Probability I - Classwork

Question 1: Events A and B are contained within a sample space S. Given that $P(A) = 0.5$, $P(B) = 0.3$, and $P(A \cap B) = 0.1$, find:

(a) $P(A \cup B)$

(b) $P(A \cap B^C)$

(c) $P[(A \cap B^C) \cup (B \cap A^C)]$

Solutions:

(a)
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.3 - 0.1 = 0.7$$

(b) We know that
$$P(A) = P(A \cap B) + P(A \cap B^C)$$

rearrange and solve:
$$P(A \cap B^C) = P(A) - P(A \cap B) = 0.5 - 0.1 = 0.4$$

(c)
$$\begin{aligned} P[(A \cap B^C) \cup (B \cap A^C)] &= P(A \cap B^C) + P(B \cap A^C) - P(A \cap B \cap B^C \cap A^C) \\ &= P(A \cap B^C) + P(B \cap A^C) - P(\emptyset) \\ &= P(A) - P(A \cap B) + P(B) - P(A \cap B) \\ &= 0.5 - 0.1 + 0.3 - 0.1 = 0.6 \end{aligned}$$

Question 2: Let $P(A) = 0.45$, $P(B) = 0.22$, and $P(C) = 0.31$,

(a) If A, B, C, and D are disjoint (mutually exclusive) and collectively exhaustive events, what is $P(D)$?

(b) If A, B, C, and D are disjoint and collectively exhaustive events what is $P(A \cup B)$?

(c) If A, B, C, and D are disjoint and collectively exhaustive events what is $P(A \cap B)$?

(d) If B and C are independent, what is $P(B \cap C)$?

(e) If $P(A \cap B) = 0.3$, what is $P(A \cup B)$?

(f) Find $P(E)$ if $P(A \cup E) = .6$, and A and E are independent events.

(g) Find $P(F)$ if $P(A \cup F) = .8$,and A and F are independent events.

Solutions:

(a) Since events A, B, C, D are disjoint and mutually exhaustive

$$P(A) + P(B) + P(C) + P(D) = 1$$

$$P(D) = 1 - 0.45 - 0.22 - 0.31 = 0.02$$

(b) Since they are disjoint: $P(A \cup B) = P(A) + P(B) = 0.45 + 0.22 = 0.67$

(c) Since A and B are disjoint, their intersection is empty: $P(A \cap B) = 0$

(d) If B and C are independent, then we know that:

$$P(B \cap C) = P(B)P(C) = 0.22 \times 0.31 = 0.0682$$

(e)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.45 + 0.22 - 0.3 = 0.37$$

(f) We know that $P(A \cup E) = P(A) + P(E) - P(A \cap E)$. But since A and E are independent, we also know that $P(A \cap E) = P(A)P(E)$. Then we have:

$$P(A \cup E) = P(A) + P(E) - P(A)P(E)$$

$$0.6 = 0.45 + P(E) - 0.45P(E)$$

$$P(E) = \frac{0.15}{0.55} \approx 0.273$$

(g) With the same logic above, we have:

$$P(A \cup F) = P(A) + P(F) - P(A)P(F)$$

$$0.8 = 0.45 + P(F) - 0.45P(F)$$

$$P(F) = \frac{0.35}{0.55} \approx 0.636$$

Question 3: Find the following probabilities. If you cannot calculate the probability, explain why:

(a) $P(Z \cap Q) = .25$, $P(Z) = .6$. What is $P(Q|Z)$?

(b) $P(A \cap B) = .3$, $P(B|A) = .4$. What is $P(A)$?

(c) $P(G \cap W) = .8$, $P(W) = .2$. What is $P(G|W)$?

(d) $P(H) = .2$, $P(D|H) = .6$. What is $P(D \cap H)$?

(e) $P(D) = .8$. Using this information, and your answer to (d), find $P(H|D)$.

(f) $P(M \cap P) = .8$, $P(P) = .81$. What is $P(M|P)$?

