Energy Flow Optimization Report

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May 8, 2025

Abstract

We optimize energy flows in a system consisting of a 1) photovoltaic (PV) system, 2) an electrical battery, 3) a connection to the external electrical grid and 4) a consumer. The objective is to meet the predicted electrical energy consumption while minimizing costs.

1 Part A

1.1 Variables

- $\mathbf{pvg}_t$: Flow from photovoltaic system to grid at time t (kW).
- $\mathbf{pvc}_t$: Flow from photovoltaic system to consumer at time t (kW).
- $\mathbf{pvb}_t$: Flow from photovoltaic system to battery at time t (kW).
- $\mathbf{gb}_t$: Flow from the grid to battery at time t (kW).
- $\mathbf{gc}_t$: Flow from the grid to the consumer at time t (kW).
- $\mathbf{bg}_t$: Flow from the battery to the grid at time t (kW).
- $\mathbf{bc}_t$: Flow from the battery to the consumer at time t (kW).
- $\mathbf{charge}_t$: Charge level of the battery at time t (kWh).

All the variables are nonnegative real values. See Figure 1.

1.2 Data

- $\mathbf{pv}_t$: Predicted photovoltaic production at time t (kW).
- $\mathbf{conso}_t$: Predicted consumption at time t (kW).
- $\mathbf{lcos}_t$: Levelized cost of storage at time t (cents/kWh).
- $\mathbf{sell}_t$: Selling price of the energy at time t (cents/kWh).
- $\mathbf{buy}_t$: Buying price of the energy at time t (cents/kWh).

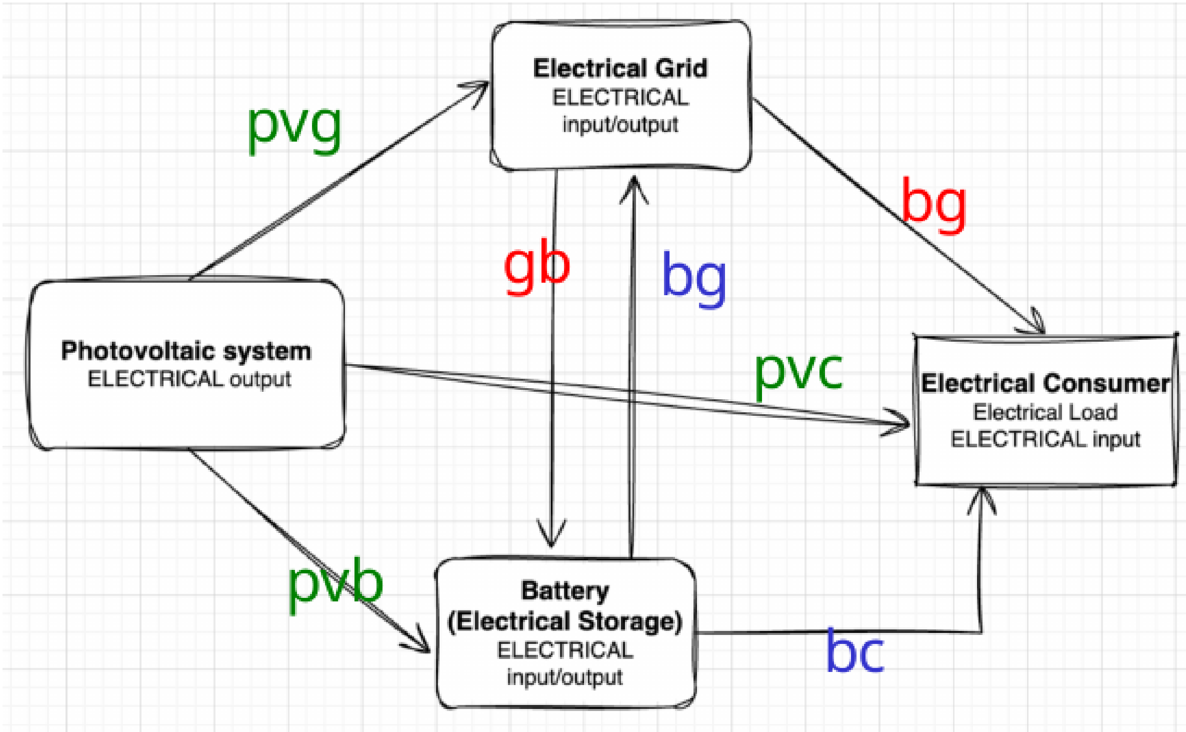

Figure 1: The different flows that represent the variables

1.3 Objective Function

$$\min C = \sum_t \mathbf{buy}_t \cdot \mathbf{eb}_t - \mathbf{sell}_t \cdot \mathbf{es}_t + \mathbf{lcos}_t \cdot \mathbf{ed}_t \tag{1}$$

where:

