0.1 Ολοκληρώματα

0.1.1 Ορισμός Αόριστου Ολοκληρώματος

$$\int f(x)\,dx = F(x) + c \Leftrightarrow F'(x) = f(x)$$

0.1.2 Ολοκληρώματα Βασικών Συναρτήσεων

$\int 0\,dx = c$
$\int 1\,dx = \int\ dx = x$
$\int x^n\,dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$
$\int \frac{1}{x}\,dx = \ln|x|$
$\int a^x\,dx = \frac{a^x}{\ln a}, \quad a > 0,\, a \neq 1$
$\int e^x\,dx = e^x$
$\int \sqrt[n]{x^m}\,dx = \int x^{\frac{m}{n}}\,dx = \frac{x^{\frac{m}{n}+1}}{\frac{m}{n}+1}$
$\int \sin x\,dx = -\cos x$
$\int \cos x\,dx = \sin x$
$\int \tan x\,dx = \ln\sec x = -\ln\cos x$
$\int \cot x\,dx = \ln\sin x$
$\int \sec x\,dx = \ln(\sec x + \tan x) = \ln\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)$
$\int \csc x\,dx = \ln(\csc x - \cot x) = \ln\tan\frac{x}{2}$
$\int \sec^2 x\,dx = \tan x$
$\int \csc^2 x\,dx = -\cot x$
$\int \tan^2 x\,dx = \tan x - x$
$\int \cot^2 x\,dx = -\cot x - x$
$\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4}$
$\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4}$
$\int \sec x\tan x\,dx = \sec x$
$\int \csc x\cot x\,dx = -\csc x$

$\int \sinh x\,dx = \cosh x$
$\int \cosh x\,dx = \sinh x$
$\int \tanh x\,dx = \ln\cosh x$
$\int \coth x\,dx = \ln\sinh x$
$\int \operatorname{sech} x\,dx = \sin^{-1}(\tanh x)$
$\int \operatorname{csch} x\,dx = \ln\tanh\frac{x}{2}$
$\int \operatorname{sech}^2 x\,dx = \tanh x$
$\int \operatorname{csch}^2 x\,dx = -\coth x$
$\int \tanh^2 x\,dx = x - \tanh x$
$\int \coth^2 x\,dx = x - \coth x$
$\int \sinh^2 x\,dx = \frac{\sinh 2x}{4} - \frac{x}{2}$
$\int \cosh^2 x\,dx = \frac{\sinh 2x}{4} + \frac{x}{2}$
$\int \operatorname{sech} x\tanh x\,dx = -\operatorname{sech} x$
$\int \operatorname{csch} x\coth x\,dx = -\operatorname{csch} x$

$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$
$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) = -\frac{1}{a}\cot^{-1}\frac{x}{a}$
$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) = \frac{1}{a}\tanh^{-1}\frac{x}{a}$
$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}$
$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x + \sqrt{x^2+a^2}\right) = \sin^{-1}\frac{x}{a}$
$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x + \sqrt{x^2-a^2}\right)$
$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|$
$\int \frac{1}{x\sqrt{x^2+a^2}}\,dx = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

0.1.3 $ax+b$

$\int \frac{dx}{ax+b}\,dx = \frac{1}{a}\ln(ax+b)$
$\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b)$
$\int \frac{dx}{x(ax+b)}\,dx = \frac{1}{b}\ln\left(\frac{x}{ax+b}\right)$
$\int \frac{1}{(ax+b)^2}\,dx = \frac{-1}{a(ax+b)}$
$\int \frac{x}{(ax+b)^2}\,dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln(ax+b)$
$\int \frac{1}{x(ax+b)^2}\,dx = \frac{1}{b^2}\ln\left(\frac{x}{ax+b}\right)$
$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}, n \neq -1$
$\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+2}}{a^2(n+2)} - \frac{b(ax+b)^{n+1}}{a^2(n+1)}, n \neq -1, -2$

$$\int x^m(ax+b)^n\,dx = \begin{cases} \frac{x^{m+1}(ax+b)^n}{m+n+1} + \frac{nb}{m+n+1}\int x^m(ax+b)^{n-1}\,dx \\ \frac{x^m(ax+b)^{n+1}}{a(m+n+1)} - \frac{mb}{a(m+n+1)}\int x^{m-1}(ax+b)^n\,dx \\ \frac{-x^{m+1}(ax+b)^{n+1}}{b(n+1)} + \frac{m+n+2}{b(n+1)}\int x^m(ax+b)^{n+1}\,dx \end{cases}$$

