MAT 3004, APPLIED LINEAR ALGEBRA, WinSem 2021–2022
DIGITAL ASSIGNMENT 2 by P Vanchinathan

(Should submit all odd-numbered questions on or before 05/04/2022, 10pm.)

Q1 Two subspaces in 4-dimensional are both dimension 3. Their basis are given by:

$$V : (1, 3, -1, 0), (3, 10, 2, 1), (1, 2, -2, 2)$$

$$W : (1, 2, 1, 3), (2, 11, 11, -4), (6, 17, -3, 5)$$

For the intersection of $V$ and $W$ find the dimension and basis.

Also find basis for $V + W$.

Q2 Find a polynomial curve $y = f(x)$ which passes through the points $(1, 10), (2, 43), (0, 3), (-2, -37)$.

Q3 What is the sum $f(5) + f(-5)$ for a polynomial $f(x)$ satisfying $f(1) = -6, f(3) = 40, f(-2) = 30$.

Q4 A triangle with vertices at $(11, -5), (2, 6), (-4, 2)$ is rotated by angle 30 degrees around the origin. What is the position of the rotated triangle? [*Extra point for a shorter answer*!]

Q5 A triangle has vertices at $(7, 2), (-2, 5), (4, -1a)$. Find the location of its vertices after it is rotated around its centroid by angle 45 degrees.

Q6 Does there exist a $4 \times 4$ orthogonal matrix whose first two columns are $(2, 1, -5, 2), (3, 3, 1, -2)$? Either construct it or prove it does not exist.

Q7 Reflection about a line L through origin moves the point $(27, 6)$ is to $(-21, 18)$. Determine the matrix of the reflection and using it find the reflected image of the point $(9, 7)$.

Q8 In the vector space $\mathbf{P}_2$ of polynomials of degree $\leq 2$, inner product between $f(t), g(t)$ is defined by

$$\int_1^2 t^2 f(t) g(t)$$

Find the matrix representation of this inner product with respect to the basis $\{1, t+1, t^2-1\}$. Further find a polynomial of degree 1 which is *orthogonal* to $t^2+1$.

Q9 Let $A = \begin{pmatrix} 4 & 2 \\ 2 & 3 \end{pmatrix}$. For two column vectors $u, v\mathbf{R}^2$ define $\langle u, v\rangle = u^t A v$. Show that this is is an innerproduct different from dot product of vectors of the plane. Find a vector orthogonal to (2,3) under this inner product and also determine its length.

Q10 By Gram-Schmidt convert the following basis into an orthogonal basis:

$$(-2, 1, 1), (-1, 1, 9), (-4, 3, 2)$$