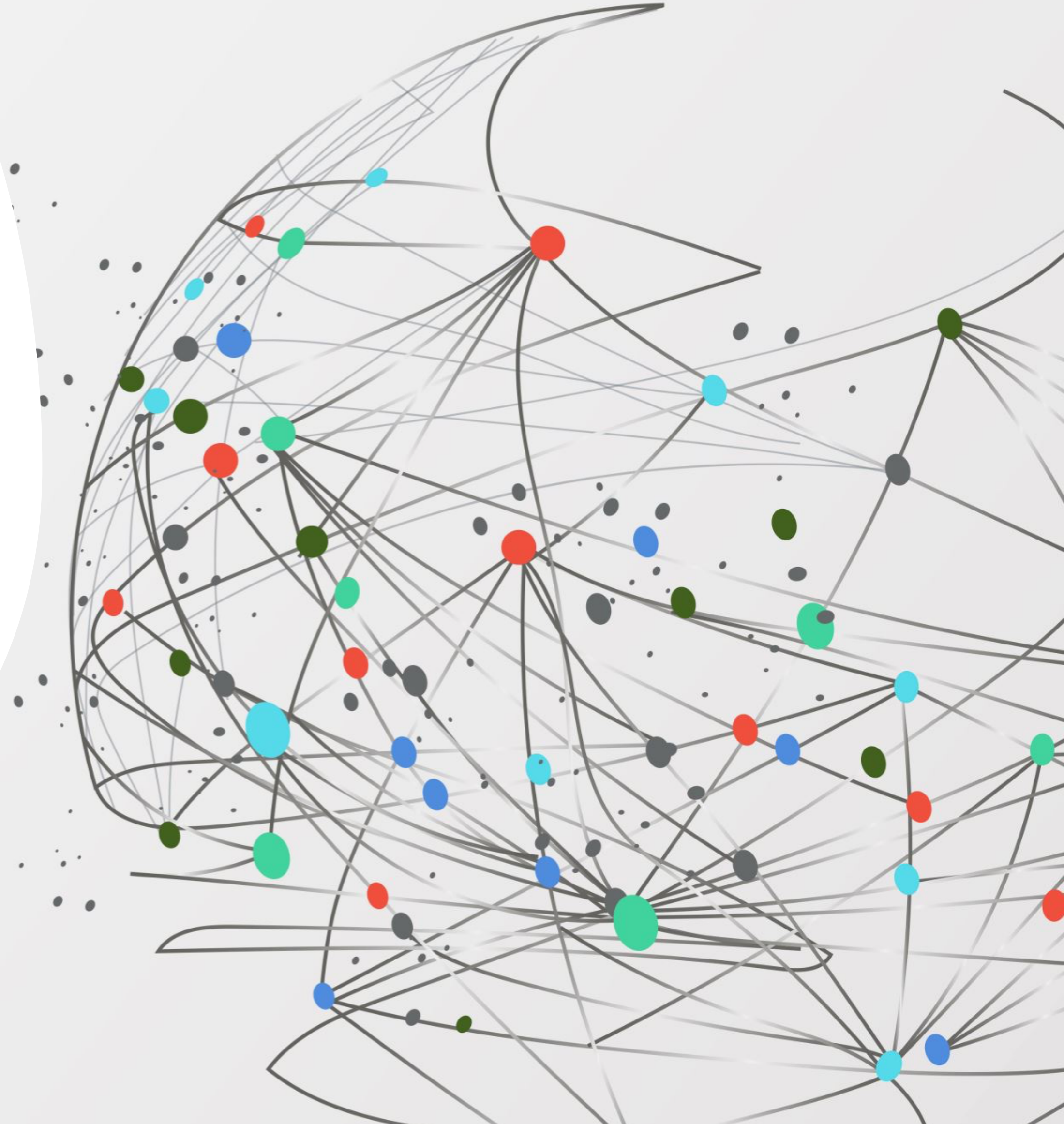
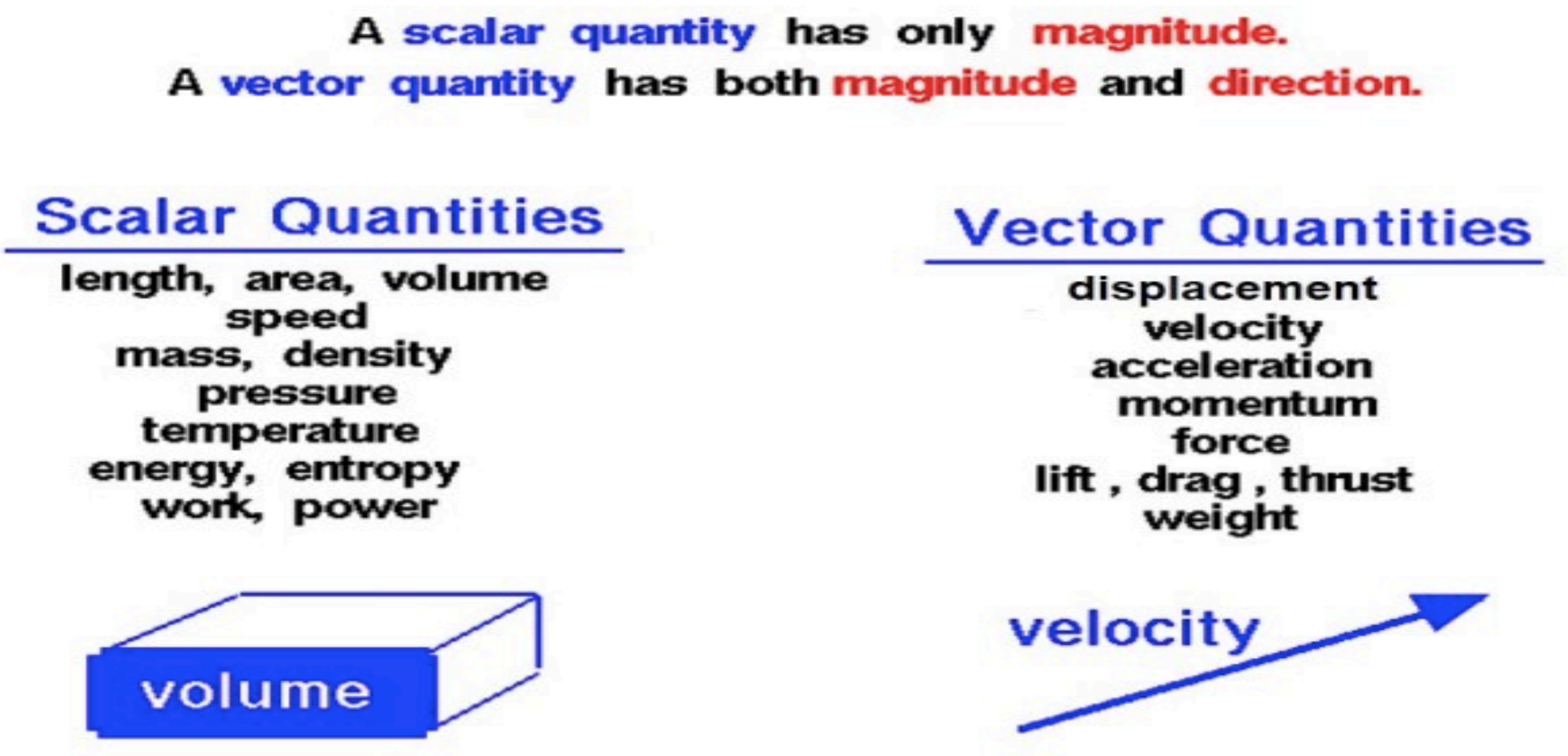
Introduction to Linear Algebra Math camp 2021