0.1.4 $\sqrt{ax+b}$

$\int \frac{dx}{\sqrt{ax+b}}\,dx = \frac{2\sqrt{ax+b}}{a}$
$\int \frac{xdx}{\sqrt{ax+b}}\,dx = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$

$$\int \frac{dx}{x\sqrt{ax+b}}\,dx = \begin{cases} \frac{1}{\sqrt{b}}\ln\left(\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right) \\ \frac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\frac{ax+b}{-b}} \end{cases}$$

$\int \sqrt{ax+b}\,dx = \frac{2\sqrt{(ax+b)^3}}{3a}$
$\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3}$
$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$
$\int \frac{x^m}{\sqrt{ax+b}}\,dx = \frac{2x^m\sqrt{ax+b}}{a(2m+1)} - \frac{2mb}{a(2m+1)}\int \frac{x^{m-1}}{\sqrt{ax+b}}\,dx$
$\int \frac{dx}{x^m\sqrt{ax+b}}\,dx = -\frac{\sqrt{ax+b}}{b(m-1)x^{m-1}} - \frac{a(2m-3)}{b(2m-2)}\int \frac{dx}{x^{m-1}\sqrt{ax+b}}$
$\int x^m\sqrt{ax+b}\,dx = \frac{2x^m}{a(2m+3)}(ax+b)^{frac32} - \frac{2mb}{a(2m+3)}\int x^{m-1}\sqrt{ax+b}\,dx$
$\int \frac{\sqrt{ax+b}}{x^m}\,dx = -\frac{\sqrt{ax+b}}{(m-1)x^{m-1}} + \frac{a}{2(m-1)}\int \frac{dx}{x^{m-1}\sqrt{ax+b}}\,dx$
$\int \frac{\sqrt{ax+b}}{x^m}\,dx = -\frac{(ax+b)^{\frac{3}{2}}}{b(m-1)x^{m-1}} - \frac{a(2m-5)}{b(2m-2)}\int \frac{\sqrt{ax+b}}{x^{m-1}}\,dx$
$\int (ax+b)^{\frac{3}{2}}\,dx = \frac{2(ax+b)^{\frac{(m+2)}{2}}}{a^2(m+2)}\,dx$
$\int x(ax+b)^{\frac{m}{2}}\,dx = \frac{2(ax+b)^{\frac{m+4}{2}}}{a^2(m+4)} - \frac{2b(ax+b)^{\frac{m+2}{2}}}{a^2(m+2)}$
$\int \frac{(ax+b)^{\frac{m}{2}}}{x}\,dx = \frac{2(ax+b)^{\frac{m}{2}}}{m} + b\int \frac{(ax+b)^{\frac{m-2}{2}}}{x}\,dx$
$\int \frac{dx}{x(ax+b)^{\frac{m}{2}}}\,dx = \frac{2}{b(m-2)(ax+b)^{\frac{m-2}{2}}} + \frac{1}{b}\int \frac{dx}{x(ax+b)^{\frac{m-2}{2}}}\,dx$

0.1.5 $(ax+b), (px+q)$

$\int \frac{dx}{(ax+b)(px+q)} = \frac{1}{bp-aq} \ln\left(\frac{px+q}{ax+b}\right)$
$\int \frac{x}{(ax+b)(px+q)}\, dx = \frac{1}{bp-aq}\{\frac{b}{a}\ln(ax+b) - \frac{q}{p}\ln(px+q)\}$
$\int \frac{1}{(ax+b)^2(px+q)}\, dx = \frac{1}{bp-aq}\{\frac{1}{ax+b} + \frac{p}{bp-aq}\ln\left(\frac{px+q}{ax+b}\right)\}$
$\int \frac{x}{(ax+b)^2(px+q)}\, dx = \frac{1}{bp-aq}\{\frac{q}{bp-aq}\ln\left(\frac{ax+b}{px+q}\right) - \frac{b}{a(ax+b)}\}$
$\int \frac{1}{(ax+b)^m(px+q)^n}\, dx = \frac{-1}{(n-1)(bp-aq)}\{\frac{1}{(ax+b)^{m-1}(px+q)^{n-1}} + a(m+n-2)\int \frac{(ax+b)^m}{(px+q)^{n-1}}\, dx\}$
$\int \frac{ax+b}{px+q}\, dx = \frac{ax}{p} + \frac{bp-aq}{p^2}\ln(px+q)$
$$\int \frac{(ax+b)^m}{(px+q)^n}\, dx = \begin{cases} \frac{-1}{(n-1)(bp-aq)}\{\frac{(ax+b)^{m+1}}{(px+q)^{n-1}} + a(n-m-2)\int \frac{(ax+b)^m}{(px+q)^{n-1}}\, dx\} \\ \frac{-1}{p(n-m-1)}\{\frac{(ax+b)^m}{(px+q)^{n-1}} + m(bp-aq)\int \frac{(ax+b)^{m-1}}{(px+q)^n}\, dx\} \\ \frac{-1}{p(n-1)}\{\frac{(ax+b)^m}{(px+q)^{n-1}} - ma\int \frac{(ax+b)^{m-1}}{(px+q)^{n-1}}\, dx\} \end{cases}$$

