_k}\left(\hat{Q}_k\right), \quad \operatorname{rank}\left(\hat{Q}\right) = \hat{\ell}, \qquad \hat{Q} = \left[\hat{Q}_1, \dots, \hat{Q}_K\right]. \tag{2}$$

This allows us to disconnect the final rank of the reconstruction from the number of particles. Furthermore, the method is able to achieve lower reconstruction errors compared to classical sPOD formulations of the same rank; the downside is that performing POD of a larger matrix is more costly.

References

[1] P. Krah, A. Marmin, B. Zorawski, J. Reiss, and K. Schneider. A robust shifted proper orthogonal decomposition: Proximal methods for decomposing flows with multiple transports, 2024.

[2] J. Reiss, P. Schulze, J. Sesterhenn, and V. Merhmann. The shifted proper orthogonal decomposition: a mode decomposition for multiple transport phenomena. *SIAM Journal on Scientific Computing*, 40:A1322–A1344, 2018.

Dynamical low rank approximation for the radiative transfer equation with non trivial boundary conditions

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Kinetic equations are important in several applications, one of them being plasma physics. Due to their high dimensionality, namely 6 dimensional space, they are often numerically too expensive to solve using standard methods. This motivates us to apply a complexity reduction technique.

In this talk we will consider dynamical low-rank approximations. Kinetic equations describe the evolution of a distribution function $f(t, x, v)$, which often depends on space x and velocity v, each of them in 3 dimensional space. The goal of dynamical low-rank methods is to separate space and velocity dependence and approximate f by low rank factors, which only depend on either x or v.
By applying dynamical low-rank we obtain dynamical evolution equations for our low rank factors. Those can then be solved by classical numerical methods. This drastically reduces the required memory and computational cost.

The main equation we will consider in this talk is the radiative transfer equation, which depends on time, space and direction of travel. We will derive evolution equations for the low rank factors and argue that our problem has a low rank structure. We also observe by conducting numerical experiments that our proposed algorithm efficiently solves the radiative transfer equation.

In addition, we will look at different boundary conditions and how to implement them numerically. In recent papers about this topic primarily periodic boundary conditions were used. Being able to handle arbitrary boundary conditions is a necessary first step to applying domain decomposition to low-rank approximation.

Certified model order reduction for parametric control systems using contour integral methods

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We provide a novel framework for evaluating the output of parametric *linear time invariant* control systems at a given time T or time window $[t_0, \Lambda t_0]$ with $\Lambda > 1$, accurately and efficiently. This is achieved by exploiting *contour integral methods* for the time integration. Those methods allow for a straightforward parallelization of the main computational burden [2, 3] and are well suited to construct a non-linear *model order reduction* (MOR) framework that can help in efficiently evaluating hardly reducible problems, such as hyperbolic problems with discontinuities; see [5, 1]. First, we deal with the non-parametric case. We show how, for fixed initial values, we can evaluate the input-output map with a computational cost that only depends on the input and output dimensions for a large variety of input functions. The approach then naturally adapts to different non-zero initial values, a remarkable difference with respect to standard system-theoretic MOR approaches. Then, we introduce the parameter dependency and we effectively couple the procedure for the non-parametric case with a certified projection-based non-linear MOR strategy based on a greedy algorithm. In this way we are able to reduce the state dimension and have an efficient evaluation of the input-output map for every parameter in the parametric domain, every input function, and initial data that can be well represented by a small dimensional subspace. This approach was exploited by [4] in the framework of source problems arsing after the discretization of parabolic partial differential equations discretization.

References

[1] C. Greif and K. Urban. Decay of the Kolmogorov N -width for wave problems. *Appl. Math. Lett.*, 96:216–222, Oct. 2019.

[2] N. Guglielmi, M. Lopez-Fernandez, and G. Nino. Numerical inverse laplace transform for convection-diffusion equations in finance. *Math. Comp.*, 89:1161–1191, Jan. 2020.

[3] N. Guglielmi, M. Lopez-Fernandez, and G. Nino. Pseudospectral roaming contour integral methods for convection-diffusion equations. *J. Sci. Comput.*, 89(1):22, 2021.

[4] N. Guglielmi and M. Manucci. Model order reduction in contour integral methods for parametric pdes. *SIAM J. Sci. Comput.*, 45(4):A1711–A1740, 2023.

[5] M. Ohlberger and S. Rave. Reduced basis methods: Success, limitations and future challenges. *Proceedings of the Conference Algoritmy*, pages 1–12, 2016.

Certified Model Order Reduction for parametric Hermitian eigenproblems

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Given a compact parameter set $\mathcal{P} \subseteq \mathbb{R}^d$ and a parametric Hermitian matrix $\mathbf{A}(\mu) = \mathbf{A}(\mu)^* \in \mathbb{C}^{N\times N}$, we propose an efficient and certified scheme to approximate the smallest eigenvalue $\lambda_1(\mu)$ and the associated eigenvector(s) $\mathbf{w}_1(\mu)$ of $\mathbf{A}(\mu)$, i.e. we are interested in the solutions of the equation

$$\mathbf{A}(\mu)\mathbf{w}_1(\mu) = \lambda_1(\mu)\mathbf{w}(\mu) \tag{1}$$

for many different $\mu \in \mathcal{P}$. To this end, we rely on projection-based model order reduction (MOR), i.e. we approximate the possibly large-scale problem (1) with a reduced problem of a much smaller dimension by projecting it on a suitable subspace. Such a subspace is constructed by eigenvectors w.r.t. some "snapshot parameters" that are chosen by a weak-greedy type strategy.
After detailing the connections to the reduced basis method for source problems, we introduce a novel a posteriori error estimate for the eigenvectors associated with the smallest eigenvalue. It turns out that the approximation of the difference between the second smallest and the smallest eigenvalues, i.e. the so-called spectral gap, is crucial for the reliability of the error estimate. Therefore, we extend the result of [3] and propose new efficiently computable upper and lower bounds for the spectral gap, which allow for an approximation through a greedy procedure.
It is worth emphasizing that our framework is well-suited to tackle the cases where the smallest eigenvalue is not simple.
Our work is motivated by a particular application, namely the repeated evaluation of the ground state of parametric quantum spin system (QSS) models, in which the ground state corresponds to the eigenspace associated with the smallest eigenvalues of the QSS Hamiltonian [2, 1]. Besides that, finding the groundstates of a parametric Hermitian eigenproblem is omni-present in physics, chemistry and engineering.

References

[1] P. Brehmer, M. F. Herbst, S. Wessel, M. Rizzi, and B. Stamm. Reduced basis surrogates for quantum spin systems based on tensor networks. *Phys. Rev. E*, 108:025306, Aug 2023.

[2] M. F. Herbst, B. Stamm, S. Wessel, and M. Rizzi. Surrogate models for quantum spin systems based on reduced-order modeling. *Phys. Rev. E*, 105:045303, Apr 2022.

[3] P. Sirkovic and D. Kressner. Subspace acceleration for large-scale parameter-dependent hermitian eigenproblems. *SIAM J. Matrix Anal. Appl.*, 37(2):1323–1353, 2016.

A priori model order reduction for free surface fluid flows

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Low Reynolds complex fluid dynamics problems such as free surface fluid flows are very rarely studied in the context of Model Order Reduction (MOR). They remain however particularly interesting for a wide range of applications ranging from breaking waves, landslides, complex fluid structure interaction and various manufacturing processes. The lack of interest can be explained by two crucial observations. First, unconventional solvers have to be used to effectively identify the free surfaces, they therefore require innovative and new MOR strategies. Secondly and most importantly, free surface fluid problems possess a slow N-Kolmogorov width decay and are hard to reduce. Mapping techniques have to be used to perform the required domain geometrical registration that ensures the reductibility of the velocity field.

Our work focuses on the Particle Finite Element Method [2], a fluid solver based on Lagrangian finite elements combined with efficient re-meshing algorithms. It automatically idetifies the interfaces/free surfaces and is tailored for this kind of problems for which it has been used extensively. Our idea is to directly adapt the Lagrangian map computed with PFEM as our non-linear reduction ansatz in our reduction scheme. We showed that using this map, we get promising reducibility for the velocity fields.

Regarding the MOR, we opted for an *a priori* reduction scheme, the Proper Generalized Decomposition [1] with a space-time decomposition. Contrary to the *a posteriori* family of reduced order modelling techniques the PGD does not require any knowledge of past solutions. The reduced solution is instead built iteratively solving several problems of lower dimension. It was shown to be equivalent to POD in a degenerate case [3] but generally has monotonic convergence with poor convergence properties. It is nonetheless cheap and can be used in a multi dimensional parametric manifolds. We will alternate between building the solution and recalculating the Lagrangian meshes from the full velocity field. Using this, we are iteratively constructing both the solution and its non-linear reduction ansatz. Convergence is then reached when the mesh velocity matches the fluid velocity.