Jinwon Lee

Scalar and Vector



Source: https://www.grc.nasa.gov/www/k-12/airplane/vectors.html

Vector

$$\vec{a} = (a_1,\ a_2,\ a_3),\ \vec{b} = (b_1,\ b_2,\ b_3)$$

$$\vec{a} = \vec{b} \iff (a_1,\ a_2,\ a_3) = (b_1,\ b_2,\ b_3) \iff \begin{cases} a_1 = b_1 & \text{and} \\ a_2 = b_2 & \text{and} \\ a_3 = b_3 \end{cases}$$

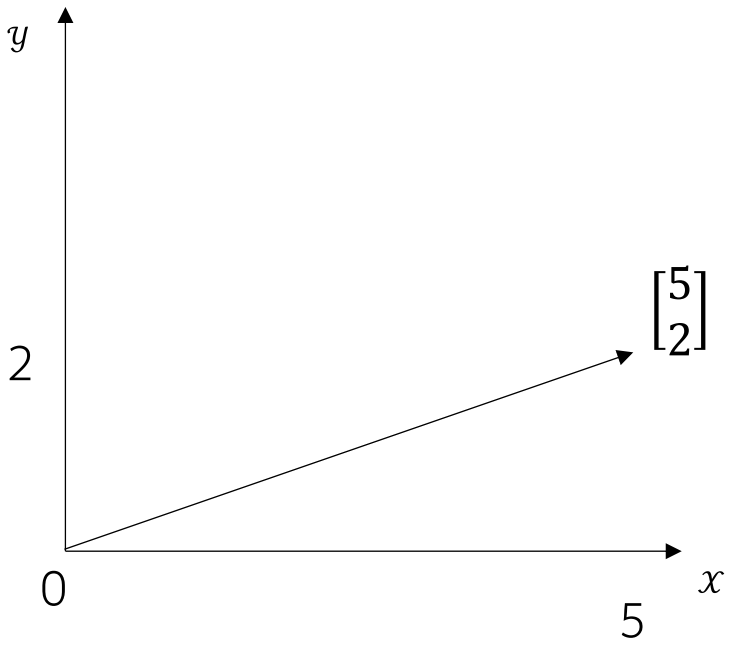
(3,1,4)
$\neq$
(3,4,1)

Urbana walmart	Date	Apple	Orange	Avocado
	August 16th	3	1	4
	August 17th	3	4	1
…..	…..			

Vector

Vector in plane : $X = (x_1, x_2)$ or $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

