Key Concepts and Formulae
(NCERT Mathematics - Class 10)

Arti Singh
Lead Instructor, MathBytes Classes

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Email: contact@mathbytes.in
Web: https://MathBytes.in

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Contents

Chapter 1 | Real Numbers

1.1 Definition and Theorems

- **(Fundamental Theorem of Arithmetic)** Every composite number can be written as a product of prime numbers, and this factorisation is unique, apart from the order in which the prime factors occur.

  Examples:
  a. $100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$ (and 100 cannot be written as product of primes other than 2 and 5)

  b. $51 = 3 \times 17$ (and 51 cannot be written as the product of primes other than 3 and 17).

- **(Theorem)** If p is prime and $a > 0$ then $p \mid a^2 \Rightarrow p \mid a$.

- If p is prime then $\sqrt{p}$ is irrational.

- $\boxed{LCM(a,b) \times HCF(a,b) = a \times b}$ where a and b are integers.

1.2 Important Problems

1. Given that HCF (306, 657) = 9, find LCM (306, 657). (Ans: 22338)
2. Check whether 6^n can end with the digit 0 for any natural number n. (Ans: No)
3. Show that $3\sqrt{2}$ is irrational.
4. Prove that $\sqrt{7}$ is irrational.
5. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p, q are primes.
6. For any positive integer n, prove that $n^3 - n$ is divisible by 6. (Hint: Factorize $n^3 - n$).

2.1 Concepts

1. **Geometrical Meaning of the Zeroes of a Polynomial** Geometrically, the zeroes of a polynomial $f(x)$ are x-coordinates of the points where the graph of $y = f(x)$ intersects the x-axis.

2. A **linear** polynomial has degree 1. For example polynomails $2x + 1, x, 1 - x$ have degree 1.

3. A **quadratic** polynomial has degree 2. Ex. $x^2 + 1, -x^2, 1 + 2x + 3x^2$ have degree 2. Quadratic polynomails are of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.

4. A **cubic** polynomial has degree 3. Examples: $5x^3 - 1, 1 + x + x^3$

5. A quadratic polynomial can have **at most** 2 zeroes and a cubic polynomial can have **at most** 3 zeroes.

6. If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c = 0$, then

$$\boxed{\alpha + \beta = \frac{-b}{a}} \quad \textit{and} \quad \boxed{\alpha\beta = \frac{c}{a}}$$

7. If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = \frac{-b}{a},$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a},$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

2.2 Important Questions

1. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients. (Ans: -2, -5)

2. Find a quadratic polynomial, the sum and product of whose zeroes are -3 and 2, respectively. (Ans : $x^2 + 3x + 2$)

Chapter 3 | Pair of Linear Equations in Two Variables

3.1 Graphical Method

Plot the graph of both lines $a_1x + b_1y + c_1 = 0,\ and\ a_2x + b_2y + c_2 = 0$. If lines are:

- Intersecting lines - we've exactly 1 solution
- Coincident/identical lines - we've infinitely many solutions
- Parallel lines - no solution

3.2 Algebraic Method

Equation of lines: $a_1x + b_1y + c_1 = 0\ and\ a_2x + b_2y + c_2 = 0$

Ratios	Agebraic Interpretation	Geometric Interpret
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Unique Solution (pair of equations is consistent)	Intersecting Lines
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Infinitely Many Solution (pair of equations is dependent (consistent))	Coincident/Identica
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	No Solution (pair of equations is inconsistent)	Parallel Lines

3.3 Problems

1. Check graphically whether the pair of equations $x + 3y = 6$ and $2x - 3y = 12$ is consistent. If so, solve them. (Ans: Consistent. $x = 6,\ y = 0$)

2. Form the pair of linear equations and find the solution graphically:
   10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz. (Ans: Girls=7, Boys=3)

3. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region. [Ans: (-1,0), (4,0), (2,3)]

4. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there? (Ans: 42 and 24)

5. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers. (Ans: 40, 48)

Chapter 4 | Quadratic Equations

4.1 Formulae & Definitions

- **Quadratic Equation -** in variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are reals and $a \neq 0$.
- **Root of Quadratic Equation** - A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$
- **Quadratic Formula -** The roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ provided\ b^2 - 4ac \geq 0.$$

- **Nature of Roots -** A quadratic equation $ax^2 + bx + c = 0$ has
  - i. Two distinct roots, if $b^2 - 4ac > 0$
  - ii. Two equal roots, if $b^2 - 4ac = 0$
  - iii. No real roots, if $b^2 - 4ac < 0$
- The number $D = b^2 - 4ac$ is called the **discriminant** of the quadratic equation.