References

[1] F. Chinesta, R. Keunings, and A. Leygue. *The Proper Generalized Decomposition for Advanced Numerical Simulations: A Primer*. SpringerBriefs in Applied Sciences and Technology. Springer International Publishing : Imprint: Springer, Cham, 1st ed. 2014 edition, 2014.

[2] M. Cremonesi, A. Franci, S. Idelsohn, and E. Oñate. A State of the Art Review of the Particle Finite Element Method (PFEM). *Archives of Computational Methods in Engineering*, 27(5):1709–1735, Nov. 2020.

[3] A. Nouy. A priori model reduction through Proper Generalized Decomposition for solving time-dependent partial differential equations. *Computer Methods in Applied Mechanics and Engineering*, 199(23-24):1603, 2010.

Optimal morphing for reduced-order modeling under non-parametrized geometrical variability

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Solving parametric partial differential equations (PDEs) for different parameter values in a given set is a common task in industrial contexts. When the evaluation of the PDE solution is computationally time-consuming, model order reduction techniques offer an effective tool for accelerating computations while maintaining accuracy.

In the present work, we deal with non-reducible problems for which a large number of modes is required to capture the behaviour of the physical system. We present a new approach to dimensionality reduction, where each snapshot is mapped to a reference configuration that is more suitable for dimensionality reduction. Such approaches are commonly known in the literature as registration problems [4]. The method we propose was first presented in [3]. The mappings are the solution to an infinite-dimensional non-linear optimization problem. Mainly, given a dataset of n fields $\{u_i\}_{1\leq i\leq n}$ that are defined on a domain Ω_0, we solve the following optimization problem

$$\text{find } \Phi^* \in \arg\max_{\Phi\in\mathbf{M}} J[\Phi] := \frac{\sum_{j=1}^{r} \lambda_j^{\Phi}}{Tr(C[\Phi])} \tag{1}$$

where λ_i are the eigenvalues of the correlation matrix defined as $C_{ij}[\Phi] = \langle u_i \circ \boldsymbol{\phi}_i, u_j \circ \boldsymbol{\phi}_j \rangle_{L^2(\Omega_0)}$.
The main difference between our method and other registration approaches is that the method we propose automatically maps the snapshots in the reference configuration, without any feature tracking, such that the energy conserved in the first r modes is maximal. In our method, we can also choose any arbitrary integer r for the number of modes.
We also study how the proposed method could be extended to the case where the geometry might change as a function of the parameter value. In this case, each geometry must first be assigned to a reference geometry where the dimensionality reduction problem is solved [2]. Finally, we show how we can apply our method to non-intrusive model order reduction techniques similar to MMGP [1] .

References

[1] F. Casenave, B. Staber, and X. Roynard. Mmgp: a mesh morphing gaussian process-based machine learning method for regression of physical problems under nonparametrized geometrical variability. *Advances in Neural Information Processing Systems*, 36, 2024.

[2] A. Kabalan, F. Casenave, F. Bordeu, V. Ehrlacher, and A. Ern. Elasticity-based mesh morphing technique with application to reduced-order modeling. *arXiv preprint arXiv:2407.02433*, 2024.

[3] A. Kabalan, F. Casenave, F. Bordeu, V. Ehrlacher, and A. Ern. Morphing techniques for model order reduction with non parametric geometrical variabilities. In *16ème Colloque National en Calcul de Structures*, 2024.

[4] T. Taddei. Compositional maps for registration in complex geometries. *arXiv preprint arXiv:2308.15307*, 2023.

KLAP: KYP lemma based low rank approximation for passivation

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We present a novel passivity enforcement (passivation) method, called KLAP, for linear time-invariant systems based on the Kalman-Yakubovich-Popov (KYP) lemma and the closely related Lur'e equations. The passivation problem in our framework corresponds to finding a perturbation to a given non-passive system that renders the system passive while minimizing the $\mathcal{H}_2$ or frequency-weighted $\mathcal{H}_2$ distance between the original non-passive and the resulting passive system. We show that this problem can be formulated as an unconstrained optimization problem whose objective function can be differentiated efficiently even in large-scale settings. We show that any minimizer of the unconstrained problem yields the same passive system. Furthermore, we prove that, in the absence of a feedthrough term, every local minimizer is also a global minimizer. For cases involving a non-trivial feedthrough term, we analyze global minimizers in relation to the extremal solutions of the Lur'e equations, which can serve as tools for identifying local minima. To solve the resulting numerical optimization problem efficiently, we propose an initialization strategy based on modifying the feedthrough term and a restart strategy when it is likely that the optimization has converged to a local minimum. Numerical examples illustrate the effectiveness of the proposed method.

References

[1] J. Nicodemus, M. Voigt, S. Gugercin, and B. Unger. KLAP: KYP lemma based low-rank approximation for $\mathcal{H}_2$-optimal passivation. *ArXiv e-print 2501.05178*, 2025.

POD-Galekin ROM for Conjugate Heat Transfer problems using Finite Volume Discretization

V. Kulkarni[1]

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Power electronics play a crucial role in electrified vehicles (EVs), where efficient thermal management is essential to prevent failures and ensure reliability. Developing advanced active thermal control strategies is key to predictive health management, as direct sensing of thermal states during operation is often limited by packaging constraints and cost. Model predictive control (MPC) is increasingly being integrated into these strategies, relying on digital twins to estimate the electrical and thermal behaviour of components. However, current thermal digital twins primarily predict pointwise temperatures, such as junction temperatures of semiconductor dies, and lack the capability to reconstruct the full temperature field in critical regions.

This work presents a Proper Orthogonal Decomposition (POD)-Galerkin Reduced Order Model (ROM) framework for conjugate heat transfer (CHT) problems, leveraging finite volume discretization [1] for high-fidelity simulations. The proposed ROM efficiently reduces computational costs while maintaining accuracy by projecting governing equations onto a reduced subspace using POD modes [3] [4]. Applied to the thermal management of inverters in EV power modules, the framework provides a comprehensive temperature field prediction, which can enhance digital twin models for real-time monitoring and optimization. The developed ROM enables more advanced predictive health management strategies by offering a detailed thermal representation, bridging the gap between existing thermal digital twins and full-field temperature prediction [2].

This study is part of a broader effort, in collaboration with L. Perrin and Prof. S. Volkwein of Department of Mathematics and Statistics, Universität Konstanz, Germany, to develop reduced-order modeling strategies for conjugate heat transfer problems involving both solid and fluid domains, where efficient and accurate ROMs are essential for handling coupled physics at the interface.

References

[1] H. Jasak. *Error analysis and estimation in the Finite Volume method with applications to fluid flows*. PhD thesis, Imperial College, University of London, 1996.

[2] V. Kulkarni and A. Kospach. Machine learning based reduced order model for full temperature prediction of power electronics component cooling. In *2024 Energy Conversion Congress & Expo Europe (ECCE Europe)*, pages 1–8. IEEE, 2024.

[3] G. Stabile, S. Hijazi, A. Mola, S. Lorenzi, and G. Rozza. POD-Galerkin reduced order methods for CFD using Finite Volume Discretisation: vortex shedding around a circular cylinder. *Communications in Applied and Industrial Mathematics*, 8(1):210–236, 2017.

[4] S. Volkwein. Model reduction on using POD. Technical report, 2018.

A Benchmark of Reduced Order Models Methods for Heat Transfer Problems

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Reduced Order Models (ROMs) offer powerful tools for efficiently solving parametrized heat transfer problems, balancing accuracy and computational cost. While many ROM techniques exist, their relative performance can vary significantly depending on the problem at hand. In this work, we present a comparative benchmark of four prominent ROM approaches –POD-Galerkin [1, 2], POD with Least-Squares Residual (POD-LSR), Greedy-POD [3], and Dynamic Mode Decomposition (DMD) [4]– applied to the same heat transfer problem. Each method is analyzed under a unified framework, enabling a fair comparison in terms of accuracy, efficiency, and robustness. Through numerical experiments, we highlight the strengths and limitations of each technique, providing insights into their suitability for different scenarios. This study is part of a broader effort, in collaboration with V. Kulkarni and his team inside *Virtual Vehicule GmbH*, to develop reduced-order modeling strategies for conjugate heat transfer problems involving both solid and fluid domains, where efficient and accurate ROMs are essential for handling coupled physics at the interface.

References

[1] C. Gräßle, M. Hinze, and S. Volkwein. *Model order reduction by proper orthogonal decomposition*, pages 47–96. De Gruyter, Berlin, Boston, 2021.

[2] M. Gubisch and S. Volkwein. *Proper orthogonal decomposition for linear-quadratic optimal control*, pages 3–63. Society for Industrial and Applied Mathematics, Philadelphia, PA, 2017.

[3] B. Haasdonk. Convergence rates of the POD-greedy method. *ESAIM: Mathematical Modelling and Numerical Analysis*, 47(3):859–873, 2013.

[4] J. Kutz, S. Brunton, B. Brunton, and J. Proctor. *Dynamic Mode Decomposition.* Society for Industrial and Applied Mathematics, Philadelphia, PA, 2016.

