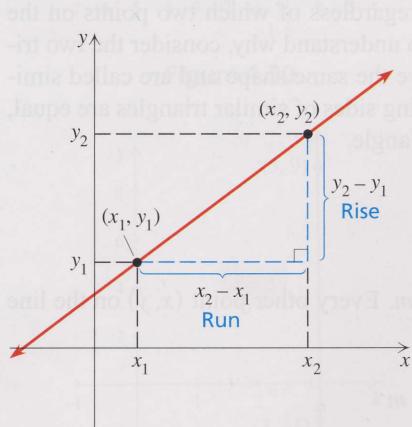


- f) Replace  $W$  and  $A$  with your actual weight and height. Now draw a graph showing how  $B$  depends on height for suitable values of  $H$ .
- g) For a fixed weight and age, does the basic energy requirement increase or decrease for taller persons? Increases
- h) For the three graphs that you have drawn, explain how to determine whether the graph is increasing or decreasing from the formula for the graph.

## 1.4

## Linear Equations in Two Variables



**Figure 1.27**

In Section 1.3 we graphed lines, including horizontal and vertical lines. We learned that every line has an equation in standard form  $Ax + By = C$ . In this section we will continue to study lines.

### Slope of a Line

A road that has a 5% grade rises 5 feet for every horizontal run of 100 feet. A roof that has a 5–12 pitch rises 5 feet for every horizontal run of 12 feet. The grade of a road and the pitch of a roof are measurements of *steepness*. The steepness or *slope* of a line in the  $xy$ -coordinate system is the ratio of the **rise** (the change in  $y$ -coordinates) to the **run** (the change in  $x$ -coordinates) between two points on the line:

$$\text{slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}} = \frac{\text{rise}}{\text{run}}$$

If  $(x_1, y_1)$  and  $(x_2, y_2)$  are the coordinates of the two points in Fig. 1.27, then the rise is  $y_2 - y_1$  and the run is  $x_2 - x_1$ :

### Definition: Slope

The **slope** of the line through  $(x_1, y_1)$  and  $(x_2, y_2)$  with  $x_1 \neq x_2$  is

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Note that if  $(x_1, y_1)$  and  $(x_2, y_2)$  are two points for which  $x_1 = x_2$  then the line through them is a vertical line. Since this case is not included in the definition of slope, a vertical line does not have a slope. We also say that the slope of a vertical line is undefined. If we choose two points on a horizontal line then  $y_1 = y_2$  and  $y_2 - y_1 = 0$ . For any horizontal line the rise between two points is 0 and the slope is 0.

### Example 1 Finding the slope

In each case find the slope of the line that contains the two given points.

- a.  $(-3, 4), (-1, -2)$    b.  $(-3, 7), (5, 7)$    c.  $(-3, 5), (-3, 8)$

#### Solution

- a. Use  $(x_1, y_1) = (-3, 4)$  and  $(x_2, y_2) = (-1, -2)$  in the formula:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{-1 - (-3)} = \frac{-6}{2} = -3$$

The slope of the line is  $-3$ .

- b. Use  $(x_1, y_1) = (-3, 7)$  and  $(x_2, y_2) = (5, 7)$  in the formula:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 7}{5 - (-3)} = \frac{0}{8} = 0$$

The slope of this horizontal line is  $0$ .

- c. The line through  $(-3, 5)$  and  $(-3, 8)$  is a vertical line and so it does not have a slope.

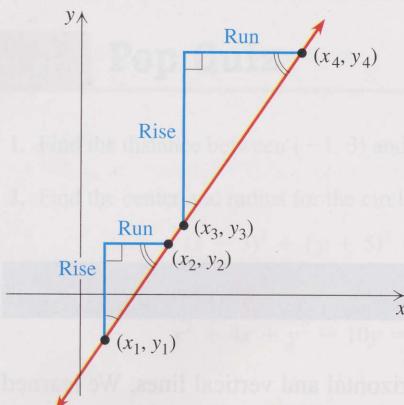


Figure 1.28

The slope of a line is the same number regardless of which two points on the line are used in the calculation of the slope. To understand why, consider the two triangles shown in Fig. 1.28. These triangles have the same shape and are called similar triangles. Because the ratios of corresponding sides of similar triangles are equal, the ratio of rise to run is the same for either triangle.

