

Math Camp 2019 - Final Math Review

Your Friendly Instructors

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Review of Mathematics Content

This part of the final assignment for Math Camp 2019 is a collection of problems produced by your instructors, designed to give you a brief review and comprehension test in a low-pressure, homework-style format. Below you will find problems for each of the math sessions from the week, in order of appearance during instruction. Please complete these exercises by Sunday and submit a .pdf of your solutions alongside the brief report from the data analysis project. Be sure to show your working. All of the forms of the solutions and working may be implemented in LaTeX fairly readily, and can be done inline or in blocked form in RMarkdown. You should feel free to collaborate with your peers, and submit one (1) document detailing your solutions per group.

Linear Algebra - Concepts and Relation to Linear Regression

1. What is a *vector*?
2. What is a *matrix*?
3. What is *regression*?
4. Consider the linear equation $Y = \beta_0 + X\beta + \epsilon$, which represents the kind of model we might estimate with OLS. Answer the following:
 - What is Y ?
 - What is β_0 ?
 - What is ϵ ?
 - What is $X\beta$?

Linear Algebra - Applying the Matrix Form of OLS

Consider the following hypothetical data:

Country	Oil Revenue (billions USD)	Dictator's time in power (years)
Kazakhstan	1	2
Iran	3	3
Libya	2	1

We can write the matrices y and X (with intercept) as follows:

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 2 \end{bmatrix}, y = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

Recall that the equation for an OLS coefficient estimator, \hat{b} , is $\hat{b} = (X'X)^{-1}X'y$.

1. Calculate \hat{b} .
2. We wish to find a and b in the equation $Survival = a + b \cdot Oil$; you have just calculated these. Rewrite the equation using the values you found.

Limits, Continuity, and Derivatives

1. What is a derivative? Provide the formal definition and also define the derivative in your own words.
2. Compute the following:
 - $\frac{d}{dx}(x^2)$
 - $\frac{d^2}{dx^2}(x^2)$
 - $\frac{d}{dx}(3x^2 + 5)$
 - $\frac{d^2}{dx^2}(3x^2 + 5)$
3. Plot the following:
 - x^2
 - $\frac{d}{dx}(x^2)$
 - $\frac{d^2}{dx^2}(x^2)$

Probability - Random Variables

Imagine rolling a six-sided fair die 50 times. The results are as follows:

x	y
1	9
2	6
3	10
4	6
5	11
6	8

1. Based on these values, compute the following probabilities:
 - $P(X = 5)$
 - $P(X \leq 3)$

Probability - Bayesian Probability

1. Models are a frequent topic in international relations, as they allow the simplification of the ways in which complex agents - such as states - interact. We might classify any given state as “good” or “bad,” having observed its behavior, but states themselves may not be able to discern whether one another will act cooperatively or defect in a political interaction. Define “good” and “bad” states such that:

$$P(\text{cooperate}|\text{Good}) = P_{gc}, P(\text{defect}|\text{Good}) = (1 - P_{gc}),$$

and

$$P(\text{defect}|\text{Bad}) = 1,$$

indicating that “bad” states always defect, while good states *may* defect with some probability (e.g. based on domestic politics).

Imagine two states. One state, A , is attempting to determine the type of the other state, B - whether it is “good” (i.e. cooperative) or “bad” (i.e. a defector). If A observes B defect, what does A believe to be the probability that B is a “good” state?

2. Many studies explore whether people engage in “partisan selective exposure” - that is, they choose to consume information aligned with their own partisan viewpoint. Answer the following questions using the information in the table below:

- What is the probability that a given member of the public watches Fox News? (*Note: assume each partisan ID given as mutually-independent and the set of partisan ID spans the population*).
- Are Fox News viewership and Democratic partisanship independent?
- Suppose we gain information concerning political interest, giving the results $P(\text{FoxNews}|\text{HighPoliticalInterest})0.2$ and $P(\text{Democrat}|\text{HighPoliticalInterest}) = 0.4$. With this information, what probability would we need to know in order to say, *conditional on high political interest*, whether Democratic partisanship and Fox News viewership are independent? What value would this probability need to take for the two to be independent?

Probabilities	
$P(\text{Democrat}) = 0.4$	$P(\text{FoxNews} \text{Democrat}) = 0.07$
$P(\text{Republican}) = 0.4$	$P(\text{FoxNews} \text{Republican}) = 0.1675$
$P(\text{Independent}) = 0.2$	$P(\text{FoxNews} \text{Independent}) = 0.05$