Adaptive Reduced Basis Trust Region Methods for Parabolic Inverse Problems

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In this work, we focus on combining model order reduction and iterative regularization methods to solve parameter identification problems governed by parabolic partial differential equations. These inverse problems are computationally demanding due to the high number of required PDE solutions (forward problems). Employing model order reduction is therefore a viable strategy to enhance computational efficiency.

Our methodology builds on ideas presented in previous work [1]. A key insight is that the high dimensionality of the parameter space makes it challenging to construct a certified reduced-order model with online-efficient error estimation. To address this, we propose an algorithm, inspired by the work in [1], adapting it to parabolic problems. This algorithm involves the adaptive construction of reduced parameter spaces as well as reduced state spaces. The enrichment of both spaces can naturally be integrated into the iterative Gauss-Newton method for solving the Tikhonov-regularized inverse problem. This, in combination with an error-aware trust region framework [2, 3], ensures the reliability and accuracy of the solutions obtained by the reduced-order model.

We demonstrate the efficiency of this new method through numerical experiments on inverse parameter identification problems with distributed reaction or diffusion coefficients and provide an outlook on our ongoing work in this area.

References

[1] M. Kartmann, T. Keil, M. Ohlberger, S. Volkwein, and B. Kaltenbacher. Adaptive Reduced Basis Trust Region Methods for Parameter Identification Problems. 2023. _eprint: 2309.07627.

[2] T. Keil, L. Mechelli, M. Ohlberger, F. Schindler, and S. Volkwein. A non-conforming dual approach for adaptive trust-region reduced basis approximation of pde-constrained parameter optimization. *ESAIM: Mathematical Modelling and Numerical Analysis*, 55(3):1239–1269, 2021.

[3] Y. Yue and K. Meerbergen. Accelerating optimization of parametric linear systems by model order reduction. *SIAM Journal on Optimization*, 23(2):1344–1370, 2013.

Data-Driven Cardiac Imaging Reconstruction: A PBDW Approach for Cardiac Mechanics

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State estimation through data assimilation plays a vital role in personalizing cardiac models, though it often requires substantial computational resources. Traditional variational methods are limited in their practical applications due to their computational intensity. Reduced-order models (ROMs) [1] have emerged as efficient alternatives to full-order models to tackle this issue. Additionally, reframing state estimation as an optimal recovery challenge provides a non-intrusive data-driven perspective that supplements conventional model-based methods.

We introduce a computational framework aimed at enhancing cardiac magnetic resonance imaging (MRI) quality through sophisticated mathematical modeling and state estimation techniques. Our solution integrates the Parameterized-Background Data-Weak (PBDW) method [2] with an advanced sensor selection approach to maximize the utility of medical imaging data in industrial applications.

The framework utilizes an efficient reduced basis method combined with an innovative mini-batch worst-case orthogonal matching pursuit (wOMP) algorithm for sensor selection optimization. Our implementation handles MRI voxel data using specialized measurement functionals based on volumetric averages, designed to align with established clinical imaging protocols.

Using our in-house finite element solver CARPentry, we generated synthetic data by modeling nonlinear orthotropic mechanics on an ellipsoidal left ventricle geometry. Our method successfully reconstructs detailed 3D displacement fields from sparse measurements, with performance evaluated through relative H^1-norm comparisons. The measurement data consists of noise-enhanced 2D slices, incorporating Gaussian noise to simulate real-world conditions.

References

[1] A. Quarteroni, A. Manzoni, and F. Negri. *Reduced Basis Methods for Partial Differential Equations.* Springer International Publishing, 2016.

[2] T. Taddei. *Model order reduction methods for data assimilation; state estimation and structural health monitoring.* PhD thesis, 2016.

Advanced Model Reduction for Microvibrations Management in Space Applications

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Advanced science missions for space and Earth exploration demand high-performance payloads with high detector resolution and extended observation durations. These requirements impose stringent pointing stability, reaching nano-radian levels [2]. According to ECSS standards, microvibrations (MV) - low-amplitude vibrations spanning a broad frequency range (a few Hz to several hundred Hz) - are a primary cause of image degradation, amplified by the lightweight, large spacecraft (SC) structures in use today. Internal sources, such as rotating components (e.g., reaction wheels), are more significant than external ones like atmospheric drag or impacts. Modeling internal sources is challenging due to frequency range variability, environmental dependence, and operational conditions, necessitating large design margins for MV control actions or precise onboard measurement, as on-ground test campaigns have limited reliability. Moreover, while flagship missions adopt tailored solutions, commercial constellations with fine-pointing requirements face substantial gaps.

In this context, high-accuracy Model Order Reduction (MOR) methods are crucial for managing the high number of degrees of freedom in structural dynamics analyses. Component Mode Synthesis (CMS) methods are widely used, dividing the SC structure into smaller substructures for separate analysis and subsequent merging. While reliable and extensively used in industry, CMS methods require manual tuning based on engineering experience, lack a-priori error bounds, and retain many degrees of freedom. Additionally, they primarily focus on structural modeling, neglecting MV sources, instrument positions, and MV transmission paths.

The presented research aims to develop a new methodology that addresses the limitations of CMS procedures by leveraging advanced model order reduction techniques. Recent advancements have introduced MOR methods from other fields that are compatible with structural system requirements, particularly second-order maintenance and stability preservation, critical features for control design [1]. These methods focus on the transfer function between MV sources and instruments, offering advantages for MV analysis over traditional CMS approaches. The use of a-priori error bounds also presents the potential for automating the order reduction process. While CMS methods have limitations, their substructuring approach remains valuable for reducing complexity and supporting independent subsystem design, a standard practice in aerospace. Therefore, integrating advanced MOR methods with substructuring [3] is a central aspect of this research.

References

[1] R. S. Beddig, P. Benner, I. Dorschky, T. Reis, P. Schwerdtner, M. Voigt, and S. W. R. Werner. *Structure-Preserving Model Reduction for Dissipative Mechanical Systems*, pages 209–230. Springer Nature Switzerland, 2024.

[2] C. J. Dennehy, A. A. Wolf, and D. K. Swanson. Spacecraft line-of-sight jitter management and mitigation lessons learned and engineering best practices. Technical Report NASA/TM-20210017871, NASA, June 2021.

[3] F. Sanfedino, D. Alazard, V. Pommier-Budinger, A. Falcoz, and F. Boquet. Finite element based n-port model for preliminary design of multibody systems. *Journal of Sound and Vibration*, 415:128–146, 2018.

Symplectic Greedy Kernel Approximation of Hamiltonian Systems

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Many non-dissipative physical phenomena can be modeled as Hamiltonian systems

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}(t) &= \mathbb{J}_{2N}\nabla_{\boldsymbol{x}}\mathcal{H}(\boldsymbol{x}(t)) \qquad \text{for all } t \in I_t, \\ \boldsymbol{x}(t_0) &= \boldsymbol{x}_0, \end{aligned} \tag{1}$$

which result in energy-conserving models, i.e. $\mathcal{H}(\boldsymbol{x}(t)) = \mathcal{H}(\boldsymbol{x}(t_0))$ for all $t \in I_t$. This conservation is reflected in the flow map of a Hamiltonian system being symplectic. In recent years, research has been conducted with the objective of incorporating a symplectic structure into machine learning models, with a particular focus on the incorporation of such a structure into neural networks [1, 2, 4]. We propose an intrinsically symplectic kernel approximation scheme for learning the dynamics of a Hamiltonian system from time series data [3]. Instead of directly approximating the time series data (which would not preserve the underlying structure), a symplectic flow map is learned based on residuals from symplectic time integration schemes, similar to the approach described in [4]. By doing so, the symplectic structure of the underlying system is preserved. To learn a symplectic flow map, we apply kernel methods using suitable generalized interpolation conditions in the framework of [5]. To achieve efficient evaluation of the flow map, we employ a kernel-based model with a small expansion size, which is attained by greedily selecting interpolation points. The time series data is then approximated by evaluating the learned symplectic flow map. We demonstrate the efficiency of our approach through numerical experiments.

References

[1] Z. Chen, J. Zhang, M. Arjovsky, and L. Bottou. Symplectic recurrent neural networks. *arXiv preprint arXiv:1909.13334*, 2019.

[2] S. Greydanus, M. Dzamba, and J. Yosinski. Hamiltonian neural networks. *Advances in neural information processing systems*, 32, 2019.

[3] R. Herkert, T. Ehring, and B. Haasdonk. Symplectic greedy kernel approximation of Hamiltonian systems. 2025. in preparation.

[4] P. Horn, V. S. Ulibarrena, B. Koren, and S. P. Zwart. A generalized framework of neural networks for Hamiltonian systems. *Journal of Computational Physics*, 521:113536, 2024.

[5] H. Wendland. *Scattered Data Approximation*, volume 17 of *Cambridge Monographs on Applied and Computational Mathematics*. Cambridge University Press, Cambridge, 2005.