### Point-Slope Form

Suppose that a line through  $(x_1, y_1)$  has slope  $m$ . Every other point  $(x, y)$  on the line must satisfy the equation

$$\frac{y - y_1}{x - x_1} = m$$

because any two points can be used to find the slope. Multiply both sides by  $x - x_1$  to get  $y - y_1 = m(x - x_1)$ , which is the **point-slope form** of the equation of a line.

#### Theorem: Point-Slope Form

The equation of the line (in point-slope form) through  $(x_1, y_1)$  with slope  $m$  is

$$y - y_1 = m(x - x_1).$$

In Section 1.3 we started with the equation of a line and graphed the line. Using the point-slope form, we can start with a graph of a line or a description of the line and write the equation for the line.

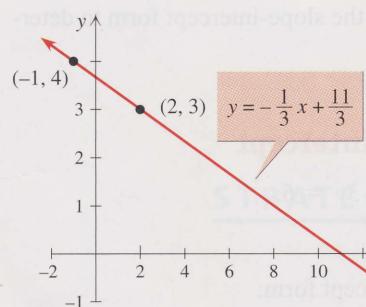


Figure 1.29

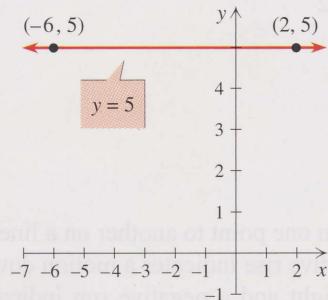


Figure 1.30

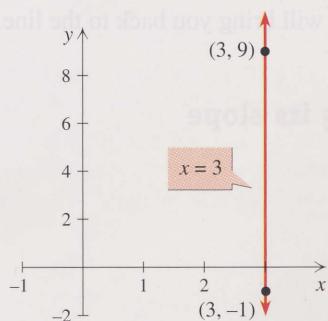


Figure 1.31

## Example 2 The equation of a line given two points

In each case graph the line through the given pair of points. Then find the equation of the line and solve it for  $y$  if possible.

- a.  $(-1, 4), (2, 3)$    b.  $(2, 5), (-6, 5)$    c.  $(3, -1), (3, 9)$

### Solution

- a. Find the slope of the line shown in Fig. 1.29 as follows:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 4}{2 - (-1)} = \frac{-1}{3} = -\frac{1}{3}$$

Now use a point, say  $(2, 3)$ , and  $m = -\frac{1}{3}$  in the point-slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{1}{3}(x - 2) \quad \text{The equation in point-slope form}$$

$$y - 3 = -\frac{1}{3}x + \frac{2}{3}$$

$$y = -\frac{1}{3}x + \frac{11}{3} \quad \text{The equation solved for } y$$

- b. The slope of the line through  $(2, 5)$  and  $(-6, 5)$  shown in Fig. 1.30 is 0. The equation of this horizontal line is  $y = 5$ .

- c. The line through  $(3, -1)$  and  $(3, 9)$  shown in Fig. 1.31 is vertical and it does not have slope. Its equation is  $x = 3$ .

## Slope-Intercept Form

The line  $y = mx + b$  goes through  $(0, b)$  and  $(1, m + b)$ . Between these two points the rise is  $m$  and the run is 1. So the slope is  $m$ . Since  $(0, b)$  is the  $y$ -intercept and  $m$  is the slope,  $y = mx + b$  is called **slope-intercept form**. Any equation in standard form  $Ax + By = C$  can be rewritten in slope-intercept form by solving the equation for  $y$  provided  $B \neq 0$ . If  $B = 0$ , then the line is vertical and has no slope.