4.2 Important Questions

1. Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$ $\quad Ans:\ \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$
2. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article. (Ans: Number of articles = 6, Cost of each article = Rs. 15)
3. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides. (Ans: 5 cm and 12 cm)
4. Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.
   (Ans: Discriminant: -8, No Real Roots)
5. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?
   (Ans: Yes. 5m from one gate and 12m from the other gate.)
6. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train. (Ans: 45 km/h)

Chapter 5 | Arithmetic Progressions

5.1 Formulae

- An **arithmetic progression** with **first term** a and **common difference** d is given by $a, a+d, a+2d, a+3d, \ldots$
- The general term (nth term) of an AP is given by

$$\boxed{a_n = a + (n-1)d}$$

- The sum of first n terms of an AP is given by

$$\boxed{S = \frac{n}{2}[2a + (n-1)d]}$$

- If an AP has n terms. If the first term is a and last term is l, then

$$\boxed{S = \frac{n}{2}(a + l)}$$

  Example: $1 + 2 + \cdots + 10 = \frac{10}{2}(1 + 10) = 55$

- (Important) The **nth term of an AP** is the difference of the sum to first n terms and the sum to first $(n-1)$ terms i.e. $a_n = S_n - S_{n-1}$

5.2 Important Problems

1. Which term of the AP:21, 18, 15, . . . is -81? Also, is any term 0? Give reason for your answer. (Ans: 35th term is -81 and 8th term is 0)
2. Determine the AP whose 3rd term is 5 and the 7th term is 9. (Ans: 3, 4, 5,6, ...)
3. How many two-digit numbers are divisible by 3? (Ans: 30)
4. A sum of Rs. 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years. (Ans: Rs. 2400)
5. How many three-digit numbers are divisible by 7? (Ans: 128)
6. How many multiples of 4 lie between 10 and 250? (Ans: 60)
7. For what value of n, are the nth terms of two APs: $63, 65, 67, \ldots$ and $3, 10, 17, \ldots$ are equal? (Ans: 13)
8. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference. (Ans: $n = 16,\ d = 8/3$)
9. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms. (Ans: $a_n = S_n - S_{n-1} = 5 - 2n$)

Chapter 6 | Triangles

6.1 Similar Figures and Similar Triangles

- **Similar Figures -** are figures that have same shape but not necessarily the same size. Congruent figures are similar but the converse is not true.

- **Similar Triangles -** Two triangles are said to be similar, if

  i. their corresponding angles are equal, and

  ii. corresponding sides are in the same ratio

  If two triangles ΔABC and ΔPQR are similar then it is denoted as $\Delta ABC \sim \Delta PQR$.

$$\boxed{\Delta ABC \sim \Delta PQR \iff \angle A = \angle P,\ \angle B = \angle Q,\ \angle C = \angle R \quad \& \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}}$$

- **(Basic Proportionality Theorem / BPT)** If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides those two sides in the same ratio.

- **Converse of BPT** If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

6.2 Criteria for Similarity of Triangles

1. **(AAA Similarity Criterion)** If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

2. **(AA Similarity Criterion)** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

3. **(SSS Similarity Criterion)** If in two triangles, sides of one triangle are in the same ratio of the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.

4. **(SAS Similarity Criterion)** If **one angle** of a triangle is equal to one angle of the other triangle and **two sides including the equal angle** are in the same ratio, then the two triangles are similar.

6.3 Important Problems

1. State and prove BPT.

2. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds. (Ans: 1.6 m)

3. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower. (Ans: 42m)

4. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$.

Chapter 7 | Coordinate Geometry

7.1 Formulae

1. The **distance** between points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. The distance of a point $P(x, y)$ from the origin (0, 0) is $\sqrt{x^2 + y^2}$

3. The coordinates of the point P(x, y) which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ **internally in the ratio** $m_1 : m_2$ are given by

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

4. The mid-point of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

7.2 Important Problems

1. Find the ratio in which the line segment joining the points (-3, 10) and (6, -8) is divided by the point (-1, 6). (Ans: 2:7)

2. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and coordinates of point B are (1, 4). [Ans: (3, -10)]

