STD-Trees: Spatio-temporal Deformable Trees for Multirotors Kinodynamic Planning

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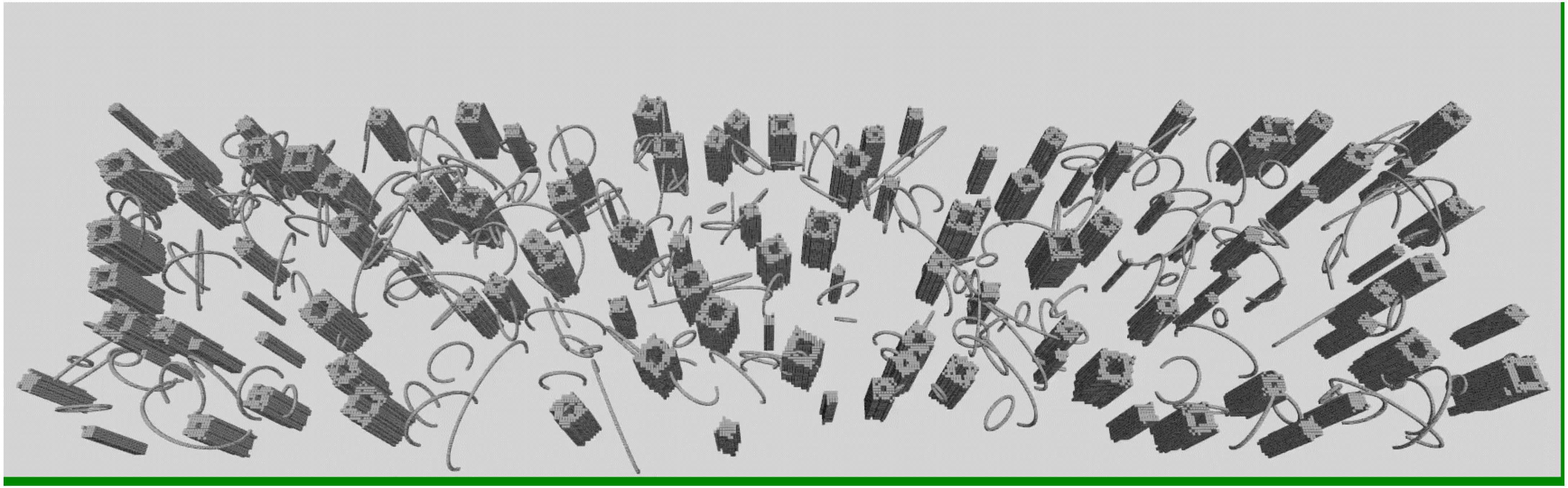
Problem



0.25x speed

Standalone sampling-based kinodynamic planners converge slowly

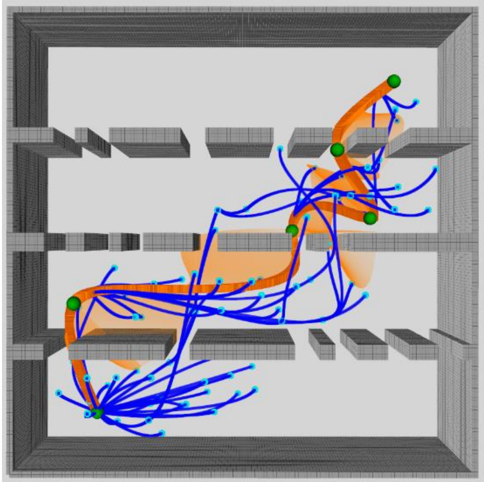
due to the difficulty of sampling states exactly near the optimal solution

Methodology

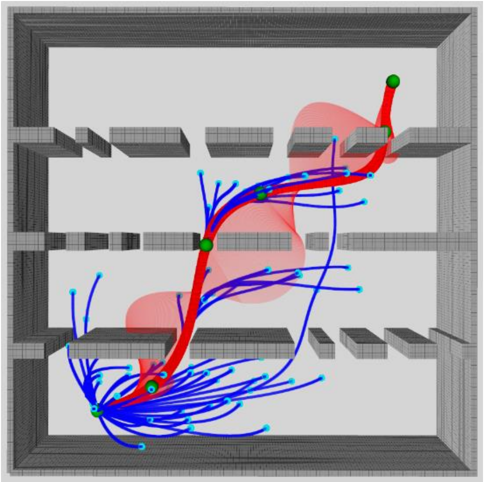
Deform the trajectory tree edges spatially and temporally

Improve overall tree quality without adding more samples

Accelerate the convergence



Without Deformation



With Deformation

High level Approach

Algorithm 1 Spatio-temporal Deformable Trees

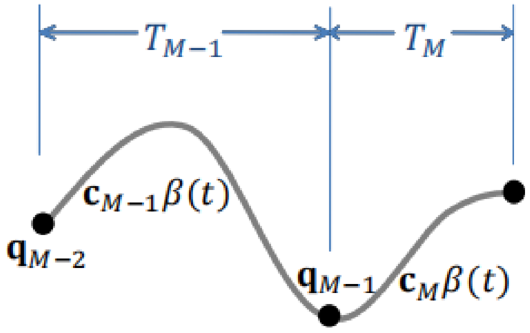
Notation: Tree $\mathcal{T}$, State $\mathbf{x}$, Deformation Units $\mathcal{U}$, Environment $\mathcal{E}$, Deform Type $\mathcal{L} \in$ {NODE, TRUNK, BRANCH, TREE}
Initialize: $\mathcal{T} \leftarrow \emptyset \cup \{\mathbf{x}_{start}\}$
while Termination condition not met **do**
  $\mathbf{x}_{new} \leftarrow$ **Sampling**($\mathcal{E}$)
  $\mathcal{X}_{backward} \leftarrow$ **BackwardNear**($\mathcal{T}$, $\mathbf{x}_{new}$)
  $\mathbf{x}_n \leftarrow$ **ChooseParent**($\mathcal{X}_{backward}$, $\mathbf{x}_{new}$)
  $\mathcal{T} \leftarrow \mathcal{T} \cup \{\mathbf{x}_n, \mathbf{x}_{new}\}$
  if **TryConnectGoal**($\mathbf{x}_{new}$, $\mathbf{x}_{goal}$) **then**
    One Solution Found.
  end if
  $\mathcal{U} \leftarrow$ **SelectDeformationUnits**($\mathbf{x}_n$, $\mathcal{L}$)
  DeformInOrder($\mathcal{U}$)
  RewireInCascade($\mathcal{T}$, $\mathbf{x}_{new}$)
end while
return $\mathcal{T}$