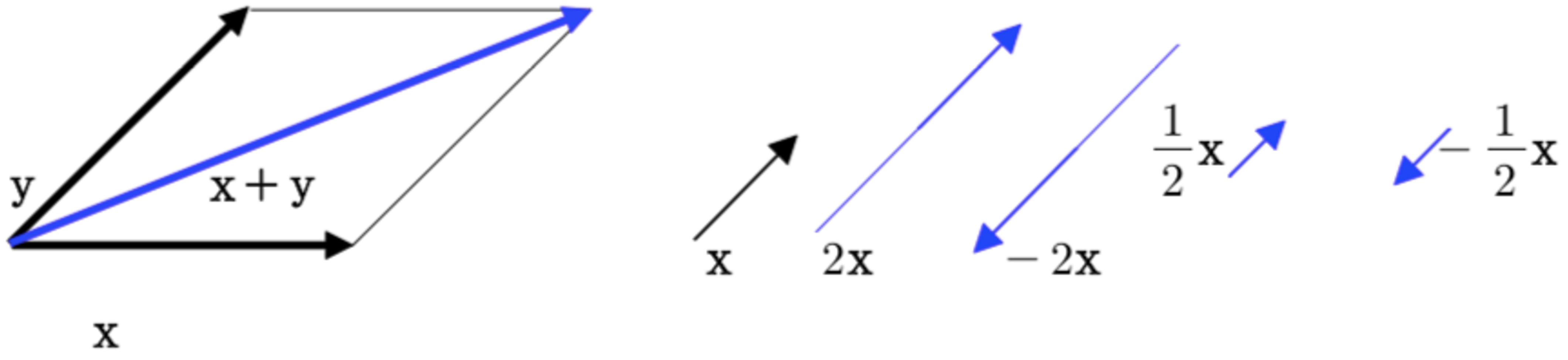
Component: x_1, x_2



A column vector = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

A row vector = $\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}$

Vector & Addition, multiplication

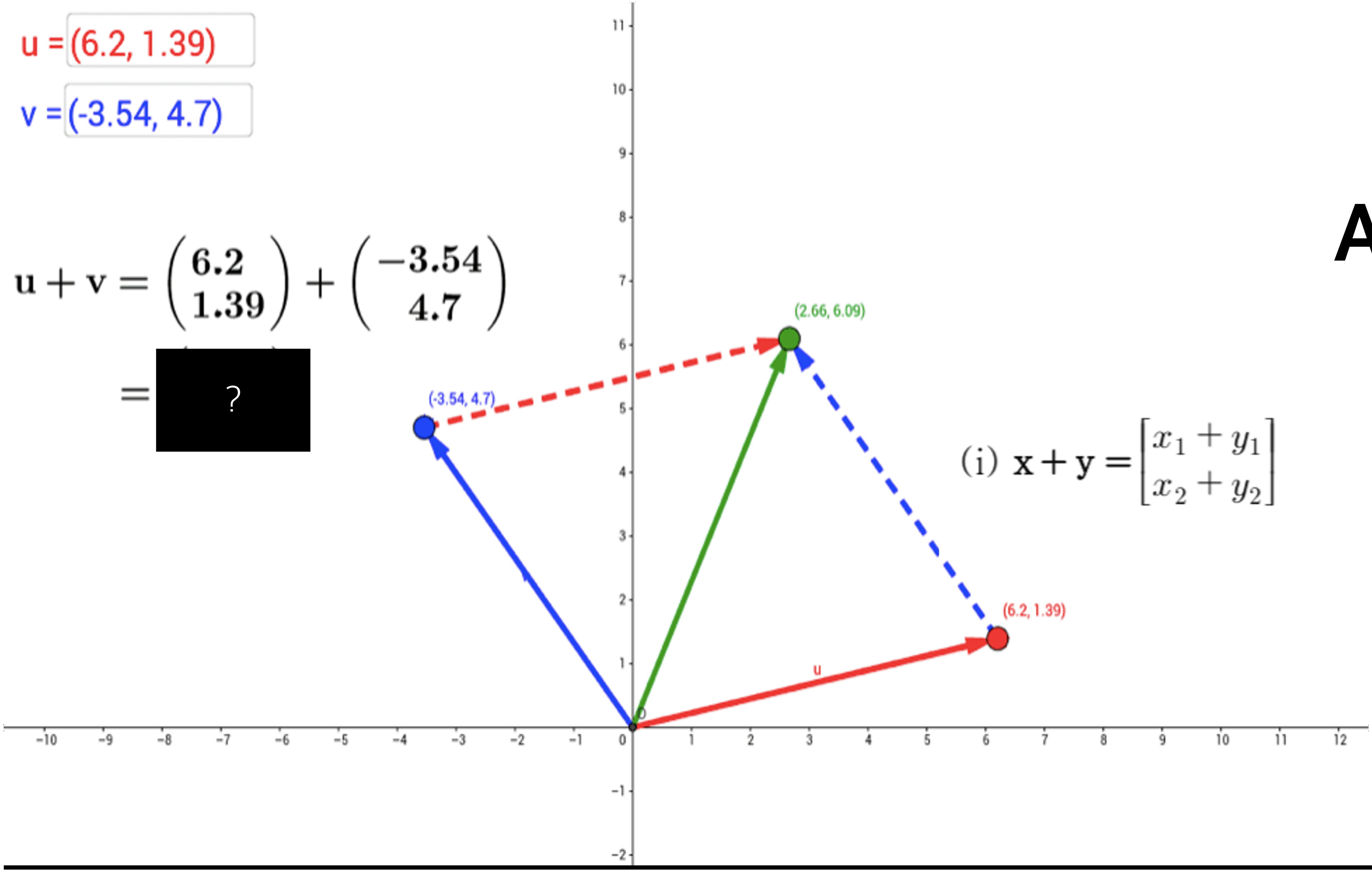


Vector Addition & multiplication

$$(1,\ 0,\ 4)+(3,\ 2,\ 5)=(1+3,\ 0+2,\ 4+5)=(4,\ 2,\ 9)$$
$$(3,\ 2,\ 4)-(1,\ 2,\ 5)=(3-1,\ 2-2,\ 4-5)=(2,\ 0,\ -1)$$
$$(-2,\ 3,\ -4)+(-1,\ 2,\ 5)=(-2+(-1),\ 3+2,\ -4+5)=(-3,\ 5,\ 1)$$
$$(2,\ -5,\ -1)-(-1,\ 2,\ -4)=(2-(-1),\ -5-2,\ -1-(-4))=(3,\ -7,\ 3)$$
$$(4,\ -2)+(-3,\ 2)=(4+(-3),\ -2+2)=(1,\ 0)$$

$$3(2,\ -1,\ 4)=(3\times 2,\ 3\times(-1),\ 3\times 4)=(6,\ -3,\ 12)$$
$$-(1,\ -4,\ 0)=\ -1(1,\ -4,\ 0)=((-1)\times 1,\ (-1)\times(-4),\ (-1)\times 0)=(-1,\ 4,\ 0)$$
$$0(3,\ 1,\ -2)=(0\times 3,\ 0\times 1,\ 0\times(-2))=(0,\ 0,\ 0)$$
$$4(0,\ 0,\ 0)=(4\times 0,\ 4\times 0,\ 4\times 0)=(0,\ 0,\ 0)$$
$$-2(1,\ -3)=((-2)\times 1,\ (-2)\times(-3))=(-2,\ 6)$$