### Theorem: Slope-Intercept Form

The equation of a line (in slope-intercept form) with slope  $m$  and  $y$ -intercept  $(0, b)$  is

$$y = mx + b.$$

Every nonvertical line has an equation in slope-intercept form.

If you know the slope and  $y$ -intercept for a line, then you can use slope-intercept form to write its equation. For example, the equation of the line through  $(0, 9)$  with

slope 4 is  $y = 4x + 9$ . In the next example we use the slope-intercept form to determine the slope and  $y$ -intercept for a line.

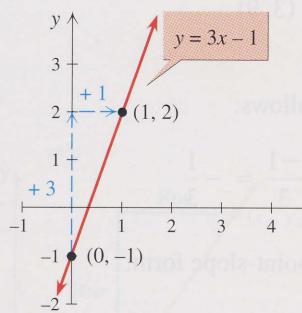


Figure 1.32

### Example 3 Find the slope and $y$ -intercept

Identify the slope and  $y$ -intercept for the line  $2x - 3y = 6$ .

#### Solution

First solve the equation for  $y$  to get it in slope-intercept form:

$$\begin{aligned} 2x - 3y &= 6 \\ -3y &= -2x + 6 \\ y &= \frac{2}{3}x - 2 \end{aligned}$$

So the slope is  $\frac{2}{3}$  and the  $y$ -intercept is  $(0, -2)$ .

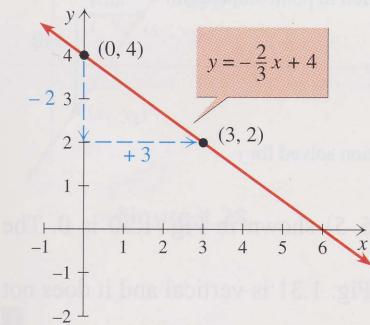


Figure 1.33

### Using Slope to Graph a Line

Slope is the ratio  $\frac{\text{rise}}{\text{run}}$  that results from moving from one point to another on a line. A positive rise indicates a motion upward and a negative rise indicates a motion downward. A positive run indicates a motion to the right and a negative run indicates a motion to the left. If you start at any point on a line with slope  $\frac{1}{2}$ , then moving up 1 unit and 2 units to the right will bring you back to the line. On a line with slope  $-3$  or  $-\frac{3}{1}$ , moving down 3 units and 1 unit to the right will bring you back to the line.

### Example 4 Graphing a line using its slope and $y$ -intercept

Graph each line.

a.  $y = 3x - 1$    b.  $y = -\frac{2}{3}x + 4$

#### Solution

a. The line  $y = 3x - 1$  has  $y$ -intercept  $(0, -1)$  and slope 3 or  $\frac{3}{1}$ . Starting at  $(0, -1)$  we obtain a second point on the line by moving up 3 units and 1 unit to the right. So the line goes through  $(0, -1)$  and  $(1, 2)$  as shown in Fig. 1.32.

b. The line  $y = -\frac{2}{3}x + 4$  has  $y$ -intercept  $(0, 4)$  and slope  $-\frac{2}{3}$  or  $-\frac{2}{3}$ . Starting at  $(0, 4)$  we obtain a second point on the line by moving down 2 units and then 3 units to the right. So the line goes through  $(0, 4)$  and  $(3, 2)$  as shown in Fig. 1.33.

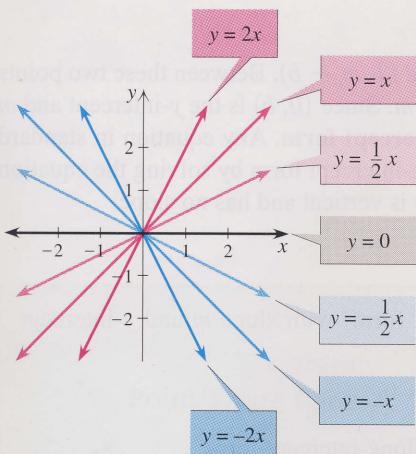
Graphs of  $y = mx$ 

Figure 1.34

As the  $x$ -coordinate increases on a line with positive slope, the  $y$ -coordinate increases also. As the  $x$ -coordinate increases on a line with negative slope, the  $y$ -coordinate decreases. Figure 1.34 shows some lines of the form  $y = mx$  with positive slopes and negative slopes. Observe the effect that the slope has on the position of the line.