0.1.6 $\sqrt{ax+b}, px+q$

$\int \frac{px+q}{\sqrt{ax+b}}\, dx = \frac{2(apx+3aq-2bp)}{3a^2}\sqrt{ax+b}$
$$\int \frac{1}{(px+q)\sqrt{ax+b}} = \begin{cases} \frac{1}{\sqrt{bp-aq}\sqrt{p}}\ln\left(\frac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\ \frac{2}{\sqrt{bp-aq}\sqrt{p}}\tan^{-1}\sqrt{\frac{p(ax+b)}{aq-bp}} \end{cases}$$
$$\int \frac{\sqrt{ax+b}}{px+q}\, dx = \begin{cases} \frac{2\sqrt{ax+b}}{p} + \frac{\sqrt{bp-aq}}{p\sqrt{p}}\ln\left(\frac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\ \frac{2\sqrt{ax+b}}{p} - \frac{2\sqrt{bp-aq}}{p\sqrt{p}}\tan^{-1}\sqrt{\frac{p(ax+b)}{aq-bp}} \end{cases}$$
$\int (px+q)^n\sqrt{ax+b}\, dx = \frac{2(px+q)^{n+1}\sqrt{ax+b}}{p(2n+3)} + \frac{bp-aq}{p(2n+3)}\int \frac{dx}{(px+q)\sqrt{ax+b}}\, dx$
$\int \frac{(px+q)^n}{\sqrt{ax+b}}\, dx = \frac{2(px+q)^n\sqrt{ax+b}}{a(2n+1)} + \frac{2n(aq-bp)}{a(2n+1)}\int \frac{(px+q)^{n-1}}{\sqrt{ax+b}}\, dx$
$\int \frac{\sqrt{ax+b}}{(px+q)^n}\, dx = \frac{-\sqrt{ax+b}}{p(n-1)(px+q)^{n-1}} + \frac{a}{2p(n-1)}\int \frac{1}{(px+q)^{n-1}\sqrt{ax+b}}\, dx$

0.1.7 $\sqrt{ax+b}, \sqrt{px+q}$

$$\int \frac{dx}{\sqrt{(ax+b)(px+q)}}\, dx = \begin{cases} \frac{2}{\sqrt{ap}}\ln\left(\sqrt{a(px+q)} + \sqrt{p(ax+b)}\right) \\ \frac{2}{\sqrt{-ap}}\tan^{-1}\sqrt{\frac{-p(ax+b)}{a(px+q)}} \end{cases}$$
$\int \frac{x}{\sqrt{(ax+b)(px+q)}}\, dx = \frac{\sqrt{(ax+b)(px+q)}}{ap} - \frac{bp+aq}{2ap}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$
$\int \sqrt{(ax+b)(px+q)}\, dx = \frac{2apx+bp+aq}{4ap}\sqrt{(ax+b)(px+q)} - \frac{(bp-aq)^2}{8ap}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$
$\int \sqrt{\frac{px+q}{ax+b}}\, dx = \frac{\sqrt{(ax+b)(px+q)}}{a} + \frac{aq-bp}{2a}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$
$\int \frac{dx}{(px+q)\sqrt{(ax+b)(px+q)}} = \frac{2\sqrt{ax+b}}{(aq-bp)\sqrt{px+q}}$