- The bought energy $\mathbf{eb}_t = \mathbf{gb}_t + \mathbf{gc}_t$ is the sum of the grid to battery flow ($\mathbf{gb}_t$) and the grid to consumer flow ($\mathbf{gc}_t$).
- The sold energy $\mathbf{es}_t = \mathbf{bg}_t + \mathbf{pvg}_t$ is the sum of the battery to grid flow ($\mathbf{bg}_t$) and the photovoltaic system to grid flow ($\mathbf{pvg}_t$).
- The discharged energy $\mathbf{ed}_t = \mathbf{bc}_t + \mathbf{bg}_t$ is the sum of the battery to consumer flow ($\mathbf{bc}_t$) and the battery to grid flow ($\mathbf{bg}_t$).

1.4 Constraints

Photovoltaic Production:

$$\mathbf{pvg}_t + \mathbf{pvc}_t + \mathbf{pvb}_t \leq \mathbf{pv}_t\,, \quad \forall t \tag{2}$$

Battery:

$$\mathbf{charge}_t \leq 160\,, \quad \forall t \tag{3}$$

$$\text{(Max discharge)} \quad \mathbf{bg}_t + \mathbf{bc}_t \leq 100\,, \quad \forall t \tag{4}$$

$$\text{(Max charge)} \quad \mathbf{gb}_t + \mathbf{pvb}_t \leq 100\,, \quad \forall t \tag{5}$$

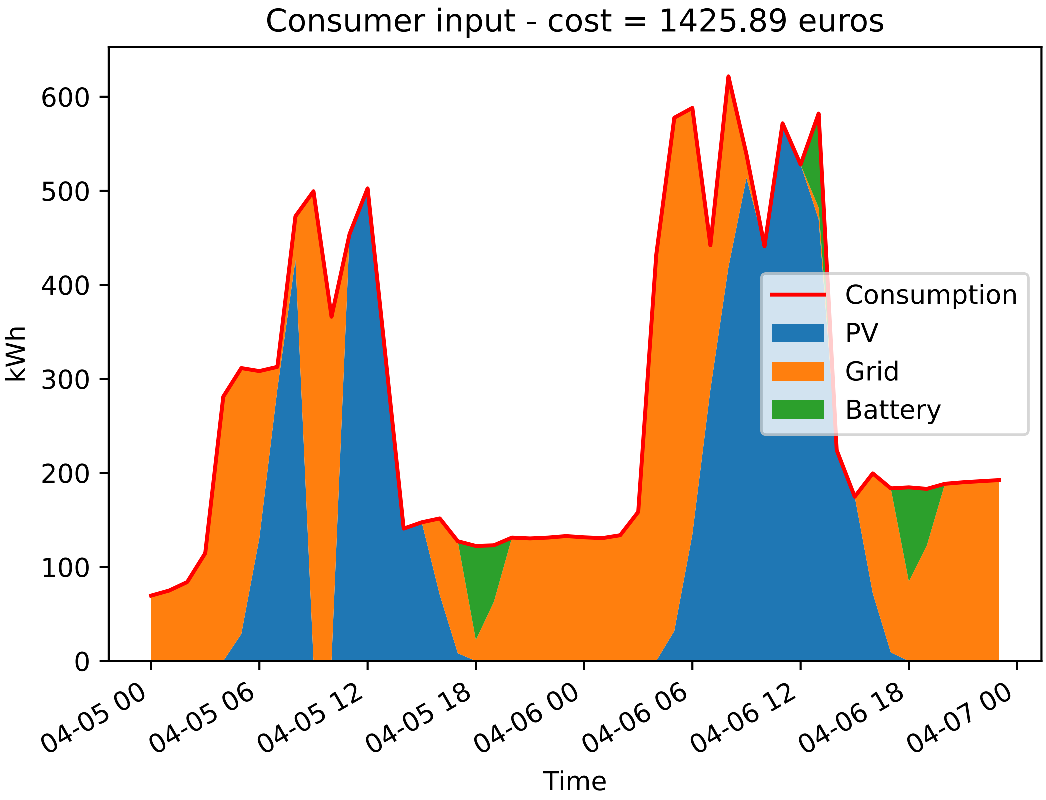

Figure 2: Results of Part A of the example from *test_data.xlsx*

Evolution of the level of charge in the battery:

$$\mathbf{charge}_t = \begin{cases} 0.92 \cdot (\mathbf{gb}_t + \mathbf{pvb}_t) - (\mathbf{bg}_t + \mathbf{bc}_t) & \text{if } t = 0 \\ \mathbf{charge}_{t-1} + 0.92 \cdot (\mathbf{gb}_t + \mathbf{pvb}_t) - (\mathbf{bg}_t + \mathbf{bc}_t) & \text{if } t > 0 \end{cases} \tag{6}$$

where the charging efficiency of the battery is 92%.
Grid:

$$\text{(Max sell power)} \quad \mathbf{bg}_t + \mathbf{pvg}_t \leq 700\,, \quad \forall t \tag{7}$$
$$\text{(Max buy power)} \quad \mathbf{gb}_t + \mathbf{gc}_t \leq 700\,, \quad \forall t \tag{8}$$

Consumer Demands:

$$\mathbf{gc}_t + \mathbf{pvc}_t + \mathbf{bc}_t = \mathbf{conso}_t\,, \quad \forall t \tag{9}$$

1.5 Results of the optimization

See Figure 2.

2 Part B

To prevent simultaneous buying and selling from the grid, we introduce two binary variables:

- $\textbf{to\_buy}_t \in \{0, 1\}$: This binary variable indicates whether we are buying energy from the grid (1) or not (0) at time t.
- $\textbf{to\_sell}_t \in \{0, 1\}$: This binary variable indicates whether we are selling energy to the grid (1) or not (0) at time t.

We add a constraint to ensure that buying and selling do not occur at the same time:

$$\textbf{to\_buy}_t + \textbf{to\_sell}_t \leq 1\,, \quad \forall t \tag{10}$$

Additionally, we introduce two constraints (through the Big-M method) to enforce the effect of the binary variables on the energy flows:

$$\mathbf{gb}_t + \mathbf{gc}_t \leq \textbf{to\_buy}_t \cdot M\,, \quad \forall t \tag{11}$$

$$\mathbf{bg}_t + \mathbf{pvg}_t \leq \textbf{to\_sell}_t \cdot M\,, \quad \forall t \tag{12}$$

where $M > 0$ is any large enough number (we took $M = 10^5$).

These constraints ensure that energy is only bought or sold when the corresponding binary variable is active.

The results of implementing these constraints are illustrated in Figures 3 and 4.