Vector Addition



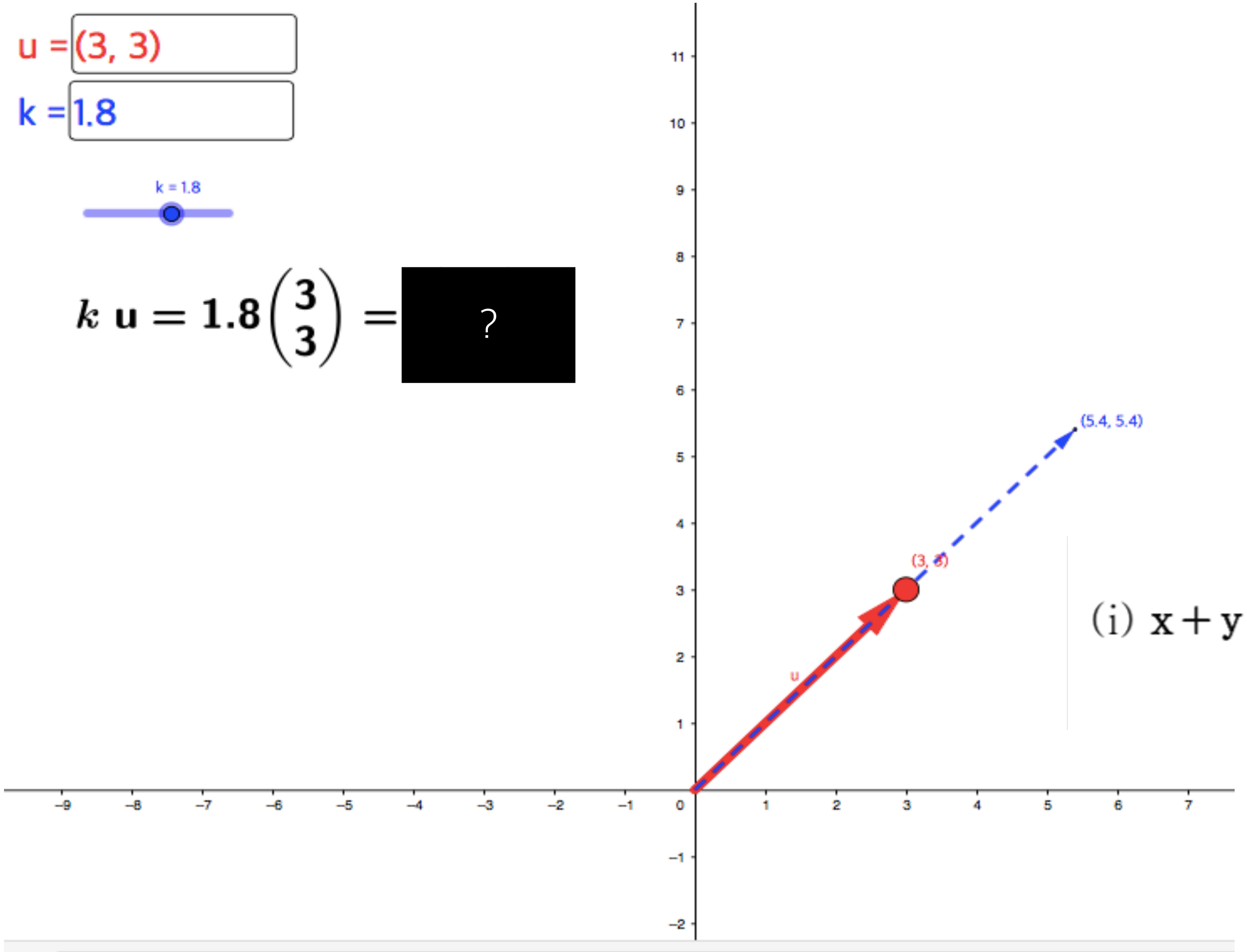
$$\mathbf{u}+\mathbf{v} = \begin{pmatrix} 6.2 \\ 1.39 \end{pmatrix} + \begin{pmatrix} -3.54 \\ 4.7 \end{pmatrix}$$

$$= \boxed{?}$$

(i) $\mathbf{x}+\mathbf{y} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix}$

(ii) $k\mathbf{x} = \begin{bmatrix} kx_1 \\ kx_2 \end{bmatrix}$

Vector multiplication

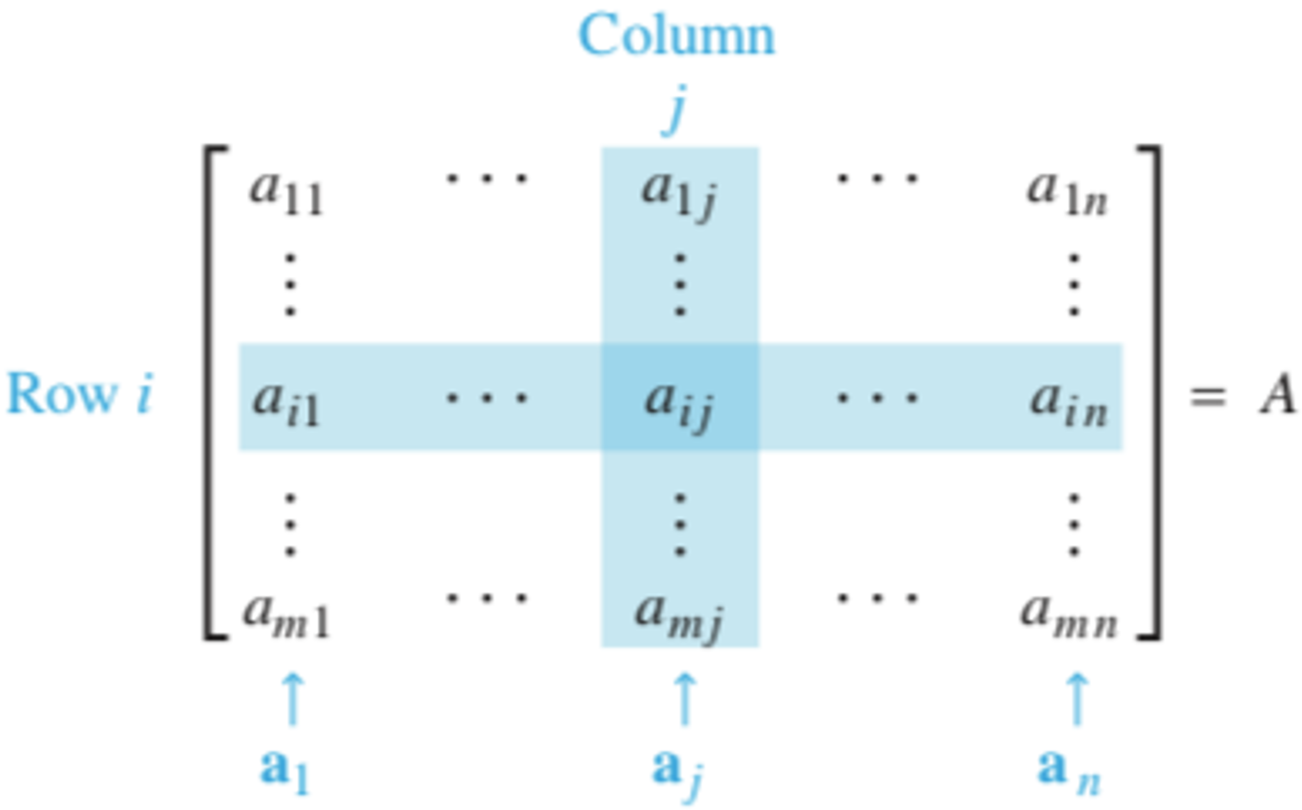


$$\boldsymbol{k}\ \mathbf{u} = \mathbf{1.8}\begin{pmatrix}\mathbf{3}\\ \mathbf{3}\end{pmatrix} = \ ?$$

$$\text{(i)}\ \mathbf{x}+\mathbf{y} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix} \qquad \text{(ii)}\ k\mathbf{x} = \begin{bmatrix} kx_1 \\ kx_2 \end{bmatrix}$$

