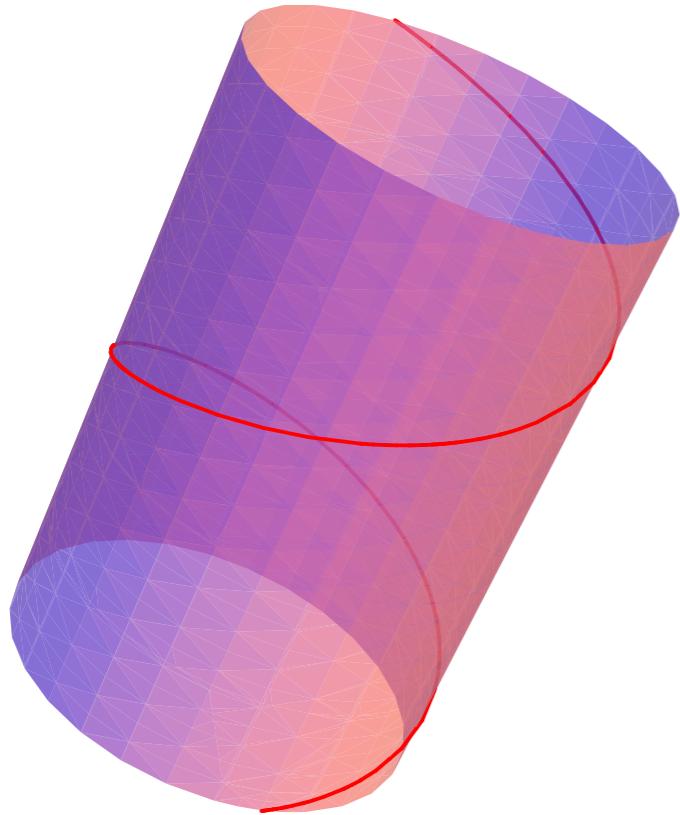
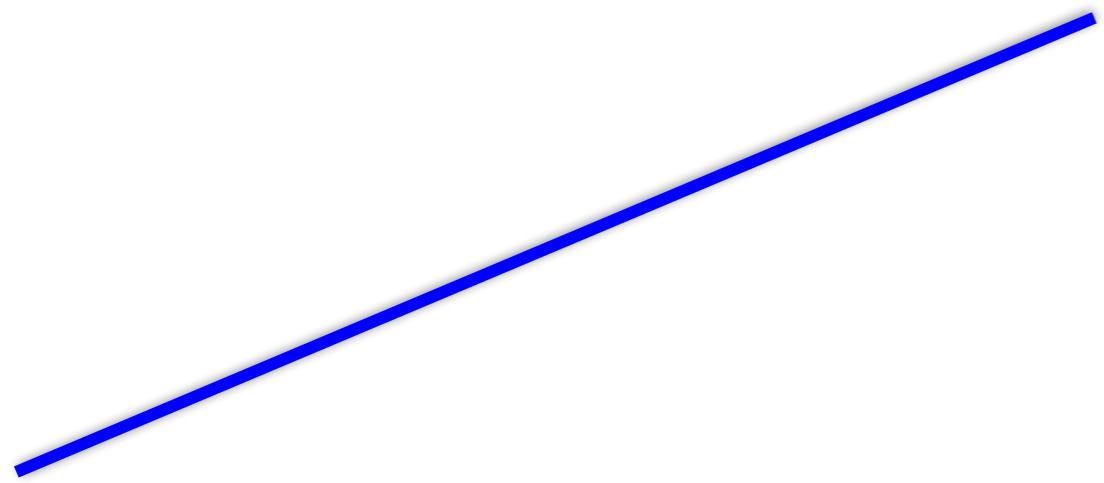