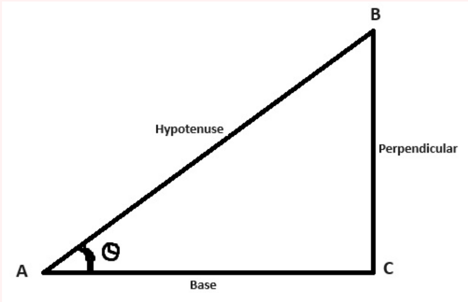
3. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order. [Hint : Area of a rhombus = $\frac{1}{2}$(product of its diagonals)] (Ans: 20 sq. units)

4. Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the point of intersection. [Ans: Ratio 5:1, POI is (0, -13/3)]

5. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4). (Ans: x - y = 2)

6. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear. (Ans: No)

8.1 Trigonometric Functions



Note: In below formulae, P=Perpendicular, B=Base, H=Hypotenuse

1. $\sin\theta = \frac{P}{H}$ (Hint: Superman is Playing Hockey)
2. $\cos\theta = \frac{B}{H}$ (Hint: Costa coffee is Boiling Hot)
3. $\tan\theta = \frac{P}{B}$ (Hint: Tanu is Playing with Barbie.)
4. $\cot\theta = \frac{1}{\tan\theta}$
5. $\sec\ \theta = \frac{1}{\cos\theta}$
6. $\text{cosec}\ \theta = \frac{1}{\sin\theta}$

8.2 Trigonometric Values at Common Angles

Angle	0	30	45	60	90
sin	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
cos	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{0}}{2} = 0$
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

Table 8.1: Trigonometric Values for Common Angles

8.3 Trigonometric Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta = 1 + \tan^2\theta$
- $\text{cosec}^2\theta = 1 + \cot^2\theta$

8.4 Important Problems

1. In a right triangle ABC, right-angled at B, if $\tan A = 1$, then verify that $2\sin A\cos A = 1$.

2. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

3. In ΔPQR, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

4. If $\sin(A - B) = \frac{1}{2}$, $\cos(A + B) = \frac{1}{2}, 0° < A + B \leq 90°,\ A > B$, find A and B. (Ans: A = 45°, B = 15°)

5. Express the ratios cos A, tan A and sec A in terms of sin A.

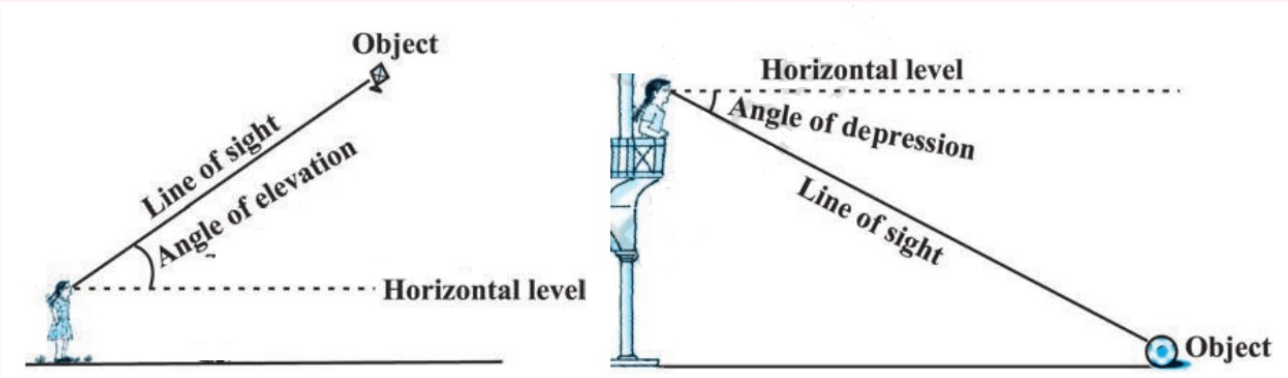
6. Prove that $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

9.1 Definitions

Line of Sight - Line drawn from the eye of an observer to the point in the object viewed by the observer.

Angle of Elevation - Angle formed by the line of sight with the horizontal when it is above the horizontal level (The observer need to raise her head up to look at the object).

Angle of Depression - The angle formed by the line of sight with the horizontal when it is below the horizontal level (the observer need to lower her head to look at the object).



9.2 Important Questions

1. (From NCERT Textbook) The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower. (Ans: $10\sqrt{3}m$)

2. (From NCERT Textbook) A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point. (Ans: 3 seconds)

3. (From NCERT Textbook) From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 m from the banks, find the width of the river. (Ans: $3(\sqrt{3}+1)m$)

Chapter 10 | Circles

10.1 Most Useful Concepts from Earlier Classes

1. Every point which is **equidistant** from two fixed points A and B lies on the **perpendicular bisector** (right bisector) of AB.
2. In an isosceles triangle, angles opposite to equal sides are also equal.
3. If two angles are equal then sides opposite to these angles are also equal (i.e. the tringle will be an isosceles triangle).
4. In a circle, angles subtended by a chord in the same segment are equal.
5. A diameter subtends $90°$ angle at each point of the circle.