## The Three Forms for the Equation of a Line

There are three forms for the equation of a line. The following strategy will help you decide when and how to use these forms.

### STRATEGY

### Finding the Equation of a Line

1. Since vertical lines have no slope, they cannot be written in slope-intercept form  $y = mx + b$  or point-slope form  $y - y_1 = m(x - x_1)$ . All lines can be described by an equation in standard form  $Ax + By = C$ .
2. For any constant  $k$ ,  $y = k$  is a horizontal line and  $x = k$  is a vertical line.
3. If you know two points on a line, then find the slope.
4. If you know the slope and a point on the line, use point-slope form. If the point is the  $y$ -intercept, then use slope-intercept form.
5. Final answers are usually written in slope-intercept or standard form. Standard form is often simplified by using only integers for the coefficients.

### Example 5 Standard form using integers

Find the equation of the line through  $(0, \frac{1}{3})$  with slope  $\frac{1}{2}$ . Write the equation in standard form using only integers.

#### Solution

Since we know the slope and  $y$ -intercept, start with slope-intercept form:

$$y = \frac{1}{2}x + \frac{1}{3} \quad \text{Slope-intercept form}$$

$$-\frac{1}{2}x + y = \frac{1}{3}$$

$$-6\left(-\frac{1}{2}x + y\right) = -6 \cdot \frac{1}{3} \quad \text{Multiply by } -6 \text{ to get integers.}$$

$$3x - 6y = -2 \quad \text{Standard form with integers}$$

Any integral multiple of  $3x - 6y = -2$  would also be standard form, but we usually use the smallest possible positive coefficient for  $x$ .

## Parallel Lines

Two lines in a plane are said to be **parallel** if they have no points in common. Any two vertical lines are parallel, and slope can be used to determine whether nonvertical lines are parallel. For example, the lines  $y = 3x - 4$  and  $y = 3x + 1$  are parallel because their slopes are equal and their  $y$ -intercepts are different.

**Theorem: Parallel Lines**

Two nonvertical lines in the coordinate plane are parallel if and only if their slopes are equal.

A proof to this theorem is outlined in Exercises 97 and 98.

**Example 6 Writing equations of parallel lines**

Find the equation in slope-intercept form of the line through  $(1, -4)$  that is parallel to  $y = 3x + 2$ .

**Solution**

Since  $y = 3x + 2$  has slope 3, any line parallel to it also has slope 3. Write the equation of the line through  $(1, -4)$  with slope 3 in point-slope form:

$$y - (-4) = 3(x - 1) \quad \text{Point-slope form}$$

$$y + 4 = 3x - 3$$

$$y = 3x - 7 \quad \text{Slope-intercept form}$$

The line  $y = 3x - 7$  goes through  $(1, -4)$  and is parallel to  $y = 3x + 2$ .

 The graphs of  $y_1 = 3x - 7$  and  $y_2 = 3x + 2$  in Fig. 1.35 support the answer. ■

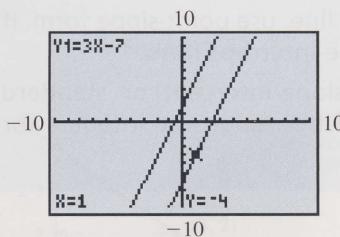


Figure 1.35

**Perpendicular Lines**

Two lines are **perpendicular** if they intersect at a right angle. Slope can be used to determine whether lines are perpendicular. For example, lines with slopes such as  $2/3$  and  $-3/2$  are perpendicular. The slope  $-3/2$  is the opposite of the reciprocal of  $2/3$ . In the following theorem we use the equivalent condition that the product of the slopes of two perpendicular lines is  $-1$ , provided they both have slopes.