# What is Curvature?

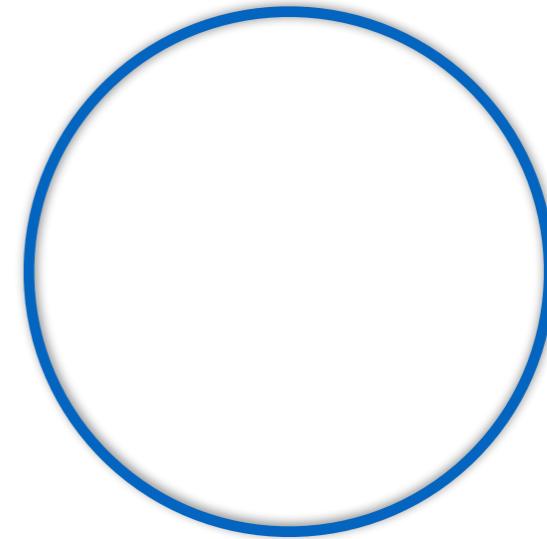


# Curvature of Simple Curves

**Rough Idea:** Curvature measures not being flat.

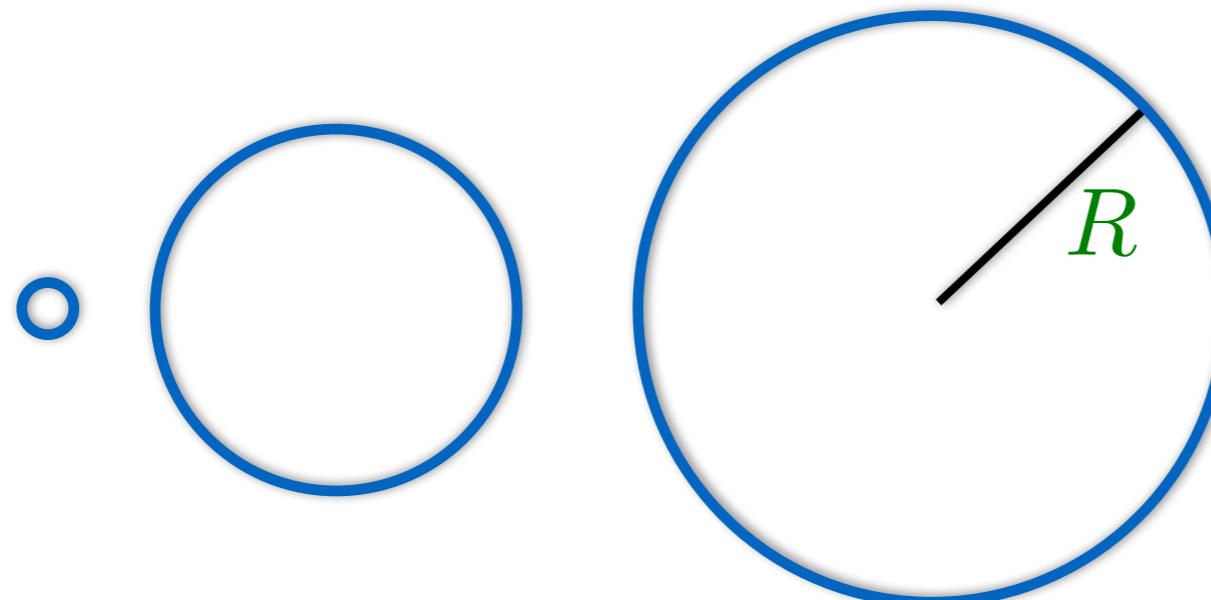


Curvature = 0



Curvature  $\neq 0$