0.1.8 $x^2 + a^2$

$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$
$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln\left(x^2+a^2\right)$
$\int \frac{dx}{x(x^2+a^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{x^2+a^2}\right)$
$\int \frac{dx}{(x^2+a^2)^2} = \frac{x}{2a^2(x^2+a^2)} + \frac{1}{2a^3}\tan^{-1}\frac{x}{a}$
$\int \frac{x}{(x^2+a^2)^2}\,dx = \frac{-1}{2(x^2+a^2)^2}$
$\int \frac{1}{x(x^2+a^2)^2}\,dx = \frac{1}{2a^2(x^2+a^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2+a^2}\right)$
$\int \frac{1}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{a^2(2n-2)}\int \frac{dx}{(x^2+a^2)^{n-1}}$
$\int \frac{x}{(x^2+a^2)^n}\,dx = \frac{-1}{2(n-1)(x^2+a^2)^{n-1}}$
$\int \frac{dx}{x(x^2+a^2)^n}\,dx = \frac{1}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x(x^2+a^2)^{n-1}}$
$\int x^m(x^2+a^2)^n = \int \frac{x^{m-2}}{(x^2+a^2)^{n-1}} - a^2\int \frac{x^{m-2}}{(x^2+a^2)^n}$
$\int \frac{1}{x^m(x^2+a^2)^n}\,dx = \frac{1}{a^2}\int \frac{dx}{x^m(x^2+a^2)^{n-1}} - \frac{1}{a^2}\int \frac{dx}{x^{m-2}(x^2+a^2)^n}$

0.1.9 $x^2 - a^2$

$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) = -\frac{1}{a}\coth^{-1}\frac{x}{a}$
$\int \frac{x}{x^2-a^2} = \frac{1}{2}\ln\left(x^2-a^2\right)$
$\int \frac{1}{x(x^2-a^2)}\,dx = \frac{1}{2a^2}\ln\left(\frac{x^2-a^2}{x^2}\right)$
$\int \frac{dx}{(x^2-a^2)^2}\,dx = \frac{-x}{2a^2(x^2-a^2)} - \frac{1}{4a^3}\ln\left(\frac{x-a}{x+a}\right)$
$\int \frac{x}{(x^2-a^2)^2}\,dx = \frac{-1}{2(x^2-a^2)}$
$\int \frac{1}{x(x^2-a^2)^2}\,dx = \frac{-1}{2a^2(x^2-a^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2-a^2}\right)$
$\int \frac{dx}{(x^2-a^2)^n} = \frac{-x}{2a^2(n-1)(x^2-a^2)^{n-1}} - \frac{2n-3}{a^2(2n-2)}\int \frac{dx}{(x^2-a^2)^{n-1}}$
$\int \frac{x}{(x^2-a^2)^n}\,dx = \frac{-1}{2(n-1)(x^2-a^2)^{n-1}}$
$\int \frac{1}{x(x^2-a^2)^n}\,dx = \frac{-1}{2a^2(n-1)(x^2-a^2)^{n-1}} - \frac{1}{a^2}\int \frac{dx}{x(x^2-a^2)^{n-1}}$
$\int \frac{x^m}{(x^2-a^2)^n}\,dx = \frac{x^{m-2}}{(x^2-a^2)^{n-1}}\,dx + a^2\int \frac{x^{m-2}}{x(x^2-a^2)^{n-1}}$
$\int \frac{dx}{x^m(x^2-a^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^{m-2}(x^2-a^2)^n} - \frac{1}{a^2}\int \frac{dx}{x^m(x^2-a^2)^{n-1}}$

0.1.10 $a^2 - x^2$

$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) = \frac{1}{a}\tanh^{-1}\frac{x}{a}$
$\int \frac{x}{a^2-x^2}\,dx = -\frac{1}{2}\ln\left(a^2-x^2\right)$
$\int \frac{dx}{x(a^2-x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2-x^2}\right)$
$\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)} + \frac{1}{4a^3}\ln\left(\frac{a+x}{a-x}\right)$
$\int \frac{x}{(a^2-x^2)^2}\,dx = \frac{1}{2(a^2-x^2)}$
$\int \frac{1}{x(a^2-x^2)^2}\,dx = \frac{1}{2a^2(a^2-x^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{a^2-x^2}\right)$
$\int \frac{dx}{(a^2-x^2)^n} = \frac{x}{2a^2(n-1)(a^2-x^2)^{n-1}} + \frac{2n-3}{a^2(2n-2)}\int \frac{dx}{(a^2-x^2)^{n-1}}$
$\int \frac{x}{(a^2-x^2)^n}\,dx = \frac{1}{2(n-1)(a^2-x^2)^{n-1}}$
$\int \frac{dx}{x(a^2-x^2)^n} = \frac{1}{2a^2(n-1)(a^2-x^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x(a^2-x^2)^{n-1}}$
$\int \frac{x^m}{(a^2-x^2)^n}\,dx = a^2\int \frac{x^{m-2}}{(a^2-x^2)^n}\,dx - \int \frac{x^{m-2}}{(a^2-x^2)^{n-1}}$
$\int \frac{dx}{x^m(a^2-x^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^m(a^2-x^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x^{m-2}(a^2-x^2)^n}$

