ΑΣΚΗΣΗ:

$$\left.\begin{aligned} x+y+z&=0\\ x+\lambda y+\lambda z&=-3\\ 2x+y+\mu z&=\lambda+1 \end{aligned}\right\}_{3\times 3}$$

$$D=\begin{vmatrix}1&1&1\\1&\lambda&\lambda\\2&1&\mu\end{vmatrix}=\begin{vmatrix}1&1&1\\0&\lambda-1&\lambda-1\\0&-1&\mu-2\end{vmatrix}=(\lambda-1)(\mu-2)+(\lambda-1)=(\lambda-1)(\mu-1)$$

$$D_x=\begin{vmatrix}0&1&1\\-3&\lambda&\lambda\\\lambda+1&1&\mu\end{vmatrix}=\begin{vmatrix}0&1&0\\-3&\lambda&0\\\lambda+1&1&\mu-1\end{vmatrix}=-\begin{vmatrix}-3&0\\\lambda+1&\mu-1\end{vmatrix}=3(\mu-1)$$

$$D_y=\begin{vmatrix}1&0&1\\1&-3&\lambda\\2&\lambda+1&\mu\end{vmatrix}=\begin{vmatrix}1&0&0\\1&-3&\lambda-1\\2&\lambda+1&\mu-2\end{vmatrix}=\begin{vmatrix}-3&\lambda-1\\\lambda+1&\mu-2\end{vmatrix}=-3(\mu-2)-(\lambda-1)(\lambda+1)$$

$$D_z=\begin{vmatrix}1&1&0\\1&\lambda&-3\\2&1&\lambda+1\end{vmatrix}=\begin{vmatrix}1&0&0\\1&\lambda-1&-3\\2&-1&\lambda+1\end{vmatrix}=\begin{vmatrix}\lambda-1&-3\\-1&\lambda+1\end{vmatrix}=(\lambda-1)(\lambda+1)-3$$

- $D\neq 0 \Leftrightarrow \lambda\neq 1$ και $\mu\neq 1$ τότε μον. λύση. $\quad x=\dfrac{D_x}{D},\ y=\dfrac{D_y}{D},\ z=\dfrac{D_z}{D}$, $\lambda,\mu\in\mathbb{R}$
- $D=0 \Leftrightarrow \lambda=1$ ή $\mu=1$.

1η περ.: $\lambda=1$ τότε: $D_x=3(\mu-1)$, $D_y=-3(\mu-2)$, $D_z=-3\neq 0$. Άρα αδύνατο

2η περ. $\mu=1$ τότε: $D_x=0$

$$D_y=3-(\lambda-1)(\lambda+1)=3-(\lambda^2-1)=3-\lambda^2+1=-\lambda^2+4=-(\lambda^2-4)=-(\lambda-2)(\lambda+2)$$

$$D_z=(\lambda-1)(\lambda+1)-3=\lambda^2-1-3=\lambda^2-4=(\lambda-2)(\lambda+2)$$

α) Αν $\lambda=2$ τότε $D_x=D_y=D_z=0$ και το σύστημα γίνεται:

$$\left.\begin{aligned} x+y+z&=0\\ x+2y+2z&=-3\\ 2x+y+z&=3 \end{aligned}\right\}$$

$\Sigma:\ \left.\begin{aligned} x+y&=-z\\ x+2y&=-3-2z\end{aligned}\right\} \overset{(-)}{\Rightarrow} -y=3+z \Rightarrow y=-3-z$, άρα $x=3$, $z\in\mathbb{R}$ } άπειρες λύσεις

Έχω: $2\cdot 3-3=3$ ✓

β) Αν $\lambda=-2$ τότε

⋮