$$\min_{u(t)} \mathcal{J} = \int_0^{\tau} (\rho + \frac{1}{2} u(t)^2) dt$$

$$\begin{aligned} s.t. \quad & \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) - \dot{\mathbf{x}}(t) = \mathbf{0}, \\ & \mathbf{x}(0) = \mathbf{x}_{start}, \ \mathbf{x}(\tau) = \mathbf{x}_{goal}, \\ & \mathcal{G}(\mathbf{x}(t), u(t)) \preceq \mathbf{0}, \ \forall t \in [0, \tau], \end{aligned}$$

Linear Quadratic Minimum Time (LQMT) problem in the area of optimal control.

Tree Edge Representation



$$\mathbf{d} = \mathbf{A}_f(T)\mathbf{c}, \;\; \mathbf{c} = \mathbf{A}_b(T)\mathbf{d}$$

By {**c**, T} as

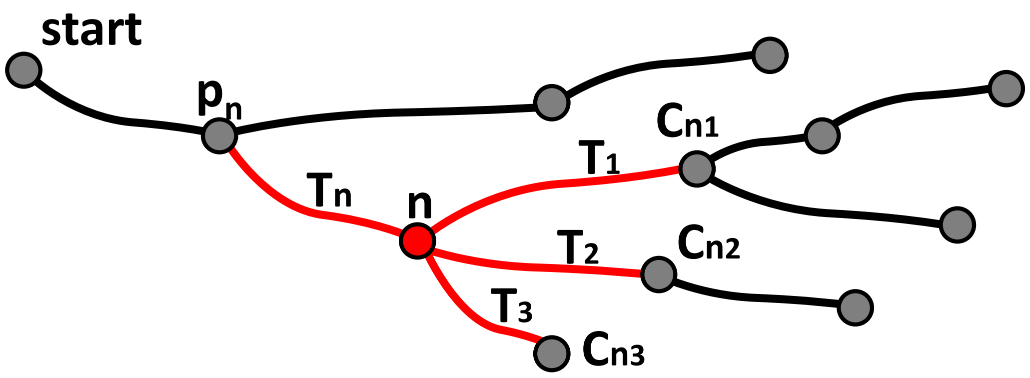
$$\begin{aligned} J_s(\mathbf{c}, T) &= \rho T + \int_0^T \frac{1}{2}\mathbf{c}^\mathsf{T}\beta^{(s)}(t)\beta^{(s)}(t)^\mathsf{T}\mathbf{c}\; dt \\ &= \rho T + \frac{1}{2}\mathbf{c}^\mathsf{T}\mathbf{Q}(T)\mathbf{c}, \end{aligned}$$

By {**d**, T} as

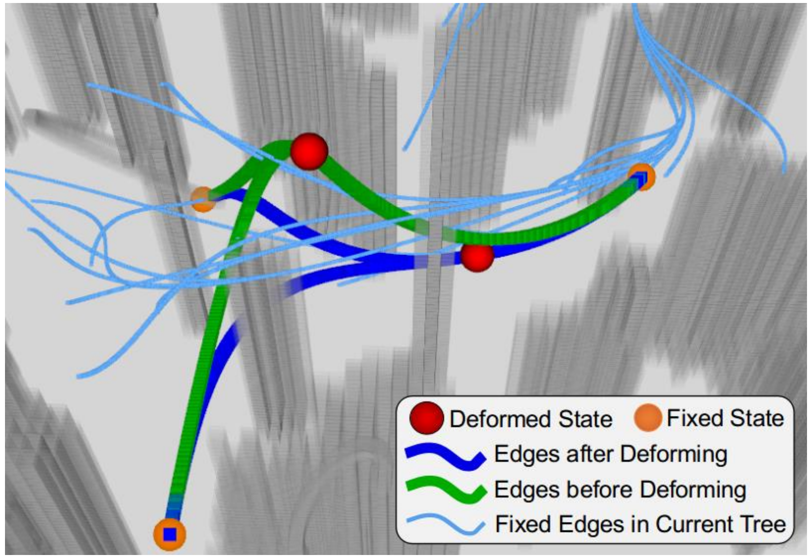
$$\begin{aligned} J_s(\mathbf{d}, T) &= J_s(\mathbf{x}(t)|_{t=0},\; \mathbf{x}(t)|_{t=T},\; T) \\ &= \rho T + \frac{1}{2}\mathbf{d}^\mathsf{T}\mathbf{M}(T)\mathbf{d}, \end{aligned}$$

$$\mathbf{d} = \begin{bmatrix} \mathbf{x}(t)|_{t=0} \\ \mathbf{x}(t)|_{t=T} \end{bmatrix}, \; \mathbf{M}(T) = \mathbf{A}_b^\mathsf{T}(T)\mathbf{Q}(T)\mathbf{A}_b(T)$$

Deformation Unit



$$\min_{\{X_n,\, T_n,\, T_1,\, T_2,\, T_3\}} g_n + \sum_{i\in\mathcal{D}_n} g_i$$