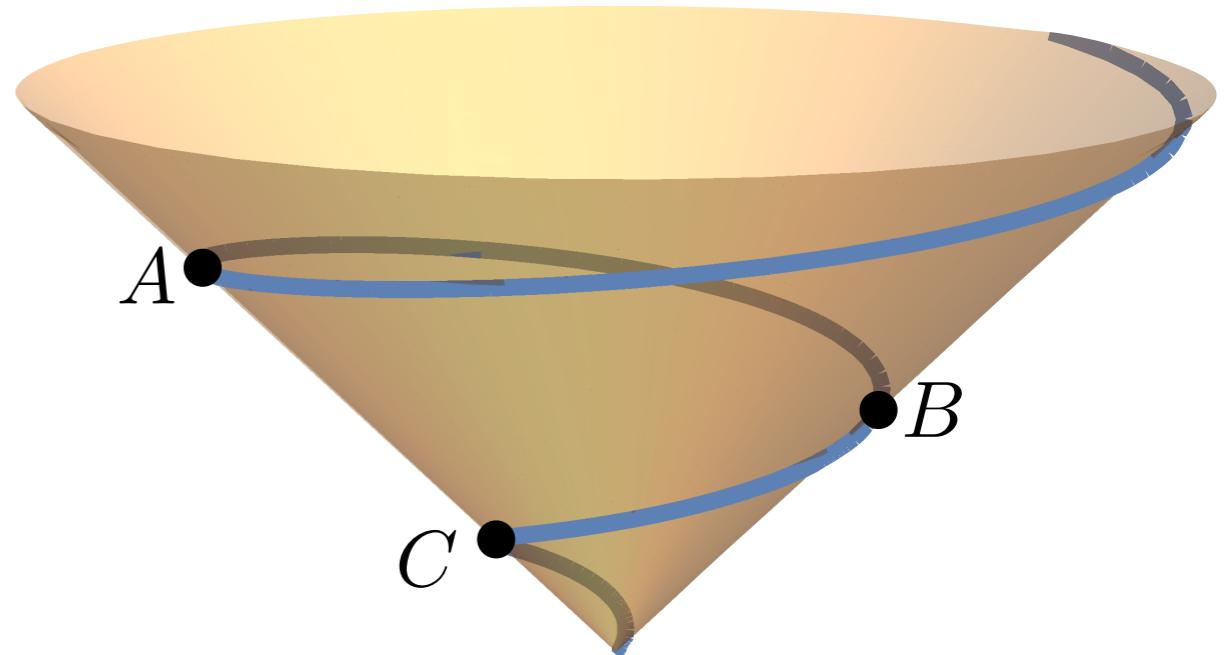
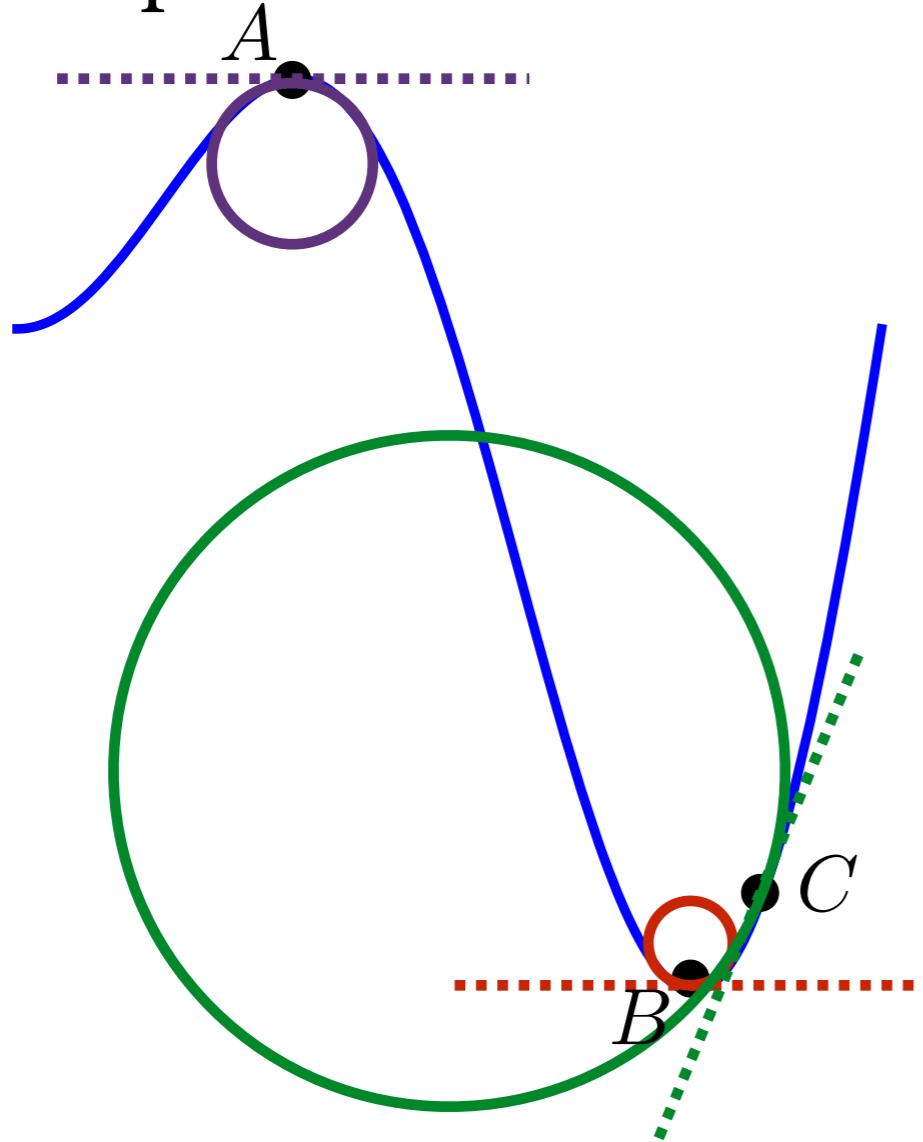
Which is more curved?



$$\text{Curvature} = \frac{1}{R}$$

# Curvature of Exotic Curves

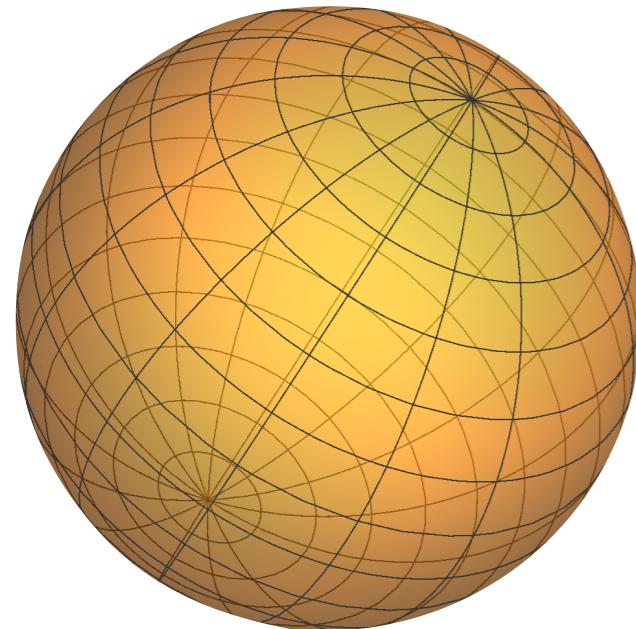
At what points is the curvature highest?