Matrix Algebra

A is an m x n matrix ? : a matrix with m rows and n columns



Matrix Algebra

A matrix is an array of numbers. It is denoted in **BOLD CAPITAL LETTERS**

$$\underset{2\times 3}{\boldsymbol{A}} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 8 \end{bmatrix}$$

Matrices have dimensions: $\#rows \times \#columns$. Dimensions are pretty important. $\boldsymbol{A}$ is 2 by 3, or 2×3. This is also called the order of a matrix. $\boldsymbol{A}$ is a matrix of order 2×3.

Individual elements of a matrix are denoted by their row number and column number:

$$\underset{m\times n}{\boldsymbol{A}} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix}$$

Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

Matrix Transposition

The transpose of a matrix is a new matrix that is formed by interchanging the rows and columns.

$$\underset{2\times 3}{\boldsymbol{A}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\underset{3\times 2}{\boldsymbol{A}'} = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

The transpose of $\boldsymbol{A}$ is denoted by $\boldsymbol{A}'$ or $\boldsymbol{A}^T$

Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

Practice: Matrix Transposition

$$A=\begin{bmatrix}1 & -2 & 3\\4 & 5 & 0\end{bmatrix},\ B=\begin{bmatrix}1 & 2 & -4\\3 & -1 & 2\\0 & 5 & 3\end{bmatrix},\ C=\begin{bmatrix}5 & 4\\-3 & 2\\2 & 1\end{bmatrix}, D=\begin{bmatrix}3 & 0 & 1\end{bmatrix},\ E=\begin{bmatrix}2\\0\\-3\end{bmatrix}$$

Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 5 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 & -4 \\ 3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix},\ C = \begin{bmatrix} 5 & 4 \\ -3 & 2 \\ 2 & 1 \end{bmatrix}, D = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix},\ E = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$$

Answers

$$A^T = \begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 3 & 0 \end{bmatrix},\ B^T = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -1 & 5 \\ -4 & 2 & 3 \end{bmatrix},\ C^T = \begin{bmatrix} 5 & -3 & 2 \\ 4 & 2 & 1 \end{bmatrix},$$

$$D^T = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix},\ E^T = \begin{bmatrix} 2 & 0 & -3 \end{bmatrix}.$$

Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

Matrix Addition(Subtraction)

Two matrices may be added (or subtracted) iff they are the same order. Simply add (or subtract) the corresponding elements. So, $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{C}$ yields:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \underset{2\times 3}{\boldsymbol{C}} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \\ a_{31} + b_{31} & a_{32} + b_{32} \end{bmatrix}$$

Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

Practice

A= $\begin{bmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}$ B= $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}$ C= $\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$

1) Dimensions?
2) A+B
3) B+A
4) A+C
5) A-2B

Answers

A= $\begin{bmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}$ B= $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}$ C= $\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$

1) Dimensions?
2) A+B
3) B+A
4) A+C
5) A-2B

1)
A= 2x3,
B=2x3,
C=2x2

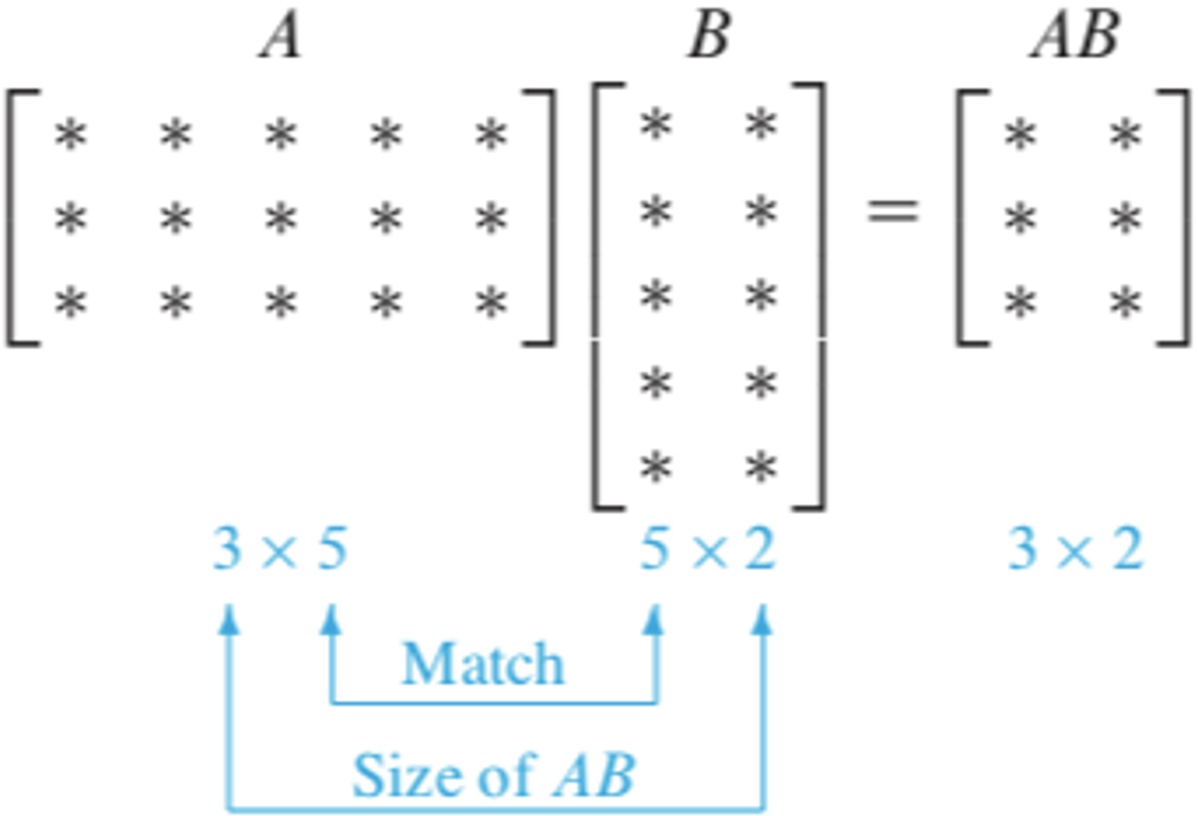
2),3) $\begin{bmatrix} 5 & 1 & 6 \\ 2 & 8 & 9 \end{bmatrix}$