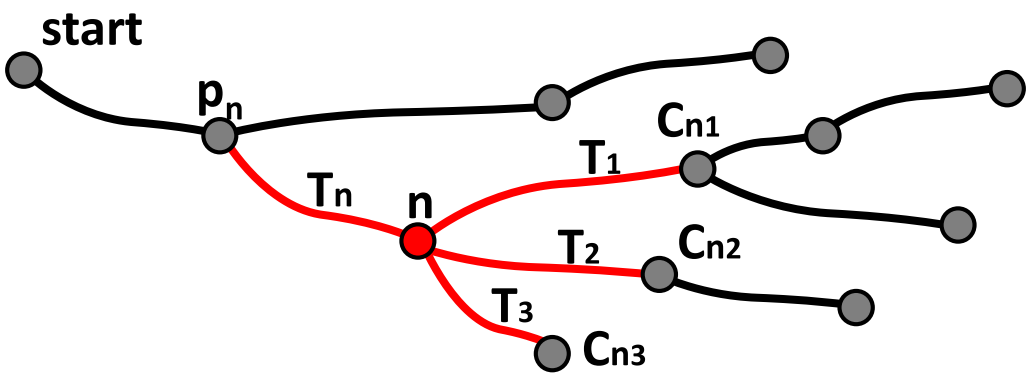


Minimize the cost-to-come values of the node and all its descendant node by optimizing the node state and the time duration of the connecting edges.

Objective Design

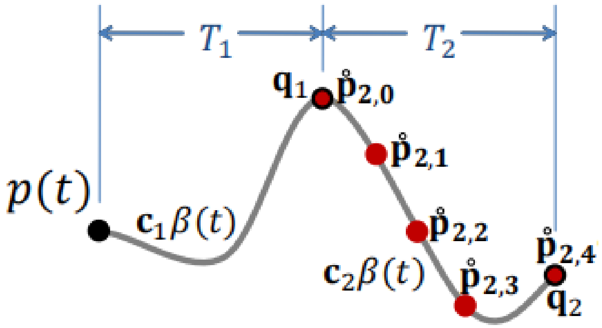
$$g_n = c(\mathbf{x}_{p_n}, \mathbf{x}_n) + g_{p_n}$$

$$\begin{aligned} g_n + \sum_{i \in \mathcal{D}_n} g_i &= \sum_{i \in \mathcal{T}_n} \sum_{j \in \mathcal{C}_i} d_j c(\mathbf{x}_i, \mathbf{x}_j) \\ &= d_n c(\mathbf{x}_{p_n}, \mathbf{x}_n) + \sum_{i \in \mathcal{C}_n} d_i c(\mathbf{x}_n, \mathbf{x}_i) + \mathbf{C} \\ &= \sum_{i \in \{n\} \bigcup \mathcal{C}_n} d_i c(\mathbf{x}_{p_i}, \mathbf{x}_i) + \mathbf{C}, \end{aligned}$$



Minimize the cost-to-come values of the node and all its descendant node by optimizing the node state and the time duration of the connecting edges.

Unconstrained Formulation



$$\min_{\mathbf{x}_n, \mathbf{T}_n} \sum_{i \in \{n\} \bigcup \mathcal{C}_n} d_i(J_s(\mathbf{c}_i, T_i) + J_f(\mathbf{c}_i, T_i, k_i))$$

$$\mathbf{c}_i = \begin{cases} \mathbf{A}_b(T_i) \left[\mathbf{x}_{p_n}^\mathsf{T}, \ \mathbf{x}_n^\mathsf{T}\right]^\mathsf{T}, & i = n \\ \mathbf{A}_b(T_i) \left[\mathbf{x}_n^\mathsf{T}, \ \mathbf{x}_i^\mathsf{T}\right]^\mathsf{T}, & i \in \mathcal{C}_n \end{cases}$$

$$J_f(\mathbf{c}_i, T_i, k_i) = \frac{T_i}{k_i} \sum_{j=0}^{k_i} \omega_j \mathcal{X}^\mathsf{T} max[\mathcal{G}(\mathbf{c}_i, T_i, t), \ \mathbf{0}]$$

$$\mathcal{G}(p^{[s]}(t)) \ \in \ \mathbb{R}^{s+1} \ \preceq \ \mathbf{0}$$

Are the functional-type constraints that consider obstacle avoidance and dynamical limitations.

Spatio-temporal Optimization

For one edge in the deformation unit, we derive the gradient of the decoupled objective w.r.t. {**x**, **T**}:

$$\frac{\partial J_s}{\partial \mathbf{x}_n} = \frac{\partial J_s}{\partial \mathbf{c}_i}\frac{\partial \mathbf{c}_i}{\partial \mathbf{x}_n} = \mathbf{Q}(T)\mathbf{c}_i, \frac{\partial J_f}{\partial \mathbf{x}_n} = \frac{\partial J_f}{\partial \mathcal{G}}\frac{\partial \mathcal{G}}{\partial \mathbf{c}_i}\frac{\partial \mathbf{c}_i}{\partial \mathbf{x}_n},$$