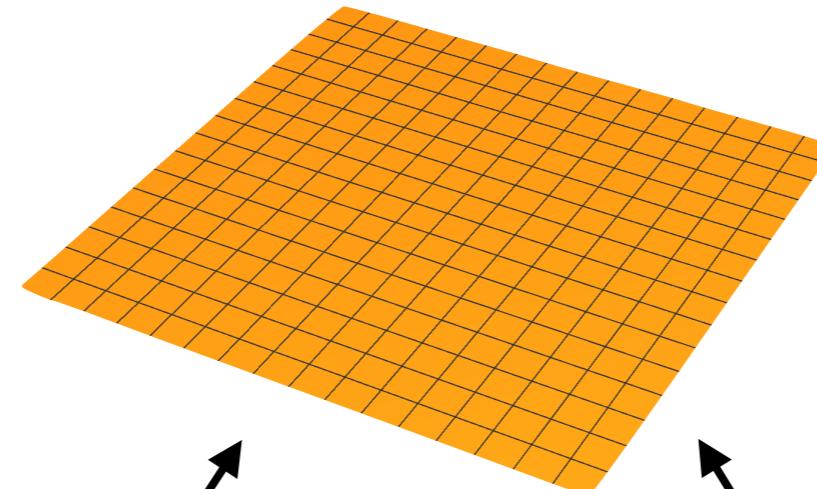
The curvature at a point on a curve is the curvature of the closest approximating circle at that point.

# Curvature of Surfaces

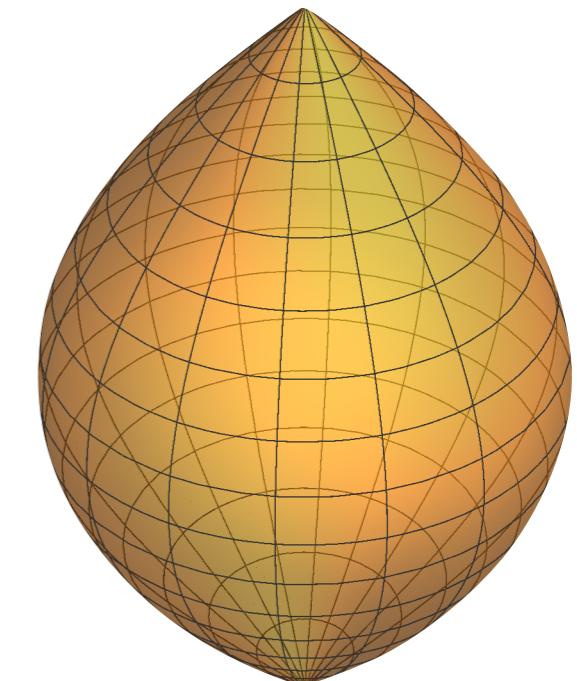
**Rough Idea:** Curvature measures not being flat. Which surfaces are flat?