4) A+C is not defined because A and C have different sizes.
A= 2x3, C=2x2

5) $\begin{bmatrix} 2 & -2 & 3 \\ -7 & -7 & -12 \end{bmatrix}$

Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

Matrix Multiplication



Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

EXAMPLE 3 Compute AB, where $A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}$.

SOLUTION Write $B = [\,\mathbf{b}_1 \quad \mathbf{b}_2 \quad \mathbf{b}_3\,]$, and compute:

$$A\mathbf{b}_1 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \quad A\mathbf{b}_2 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \quad A\mathbf{b}_3 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 \\ -1 \end{bmatrix} \qquad = \begin{bmatrix} 0 \\ 13 \end{bmatrix} \qquad = \begin{bmatrix} 21 \\ -9 \end{bmatrix}$$

Then

$$AB = A[\,\mathbf{b}_1 \quad \mathbf{b}_2 \quad \mathbf{b}_3\,] = \begin{bmatrix} 11 & 0 & 21 \\ -1 & 13 & -9 \end{bmatrix}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ A\mathbf{b}_1 & A\mathbf{b}_2 & A\mathbf{b}_3 \end{array}$$

Matrix Multiplication

Practice

EXAMPLE 7 Let $A = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$. Show that these matrices do not commute. That is, verify that $AB \neq BA$.

SOLUTION

$$AB = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 14 & 3 \\ -2 & -6 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 10 & 2 \\ 29 & -2 \end{bmatrix}$$

Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

Question

In Exercises 1 and 2, compute each matrix sum or product if it is defined. If an expression is undefined, explain why. Let

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

1. $-2A$, $B - 2A$, AC, CD

2. $A + 2B$, $3C - E$, CB, EB

If a matrix A is 5×3 and the product AB is 5×7, what is the size of B?

Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any, will make $AB = BA$?

1. $\begin{bmatrix} -4 & 0 & 2 \\ -8 & 10 & -4 \end{bmatrix}$, $\begin{bmatrix} 3 & -5 & 3 \\ -7 & 6 & -7 \end{bmatrix}$, not defined, $\begin{bmatrix} 1 & 13 \\ -7 & -6 \end{bmatrix}$

2. $\begin{bmatrix} 16 & -10 & 1 \\ 6 & -13 & -4 \end{bmatrix}$, not defined, $\begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -2 \end{bmatrix}$, not defined

3. 3x7

4. κ=5

Identity Matrix

The identity matrix is a square matrix that has 1’s along the main diagonal and 0’s for all other entries.

This matrix is often written simply as *I*.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$I_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The Inverse of a Matrix

If A is a non-singular square matrix, there is an existence of n x n matrix A^{-1}, which is called the **inverse matrix** of A such that it satisfies the property:

$AA^{-1} = A^{-1}A = I$, where I is the Identity matrix; The identity matrix for the 2 x 2 matrix is given by $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The inverse of a matrix is found using the following formula:

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Source: https://byjus.com/maths/inverse-matrix/

Practice : The Inverse of a Matrix

EXAMPLE 1 If $A = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix}$ and $C = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}$, then

$$AC = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix}\begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and}$$

$$CA = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus $C = A^{-1}$.

Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

Q2 : the Inverse of a Matrix

1. Find the inverse of A = $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

Find the inverses of the matrices in Exercises 1–4.

1. $\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 2 \\ 7 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 8 & 5 \\ -7 & -5 \end{bmatrix}$

4. $\begin{bmatrix} 3 & -4 \\ 7 & -8 \end{bmatrix}$

Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

Solution : the Inverse of a Matrix

1. Find the inverse of A = $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$$\begin{bmatrix} -3 & 2 \\ 5/2 & -3/2 \end{bmatrix}$$

Find the inverses of the matrices in Exercises 1–4.

1. $\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$ $\begin{bmatrix} 2 & -3 \\ -5/2 & 4 \end{bmatrix}$ **2.** $\begin{bmatrix} 3 & 2 \\ 7 & 4 \end{bmatrix}$ $\begin{bmatrix} -2 & 1 \\ 7/2 & -3/2 \end{bmatrix}$

3. $\begin{bmatrix} 8 & 5 \\ -7 & -5 \end{bmatrix}$ **4.** $\begin{bmatrix} 3 & -4 \\ 7 & -8 \end{bmatrix}$ $\begin{bmatrix} -2 & 1 \\ -7/4 & 3/4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ -7/5 & -8/5 \end{bmatrix}$

Source: https://math.berkeley.edu/~yonah/files/Linear%20Algebra.pdf

Application of Inverse Matrix

$$
\begin{aligned}
&a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n=b_1\\
&a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n=b_2\\
&\vdots\\
&a_{n1}x_1+a_{n2}x_2+\cdots+a_{nn}x_n=b_n
\end{aligned}
$$

$$
\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}
\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}
=
\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} \quad \text{--- (4)}
$$

$$AX=B \quad \text{--- (5)}$$

$$X= A^{-1} B$$

Source: https://byjus.com/maths/inverse-matrix/

Application of Inverse Matrix

* Consider the following system of equations,

2x – y = 3

x + 3y = -2

* Matrix

$$\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

* Inversion

$$A^{-1} = \frac{1}{7}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \rightarrow \frac{1}{7}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} * \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Source: https://byjus.com/maths/inverse-matrix/