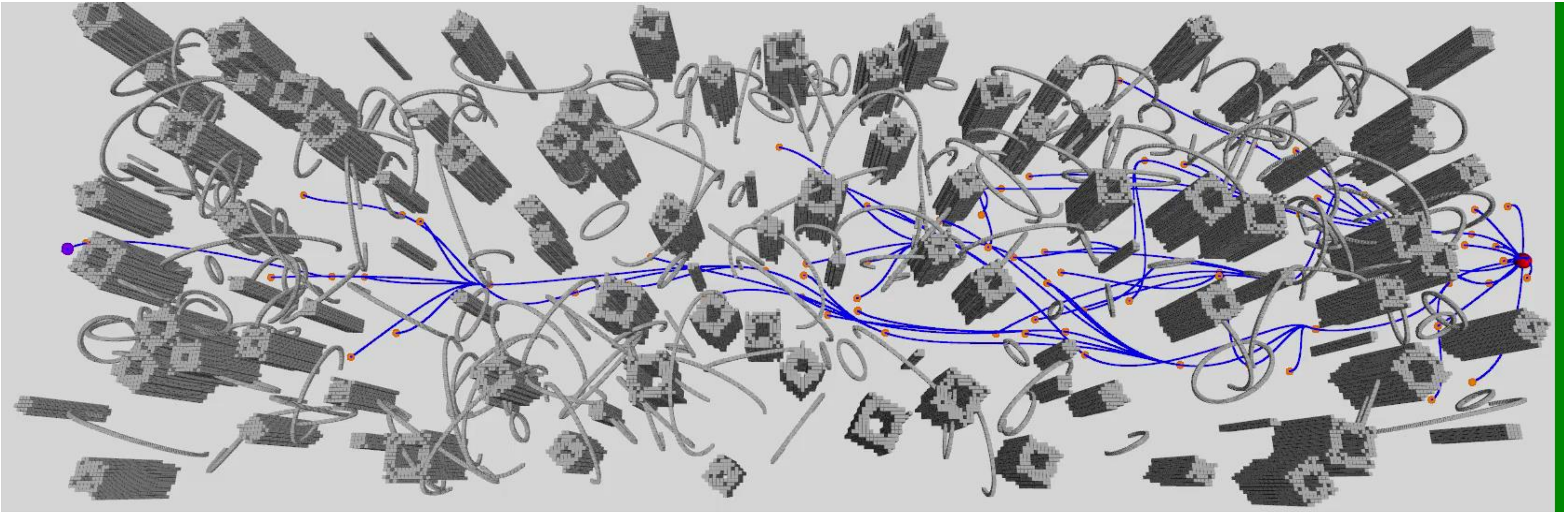
$$\frac{\partial J_s}{\partial T_i} = \rho + \frac{1}{2}\mathbf{c}_i^{\mathsf{T}}\dot{\mathbf{Q}}(T_i)\mathbf{c}_i, \frac{\partial J_f}{\partial T_i} = \frac{J_f}{T_i} + \frac{\partial J_f}{\partial \mathcal{G}}\frac{\partial \mathcal{G}}{\partial t}\frac{j}{k_i},$$

$$\frac{\partial J_f}{\partial \mathcal{G}} = \frac{T_i}{k_i}\sum_{j=0}^{k_i}\omega_j \mathcal{X} \odot max[Sign[\mathcal{G}(\mathbf{c}_i, T_i, \frac{j}{k_i})],\ \mathbf{0}],$$

$$\frac{\partial \mathbf{c}_i}{\partial \mathbf{x}_n} = \begin{cases} \left[\mathbf{A}_b^{01}(T_i)^{\mathsf{T}}\ \mathbf{A}_b^{11}(T_i)^{\mathsf{T}}\right]^{\mathsf{T}}, & i = n \\ \left[\mathbf{A}_b^{00}(T_i)^{\mathsf{T}}\ \mathbf{A}_b^{10}(T_i)^{\mathsf{T}}\right]^{\mathsf{T}}, & i \in \mathcal{C}_n, \end{cases}$$

Adopt Limited Memory Bundle Method (LMBM) to address the non-smoothness introduced by the interpolation of the distance fields.