POD-Galerkin Reduced Order Modeling of El Niño Southern Oscillation

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Simulations of complex climate models, such as those for the El Niño Southern Oscillation (ENSO), often require weeks of computation even on advanced supercomputers. To address these challenges, reduced-order modeling (ROM) offers a cost-efficient alternative by simplifying high-dimensional state spaces. In this study, we demonstrate the efficiency, accuracy, and stability of proper orthogonal decomposition (POD) combined with Galerkin projection when applied to an ENSO model that couples atmosphere, ocean, and sea surface temperature (SST) dynamics in the equatorial Pacific. The governing nonlinear coupled PDEs, introduced by Majda and co-workers [1], are expressed as

$$\mathcal{M}\frac{\partial u}{\partial t}(t,x) = \mathcal{A}(\kappa)u(t,x) + \mathcal{N}(u,u,\mu)(t,x) \tag{1}$$

where $u(t,x) = [K^A(t,x), R^A(t,x), K^O(t,x), R^O(t,x), T(t,x)]^T$ represents atmospheric and oceanic equatorial Kelvin and Rossby waves alongside SST, $\mathcal{M}$ is a 5×5 constant matrix with diagonals $[0,0,1,1,1]$, $A(\kappa)$ is a linear operator dependent on the wind stress parameter κ, and $\mathcal{N}(u,u,\mu)$ denotes the nonlinearity with advection parameter μ. Using POD modes $\{\psi_1, ..., \psi_n\}$ derived from numerical snapshots [2] of the ENSO model (1) , we represent the solution as $u(t,x) \approx \sum_{\alpha=1}^{n} a_\alpha(t)\psi_\alpha(x)$, where $a_\alpha(t)$ are time dependent coefficients. Substituting this into the governing equations and applying Galerkin projection, we construct a reduced-order model in a finite-dimensional subspace, yielding:

$$\dot{a}_\gamma(t) = \sum_{\alpha=1}^{n} a_\alpha(t)\langle \mathcal{A}\psi_\alpha, \psi_\gamma\rangle + \sum_{\alpha,\beta=1}^{n} a_\alpha(t)a_\beta(t)\langle \mathcal{N}(\psi_\alpha,\psi_\beta),\psi_\gamma\rangle \qquad \gamma = 1, ..., n \tag{2}$$

We show that the reduced-order equations (2) achieve remarkable computational efficiency, requiring only four POD modes ($n = 4$) to accurately replicate the dynamics of the full-order model (FOM). Our approach leverages the unique coupling properties of the ENSO model, delivering stability and accuracy over 95% compared to the high-dimensional FOM. To evaluate the method's robustness, we also introduce stochastic noise into the model and demonstrate that the POD-Galerkin approach maintains high stability and accuracy under perturbations. These results highlight the potential of POD-Galerkin reduced order modeling for efficient, accurate and robust climate simulations.

References

[1] N. Chen et al. Observations and mechanisms of a simple stochastic dynamical model capturing el niño diversity. *Journal of Climate*, 31(1):449–471, 2018.

[2] L. Sirovich. Turbulence and the dynamics of coherent structures. i. coherent structures. *Quarterly of Applied mathematics*, 45(3):561–571, 1987.

Certified Reduced-Order Model Predictive Control for Evolution Equations

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In this talk, we are concerned with the solution of infinite-horizon optimal control problems of the form

$$\min_{y \in L^2((0,\infty),\mathbb{R}^p)} \mathcal{J}(u) = \int_0^\infty \tfrac{1}{2}\|y(t) - y_d(t)\|_H^2 + \tfrac{\lambda}{2}\|u(t)\|_2^2 + \mu\|u(t)\|_{L^1(\mathbb{R}^p)}\,\mathrm{d}t, \tag{1}$$

subject to (u, y) solving the PDE

$$\begin{cases} \dot{y}(t) = f(y(t)) + Bu(t) \quad t \geq 0, \\ y(0) = y_\circ \end{cases} \tag{2}$$

for control constraints $u \in U_{ad} \subset L^2((0,\infty),\mathbb{R}^p)$.
To approximate the solution of (1), we apply Model Predictive Control (MPC): the optimal control problem is solved over smaller, receding time intervals $(t_n, t_n + T)$ for some prediction horizon $T > 0$ and the solutions are concatenated in the sampling interval (t_n, t_{n+1}) for $0 < t_{n+1} < t_n + T$. First, we derive optimality conditions for these small-horizon problems and discuss their suboptimality w.r.t. (1). The repeated solution of small-horizon optimal control problems motivates model reduction: we consider (Petrov-)Galerkin reduced-order models for (2) to speed up the MPC process. To quantify the error in the case of f being a linear function, we do a full a posteriori error analysis for the optimal control, optimal state, and optimal value function of the small-horizon problems, which allows us to control the evolving error through the MPC iterations. These estimates are then used to construct two certified ROM-MPC algorithms for the solution of (1). We provide some extensions for certain classes of nonlinear functions f.

Model order reduction of bilinear port-Hamiltonian systems

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Large-scale and complex dynamical systems increasingly play a major role in mathematical modeling today. In many instances, the underlying dynamics have a physical structure that plays a fundamental role in the modeling stage. Port-Hamiltonian(pH) structure has gained prominence in recent years in modeling various dynamical systems with examples from mechanics to electrical circuits to flow dynamics (see, e.g., [3]). The pH formulation is a general structure based on a Hamiltonian function associated with the total stored energy. Furthermore, the formulation allows one to seamlessly integrate dissipation, interconnection, and control terms into the (possibly implicit) differential equations. For high-dimensional pH systems, the task of structure-preserving model order reduction arises, i.e., the goal is to ensure that the reduced model retains the pH structure. For linear dynamical systems, there are various methods to achieve this goal (see, e.g.,[1, 2, 4]). In this talk, we extend the pH-preserving model reduction to a special class of nonlinear systems, namely the bilinear pH systems described by the differential equations of the form

$$\begin{aligned}\dot{x}(t) &= (J-R)Qx(t) + \left(B + \sum_{k=1}^{n} N_k x_k(t)\right) u(t),\\ y(t) &= \left(B + \sum_{k=1}^{n} N_k x_k(t)\right)^T Qx(t)\end{aligned}$$

with state $x(t) \in \mathbb{R}^n$, input $u(t) \in \mathbb{R}^m$, output $y(t) \in \mathbb{R}^n$ and matrices $J = -J^T \in \mathbb{R}^{n\times n}$, $R = R^T \geq 0$, $R \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $N_k \in \mathbb{R}^{n\times n}$, $Q = Q^T \geq 0, Q \in \mathbb{R}^{n\times n}$ with Hamiltonian $\mathcal{H}(x) = \frac{1}{2}x^T Qx$.
In this talk, we will first establish certain theoretical properties of such bilinear pH systems and then develop structure-preserving model order reduction in this case.

References

[1] T. Breiten and B. Unger. Passivity preserving model reduction via spectral factorization. *Automatica*, 142:110368, Aug. 2022.

[2] S. Gugercin, R. V. Polyuga, C. Beattie, and A. van der Schaft. Structure-preserving tangential interpolation for model reduction of port-Hamiltonian systems. *Automatica*, 48(9):1963–1974, Sept. 2012.

[3] V. Mehrmann and B. Unger. Control of port-Hamiltonian differential-algebraic systems and applications. *Acta Numerica*, 32:395–515, May 2023.

[4] R. V. Polyuga and A. J. van der Schaft. Effort- and flow-constraint reduction methods for structure preserving model reduction of port-Hamiltonian systems. *Systems & Control Letters*, 61(3):412–421, Mar. 2012.

Structure-preserving model reduction on manifolds for port-Hamiltonian systems

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For many physical phenomena, it is important to find suitable systems to represent them such that the change of system internal energy remains less than or equal to the energy exchange to the environment. Port-Hamiltonian (pH) systems provide a robust framework for achieving this as they can be connected by their ports and naturally have stability and passivity [4]. However, the simulation cost of a high dimensional pH system is high. To overcome this bottleneck, structure-preserving model order reduction (MOR) methods are needed. Although lots of algorithms have been proposed for structure-preserving MOR recently, most of them are linear subspace-based algorithms for linear pH systems [2, 3]. In this work, we propose a new structure-preserving MOR method for linear (nonlinear) pH systems. The full-order model is reduced to a reduced-order model (ROM) via the generalized manifold Galerkin projection [1], where the projection basis is chosen based on a generalized matrix inverse. Later, we discuss the moment-matching ability and an a priori error bound of the ROMs. In the numerical example, both the mass-spring-damper system and the nonlinear Toda lattice network are investigated. The results show that the proposed algorithm has lower errors in both the time domain and the frequency domain compared to classical structure-preserving MORs.

References

[1] P. Buchfink, S. Glas, B. Haasdonk, and B. Unger. Model reduction on manifolds: A differential geometric framework. *Physica D: Nonlinear Phenomena*, 468:134299, 2024.