0.1.11 $\sqrt{x^2+a^2}$

$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\bigl(x+\sqrt{x^2+a^2}\bigr) = \sin^{-1}\frac{x}{a}$
$\int \frac{x}{\sqrt{x^2+a^2}}\,dx = \sqrt{x^2+a^2}$
$\int \frac{1}{x\sqrt{x^2+a^2}}\,dx = -\frac{1}{a}\ln\biggl(\frac{a+\sqrt{x^2+a^2}}{x}\biggr)$
$\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln\bigl(x+\sqrt{x^2+a^2}\bigr)$
$\int x\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{\frac{3}{2}}}{3}$
$\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\ln\biggl(\frac{a+\sqrt{x^2+a^2}}{x}\biggr)$

0.1.12 $\sqrt{x^2-a^2}$

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\bigl(x+\sqrt{x^2-a^2}\bigr)$
$\int \frac{x}{\sqrt{x^2-a^2}}\,dx = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2}\ln\bigl(x+\sqrt{x^2-a^2}\bigr)$
$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|$
$\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\ln\bigl(x+\sqrt{x^2-a^2}\bigr)$
$\int x\sqrt{x^2-a^2}\,dx = \frac{(x^2-a^2)^{\frac{3}{2}}}{3}$
$\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\sec^{-1}\left|\frac{x}{a}\right|$

0.1.13 $\sqrt{a^2-x^2}$

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = sin^{-1}\frac{x}{a}$
$\int \frac{x}{\sqrt{a^2-x^2}}\,dx = -\sqrt{a^2-x^2}$
$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{1}{a}\ln\biggl(\frac{a+\sqrt{a^2-x^2}}{x}\biggr)$
$\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$
$\int x\sqrt{a^2-x^2}\,dx = -\frac{x(a^2-x^2)^{\frac{3}{2}}}{3}$
$\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln\biggl(\frac{a+\sqrt{a^2-x^2}}{x}\biggr)$

0.1.14 ax^2+bx+c

$\int \frac{1}{ax^2+bx+c}\,dx = \begin{cases} \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \\ \frac{1}{\sqrt{b^2-4ac}}\ln\biggl(\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\biggr) \end{cases}$
$\int \frac{x}{ax^2+bx+c}\,dx = \frac{1}{2a}\ln\bigl(ax^2+bx+c\bigr) - \frac{b}{2a}\int \frac{1}{ax^2+bx+c}\,dx$
$\int \frac{x^m}{ax^2+bx+c}\,dx = \frac{x^{m-1}}{a(m-1)} - \frac{c}{a}\int \frac{x^{m-2}}{ax^2+bx+c}\,dx - \frac{b}{a}\int \frac{x^{m-1}}{ax^2+bx+c}\,dx$
$\int \frac{1}{x(ax^2+bx+c)}\,dx = \frac{1}{2c}\ln\biggl(\frac{x^2}{ax^2+bx+c}\biggr) - \frac{b}{2c}\int \frac{1}{ax^2+bx+c}\,dx$
$\int \frac{1}{x^n(ax^2+bx+c)}\,dx = -\frac{1}{c(n-1)x^{n-1}} - \frac{b}{c}\int \frac{1}{x^{n-1}(ax^2+bx+c)}\,dx - \frac{a}{c}\int 1x^{n-2}(ax^2+bx+c)\,dx$
$\int \frac{x^m}{(ax^2+bx+c)^n}\,dx = -\frac{x^{m-1}}{a(2n-m-1)(ax^2+bx+c)^{n-1}} + \frac{c(m-1)}{a(2n-m-1)}\int \frac{x^{m-2}}{(ax^2+bx+c)^n} - \frac{b(n-m)}{a(2n-m-1)}\int \frac{x^{m-1}}{(ax^2+bx+c)^n}\,dx$
$\int \frac{1}{x^m(ax^2+bx+c)^n} = -\frac{1}{c(m-1)x^{n-1}(ax^2+bx+c)^{n-1}} - \frac{a(m+2n-3)}{c(m-1)}\int \frac{1}{x^{m-2}(ax^2+bx+c)^n}\,dx - \frac{b(m+n-2)}{c(m-1)}\int \frac{1}{x^{m-1}(ax^2+bx+c)^{n-1}}\,dx$