Deform One Unit



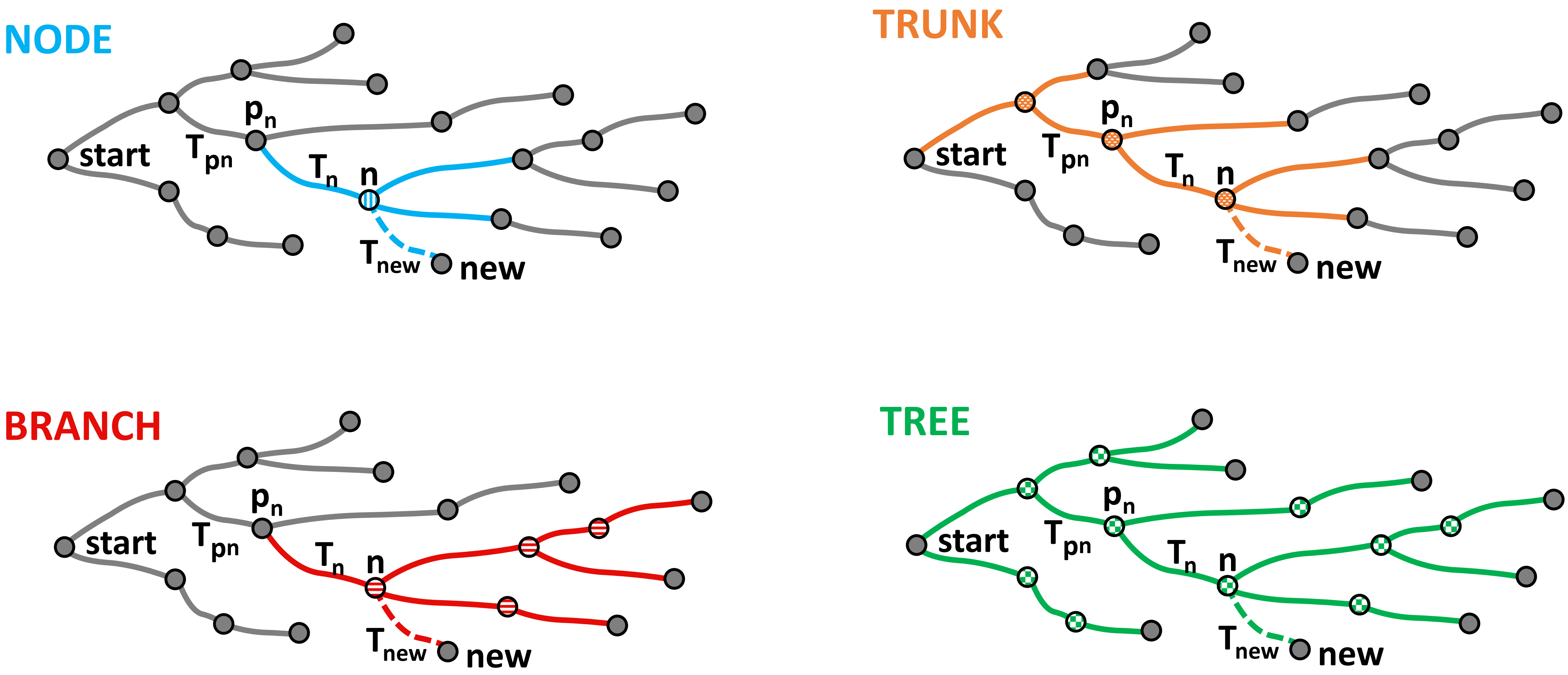
Green: before deforming

Red: after deforming

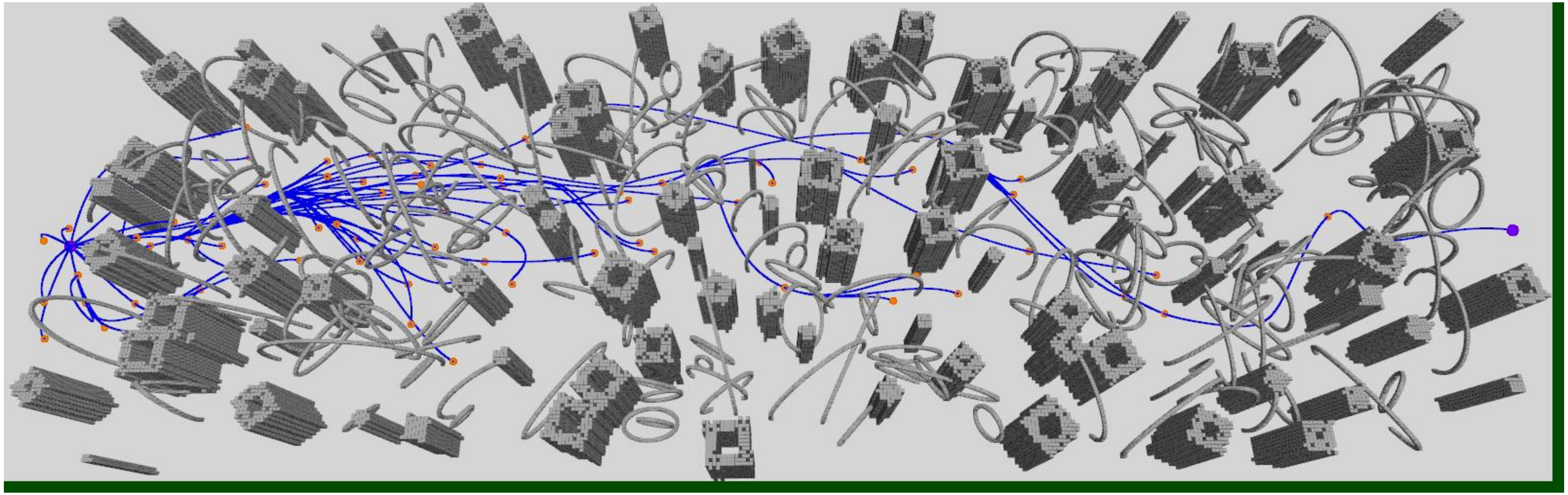
Visualization of deforming one deformation unit each time.