[2] S. Gugercin, R. V. Polyuga, C. A. Beattie, and A. J. van der Schaft. Interpolation-based H_2 model reduction for port-hamiltonian systems. In *Proceedings of the 48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference*, pages 5362–5369. IEEE, 2009.

[3] R. V. Polyuga and A. J. van der Schaft. Effort-and flow-constraint reduction methods for structure preserving model reduction of port-hamiltonian systems. *Systems & Control Letters*, 61(3):412–421, 2012.

[4] A. Van Der Schaft, D. Jeltsema, et al. Port-hamiltonian systems theory: An introductory overview. *Foundations and Trends® in Systems and Control*, 1(2-3):173–378, 2014.

The Kolmogorov N-width for linear transport: Exact representation and the influence of the data

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The Kolmogorov N-width describes the best possible error one can achieve by elements of an N-dimensional linear space. Its decay has extensively been studied in Approximation Theory and for the solution of Partial Differential Equations (PDEs). Particular interest has occurred within Model Order Reduction (MOR) of parameterized PDEs e.g. by the Reduced Basis Method (RBM).
While it is known that the N-width decays exponentially fast (and thus admits efficient MOR) for certain problems, there are examples of the linear transport and the wave equation, where the decay rate deteriorates to $N^{-1/2}$. On the other hand, it is widely accepted that a smooth parameter dependence admits a fast decay of the N-width. However, a detailed analysis of the influence of properties of the data (such as regularity or slope) on the rate of the N-width seems to lack.
In this paper, we state that the optimal linear space is a direct sum of shift-isometric eigenspaces corresponding to the largest eigenvalues, yielding an exact representation of the N-width as their sum. For the linear transport problem, which is modeled by half-wave symmetric initial and boundary conditions g, we obtain such an optimal decomposition by sorted trigonometric functions with eigenvalues that match the Fourier coefficients of g. Further the sorted eigenfunctions give for normalized $g \in H^r$ of broken order $r > 0$ the sharp upper bound of the N-width, which is a reciprocal of a certain power sum. Yet for ease, we also provide the decay $(\pi N)^{-r}$, obtained by the non-optimal space of ordering the trigonometric functions by frequency rather than by eigenvalue.
Our theoretical investigations are complemented by numerical experiments which confirm the sharpness of our bounds and give additional quantitative insight.

References

[1] Andrei Kolmogoroff. Uber Die Beste Annaherung Von Funktionen Einer Gegebenen Funktionenklasse. *Annals of Mathematics*, 1936.

[2] Alan Pinkus. N-Widths in Approximation Theory. *Springer*, 1985.

[3] Mario Ohlberger and Stephan Rave. Reduced Basis Methods: Success, Limitations and Future Challenges. *Proceedings of the Conference Algoritmy*, pp. 1–12, 2016.

[4] Constantin Greif and Karsten Urban. Decay of the Kolmogorov N-width for wave problems. *Applied Mathematics Letters*, 2019.

Geometric model order reduction applied to brain geometries

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In recent years, computational mechanics has emerged as a key tool for understanding the dynamics of how neurodegenerative diseases like dementia, Alzheimer's dementia and Parkinson develop in the human brain. These illnesses are thought to be tied to insufficient clearance of toxic proteins from the brain through the so-called glymphatic system. Tools have therefore been developed for solving computational problems on brain geometries derived from magnetic resonance imaging (MRI)[3]. However, the applicability of this pipeline for high-throughput patient-specific simulations remains limited, due to the considerable computational cost involved in performing simulations on geometries as complex as the human brain. We therefore propose an approach for transforming solutions from one brain to another, treating the geometry as a parameter for the construction of a reduced basis on a target brain. The particular challenge of brain geometries is their incredible complexity, making exact brain-to-brain mappings impossible. Our approach therefore takes advantage of image registration tools[1], which compute deformation fields by iteratively matching one MRI to another. These deformation fields are subsequently used to map solutions between brains, thus avoiding issues which can arise in mesh-to-mesh deformations. We first illustrate this approach on simple geometries and then proceed to apply the method to a dataset of more than 100 real human brains, using a simplified multi-compartmental poroelastic model of glymphatic dynamics[2].

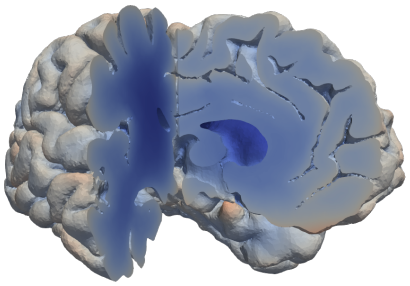

Figure 1: Example profile computed from a glymphatic model on a human brain

References

[1] B. Avants, C. Epstein, M. Grossman, and J. Gee. Symmetric diffeomorphic image registration with cross-correlation: Evaluating automated labeling of elderly and neurodegenerative brain. *Medical Image Analysis*, 12(1):26–41, 2008. Special Issue on The Third International Workshop on Biomedical Image Registration WBIR 2006.

[2] L. W. Dreyer, A. Eklund, M. E. Rognes, J. Malm, S. Qvarlander, K.-H. Støverud, K.-A. Mardal, and V. Vinje. Modeling csf circulation and the glymphatic system during infusion using subject specific intracranial pressures and brain geometries. *Fluids and Barriers of the CNS*, 21(1):82, 2024.

[3] K.-A. Mardal, M. Rognes, T. Thompson, and L. Valnes. *Mathematical Modeling of the Human Brain: From Magnetic Resonance Images to Finite Element Simulation*. 01 2022.

Model order reduction in 4D-var data assimilation: An optimal control based reformulation and its advantages for model order reduction methods

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The goal of data assimilation is to update a mathematical model with observations from the real world. In this talk, the application of model order reduction techniques to 4D-var data assimilation problems for parabolic systems will be addressed. To this end, we consider the data assimilation problem in the context of optimal control and examine the initial control problem

$$\operatorname*{argmin}_{v\in U_{\mathrm{ad}}}\Big\{ J(v) := \frac{1}{2}\left\|y(v)-y^{(d)}\right\|^2_{L^2(\Omega_T)} + \frac{\alpha}{2}\left\|v-y^{(b)}\right\|^2_{L^2(\Omega)} \Big\}, \tag{1}$$

subject to

$$\begin{cases} y_t + \mathcal{A}y &= f \quad \text{in } \Omega_T, \\ y &= 0 \quad \text{on } \Sigma_T, \\ y(0) &= v \quad \text{in } \Omega, \end{cases} \tag{2}$$

where $\Omega \subseteq \mathbb{R}^d$ for which we assume that it is open, bounded and has a sufficiently smooth boundary $\Gamma := \partial\Omega$ and $\Sigma_T := (0,T]\times\Gamma$. We assume the second order elliptic differential operator $\mathcal{A} : L^2(\Omega_T) \to L^2(\Omega_T)$ to be self-adjoint and choose $f, y^{(d)} \in L^2(\Omega_T)$, $y^{(b)} \in L^2(\Omega)$, $\alpha > 0$ as well as the set of admissible controls $U_{\mathrm{ad}} \subseteq L^2(\Omega)$.

While standard model order reduction techniques are straight forward to apply to this control problem, we seek a new foundation for model order reduction by rewriting the optimality conditions as a second order in time, fourth order in space elliptic system. While such reformulations have already proven to be effective to solve control problems with distributed control (see among others [1, 2]), it has recently been adapted and studied in the context of initial control in [3].

In this presentation, the application of model order reduction techniques to the reformulated systems will be discussed. For that purpose, we investigate the possibilities of proper orthogonal decomposition (POD), dynamic mode decomposition (DMD) and related techniques. Furthermore, we discuss the possibility to introduce adaptive time grids and demonstrate how reduced order models can benefit from such adaptive grids (see [4] for further reference).

References

[1] A. Alla, C. Gräßle, and M. Hinze. Time adaptivity in model predictive control. *Journal of Scientific Computing*, 90(1):12, 2021.

[2] W. Gong, M. Hinze, and Z. Zhou. Space-time finite element approximation of parabolic optimal control problems. *Journal of Numeral Mathematics*, 20(2):111–146, 2012.

[3] C. Gräßle and J. Marquardt. A time adaptive optimal control approach for 4D-var data assimilation problems governed by parabolic PDEs, 2024. arXiv:2407.20866, submitted.

[4] C. Gräßle and J. Marquardt. Towards time adaptive observations for model order reduction in data assimilation. In *11th Vienna International Conference on Mathematical Modelling (MATHMOD 2025)*, 2025.