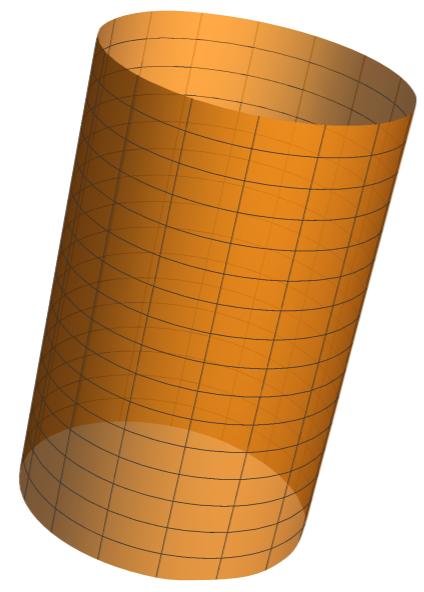
Not Flat



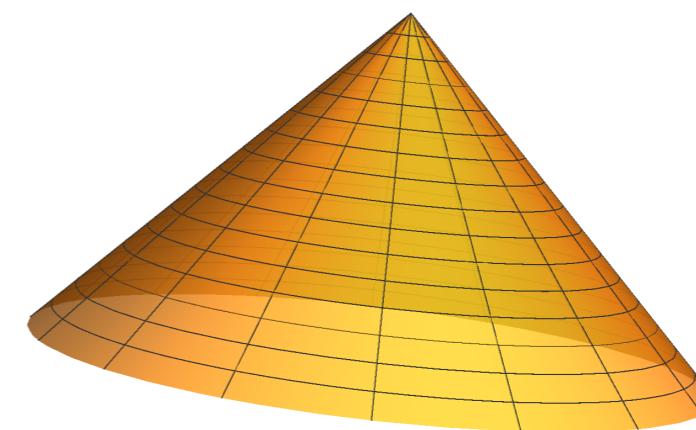
Flat



Not Flat



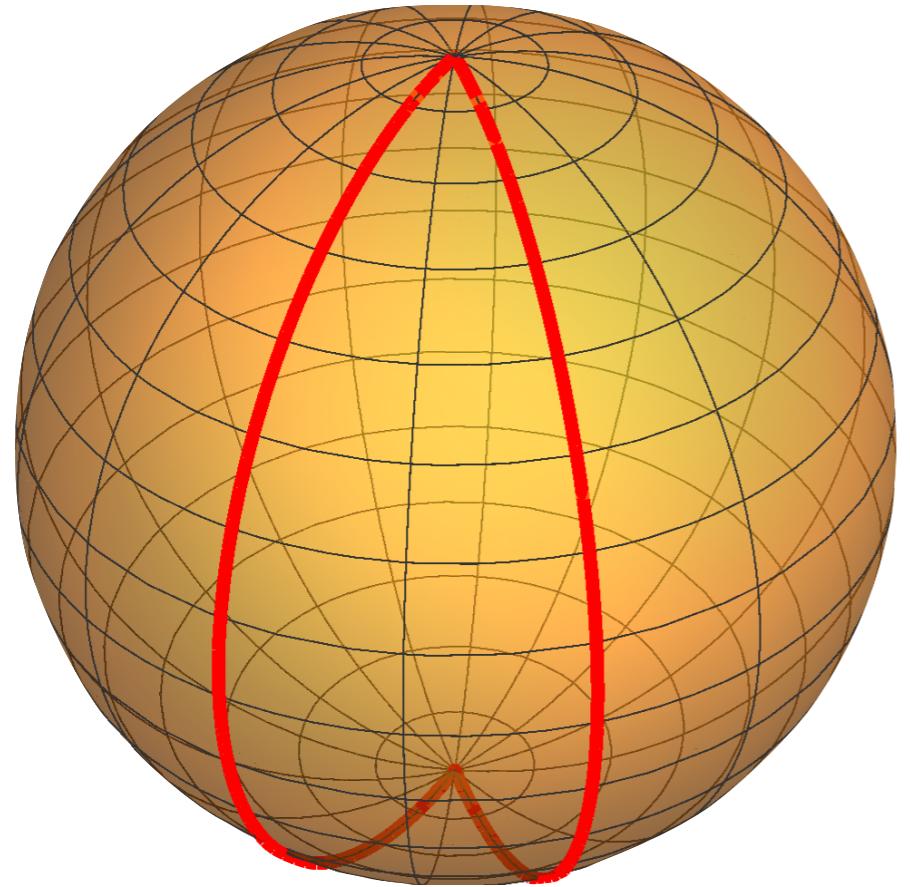
Flat



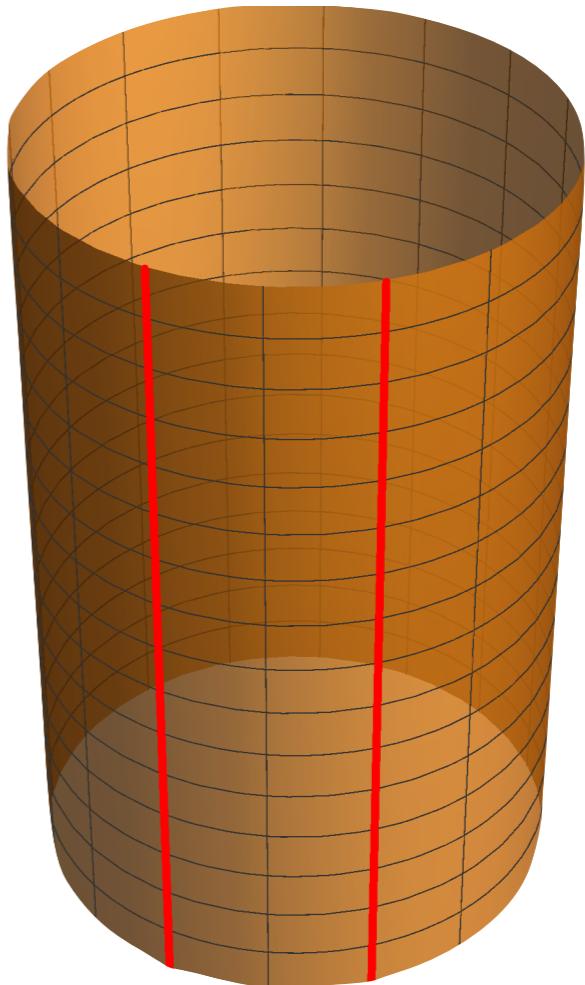
Flat

# What Does it Mean to be Flat?

A surface is flat if straight lines on the surface remain parallel.