**Theorem: Perpendicular Lines**

Two lines with slopes  $m_1$  and  $m_2$  are perpendicular if and only if  $m_1m_2 = -1$ .

**PROOF** The phrase “if and only if” means that there are two statements to prove. First we prove that if  $l_1$  with slope  $m_1$  and  $l_2$  with slope  $m_2$  are perpendicular, then  $m_1m_2 = -1$ . Assume that  $m_1 > 0$ . At the intersection of the lines draw a right triangle using a rise of  $m_1$  and a run of 1, as shown in Fig. 1.36. Rotate  $l_1$  (along with the right triangle) 90 degrees so that it coincides with  $l_2$ . Now use the triangle in its new position to determine that  $m_2 = \frac{1}{-m_1}$  or  $m_1m_2 = -1$ .

The second statement to prove is that  $m_1m_2 = -1$  or  $m_2 = \frac{1}{-m_1}$  implies that the lines are perpendicular. Start with the two intersecting lines and the two congruent right triangles, as shown in Fig. 1.36. It takes a rotation of 90 degrees to get the vertical side marked  $m_1$  to coincide with the horizontal side marked  $m_1$ . Since the triangles are congruent, rotating 90 degrees makes the triangles coincide and the lines coincide. So the lines are perpendicular. ■

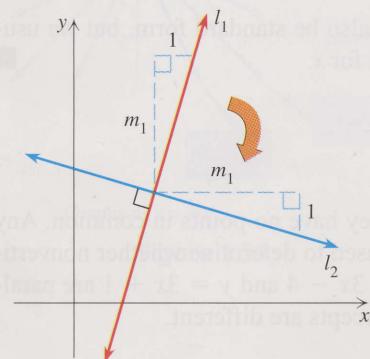


Figure 1.36

**Example 7** Writing equations of perpendicular lines

Find the equation of the line perpendicular to the line  $3x - 4y = 8$  and containing the point  $(-2, 1)$ . Write the answer in slope-intercept form.

**Solution**

Rewrite  $3x - 4y = 8$  in slope-intercept form:

$$-4y = -3x + 8$$

$$y = \frac{3}{4}x - 2 \quad \text{Slope of this line is } \frac{3}{4}.$$

Since the product of the slopes of perpendicular lines is  $-1$ , the slope of the line that we seek is  $-4/3$ . Use the slope  $-4/3$  and the point  $(-2, 1)$  in the point-slope form:

$$y - 1 = -\frac{4}{3}(x - (-2))$$

$$y - 1 = -\frac{4}{3}x - \frac{8}{3}$$

$$y = -\frac{4}{3}x - \frac{5}{3}$$

The last equation is the required equation in slope-intercept form. The graphs of these two equations should look perpendicular.

If you graph  $y_1 = \frac{3}{4}x - 2$  and  $y_2 = -\frac{4}{3}x - \frac{5}{3}$  with a graphing calculator, the graphs will not appear perpendicular in the standard viewing window because each axis has a different unit length. The graphs appear perpendicular in Fig. 1.37 because the unit lengths were made equal with the ZSquare feature of the TI-83. ■

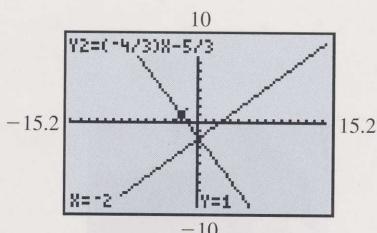


Figure 1.37

**Applications**

In Section 1.3 the distance formula was used to establish some facts about geometric figures in the coordinate plane. We can also use slope to prove that lines are parallel or perpendicular in geometric figures.

**Example 8** The diagonals of a rhombus are perpendicular

Given a rhombus with vertices  $(0, 0)$ ,  $(5, 0)$ ,  $(3, 4)$ , and  $(8, 4)$ , prove that the diagonals of this rhombus are perpendicular. (A rhombus is a four-sided figure in which the sides have equal length.)