0.1.15 $\sqrt{ax^2+bx+c}$

$\int \frac{1}{\sqrt{ax^2+bx+c}} = \begin{cases} \frac{1}{\sqrt{a}} \ln\left(2\sqrt{a}\sqrt{ax^2+bx+c}+2ax+b\right) \\ -\frac{1}{\sqrt{-a}} \sin^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) = \frac{1}{\sqrt{a}} \sinh^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) \end{cases}$
$\int \frac{x}{\sqrt{ax^2+bx+c}}\,dx = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a}\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx$
$\int \frac{1}{x\sqrt{ax^2+bx+c}}\,dx = \begin{cases} -\frac{1}{\sqrt{c}} \ln\left(\frac{2\sqrt{c}\sqrt{ax^2+bx+c}+bx+2c}{x}\right) \\ \frac{1}{\sqrt{-c}} \sin^{-1}\left(\frac{bx+2c}{|x|\sqrt{b^2-4ac}}\right) = -\frac{1}{\sqrt{c}} \sinh^{-1}\left(\frac{bx+2c}{|x|\sqrt{4ac-b^2}}\right) \end{cases}$
$\int \sqrt{ax^2+bx+c}\,dx = \frac{(2ax+b)\sqrt{ax^2+bx+c}}{4a} + \frac{4ac-b^2}{8a}\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx$
$\int x\sqrt{ax^2+bx+c}\,dx = \frac{\sqrt{ax^2+bx+c)}^{\frac{3}{2}}}{3a} - \frac{b(2ax+b)}{8a^2}\sqrt{ax^2+bx+c} - \frac{b(4ac-b^2)}{16a^2}\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx$
$\int \frac{\sqrt{ax^2+bx+c}}{x}\,dx = \sqrt{ax^2+bx+c} + \frac{b}{2}\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx + c\int \frac{1}{x\sqrt{ax^2+bx+c}}\,dx$
$\int (ax^2+bx+c)^{n+\frac{1}{2}}\,dx = \frac{(2ax+b)(ax^2+bx+c)^{n+\frac{1}{2}}}{4a(n+1)} + \frac{(2n+1)(4ac-b^2)}{8a(n+1)}\int (ax^2+bx+c)^{n-\frac{1}{2}}\,dx$
$\int x(ax^2+bx+c)^{n+\frac{1}{2}}\,dx = \frac{(ax^2+bx+c)^{n+\frac{3}{2}}}{a(2n+3)} - \frac{b}{2a}\int (ax^2+bx+c)^{n+\frac{1}{2}}\,dx$
$\int \frac{1}{(ax^2+bx+c)^{n+\frac{1}{2}}}\,dx = \frac{2(2ax+b)}{(2n-1)(4ac-b^2)(ax^2+bx+c)^{n-\frac{1}{2}}} + \frac{8a(n-1)}{(2n-1)(4ac-b^2)}\int \frac{1}{(ax^2+bx+c)^{n-\frac{1}{2}}}$
$\int \frac{1}{x(ax^2+bx+c)^{n+\frac{1}{2}}}\,dx = \frac{1}{c(2n-1)(ax^2+bx+c)^{n-\frac{1}{2}}} + \frac{1}{c}\int \frac{1}{x(ax^2+bx+c)^{n-\frac{1}{2}}}\,dx - \frac{b}{2c}\int \frac{1}{(ax^2+bx+c)^{n+\frac{1}{2}}}\,dx$

0.1.16 e^{ax}

$\int e^{ax}\,dx = \frac{e^{ax}}{a}$
$\int xe^{ax}\,dx = \frac{e^{ax}}{a}(x - frac1a)$
$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx = \frac{e^{ax}}{a}\left(x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \cdots \frac{(-1)^n n!}{a^n}\right)$
$\int \frac{e^{ax}}{x}\,dx = \ln x + \frac{ax}{1\cdot 1!} + \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^3}{3\cdot 3!} + \cdots$