Surrogate to Poincare inequalities on manifolds for dimension reduction in nonlinear feature spaces

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This work focuses on approximating a differentiable function $u : \mathbb{R}^d \to \mathbb{R}$ with $d \gg 1$ by a composition of functions $f \circ g$ where $g : \mathbb{R}^d \to \mathbb{R}^m$ and $f : \mathbb{R}^m \to \mathbb{R}$. The approximation error is assessed in the L^2_μ-norm where μ is some probability measure on $\mathbb{R}^d$. The approach considered is two-staged. Firstly the feature map g is selected among some prescribed functional class by minimizing some function $\mathcal{J}$ involved in the upper bound of the approximation error

$$\min_{f:\mathbb{R}^m\to\mathbb{R}} \mathbb{E}_\mu(|u - f \circ g|^2) \leq C_\mu \mathcal{J}(g), \tag{1}$$

which is based on Poincare inequalities and requires evaluations of ∇u.
Secondly the function f is built using classical regression methods. Until recently, bounds of the form (1) were only available for linear feature maps g. This framework has been extensively studied under the name Active Subspace and the solution is given by the eigenvectors of the matrix $\mathbb{E}(\nabla u \nabla u^T) \in \mathbb{R}^d$. This approach is easy to implement, computationally efficient, has robust theoretical guarantees for some classical probability laws μ, and showed good performances in various numerical applications. However, there are many functions u for which such an approximation with $m < d$ is known to be not efficient.
Therefore, recent works consider non-linear feature maps in order to produce better dimension reduction. More especially, we will focus on the work from [1, 2] in which authors leverage Poincare inequalities on smooth manifolds to obtain a bound of the form (1) for non-linear g. Although there are less theoretical guarantees, their numerical experiments showed improved performances compared with linear featuring. However, minimizing $\mathcal{J}$ is now much harder than finding eigenvectors of some matrix, and can only be done using iterative descent methods.
In this work we consider feature maps as in [1], of the form $g(x) = G^T\Phi(x)$ with $G \in \mathbb{R}^{K\times m}$ and where $\Phi : \mathbb{R}^d \to \mathbb{R}^K$, $K \geq d$, is fixed. We study a new quantity, denoted $\mathcal{L}(g)$, which can be expressed as the minimal singular value of some positive semi-definite matrix. We show that for a compact set of polynomial feature maps with $m = 1$, for some class of probability distributions, any minimizer g^* of $\mathcal{L}$ satisfies the sub-optimality result

$$\mathcal{J}(g^*) \lesssim \min_g \mathcal{J}(g)^\beta,$$

where $0 < \beta \leq 1$ is some constant which depends on the degree. We also extend this approach to the case $m > 1$ as well as for simultaneously learning a parametrized family of functions $u_y \in L^2_\mu$ by $y \in \mathcal{Y}$, although the theoretical results are weaker. Finally, we provide numerical examples to illustrate the performances of g^*, both as the feature map used in the regression step, or as the initializer for some iterative descent algorithm for minimizing $\mathcal{J}$.

References

[1] D. Bigoni, Y. Marzouk, C. Prieur, and O. Zahm. Nonlinear dimension reduction for surrogate modeling using gradient information. *Information and Inference: A Journal of the IMA*, 11(4):1597–1639, Dec. 2022.

[2] R. Verdière, C. Prieur, and O. Zahm. Diffeomorphism-based feature learning using Poincaré inequalities on augmented input space, Dec. 2023.

Time-adaptive Optimisation-Based Domain-Decomposed Reduced Order Models for Fluid-Structure Interaction Problems

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Fluid-Structure Interaction (FSI) problems, which describe the dynamic interplay between a fluid and a solid, present significant mathematical challenges due to the complex coupling at the FSI interface. This interface, shared by the fluid and solid subdomains, evolves based on the dynamics of both, making its profile unknown a priori. Despite extensive research, a unified mathematical approach to FSI remains elusive, partly due to the intricate nature of the Navier-Stokes and elastic solid equations that govern the problem.

Traditionally, FSI problems are tackled using either partitioned or monolithic approaches. Partitioned algorithms leverage existing computational tools for fluid and structural dynamics, coupled through iterative procedures, while monolithic algorithms solve the coupled problem simultaneously by imposing global fluid-structure spaces. Although partitioned approaches are computationally advantageous and highly parallelisable, they often suffer from stability and convergence issues, particularly due to the "added-mas" effect common in cardiovascular applications.

This work aims to introduce a domain–decomposition framework for the construction of stable localised reduced order models for FSI problems. In particular, we employ the optimisation-based DD approach, which ensures physical consistency across subdomains [1, 3]. Given the complexity of FSI problems and the need for small time steps to ensure numerical convergence, we will explore initial attempts to incorporate time-adaptivity [4] into the domain-decomposed model order reduction framework. This requires more effective optimisation algorithms to overcome the slow, local convergence of classical minimisation methods. In particular, we will discuss a trust-region approach to address this challenge and get a look at the preliminary numerical results.

The talk will conclude with a brief discussion of the heterogenous optimisation–based coupling of reduced order models [2] as well as ongoing endeavours in the development of efficient multi-domain decomposition techniques.

References

[1] I. Prusak, M. Nonino, D. Torlo, F. Ballarin, and G. Rozza. An optimisation-based domain-decomposition reduced order model for the incompressible Navier-Stokes equations. *Computers & Mathematics with Applications*, 151:172–189, 2023.

[2] I. Prusak, D. Torlo, M. Nonino, and G. Rozza. Optimisation-based coupling of finite element model and reduced order model for computational fluid dynamics, 2024. (accepted).

[3] I. Prusak, D. Torlo, M. Nonino, and G. Rozza. An optimisation-based domain-decomposition reduced order model for parameter-dependent non-stationary fluid dynamics problems. *Computers & Mathematics with Applications*, 166:253–268, 2024.

[4] I. Prusak, D. Torlo, M. Nonino, and G. Rozza. A time-adaptive algorithm for pressure dominated flows: a heuristic estimator, 2024. (accepted).

Model order reduction for elastodynamic boundary element formulations

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Addressing soil-structure interaction and wave propagation challenges is essential in dynamic civil engineering analyses, facilitating applications such as environmental vibration prediction and seismic wave modeling. While these problems are inherently high-dimensional, complex and computationally demanding, boundary element (BE) methods have proven to be indispensable for making these analyses feasible. Advancements in fast BE methods such as the fast multiple method [1] and methods utilizing hierarchical matrices [2] have further improved their applicability to large-scale problems. However, the computation of fundamental solutions, or Greens functions, across multiple source-receiver configurations remains a significant computational bottleneck.

We demonstrate the potential of model order reduction (MOR) techniques within the BE framework by deriving Green's functions in a decomposed form using Proper Generalized Decomposition (PGD) [3, 4]. PGD assumes a separable form of the multi-dimensional field, where each separated component is computed iteratively in a greedy manner. The algorithm convergence is enhanced by employing a greedy Tucker algorithm [5]. The formulation incorporates source/receiver positions, frequency, wavenumber, and material properties as coordinates. By leveraging low-rank approximations, the boundary integral equations are reduced to a series of decoupled one-dimensional problems. Iterative solvers significantly boost computational efficiency by operating on these decoupled functions, bypassing the assembly of densely populated BE system matrices.

We demonstrate this approach through proof-of-concept examples involving a surface foundation on a layered halfspace subjected to various boundary conditions. The study extends to a relatively complex problem with an embedded foundation, incorporating a regularization approach. The conclusions highlight a comparative analysis of solution accuracy and computational efficiency, focusing on memory requirements and CPU time, between the reduced order BE formulation and the prevailing BE method.

References

[1] M. Bonnet, S. Chaillat, and J. Semblat. Multi-level fast multipole BEM for 3-D elastodynamics. *Recent Advances in Boundary Element Methods*, pages 15–28, 2005.

[2] W. Hackbusch. A sparse matrix arithmetic based on H-matrices. Part I: Introduction to H-matrices. *Computing 62(2)*, pages 89–108, 1999.

[3] F. Chinesta, R. Keunings, and A. Leygue. The Proper Generalized Decomposition for advanced numerical simulations: a Primer. *Springer Briefs in Applied Sciences and Technology*, 2013.

[4] A. Nouy. A priori model reduction through Proper Generalized Decomposition for solving time-dependent partial differential equations. *Computer Methods in Applied Mechanics and Engineering*, 199(23–24)(23–24):1603–1626, 2010.

[5] I. Georgieva and C. Hofreither. Greedy low-rank approximation in Tucker format of solutions of tensor linear systems. *Journal of Computational and Applied Mathematics*, 358:206–220, 2019.

Trust-Region Optimization using Hermite Kernel Interpolation for high-dimensional problems

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In this work we propose a Trust-Region (TR) optimization scheme using Hermite kernel interpolation to address high-dimensional optimization tasks with computationally expensive objective functions. Direct optimization of such functions often becomes impractical due to the high cost of function (and gradient) evaluations. A common instance of such an optimization problem can be a PDE-constrained optimization problem of the form

$$\min_{\mu \in \mathcal{P}} \mathcal{J}(u(\mu); \mu),$$

where $u(\cdot; \mu) \in H$ is the solution of the primal equation

$$a(u(\mu), v; \mu) = f(v; \mu) \qquad \forall v \in H.$$

with an appropriate function space H. To enhance computational efficiency, we leverage TR methods, which restrict the search for the next iterate to a region around the current iterate, called Trust-Region, and approximate the objective function using an efficient surrogate model. In our approach, the surrogate is constructed using Hermite kernel interpolation [1, 3]. Kernel-based methods, particularly as mesh-free techniques, are well-suited for medium- to high-dimensional problems due to their robustness against the curse of dimensionality. By adopting Hermite interpolation, which approximates both function values and gradients, we improve the accuracy and effectiveness of the surrogate model.

