

Integration

Problem Set

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Below are problems related to integrals. You may either work out your responses in Latex or R Markdown and submit a rendered pdf, or you may write out your answers in a word doc that you save as a pdf, or you can write out your scanned answers. Send your answers to milesdw2@illinois.edu.

Problem 1. The area under *and over* the curve?

Recall the example of the mid-sized paper supply company in northeast Pennsylvania. Say you've invested \$50 in this company and want to know how much money you've netted by week 15. The function describing the rate of ROI at week t , remember, is given as

$$k(t) = -2t.$$

The indefinite integral is

$$\int k(t)dt = C - t^2.$$

Part (A)

What will be your net ROI between when you invested and week 15?

Part (B)

Hint for **Part (A)**: your answer should have been *negative*. What does a negative value returned by the definite integral of $k(t)$ imply? Are we calculating the area *below* the curve of $k(t)$, or *above*?

Problem 2. A unique fact about circles.

If you've taken a class on geometry, you're probably familiar with the equation for the circumference of a circle:

$$C = \pi d = 2\pi r,$$

where π is, well, the value of pi (3.14...), and where d is the diameter of the circle and r the circumference.

Use integration to find the area of a circle with $r = 5$.

Hint: imagine that r varies from 0 at the origin of the circle to $r = 5$.

Problem 3. Practice with Integrals

In today's age of Google where information is readily at our fingertips, rote memorization is pretty pointless. *But*, the ability to leverage tools to solve complex problems is incredibly valuable.

With that in mind, below are some functions that I want you to integrate—but not by hand...

Instead, use [Wolfram Alpha](#) to get the indefinite integrals for these functions.

Here's an example of how it works. Say you have the function

$$q(x) = \frac{1}{x}.$$

When you follow the link to Wolfram Alpha above, go to the search bar at the top of the page that says *Enter what you want to calculate or know about*.

To get the integral of $q(x)$, all we need to do is write in the search bar integrate $1/x$, then hit enter.

Some software will do all the work for you behind the scenes and return the integral of the function along with some plots.

For the above function, it should return the following indefinite integral:

$$\int \frac{1}{x} dx = \ln(x) + C.$$

Try it yourself to see.

Using this tool, find the integrals for the following functions:¹

1. x^4 ,
2. $\ln(w)$,
3. e^y ,
4. 10^v ,
5. $1/(x^2 + 2x)$,
6. $\sqrt{3 + 2x}$.

¹*Note:* To indicate that you want to multiply a variable by a constant, either write 'x*6', for example, or 'x 6' with a space. Otherwise, Wolfram's language will think you're specifying a function (e.g., 'x4' = $x(4)$).