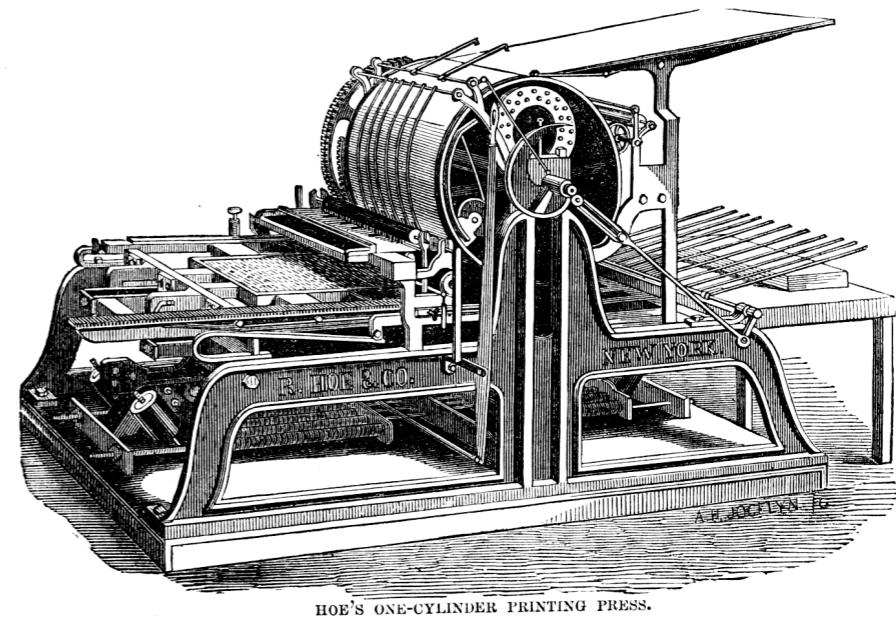
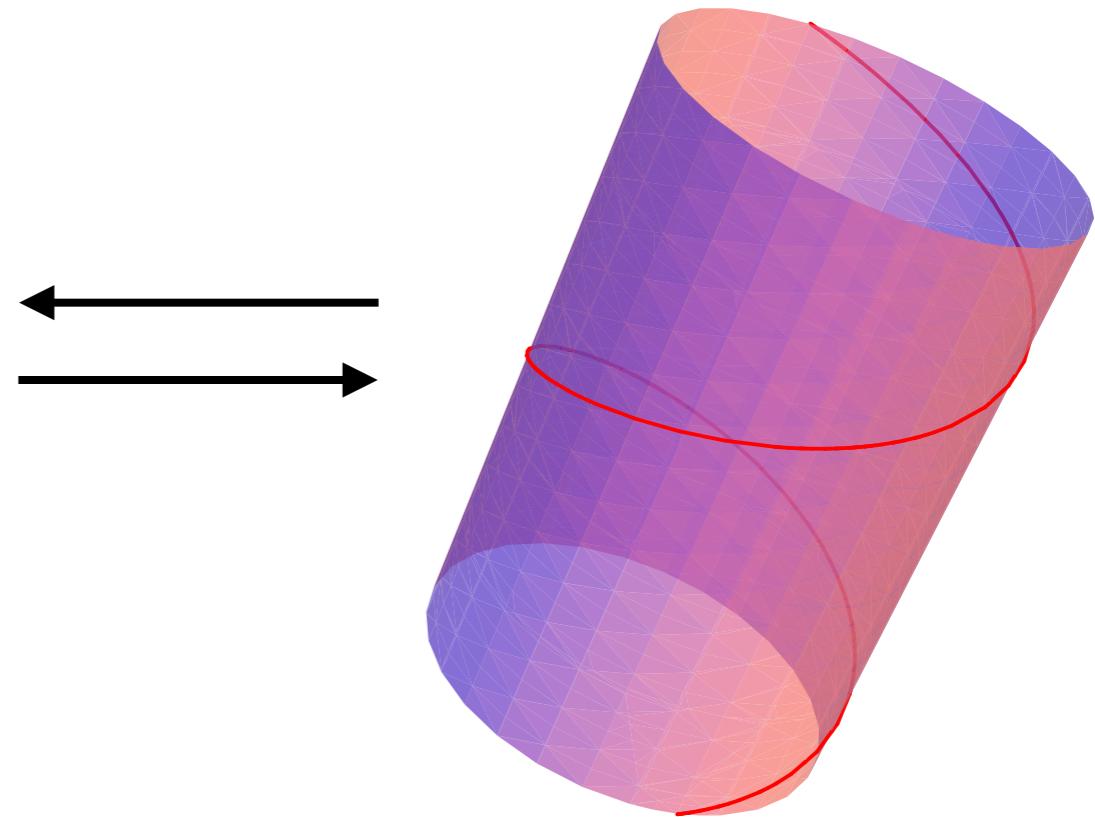
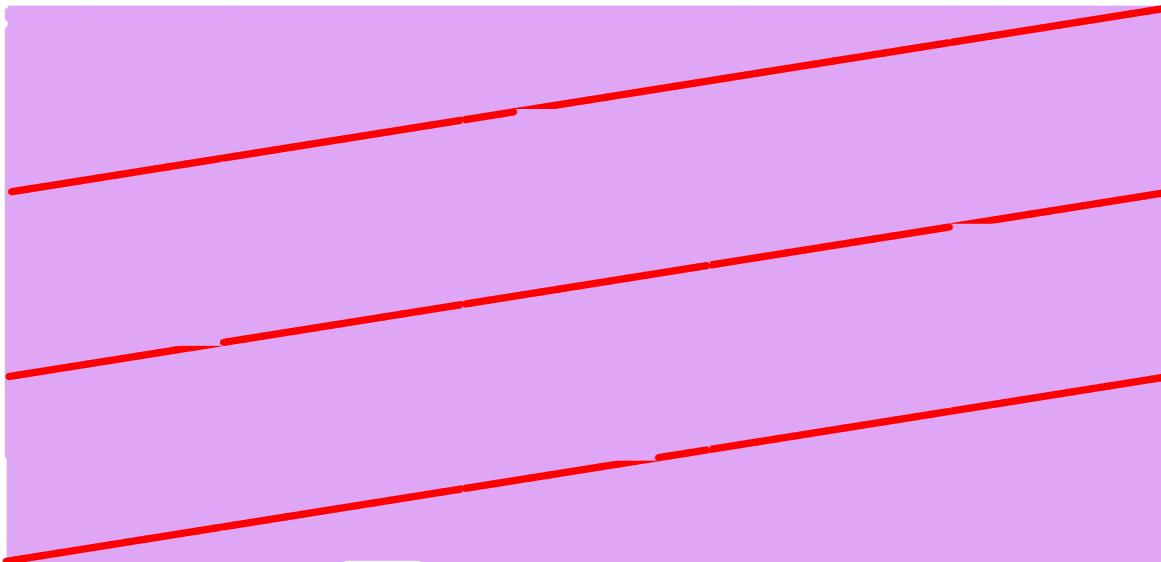