The algorithmic framework for the Hermite kernel TR algorithm is motivated by [2], which uses Reduced Basis surrogate models. To establish convergence under reasonable assumptions for the Hermite kernel TR algorithm, we extend the convergence analysis provided in [4], aligning it with the Hermite kernel framework. In addition, we introduce a scheme to optimize the shape of the kernel during the optimization process. To evaluate the performance of the Hermite kernel TR algorithm, we conduct numerical experiments on various low- and high-dimensional optimization problems.

References

[1] T. Ehring and B. Haasdonk. Hermite kernel surrogates for the value function of high-dimensional nonlinear optimal control problems, 2023.

[2] T. Keil, L. Mechelli, M. Ohlberger, F. Schindler, and S. Volkwein. A non-conforming dual approach for adaptive Trust-Region reduced basis approximation of PDE-constrained parameter optimization. *ESAIM: Mathematical Modelling and Numerical Analysis*, 55(3):1239–1269, May 2021.

[3] H. Wendland. *Scattered Data Approximation*. Cambridge Monographs on Applied and Computational Mathematics. Cambridge University Press, 2004.

[4] Y. Yue and K. Meerbergen. Accelerating optimization of parametric linear systems by model order reduction. *SIAM Journal on Optimization*, 23(2):1344–1370, 2013.

A Novel Model Order Reduction Method for Bayesian Linear Inverse Problems

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The Bayesian approach to solving inverse problems updates a prior distribution to a posterior distribution of the parameters through data [1]. The likelihood of the data needed to obtain the posterior distribution depends on a computational model for the numerical solution of a partial differential equation. Standard procedures for solving the Bayesian inverse problem are based on multiple evaluations of the likelihood function and, hence, the underlying computational model. The application of model order reduction enables a cheaper solution of the inverse problem for resource demanding computational models.

We focus on Bayesian linear inverse problems with Gaussian parameter priors and sparse data. As limited data is only informative in a low-dimensional subspace of the parameter space, the posterior in such problems can be expressed as a low-rank update of the prior distribution [2]. In this work, we introduce a framework that leverages the low-rank mapping between the data and the parameter itself to construct a reduced-order model for inverse problems. This approach is based on the leading eigendirections of the Fisher information matrix and the prior precision and is referred to as likelihood-informed subspace (LIS). Through numerical examples, we compare the LIS reduced-order model with reduced-order models obtained using proper orthogonal decomposition (POD).

References

[1] M. Dashti and A. M. Stuart. *The Bayesian Approach to Inverse Problems.* Springer International Publishing, 2017.

[2] A. Spantini et al. Optimal Low-Rank Approximations of Bayesian Linear Inverse Problems. *SIAM Journal on Scientific Computing*, 37:A2451–A2487, 2015.

Computing Chaotic Time-Averages from Few Periodic Orbits

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Chaotic systems challenge traditional methods of predicting long-term behavior due to their sensitivity to initial conditions. In contrast, infinite-time averages over the chaotic dynamics offer robust characterizations of the system that do not depend on initialization. Under very restrictive assumptions, it can be shown that Axiom A [4] dynamical systems admit a reduced order statistical model in which infinite-time averages can be expanded as a weighted sum of finite-time averages over unstable periodic orbits (UPOs) [3]. For decades, practitioners have attempted to generalized this Periodic Orbit Theory (POT) to high-dimensional systems of physical significance [1]. However, the assumptions of POT are rarely satisfied in such systems and in high dimensions it is computationally intractable to compute sufficiently many periodic orbits for the expansion to converge. Nevertheless, a reduced order description of chaotic time-averages has remained enticing, and recent literature has begun seeking alternatives to POT that, in particular, work in high-dimensions where periodic orbits are sparse [8, 5]. This work introduces a data-driven, interpretable approach to computing optimal weights. The method utilizes Kriging [2], which is known for its accuracy in applications to high-dimensional data sets [7, 6]. The method is also quite simple. Namely, for any finite collection of P periodic orbits, we derive a linear system, $A\vec{w} = \vec{b}$, that optimal weights $\vec{w} \in \mathbb{R}^P$ must approximately satisfy. We refer to the least-squares solution of this system as the Least Squares Weighting (LSW) of periodic orbits. We find that the LSW outperforms POT when P is small, and converges more quickly as a function of increasing P. Moreover, we show that the solution space to the linear system is degenerate and, for fixed P, can be solved by multiple, qualitatively distinct choices of $\vec{w}$. These results suggest that not only is an expansion of infinite-time averages in terms of few periodic orbits *possible*, but that multiple such expansions exist. While inspired by UPO expansions, this method can be used to expand infinite-time averages in terms of *any* subsets of the chaotic set. Hence, this method opens exciting new opportunities to derive statistical models of chaotic systems from non-linear modes or sub-manifolds of the deterministic dynamics, thereby connecting simplified statistical and deterministic models of chaos.

References

[1] G. J. Chandler and R. R. Kerswell. Invariant recurrent solutions embedded in a turbulent two-dimensional kolmogorov flow. *J. Fluid Mech.*, 722:554–595, 2013.

[2] N. Cressie. The origins of kriging. *Mathematical Geology*, 22(3):239â252, Apr. 1990.

[3] P. Cvitanović, R. Artuso, R. Mainieri, G. Tanner, and G. Vattay. *Chaos: Classical and Quantum.* Niels Bohr Institute, 2020.

[4] J. P. Eckmann and D. Ruelle. Ergodic theory of chaos and strange attractors. *Rev. Mod. Phys.*, 57(3):617–656, 1985.

[5] J. Page, P. Norgaard, M. P. Brenner, and R. R. Kerswell. Recurrent flow patterns as a basis for two-dimensional turbulence: Predicting statistics from structures. *Proc. Natl. Acad. Sci. U. S. A.*, 121(23):e2320007121, June 2024.

[6] Z. Song, Z. Liu, H. Zhang, and P. Zhu. An improved sufficient dimension reduction-based kriging modeling method for high-dimensional evaluation-expensive problems. *Computer Methods in Applied Mechanics and Engineering*, 418:116544, Jan. 2024.

[7] T. Wang, Z. Chen, G. Li, J. He, C. Liu, and X. Du. A novel method for high-dimensional reliability analysis based on activity score and adaptive kriging. *Reliability Engineering & System Safety*, 241:109643, Jan. 2024.

[8] G. Yalnız, B. Hof, and N. B. Budanur. Coarse graining the state space of a turbulent flow using periodic orbits. *Phys. Rev. Lett.*, 126(24):244502, 2021.

Non-intrusive reduced-order modeling for dynamical systems with spatially localized features

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In this talk we present a non-intrusive reduced-order modeling framework for dynamical systems with spatially localized features characterized by slow singular value decay [1]. The proposed approach builds upon two existing methodologies for reduced and full-order non-intrusive modeling, namely Operator Inference (OpInf) and sparse Full-Order Model (sFOM) inference. We decompose the domain into two complementary subdomains which exhibit fast and slow singular value decay, accordingly. The dynamics on the subdomain exhibiting slow singular value decay are learned using sFOM while the dynamics with intrinsically low dimensionality on the complementary subdomain are learned using OpInf. The resulting, coupled OpInf-sFOM formulation leverages the computational efficiency of OpInf and the high spatial resolution of sFOM and thus enables fast non-intrusive predictions for localized features with slow singular value decay, such as transport-dominated dynamics. Furthermore, we propose a novel, stability-promoting regularization technique with a closed-form solution based on the Gershgorin disk theorem for both sFOM and OpInf models and evaluate the efficiency of the coupled approach in terms of offline and online speedup. Finally, we demonstrate the capabilities of the coupled OpInf-sFOM formulation for testcases such as the one-dimensional, viscous Burgers' equation and a two-dimensional parametric model for the Pine Island Glacier ice thickness dynamics.

References

[1] L. Gkimisis, N. Aretz, M. Tezzele, T. Richter, P. Benner, and K. E. Willcox. Non-intrusive reduced-order modeling for dynamical systems with spatially localized features. Research article in preparation, Max Planck Institute, Magdeburg, July 2024.

Sparse Identification for bifurcating phenomena in Computational Fluid Dynamics

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This work investigates model reduction techniques for complex parameterized PDEs, specifically focusing on bifurcating phenomena in Computational Fluid Dynamics (CFD). We develop an interpretable, non-intrusive reduced-order model capable of capturing nonlinear dynamics associated with bifurcations by identifying a minimal set of coordinates.

Our methodology combines the Sparse Identification of Nonlinear Dynamics (SINDy) method with a deep learning framework based on Autoencoder (AE) architectures. To enhance dimensionality reduction, we integrate a two-level Proper Orthogonal Decomposition (POD) with the SINDy-AE architecture. This novel combination enables efficient discovery of system dynamics while maintaining interpretability of the reduced model.