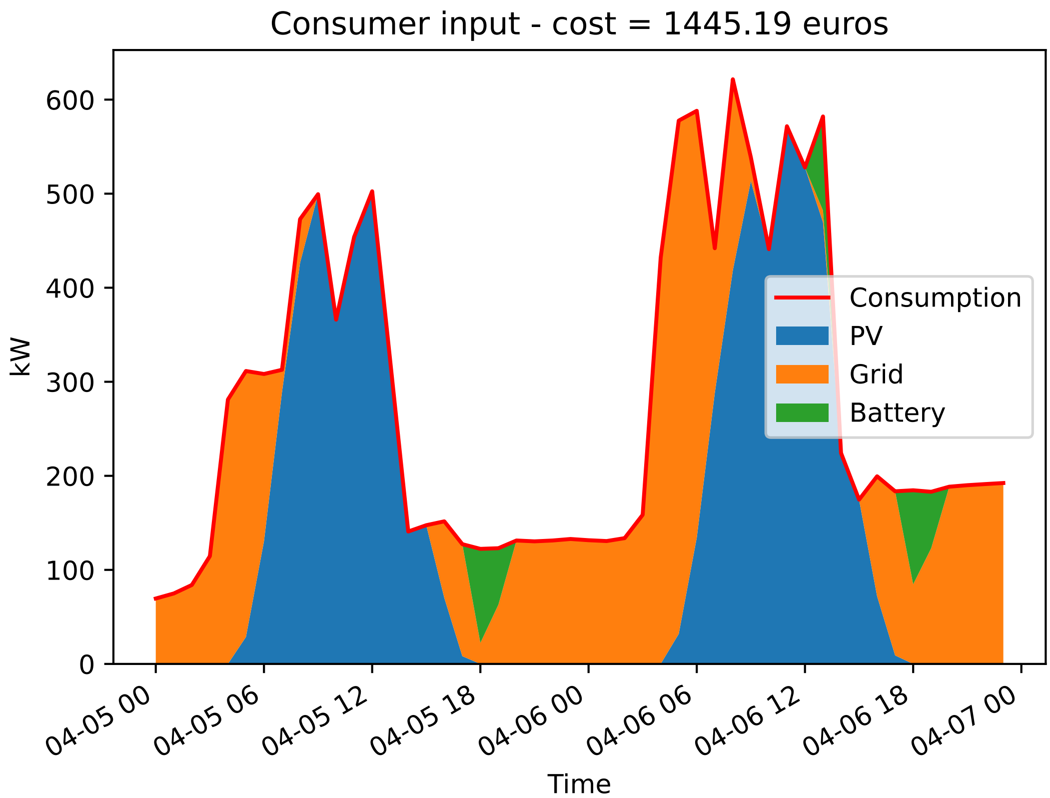

Figure 3: Results of Part B of the example from *test_data.xlsx*

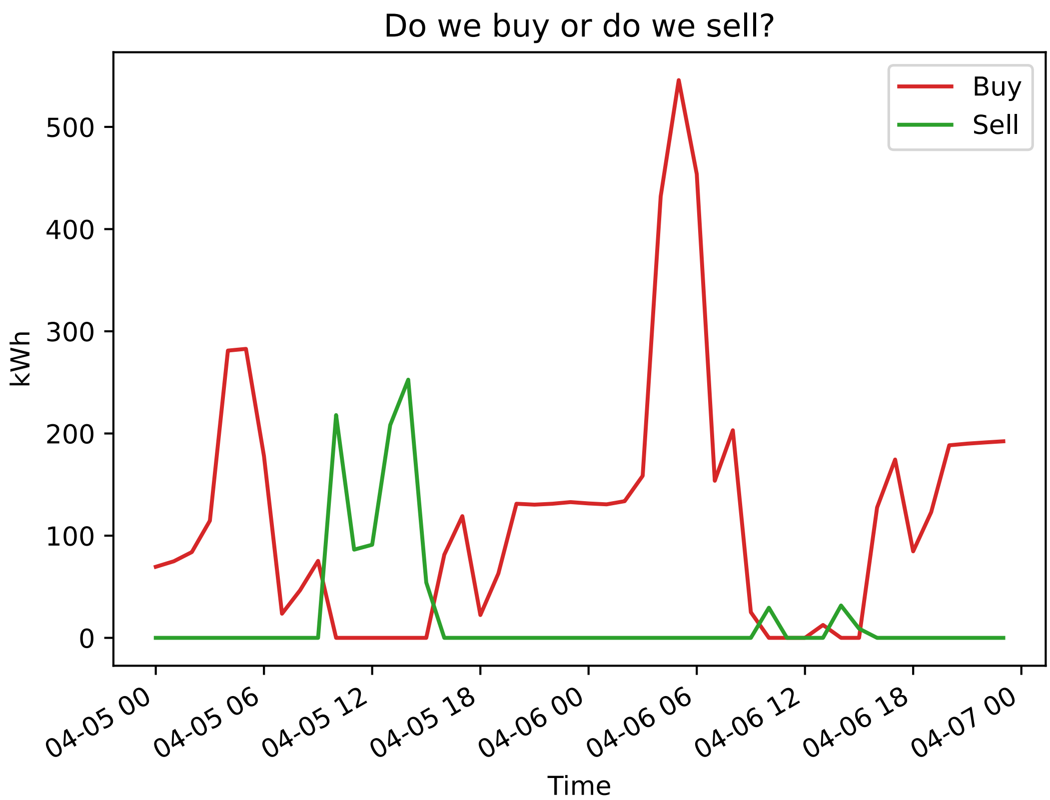

Figure 4: The amount of energy bought and sold over time (Part B). We do ensure that we do not buy and sell simultaneously.