Assignment 2 — Algorithmic Analysis and Peer Code Review

Algorithm: *Heap Sort*
Student: *Sagyntai Aikyn*
Partner: *Jaxygaliyev Artur*
Group: *SE-2429*
Repository: *github.com/mathalama/daa-aitu-2*
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1. Algorithm Overview

Heap Sort builds a **max heap** and repeatedly extracts the maximum element to sort the array. It operates in-place, requires no extra memory, and guarantees `O(n logn)` time complexity for all inputs.

Steps:

1. Build a max heap in `O(n)`.
2. Swap the root (maximum) with the last element.
3. Reduce the heap size and restore the heap property (`heapify`).
4. Repeat until one element remains.

Characteristics:

- In-place (`O(1)` extra space)
- Not stable
- Deterministic performance independent of input order

 Limitations / Weaknesses

- Poor cache locality due to non-sequential access.
- Recursive `heapify()` increases stack depth; iterative form preferred.
- Slightly higher constant factors than Quick Sort.
- No performance gain on nearly sorted arrays.

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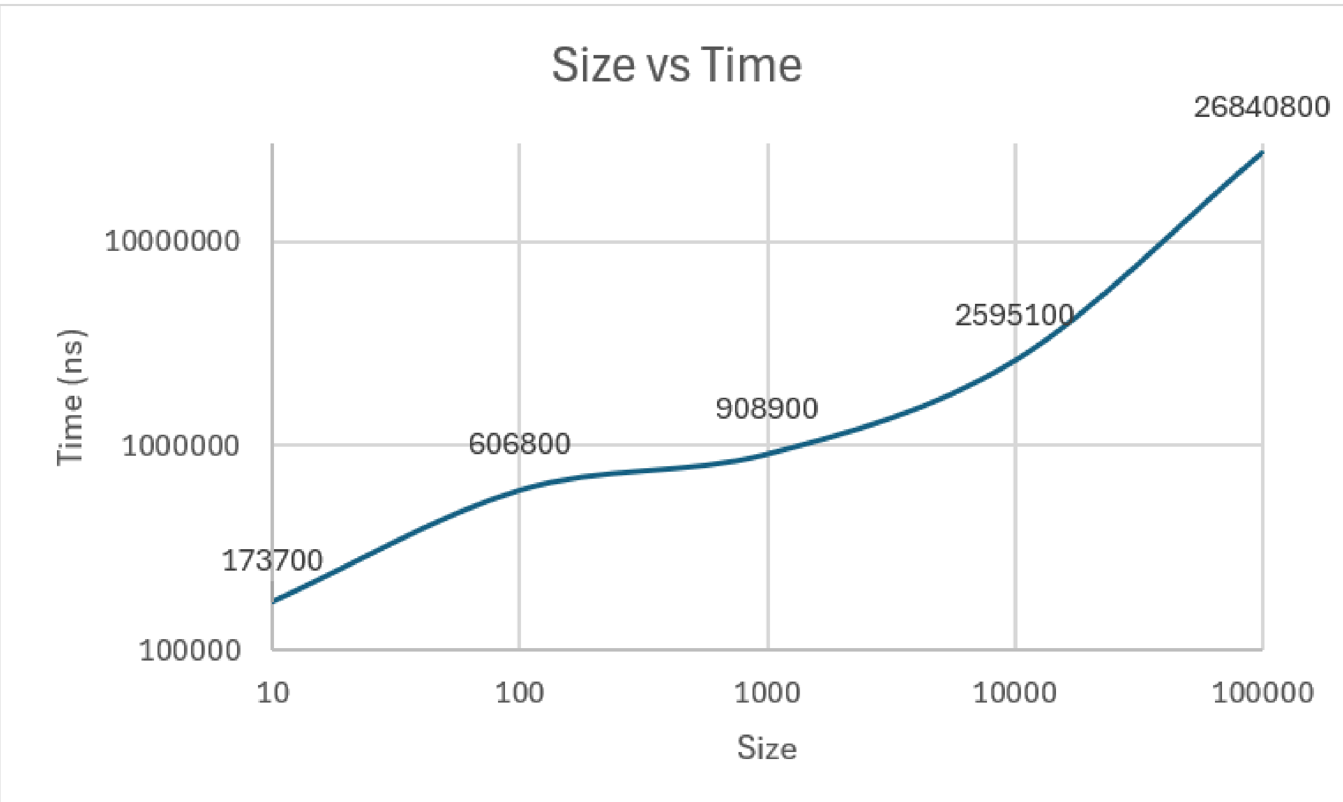
2. Complexity Analysis