Deformation Variants

Different variants to balance optimization level and computation burden



BRANCH Deformation

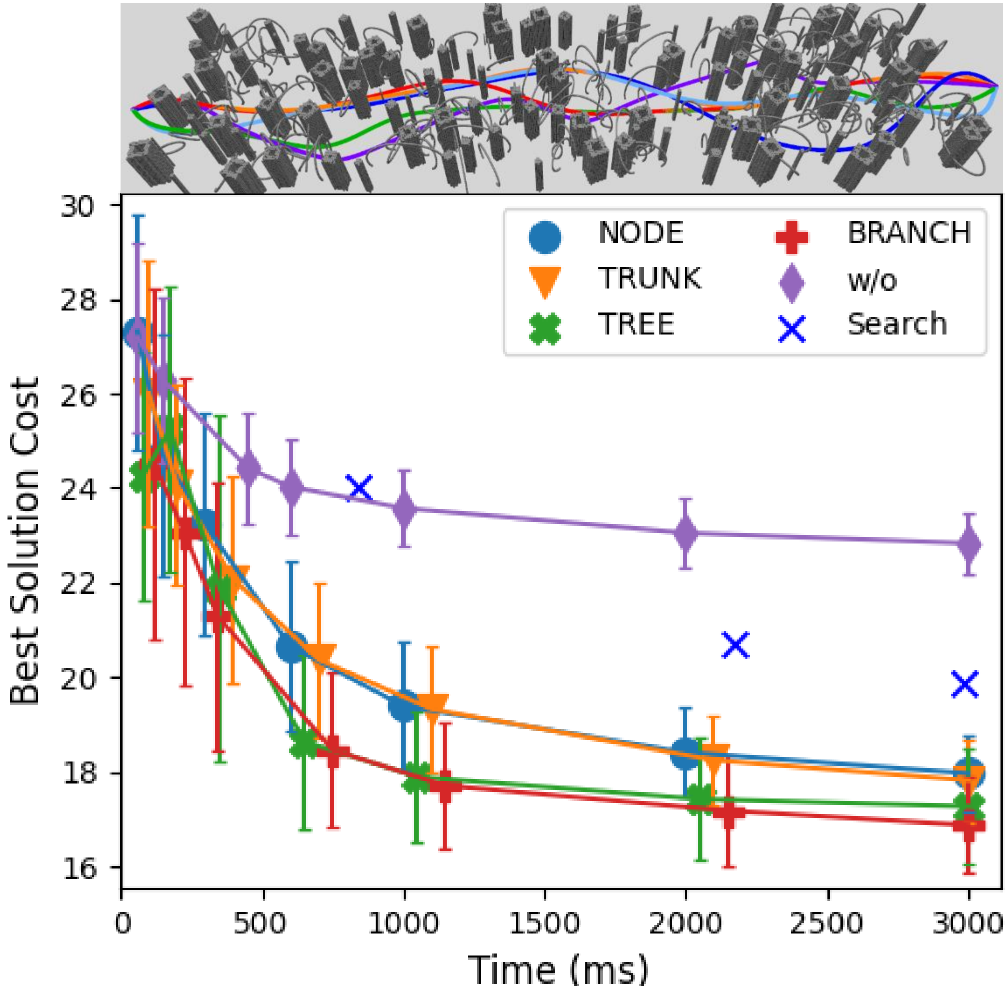


Green: before deforming Red: after deforming

Visualization of deforming by variant BRANCH.

Convergence Comparison

Different variants to balance optimization level and computation burden

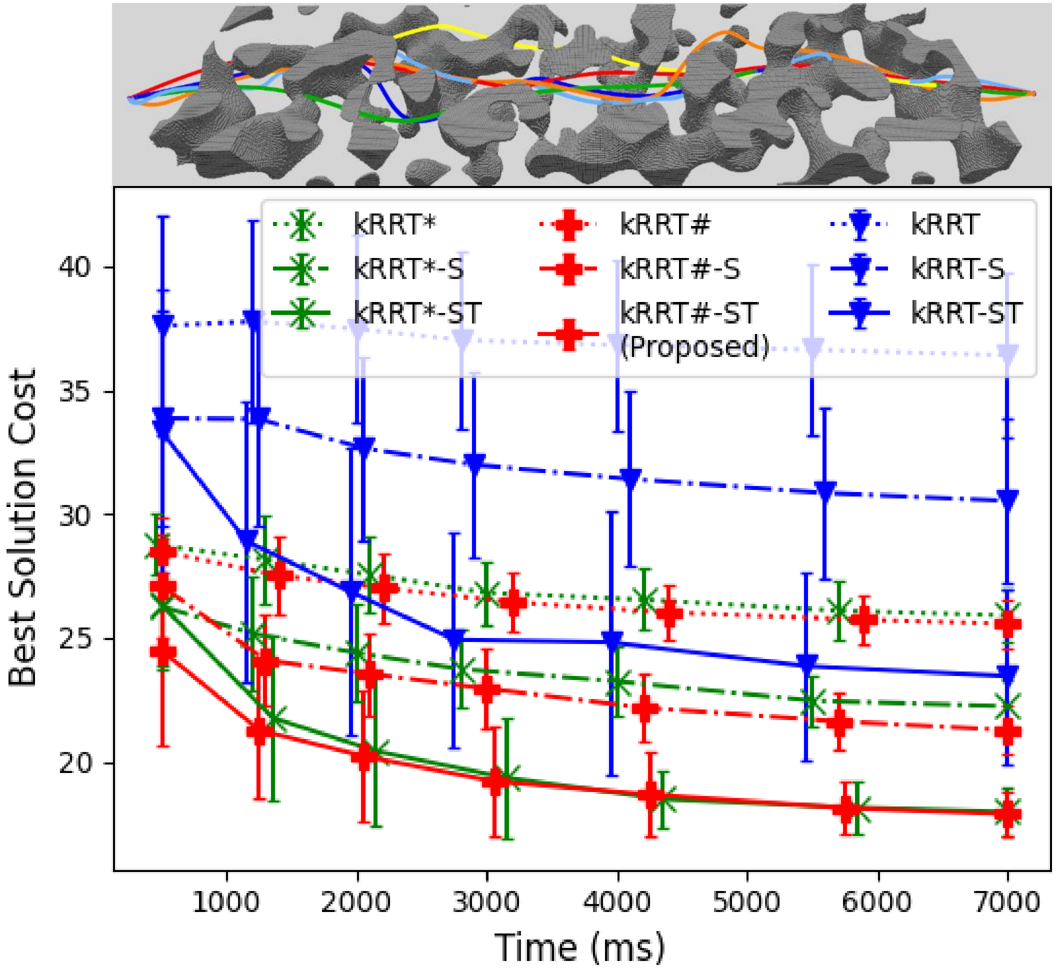


Variant BRANCH outperforms others

A new sample brings potential improvements mostly on the sub-tree while other tree parts are less likely influenced.