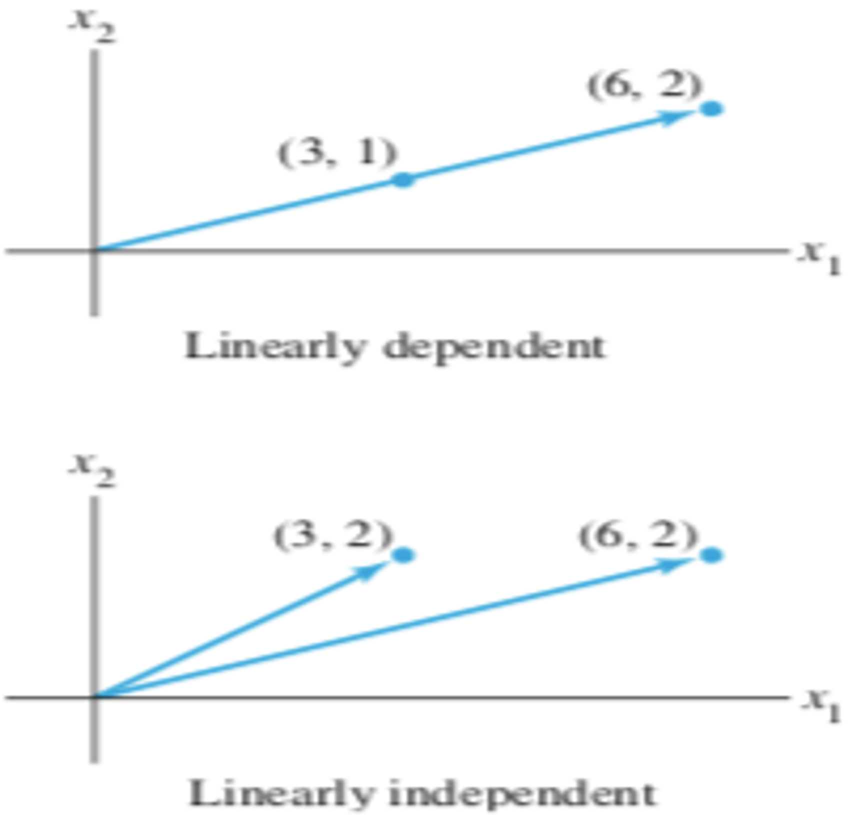
Linear Interdependence
: set of two vectors

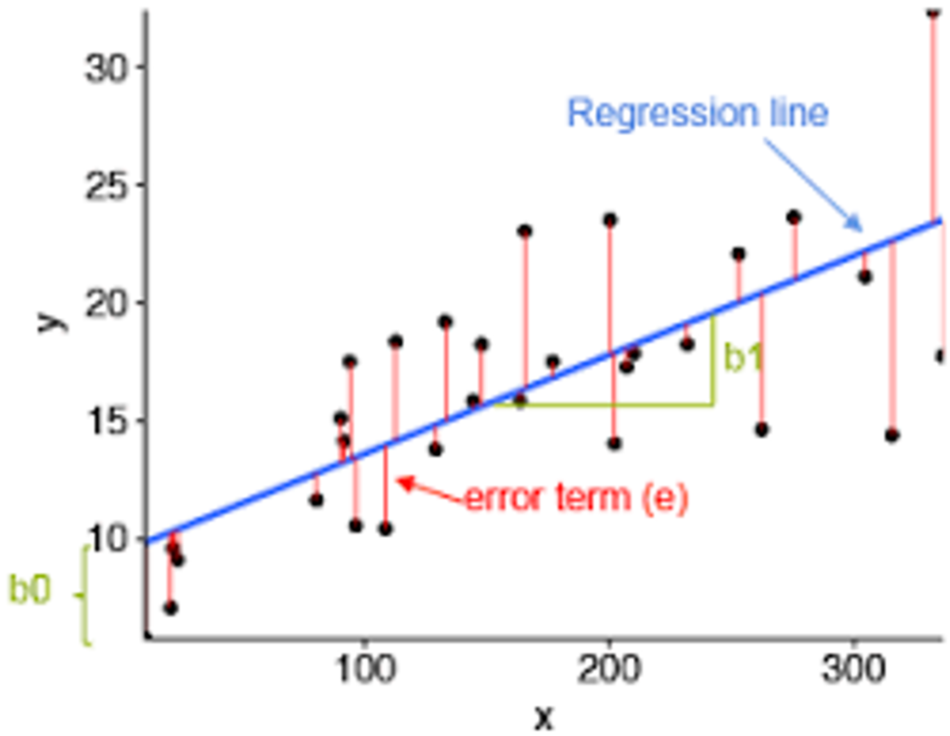
EXAMPLE 3 Determine if the following sets of vectors are linearly independent.

a. $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ b. $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

Source: https://byjus.com/maths/inverse-matrix/

Linear Interdependence : set of two vectors and…



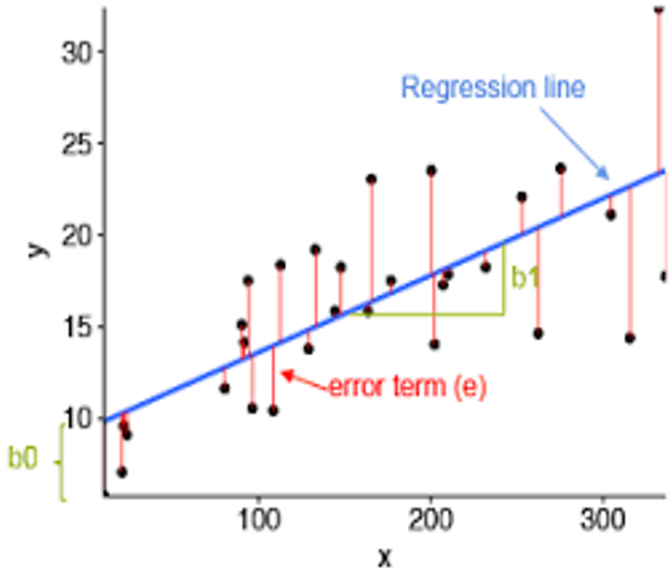


Source: https://byjus.com/maths/inverse-matrix/

Linear Regression in Matrix Form

Consider now writing an equation for each observation:

$$
\begin{aligned}
Y_1 &= \beta_0 + \beta_1 X_1 + \epsilon_1 \\
Y_2 &= \beta_0 + \beta_1 X_2 + \epsilon_2 \\
\vdots \;\; & \;\; \vdots \;\;\;\; \vdots \\
Y_n &= \beta_0 + \beta_1 X_n + \epsilon_n
\end{aligned}
$$