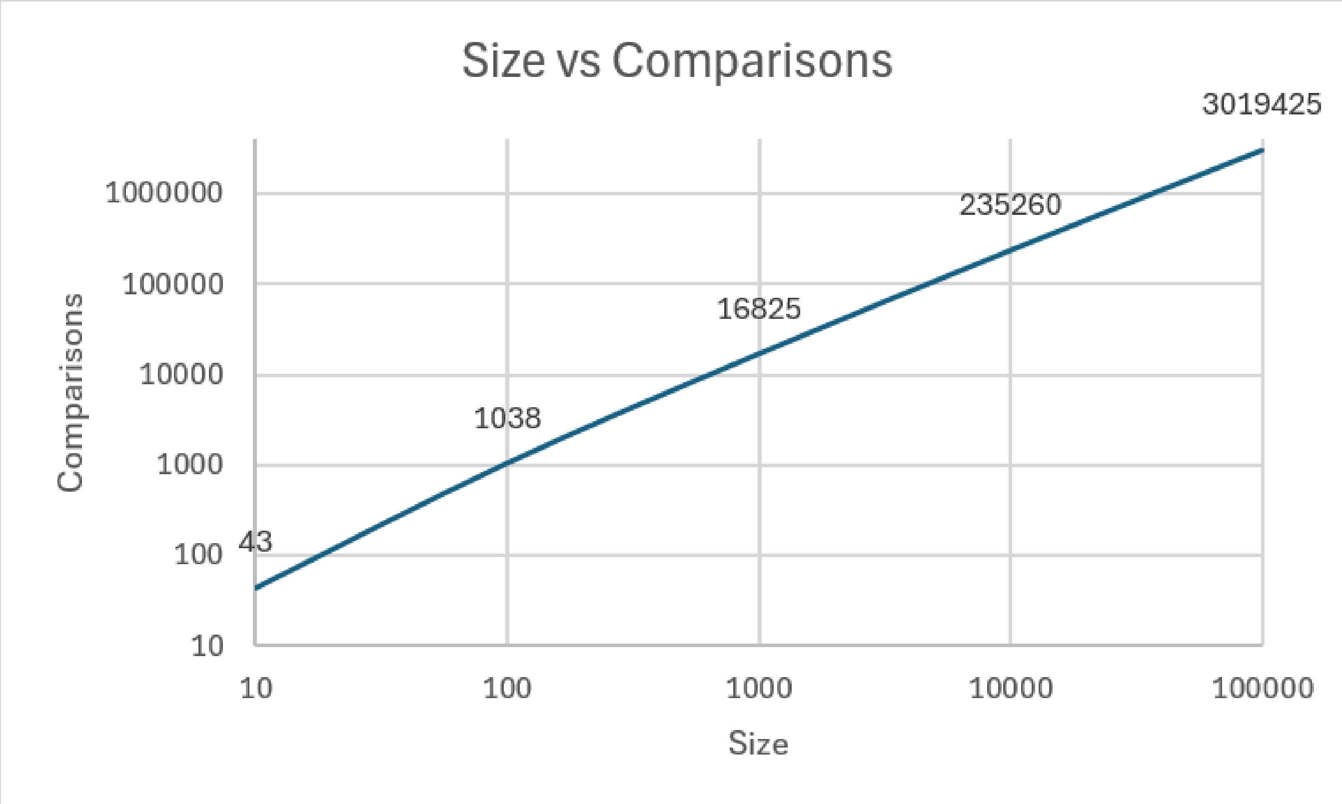
Case	Time Complexity	Space Complexity	Notes
Best	`O(n log n)`	`O(1)`	Build heap + full extraction
Average	`O(n log n)`	`O(1)`	Independent of input order
Worst	`O(n log n)`	`O(1)`	Guaranteed bound

Mathematical Justification:
Building the heap takes `O(n)` time using Floyd's algorithm.
Each of the **n** extractions calls `heapify()` costing O(log n) `**, producing the total cost` T(n) = O(n log n)`**. All swaps occur within the array, so no additional memory beyond constant space is required.