(g) $P(L \cap E) = .6$, $P(L|E) = .05$. What is $P(L)$?

Solutions:

(a)

$$P(Q|Z) = \frac{P(Z \cap Q)}{P(Z)} = \frac{.25}{.6} \approx .417$$

(b)

$$P(A) = \frac{P(A \cap B)}{P(B|A)} = \frac{.3}{.4} = .75$$

(c) The given probabilities must be off in some way. If we try to apply the conditional probability rule we get a number greater than 1. The issue is that the probability of the intersection of one event with another cannot be larger than the marginal probability of that event to begin with (e.g., $P(A \cap B) \leq P(A)$)

(d)

$$P(D \cap H) = P(D|H)P(H) = .6 \times .2 = .12$$

(e)

$$P(H|D) = \frac{P(D \cap H)}{P(D)} = \frac{.12}{.8} = .15$$

(f)

$$P(M|P) = \frac{P(M \cap P)}{P(P)} = \frac{.8}{.81} \approx .99$$

(g) Setting up our problem, we could try to find $P(L)$ in the following manner.

$$P(E|L) = \frac{P(L \cap E)}{P(L)}$$

$$P(L) = \frac{P(L \cap E)}{P(E|L)}$$

We cannot find $P(L)$ because we dont have the right set of probabilities. To do so we would need additional information (e.g., $P(E|L)$).

Question 3: A bag contains 6 black marbles and 4 blue marbles. Two marbles are drawn from the bag, without replacement. What is the probability that both marbles are blue?

Question 4: 80% of people attend their primary care physician regularly; 35% of those people have no health problems crop up during the following year. Out of the 20% of people who dont see their doctor regularly, only 5% have no health issues during the following year. What is the probability a random person will have no health problems in the following year?

Question 5: (On the slide) A test for cancer correctly detects it 90% of the time, but incorrectly identifies a person as having cancer 10% of the time. If 10% of all people have cancer at any given time, what is the probability that a person who tests positive actually has cancer?

Question 6 The Monty Hall Game Show Problem Question :

In a TV Game show, a contestant selects one of three doors; behind one of the doors there is a prize, and behind the other two there are no prizes. After the contestant selects a door, the game-show host opens one of the remaining doors, and reveals that there is no prize behind it. The host then asks the contestant whether they want to SWITCH their choice to the other unopened door, or STICK to their original choice. Is it probabilistically advantageous for the contestant to SWITCH doors, or is the probability of winning the prize the same whether they STICK or SWITCH? (Assume that the host selects a door to open, from those available, with equal probability).

Question 7: Suppose a researcher conducted an experiment in which they measured public opinion towards a new policy. Before they offered their opinion, 45% of the respondents in the study were presented with an argument against the proposed policy while the other 55% of study respondents provided their opinion without hearing the argument. Overall, 40% of the individuals in the study supported the policy. However, among individuals who encountered the argument against the policy, only 20% supported the policy.

(a) Given this information, what is the probability an individual encountered the argument (i.e., was assigned to the argument condition) given that they support the policy?

(b) Suppose instead that encountering the argument did not influence respondent opinion. That is, suppose 40% of the subjects in the study supported the policy and 40% of the subjects who encountered the argument support the policy. How does this change your answer to the previous question?

Solutions:

(a)

$$\begin{aligned} P(ArgumentGroup|SupportPolicy) &= \frac{P(SupportPolicy|ArgumentGroup)P(ArgumentGroup)}{P(SupportPolicy)} \\ &= \frac{0.2 \times 0.45}{0.4} = 0.225 \end{aligned}$$

(b)

$$\begin{aligned} P(ArgumentGroup|SupportPolicy) &= \frac{P(SupportPolicy|ArgumentGroup)P(ArgumentGroup)}{P(SupportPolicy)} \\ &= \frac{0.4 \times 0.45}{0.4} = 0.45 \end{aligned}$$