**Solution**

Plot the four points  $A$ ,  $B$ ,  $C$ , and  $D$  as shown in Fig. 1.38. You should show that each side of this figure has length 5, verifying that the figure is a rhombus. Find the slopes of the diagonals and their product:

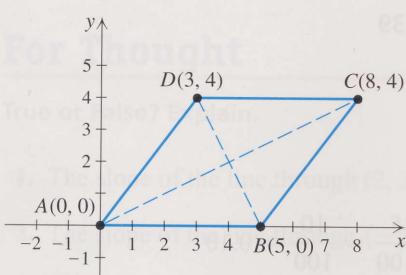


Figure 1.38

$$m_{AC} = \frac{4 - 0}{8 - 0} = \frac{1}{2} \quad m_{BD} = \frac{0 - 4}{5 - 3} = -2$$

$$m_{AC} \cdot m_{BD} = \frac{1}{2}(-2) = -1$$

Since the product of the slopes of the diagonals is  $-1$ , the diagonals are perpendicular.

If the value of one variable can be determined from the value of another variable, then we say that the first variable is a **function** of the second variable. Because the area of a circle can be determined from the radius by the formula  $A = \pi r^2$ , we say that  $A$  is a function of  $r$ . If  $y$  is determined from  $x$  by using the slope-intercept form of the equation of a line  $y = mx + b$ , then  $y$  is a **linear function** of  $x$ . The formula  $F = \frac{9}{5}C + 32$  expresses  $F$  as a linear function of  $C$ . We will discuss functions in depth in Chapter 2.

### Example 9 A linear function

From ABC Wireless the monthly cost for a cell phone with 100 minutes per month is \$35, or 200 minutes per month for \$45. See Fig. 1.39. The cost in dollars is a linear function of the time in minutes.

- Find the formula for  $C$ .
- What is the cost for 400 minutes per month?

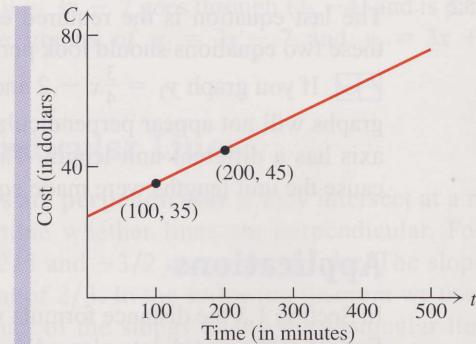


Figure 1.39

### Solution

- First find the slope:

$$m = \frac{C_2 - C_1}{t_2 - t_1} = \frac{45 - 35}{200 - 100} = \frac{10}{100} = 0.10$$

The slope is \$0.10 per minute. Now find  $b$  by using  $C = 35$ ,  $t = 100$ , and  $m = 0.10$  in the slope-intercept form  $C = mt + b$ :

$$35 = 0.10(100) + b$$

$$35 = 10 + b$$

$$25 = b$$

So the formula is  $C = 0.10t + 25$ .

Figure 1.35

- b. Use  $t = 400$  in the formula  $C = 0.10t + 25$ :

$$C = 0.10(400) + 25 = 65$$

The cost for 400 minutes per month is \$65.

Note that in Example 9 we could have used the point-slope form  $C - C_1 = m(t - t_1)$  to get the formula  $C = 0.10t + 25$ . Try it.

The situation in the next example leads naturally to an equation in standard form.

### Example 10 Interpreting slope

A manager for a country market will spend a total of \$80 on apples at \$0.25 each and pears at \$0.50 each. Write the number of apples she can buy as a linear function of the number of pears. Find the slope and interpret your answer.

#### Solution

Let  $a$  represent the number of apples and  $p$  represent the number of pears. Write an equation in standard form about the total amount spent:

$$0.25a + 0.50p = 80$$

$$0.25a = 80 - 0.50p$$

$$a = 320 - 2p \quad \text{Solve for } a.$$

The equation  $a = 320 - 2p$  or  $a = -2p + 320$  expresses the number of apples as a function of the number of pears. Since  $p$  is the first coordinate and  $a$  is the second, the slope is  $-2$  apples per pear. So if the number of apples is decreased by 2, then the number of pears can be increased by 1 and the total is still \$80. This makes sense because the pears cost twice as much as the apples.