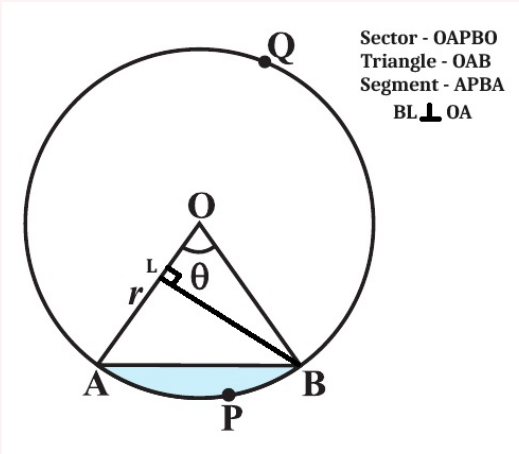
10.2 Circle Related Concepts

- A **Secant Line** is line that intersects a given circle at two points.
- A line that **touches** the circle (i.e. intersects at 1-point) is called a **tangent** line. The common point is called the **point of contact**.
- (Theorem 10.1) The **tangent** at any point of a circle is **perpendicular** to the **radius** (through the point of contact).
- (Theorem 10.2) The lengths of tangents drawn from an external point to a circle are equal.

10.3 Important Questions

1. Prove that the parallelogram circumscribing a circle is a rhombus.
2. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
3. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.
4. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP. (Ans: 20/3 cm)
5. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.
6. Prove that, the lengths of tangents drawn from an external point to a circle are equal.

11.1 Important Formulae



In above figure, $OA = OB = r$ (radius), $\angle AOB = \theta$ and BL is perpendicular to OA.

1. **Length of an Arc** of a sector of a circle with radius r and angle with degree measure $\theta = \frac{\theta}{360} \times 2\pi r$
2. **Area of a Sector** of a circle with radius r and angle with degree measure $\theta = \frac{\theta}{360} \times \pi r^2$
3. **Area of Segment** = (Area of the corresponding sector) - (Area of the corresponding triangle).
   Note that in ΔOAB, $\sin\theta = \frac{BL}{OB} = \frac{BL}{r} \Rightarrow BL = r\sin\theta$. Therefore
   Area of Triangle OAB $= \frac{1}{2} \times OA \times BL = \frac{1}{2} \times r \times r\sin\theta = \frac{1}{2}r^2\sin\theta$. Hence,

$$\boxed{ar(Segment\ APB) = \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \times \sin\theta}$$

   Remark: If $\theta > 90°$, use $ar(\Delta OAB) = r^2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2}$.

11.2 Important Problems

1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°. (Ans: $\frac{132}{7}cm^2$)
2. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. Use $\pi = 3.14$ and $\sqrt{3} = 1.73$. (Ans: 20.44 cm^2, 686.06 cm^2)
3. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 11.8). Find the area of that part of the field in which the horse can graze.
4. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Chapter 12 | Surface Areas and Volumes

12.1 Perimeter and Area of 2D/Plane Objects

2D Object	Perimeter / Cicumference	Area
Circle (radius r)	$2\pi r$	πr^2
Square (side a)	$4a$	a^2
Rectangle (sides a and b)	$2(a+b)$	$a*b$
Equilateral Triangle (side a)	$3a$	$\frac{\sqrt{3}}{4}a^2$

12.2 Area and Volume of Useful 3D Objects

S.N.	3D Object	Total Surface Area	Volume
1.	Cube (side a)	$6a^2$	a^3
2.	Cuboid (sides l, b, h)	$2(lb+bh+hl)$	$l*b*h$
3.	Cylinder (radius=r, height=h)	$2\pi r^2+2\pi rh$ OR $2\pi r(r+h)$	$\pi r^2 h$
4.	Right Circular Cone	$\pi r^2+\pi rl$ OR $\pi r(r+l)$	$\frac{1}{3}\pi r^2 h$
5.	Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$
6.	Semi/Hemi Sphere	$2\pi r^2$	$\frac{2}{3}\pi r^3$

Note: In S.N. 4 above r, h, and l are respectively the radius, height and slant height of the right circular cone and these are connected by the equation $\boxed{l^2=r^2+h^2}$

12.3 Some Important Problems

1. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2. (Ans: 18 cm^2)

2. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs. 500 per m^2. (Note that the base of the tent will not be covered with canvas.) (Ans: $44m^2$, Rs. 22,000)

3. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$) (Ans: 25.12 cm^3)

4. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. (Ans: 1.131 m^3 approx.)

5. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm3 of iron has approximately 8g mass. (Use $\pi = 3.14$) (Ans: 892.26 kg)

6. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm. (Ans: 2 hours)

7. 1000 persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by apersion is 0.04 m^3? (Ans: 1 cm)

13.1 Mean

1. Direct Method

$$\boxed{\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}}$$

2. Assumed Mean Method

$$\boxed{\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}}$$

   where a is assumed mean and $d_i = x_i - a$

3. Step Deviation Method

$$\boxed{\bar{x} = a + h \times \frac{\Sigma f_i u_i}{\Sigma f_i},\ where\ u_i = \frac{x_i - a}{h}}$$

13.2 Mode

Locate the **modal class** (a class with the maximum frequency) and use the formula

$$\boxed{Mode = l + h \times \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)}$$

where,
l = **lower limit** of the **modal class**,
h = class interval size,
f_1 = frequency of the **modal class**,
f_0 = frequency of the class **preceding** the modal class,
f_2 = frequency of the class **succeeding** the modal class.

13.3 Median

Locate the **medean class** which is the class whose cumulative frequency is greater than (and nearest to) $\frac{n}{2}$ and apply the formula:

$$\boxed{Median = l + h \times \left(\frac{\frac{n}{2} - cf}{f}\right)}$$

where,
l = lower limit of **median class**,
n = total number of observations,
cf = cumulative frequency of class **preceding** the median class,
f = frequency of **median class**,
h = class size

13.4 Empirical Relation

$$\boxed{3Median = Mode + 2Mean}$$

Trick: Median is the longest word so it's kept on the left with weight 3. There are 4 common letters between 'Median' and 'Mean' which is more than 3 common letters between Median and Mode so we assign weight 2 to Mean and 1 to Mode in the right hand side of above formula.

13.5 Problems

1. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of Mangoes	50 - 52	53 - 55	56 - 58	59 - 61	62 - 64
Number of Boxes	15	110	135	115	25

   Find the mean number of mangoes kept in a packing box. (Hint: Is data continuous?, Ans: 57.19)

2. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

   (Ans: 44.7 cars)

3. If the median of the distribution given below is 28.5, find the values of x and y.

Class Interval	**Frequency**
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

   (Ans: $x = 8$, $y = 7$)

4. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of students	2	3	8	6	6	3	2

   (Ans: 56.67 kg)

Chapter 14 | Probability

14.1 Definitions and Properties

Theoretical(Classical) Definition of Probability - The classical probability of an event E, denoted by $P(E)$, is defined as

$$\boxed{P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes}}$$

Properties

- The probability of a **sure event** (or certain event) is 1.
- The probability of an **impossible event** is 0.
- The probability $P(E)$ of every event E always such that $0 \leq P(E) \leq 1$.
- **Elementary Event** is an event consisting of only one possible outcome of the experiment.

  Example: In the experiment of rolling **one** die, 'the number 3 will come up' is an elementary event while the event 'even number will show up' is not an elementary event (events consisting of more than 1 possible outcomes of the experiment are called compound events. You will study these events in Class 11).

- The **sum** of the probabilities of **all** the elementary events of an experiment is **1**.

  Example: Experiment: Tossing one coin . Possible outcomes = {Head, Tail}
  P(Head) + P(Tail) = 1/2 + 1/2 = 1

- **('Not E' Event)** If E is an event, then the event 'Not E' denoted of E' is the event consiting of all possible outcomes that are not in E. We always have $\boxed{P(E) + P(E') = 1}$.

  Example: Experiment: Rolling a die. Event E = "Odd number will show up" then 'Not E' is the event "Odd number doesn't show".

14.2 Some Important Problems

1. Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is:
   (i) 8 (ii) 13? (iii) less than or equal to 12 (Ans: 5/36, 0, 1)

2. If P(E) = 0.05, what is the probability of 'not E'? (Ans: 0.95)

3. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one. (Ans: $\frac{11}{12}$)

4. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5. (Ans: (i) $\frac{9}{10}$, (ii) $\frac{1}{10}$, (iii) $\frac{1}{5}$)