Convergence Comparison

Integrate deformation on RRT-family kinodynamic planners



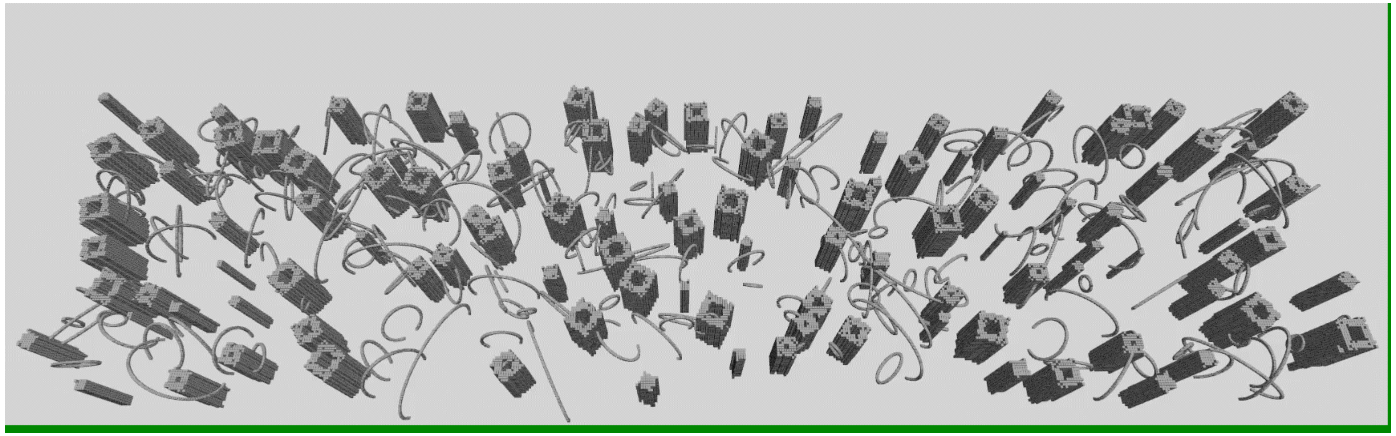
Spatio-temporal (-ST) deformation

outperforms

Spatial-only (-S) deformation

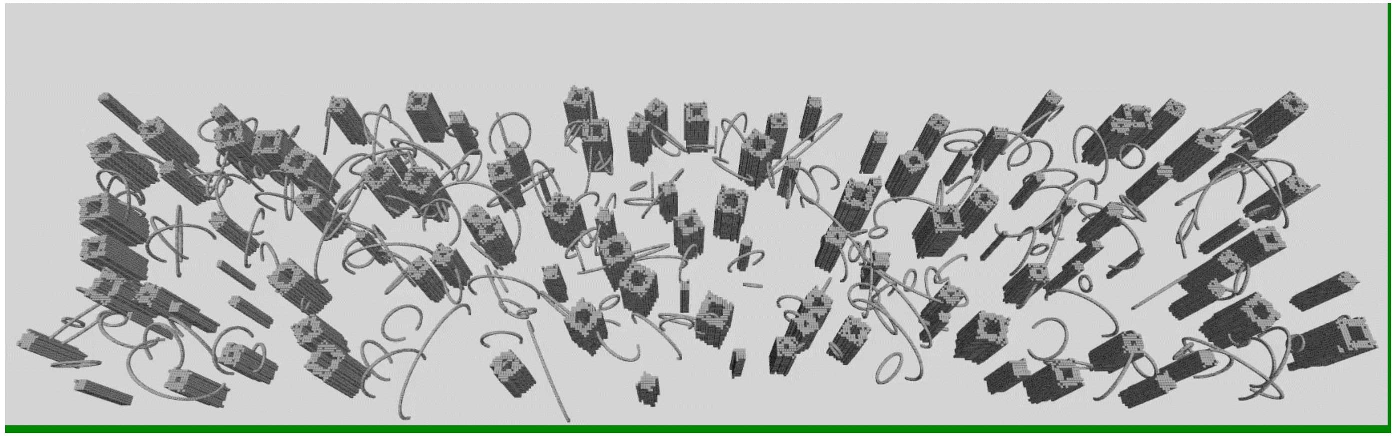
outperforms

No deformation



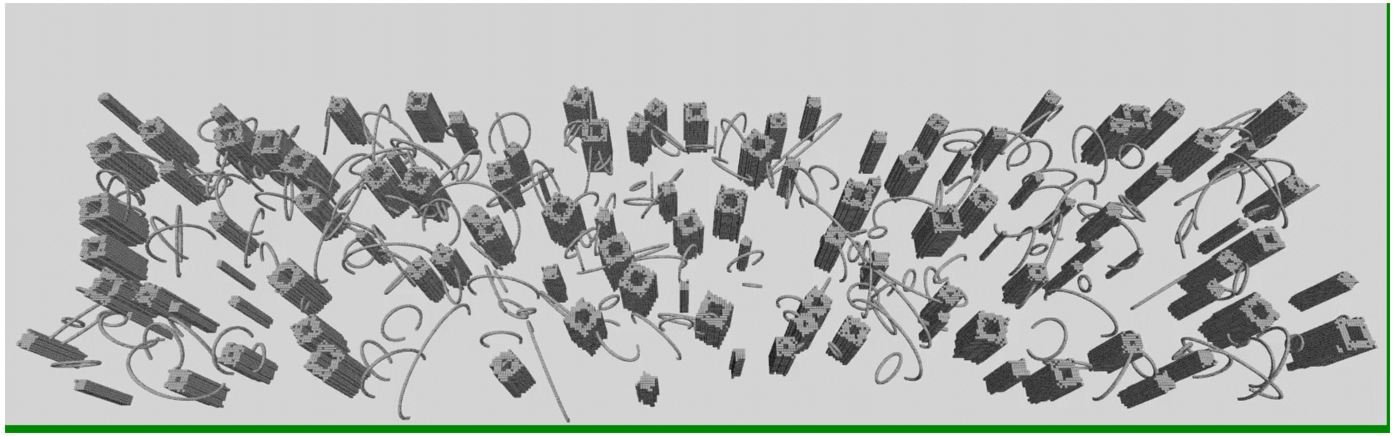
No Deformation

Slow Convergence