## For Thought

**True or False? Explain.**

1. The slope of the line through  $(2, 2)$  and  $(3, 3)$  is  $3/2$ . **F**
2. The slope of the line through  $(-3, 1)$  and  $(-3, 5)$  is  $0$ . **F**
3. Any two distinct parallel lines have equal slopes. **F**
4. The graph of  $x = 3$  in the coordinate plane is the single point  $(3, 0)$ . **F**
5. Two lines with slopes  $m_1$  and  $m_2$  are perpendicular if  $m_1 = -1/m_2$ . **T**
6. Every line in the coordinate plane has an equation in slope-intercept form. **F**
7. The slope of the line  $y = 3 - 2x$  is  $3$ . **F**
8. Every line in the coordinate plane has an equation in standard form. **T**
9. The line  $y = 3x$  is parallel to the line  $y = -3x$ . **F**
10. The line  $x - 3y = 4$  contains the point  $(1, -1)$  and has slope  $1/3$ . **T**

## 1.4 Exercises

Find the slope of the line containing each pair of points. (Example 1)

1.  $(-2, 3), (4, 5) \frac{1}{3}$

2.  $(-1, 2), (3, 6) 1$

3.  $(1, 3), (3, -5) -4$

4.  $(2, -1), (5, -3) -\frac{2}{3}$

5.  $(5, 2), (-3, 2) 0$

6.  $(0, 0), (5, 0) 0$

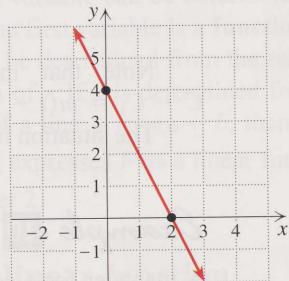
7.  $\left(\frac{1}{8}, \frac{1}{4}\right), \left(\frac{1}{4}, \frac{1}{2}\right) 2$

8.  $\left(-\frac{1}{3}, \frac{1}{2}\right), \left(\frac{1}{6}, \frac{1}{3}\right) -\frac{1}{3}$

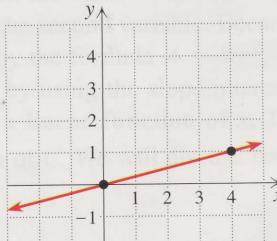
9.  $(5, -1), (5, 3)$  No slope

10.  $(-7, 2), (-7, -6)$  No slope

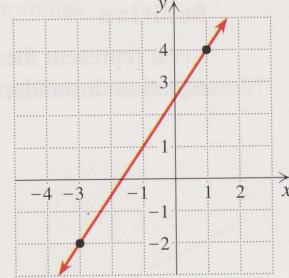
23.  $y = -2x + 4$



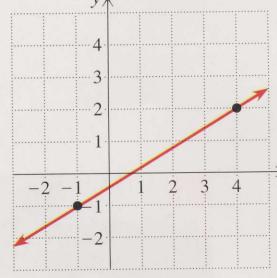
24.  $y = \frac{1}{4}x$



25.  $y = \frac{3}{2}x + \frac{5}{2}$



26.  $y = \frac{3}{5}x - \frac{2}{5}$



Find the equation of the line through the given pair of points. Solve it for  $y$  if possible. (Example 2)