The SLR Model in Matrix Form

$$
\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 X_1 \\ \beta_0 + \beta_1 X_2 \\ \vdots \\ \beta_0 + \beta_1 X_n \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}
$$

$$
\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ \vdots & \vdots \\ 1 & X_n \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}
$$

Source: https://www.stat.purdue.edu/~boli/stat512/lectures/topic3.pdf

Regression Matrix

- Consider example with $n = 4$

$$\begin{aligned} Y_1 &= \beta_0 + \beta_1 X_1 + \varepsilon_1 \\ Y_2 &= \beta_0 + \beta_1 X_2 + \varepsilon_2 \\ Y_3 &= \beta_0 + \beta_1 X_3 + \varepsilon_3 \\ Y_4 &= \beta_0 + \beta_1 X_4 + \varepsilon_4 \end{aligned}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 X_1 \\ \beta_0 + \beta_1 X_2 \\ \beta_0 + \beta_1 X_3 \\ \beta_0 + \beta_1 X_4 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} = \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ 1 & X_3 \\ 1 & X_4 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Source: https://www.stat.purdue.edu/~bacraig/notes525/topic7.pdf

Linear Regression in Matrix Form

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

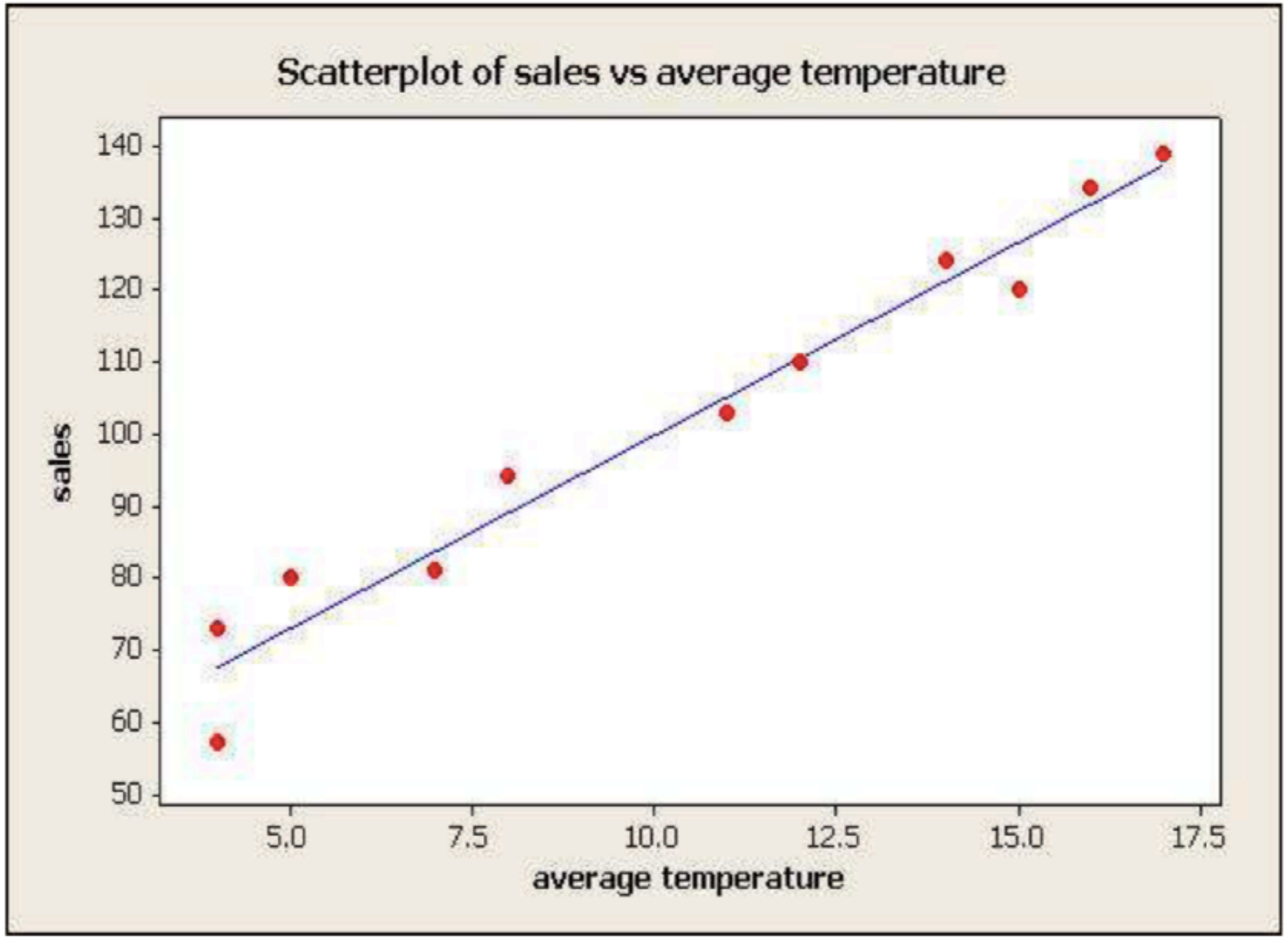
$$Y = X\beta + \varepsilon$$

That is, instead of writing out the n equations, using matrix notation, our simple linear regression function reduces to a short and simple statement:

$$Y = X\beta + \epsilon$$

Example: Linear Regression (ice cream sales)

Month	Average Temp (°C)	Sales (£ 000's)
January	4	73
February	4	57
March	7	81
April	8	94
May	12	110
June	15	124
July	16	134
August	17	139
September	14	124
October	11	103
November	7	81
December	5	80



Source: http://www.mas.ncl.ac.uk/~nag48/teaching/MAS1403/notes7slr.pdf

Your own project or interesting example?