Spatial-only Deformation

Fast Convergence

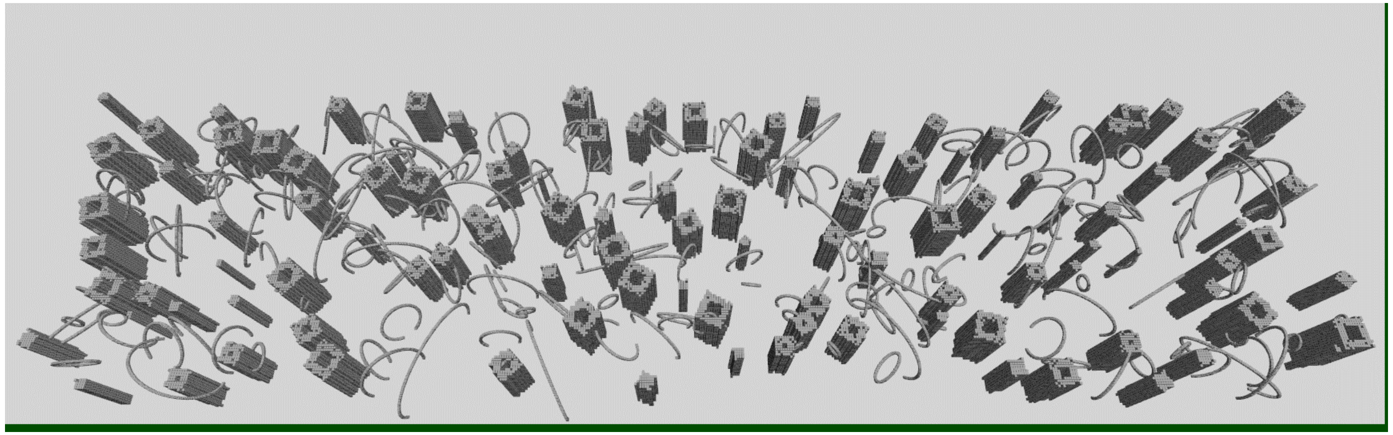


Spatio-temporal Deformation

Faster Convergence

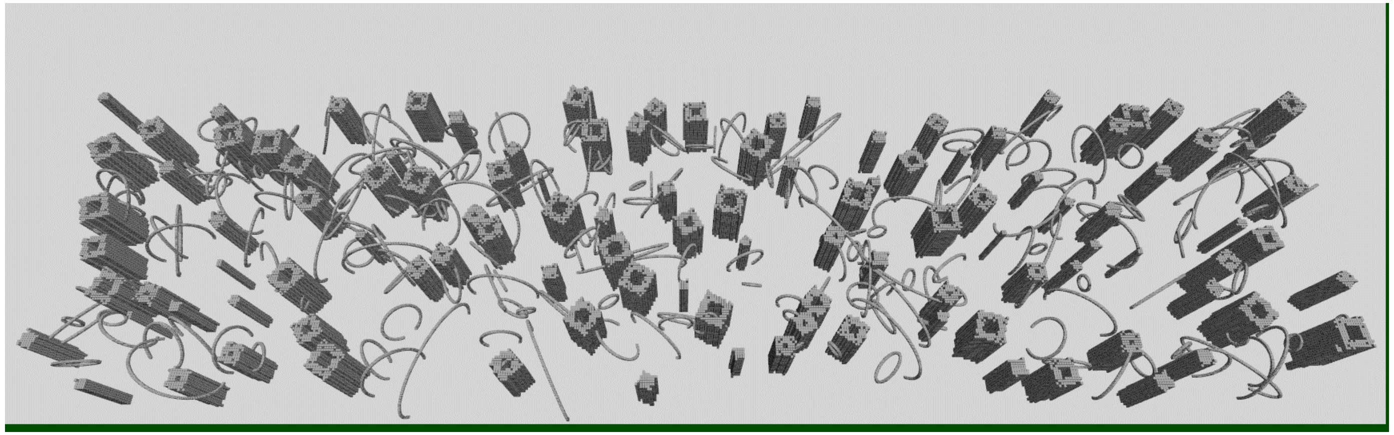
0.25x speed

No Deformation



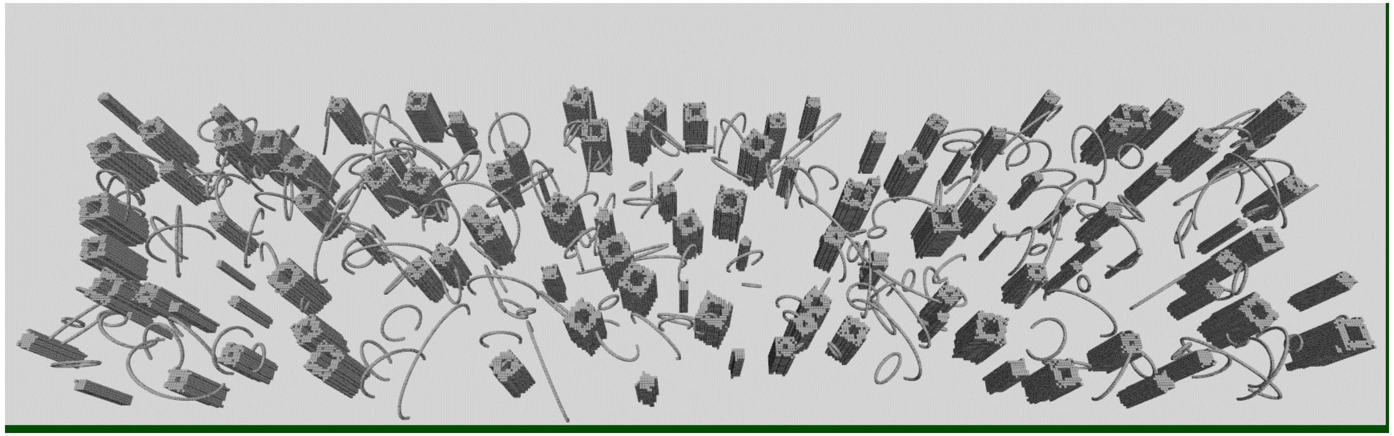
Spatial-only Deformation

Smoother



Spatio-temporal Deformation

Smoother & Shorter



Thanks for Watching!