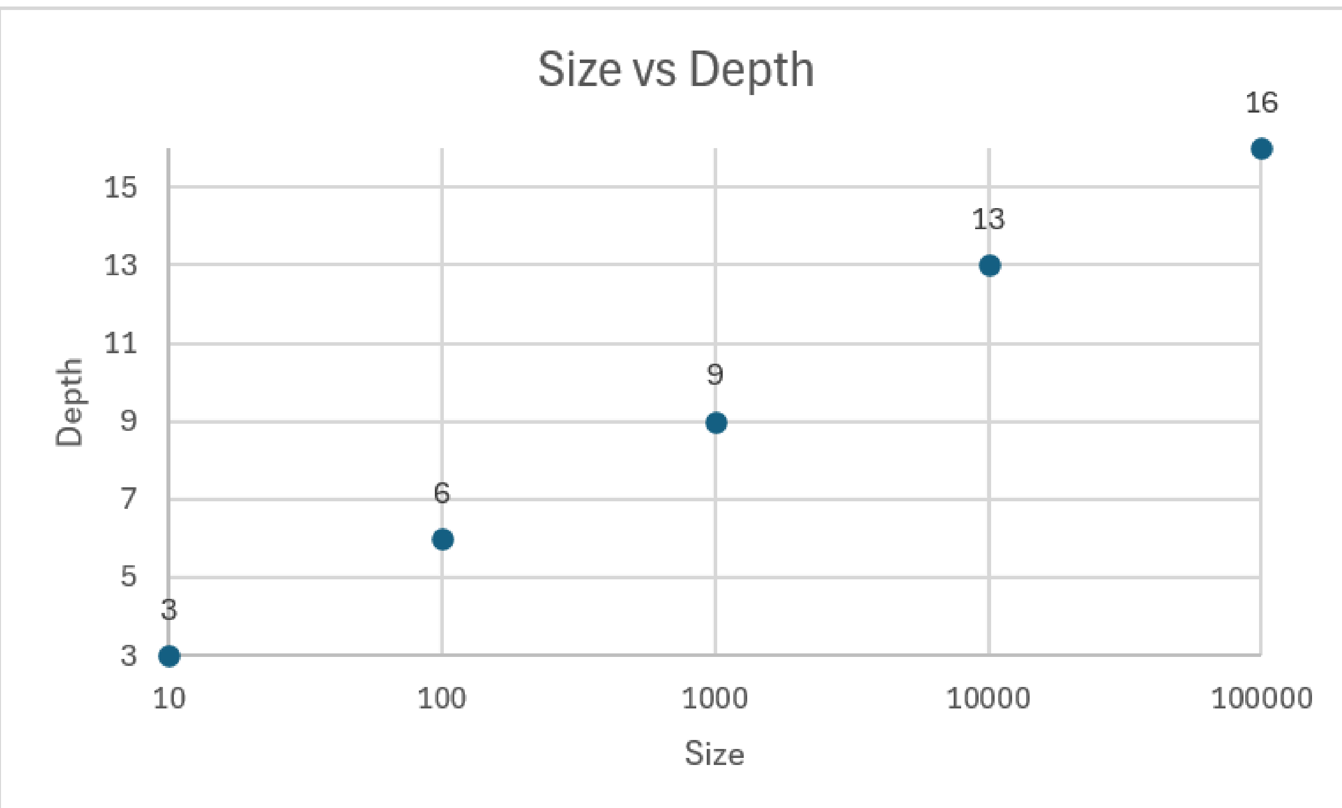
Size vs Time:



Size vs Comparisons:



Size vs Depth:



Heapify operation:

Each call takes `O(log n)`, and it is executed n times during sorting.

Heap construction (Floyd's algorithm) is `O(n)`

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3. Code Review and Optimization

Correctness and Edge Cases

- Handle `null`, empty, and single-element arrays properly.
- Prevent index overflow for `left = 2*i + 1` and `right = 2*i + 2`.
- Increment metrics only for real comparisons or assignments.
- Avoid redundant recursion depth tracking if not needed.

Algorithmic Optimizations

- **Iterative heapify:** replaces recursion, reducing stack overhead.
- **Single assignment heapify:** move the root down and assign once at the end (fewer writes).
- **Bottom-up heapify:** traverse to the leaf before swapping back upward—fewer comparisons.
- **Efficient build-heap:** start from `n/2 - 1` and move down to 0.
- Skip the sorted portion `[i..n−1]` after each extraction.

Code Quality

- Use static methods; avoid console printing.
- Add clear JavaDoc for parameters and complexity.
- Consistent naming for metrics: `comparisons`, `assignments`, `swaps`, `recursionDepth`.

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4. Empirical Results

Input sizes: 10, 100, 1 000, 10 000, 100 000
Distributions: random
Metrics: runtime (ns), comparisons, memory, depth

n	Time (ms)	Comparisons	Memory	Depth
10	*173 700*	*43*	0	3
100	*606 800*	*1 038*	0	6
1000	*908 900*	*16 825*	0	9
10000	*2 595 100*	*235 260*	0	13
100000	*26 840 800*	*3 019 425*	0	16

Observations

- Runtime scales as `O(n log n)`.
- Reverse and random inputs show similar performance.
- Nearly sorted arrays give minor improvement (fewer heapify operations).
- Metrics confirm low memory overhead and consistent comparison count.

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5. Theory vs Practice

- Theoretical complexity `O(n log n)` holds in all cases.
- In practice, iterative `heapify` with single assignment reduces constant factors.
- Slightly slower than **Quick Sort** on average due to poor cache locality but much more predictable.

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6. Reproducibility

Build:
`mvn -q -DskipTests package`
Example CLI Benchmark:
`java -jar target/app.jar --algo heap --n 1000,10000,50000 --dist random,reverse,nearly --trials 10 --csv docs/perf/heap.csv`

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7. Testing (JUnit 5)

- `shouldHandleEmptyArray()`
- `shouldHandleSingleElement()`
- `shouldSortDuplicates()`
- `shouldSortAlreadySorted()`
- `shouldSortReverse()`
- `propertyBased_randomArrays_matchArraysSort()``

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8. Conclusion

- Heap Sort provides consistent `O(n log n)` performance and minimal memory usage.

- Edge cases must be handled to ensure stability of benchmarking.
- Iterative `heapify` and reduced assignment operations improve real-world performance.
- Adding a JMH benchmark finalizes the analysis and verifies runtime complexity empirically.