Probability I - Questions and Solutions for Final Project

Question 1: In international relations, we use models a lot. Sometimes we will simplify the world where two states interact. They may be good or bad states, having observed how they play in the game. They may cooperate or defect. A state can never know whether it encounters a good or bad state because the intention is unobservable.

Suppose there is one state attempting to determine the type of another state, where the type could be good or bad. Good types make up half of the population, and the bad types make up the other half. They are playing games in which they may operate or defect. Good types defect only when forced by domestic factors, which happens with probability . Bad types defect every time. Lets say the state has observed a defection. What would it believe about the probability that it faces a good type?

Solutions:

Let A stand of the event: the other player being good. A^c stands for the other player being a bad type. Let B stand for the event of seeing a defection.

The likelihood of seeing a defection if the other side is good is $\frac{1}{4}$, so the conditional probability, $P(B|A) = \frac{1}{4}$. The likelihood of seeing a defection if the other side is bad is 1, so $P(B|A^c) = 1$.

According to Bayes Rule, the posterior belief that the other side is good, having observed a defection, is given by the following formula.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

.

Therefore, we can get,

$$P(A|B) = \frac{\frac{1}{4} * \frac{1}{2}}{\frac{1}{4} * \frac{1}{2} + 1 * \frac{1}{4}} = \frac{\frac{1}{8}}{\frac{5}{8}} = \frac{1}{5}$$

This means the probability that the other side is the good type is now only 20 percent. The defection makes us more suspicious of the others intention.

Question 2: Many studies explore whether people engage in "partisan selective exposure." That is, do people choose information to consume based on what aligns with their partisan views?

(a) Given the information below, what is the probability a given member of the public watches Fox News? (For the purposes of this question, consider Democrat, Republican and Independent as mutually exclusive categories that include every member of the public.)

$$P(Democrat) = .4 \qquad P(FoxNews|Democrat) = .07$$

$$P(Republican) = .4 \qquad P(FoxNews|Republican) = .1675$$

$$P(Independent) = .2 \qquad P(FoxNews|Independent) = .05$$

(b) Are Fox News viewership and Democratic partisanship independent? Use your answer to the previous question to help you determine this.

(c) Now suppose we get additional information that lets us determine which individuals have high levels of political interest. We then find the following two quantities:

$$P(FoxNews|HighPoliticalInterest) = .2 \quad \text{and} \quad P(Democrat|HighPoliticalInterest) = .4.$$

Given this information, what probability would we need to know in order to say that, conditional on high levels of political interest, Democratic partisanship and Fox News viewership are independent? What value would this probability need to take for the two variables to be independent conditional on high levels of political interest?

Solutions:

(a) Using the law of total probability:

$$\begin{aligned}
P(WatchFoxNews) &= P(WatchFoxNews|Democrat)P(Democrat) \\
&+ P(WatchFoxNews|Republican)P(Republican) \\
&+ P(WatchFoxNews|Independent)P(Independent) \\
&= (.4 \times .07) + (.4 \times .1675) + (.2 \times .05) = 0.105
\end{aligned}$$

(b) If $P(A|B) \neq P(A)$ than A and B are not independent. In this case this equality doesn't hold for Democratic Partisanship and Fox News Viewership so the two events are not independent.

$$P(WatchFoxNews) = .105; P(WatchFoxNews|Democrat) = .07$$
$$P(WatchFoxNews) \neq P(WatchFoxNews|Democrat)$$

(c) We would need to find $P(FoxNews \cap Democrat|HighPoliticalInterest)$. For conditional independence to hold, we need:

$$P(FoxNews \cap Democrat|HighPoliticalInterest) = P(FoxNews|HighPolInt)P(Democrat|HighPolInt)$$
$$P(FoxNews \cap Democrat|HighPoliticalInterest) = 0.2 \times 0.4 = 0.08$$