**Not Flat**



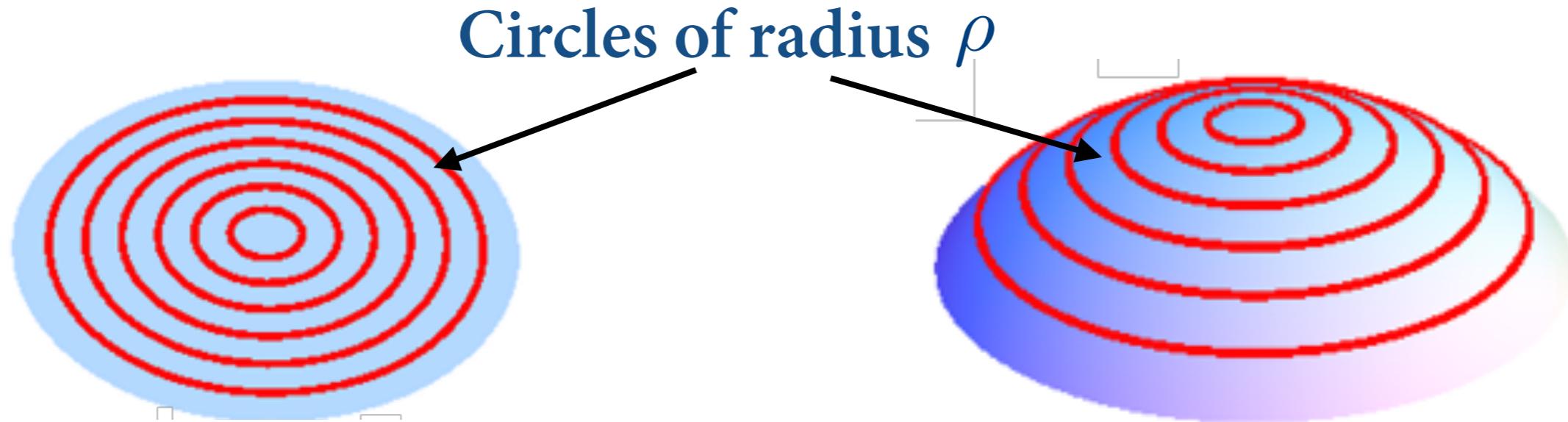
**Flat**

# Flat Geometry



Printing Press

# Gaussian Curvature



$$\text{Circumference} = 2\pi\rho$$

$$\text{Circumference} = C(\rho) < 2\pi\rho$$

The Gaussian curvature  $K$  measures the change in circumference on the surface:

$$\lim_{\rho \rightarrow 0} 3 \frac{2\pi\rho - C(\rho)}{\pi\rho^3}$$

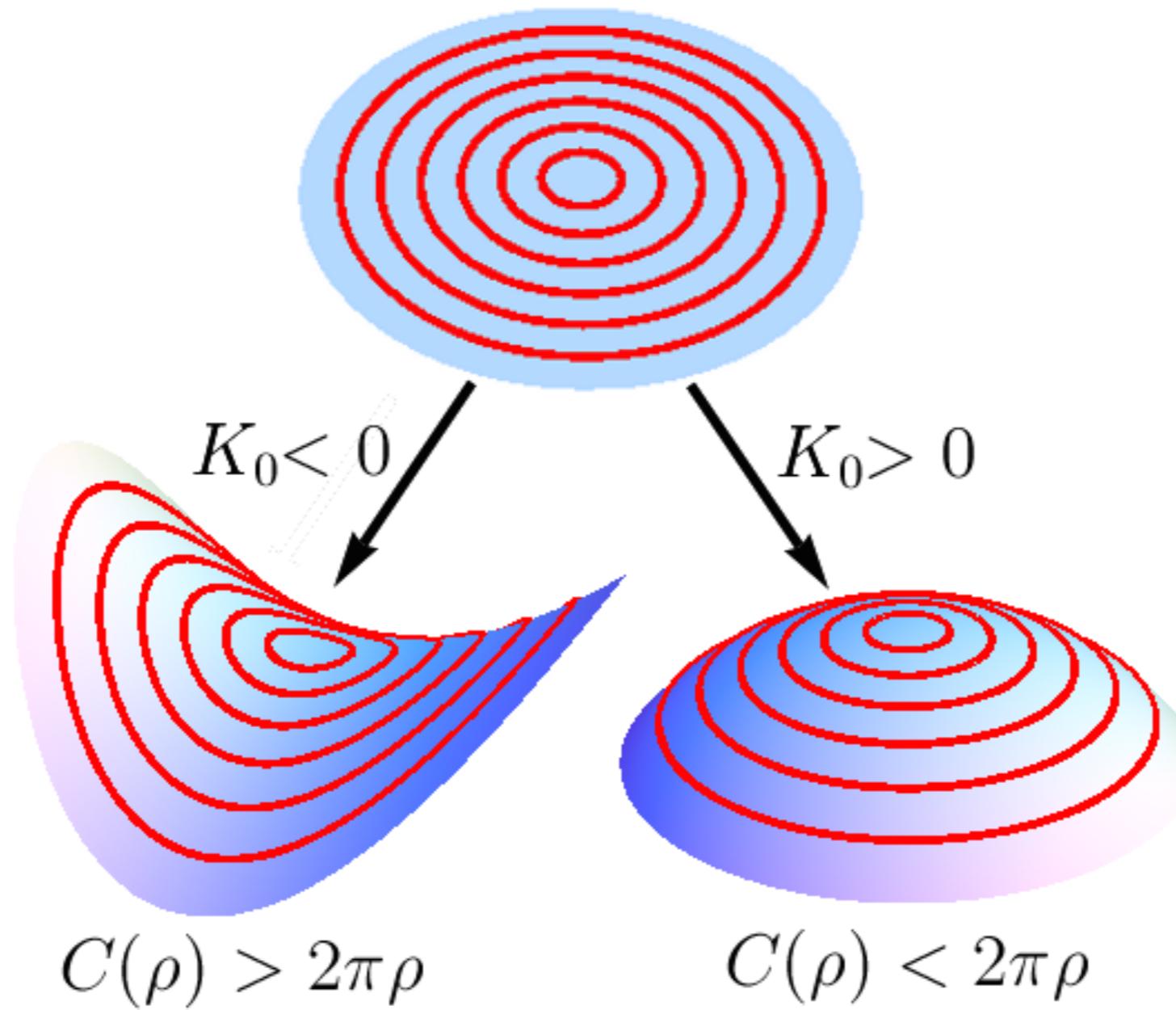
Flat surfaces have zero Gaussian curvature!

# A Sphere is not Flat



There is no perfect map of the Earth!

# Curvature Can be Negative



$$K = \lim_{r \rightarrow 0} 3 \frac{2\pi\rho - C(\rho)}{\pi\rho^3}$$

# Wavy Surfaces have Negative Curvature