We demonstrate our approach through two challenging test cases: a pitchfork bifurcation in a sudden-expansion channel and a Hopf bifurcation in a contraction-expansion channel. For the sudden-expansion case, we present a comprehensive analysis of the full-order behavior of solutions. These benchmarks validate the effectiveness of our Reduced Order Models (ROMs) in terms of both accuracy and computational efficiency. The results show successful reconstruction of steady-state solutions, accurate prediction of system evolution for unseen parameter values, and significant speed-up compared to full-order methods.

Keywords: Reduced Order Modeling; Sparse Identification; Bifurcation Problems; Computational Fluid Dynamics; Nested Proper Orthogonal Decomposition; Coand? Effect; Autoencoders.

Accelerating Kernel-Based Dynamic Mode Decomposition through Randomized Column Subsampling

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Dynamic Mode Decomposition (DMD) and the underpinning Koopman operator theory enjoy large popularity in scientific applications with unknown or unavailable operators. For a discrete dynamical system on a manifold $\mathcal{M}$ given by $y_i = F(x_i)$ the Koopman operator $\mathcal{K}$ is the linear operator which maps the observable $\psi\colon \mathcal{M} \to \mathbb{C}$ to a function $\mathcal{K}[\psi]$ such that $\mathcal{K}[\psi](x_i) = \psi(F(x_i))$. Numerical applications are interested in approximations of the eigenpairs of $\mathcal{K}$ to compute physical quantities, make predictions of future states, or investigate low-rank representations of trajectories from the input data $X = [x_1, x_2, \dots, x_S], Y = [y_1, y_2, \dots, y_S]$.

In the case of nonlinear observables there exist two dominating frameworks to compute approximate eigenpairs: Extended DMD (EDMD) and kernel-based DMD (KDMD), both of which construct a surrogate operator $\hat{K} = G^{\dagger}A$ for finding eigenpairs. EDMD uses an explicit dictionary $(\psi)_{i=1}^{B}$ of observables. Here, the computational costs associated with the assembly of $\hat{K}$ are dominated by the number of observables $B \gg 0$. KDMD, on the other hand, utilizes the kernel trick with a fixed choice of kernel function $k(x, x')$. This affects the cost of assembly of $\hat{K}$ such that the number of available data snapshots $S \gg 0$ dominate the computations.

In an effort to allow both large numbers of snapshots $S \gg 0$ and large numbers of observables $B \gg 0$, DeGennaro and Urban [2] used Bochner's theorem for positive definite and translation-invariant kernels to transform the KDMD approach back into the EDMD setting with a reduced number of observables $b \ll B$. Bochner's theorem states that any such kernel k is the Fourier transform of a probability measure μ. To approximate this Fourier transform one applies randomized Fourier features (RFF) [3], which results in a Monte Carlo-like quadrature where the samples are drawn from the distribution given by μ. Similar to other Monte Carlo methods, the RFF maximal error's expectation converges with an order of $\mathcal{O}(R^{-1/2})$, cf. [4], where $R > 0$ is the chosen number of random features.

Instead of an RFF approximation we envisage a low-rank approximation of the kernel matrix $G_{ij} = k(x_i, x_j)$ using RPCholesky [1]. For smooth kernels the singular values of the kernel matrix G will decay, therefore justifying this approach. This approach has the major advantage that RPCholesky is spectrally accurate, whence it can be expected that it produces a better approximation than RFF.

References

[1] Y. Chen, E. N. Epperly, J. A. Tropp, and R. J. Webber. Randomly pivoted Cholesky: Practical approximation of a kernel matrix with few entry evaluations. *Comm. Pure Appl. Math.*, 2024.

[2] A. M. DeGennaro and N. M. Urban. Scalable extended dynamic mode decomposition using random kernel approximation. *SIAM J. Sci. Comput.*, 41(3):A1482–A1499, 2019.

[3] A. Rahimi and B. Recht. Random features for large-scale kernel machines. In *Advances in Neural Information Processing Systems*, volume 20. Curran Associates, Inc., 2007.

[4] D. J. Sutherland and J. Schneider. On the error of random Fourier features. In *Proceedings of the Thirty-First Conference on Uncertainty in Artificial Intelligence*, UAI'15, pages 862–871, Arlington, Virginia, USA, 2015. AUAI Press.

Generalizing the optimal interpolation points for IRKA

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In this work, we consider systems with poles in general domains, allowing for non-asymptotically stable systems. We build upon an extended version of the iterative rational Krylov algorithm (IRKA) introduced in [1] and propose an efficient method for computing the interpolation points with respect to simply connected analytic closed curves. Specifically, we show that the optimal interpolation conditions for the extended IRKA are related to the Schwarz function. Additionally, we employ the adaptive Antoulas-Anderson (AAA) algorithm introduced in [2] to compute a rational approximation of the interpolation points given only boundary samples of user-defined domains.

References

[1] A. Borghi and T. Breiten. $\mathcal{H}_2$ optimal rational approximation on general domains. *Advances in Computational Mathematics*, 50:28, 2024.

[2] Y. Nakatsukasa, O. Sète, and L. N. Trefethen. The AAA algorithm for rational approximation. *SIAM Journal on Scientific Computing*, 40(3):A1494–A1522, 2018.

Three Ways to Treat Time in Reduced Basis Methods

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Similar to [1] we compare three ways to treat time within reduced basis methods for instationary parameterized partial differential equations. More precisely, we look at parabolic problems where we are only interested in the state (or a function of the state) at the final time $t = T$ or in a partial time window $[T\Lambda^{-1}, T]$ with $\Lambda > 1$.
The first method in our comparison can be considered the standard approach. We solve our instationary problem with a time stepping scheme after descretizing it in time. The reduced basis is then formed in space by the POD-Greedy method, [3].
The second considered approach is based upon a space-time variational formulation of the parabolic problem. Here, we consider time as an additional variable and solve a $d + 1$ dimensional Petrov-Galerkin problem (where d is our spatial dimension). The reduced basis can then be formed by a standard greedy approach, [4].
For the third method we transform our problem via the Laplace transform from the time to the frequency domain. The numerical inversion is done by a contour integral method, while the reduced basis is formed in the frequency domain by a greedy method, [2].
We provide numerical investigations focusing on how the choices of T and Λ influence the approximation quality and efficiency of the reduced basis, as well as the reliability of the error estimator. Furthermore, the advantages and disadvantages of the three presented approaches are discussed.

References

[1] S. Glas, A. Mayerhofer, and K. Urban. Two ways to treat time in reduced basis methods. *Model reduction of parametrized systems*, pages 1–16, 2017.

[2] N. Guglielmi and M. Manucci. Model order reduction in contour integral methods for parametric pdes. *SIAM Journal on Scientific Computing*, 45(4):A1711–A1740, 2023.

[3] B. Haasdonk. Convergence rates of the pod–greedy method. *ESAIM: Mathematical modelling and numerical Analysis*, 47(3):859–873, 2013.

[4] K. Urban and A. Patera. An improved error bound for reduced basis approximation of linear parabolic problems. *Mathematics of Computation*, 83(288):1599–1615, 2014.

Optimal Transport-based Displacement Interpolation with Data Augmentation for Reduced Order Modeling of Nonlinear Dynamical Systems

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We present a novel reduced-order Model (ROM) that leverages optimal transport (OT) theory [2] and displacement interpolation to enhance the representation of nonlinear dynamics in complex systems. While traditional ROM techniques face challenges in this scenario, especially when data (i.e., observational snapshots) are limited, our method addresses these issues by introducing a data augmentation strategy based on OT principles [1]. The proposed framework generates interpolated solutions tracing geodesic paths in the space of probability distributions, enriching the training dataset for the ROM. A key feature of our approach is its ability to provide a continuous representation of the solution's dynamics by exploiting a virtual-to-real time mapping. This enables the reconstruction of solutions at finer temporal scales than those provided by the original data.

To further improve prediction accuracy, we employ Gaussian Process Regression to learn the residual and correct the representation between the interpolated snapshots and the physical solution. We demonstrate the effectiveness of our methodology with atmospheric mesoscale benchmarks characterized by highly nonlinear, advection-dominated dynamics. Our results show improved accuracy and efficiency in predicting complex system behaviors, indicating the potential of this approach for a wide range of applications in computational physics and engineering. Additionally, we propose an extension to the framework incorporating semi-discrete optimal transport regularization, which promises to further enhance the method's robustness and computational efficiency.

References

[1] M. Khamlich, F. Pichi, M. Girfoglio, A. Quaini, and G. Rozza. Optimal transport-based displacement interpolation with data augmentation for reduced order modeling of nonlinear dynamical systems. *Journal of Computational Physics*, 531:113938, 2025.

[2] G. Peyré and M. Cuturi. Computational optimal transport. *Foundations and Trends in Machine Learning*, 11 (5-6):355–602, 2019.