11.  $(-1, -1), (3, 4) y = \frac{5}{4}x + \frac{1}{4}$

12.  $(-2, 1), (3, 5) y = \frac{4}{5}x + \frac{13}{5}$

13.  $(-2, 6), (4, -1) y = -\frac{7}{6}x + \frac{11}{3}$

14.  $(-3, 5), (2, 1) y = -\frac{4}{5}x + \frac{13}{5}$

15.  $(3, 5), (-3, 5) y = 5$

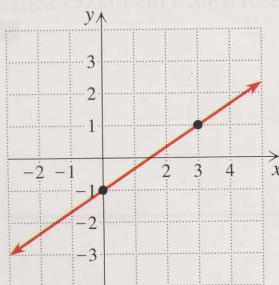
16.  $(-6, 4), (2, 4) y = 4$

17.  $(4, -3), (4, 12) x = 4$

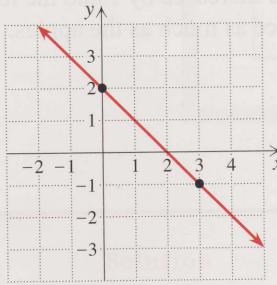
18.  $(-5, 6), (-5, 4) x = -5$

Write an equation in slope-intercept form for each of the lines shown. (Examples 2 and 3)

19.  $y = \frac{2}{3}x - 1$



20.  $y = -x + 2$



Write each equation in slope-intercept form and identify the slope and  $y$ -intercept of the line. (Example 3)

27.  $3x - 5y = 10$

$y = \frac{3}{5}x - 2, \frac{3}{5}, (0, -2)$

29.  $y - 3 = 2(x - 4)$   
 $y = 2x - 5, 2, (0, -5)$

31.  $y + 1 = \frac{1}{2}(x - (-3))$   
 $y = \frac{1}{2}x + \frac{1}{2}, \frac{1}{2}, (0, \frac{1}{2})$

33.  $y - 4 = 0$   
 $y = 4, 0, (0, 4)$

35.  $y - 0.4 = 0.03(x - 100)$   
 $y = 0.03x - 2.6, 0.03, (0, -2.6)$

36.  $y + 0.2 = 0.02(x - 3)$   
 $y = 0.02x - 0.26, 0.02, (0, -0.26)$

28.  $2x - 2y = 1$

$y = x - \frac{1}{2}, 1, (0, -1/2)$

30.  $y + 5 = -3(x - (-1))$   
 $y = -3x - 8, -3, (0, -8)$

32.  $y - 2 = -\frac{3}{2}(x + 5)$   
 $y = -\frac{3}{2}x - \frac{11}{2}, -\frac{3}{2}, (0, -\frac{11}{2})$

34.  $-y + 5 = 0$   
 $y = 5, 0, (0, 5)$

Use the  $y$ -intercept and slope to sketch the graph of each equation. (Example 4)

37.  $y = \frac{1}{2}x - 2$

38.  $y = \frac{2}{3}x + 1$

39.  $y = -3x + 1$

40.  $y = -x + 3$

41.  $y = -\frac{3}{4}x - 1$

42.  $y = -\frac{3}{2}x$

<sup>†</sup>Due to space constraints, answers to these exercises may be found in the complete Answers beginning on page A-1 in the back of the book.

43.  $x - y = 3$

44.  $2x - 3y = 6$

45.  $y - 5 = 0$

46.  $6 - y = 0$

Find the equation of the line through the given pair of points in standard form using only integers. See the strategy for finding the equation of a line on page 43. (Example 5)

47. (3, 0) and (0, -4)

$$4x - 3y = 12$$

49. (2, 3) and (-3, -1)

$$4x - 5y = -7$$

51. (-4, 2) and (-4, 5)  $x = -4$

53.  $\left(2, \frac{2}{3}\right), \left(-\frac{1}{2}, -2\right)$   
$$16x - 15y = 22$$

55.  $\left(\frac{1}{2}, \frac{1}{4}\right), \left(-\frac{1}{3}, \frac{1}{5}\right)$   
$$3x - 50y = -11$$

48. (-2, 0) and (0, 3)

$$3x - 2y = -6$$

50. (4, -1) and (-2, -6)

$$5x - 6y = 26$$

52. (-3, 6) and (9, 6)  $y = 6$

54.  $\left(\frac{3}{4}, -3\right), \left(-5, \frac{1}{8}\right)$   
$$100x + 184y = -477$$

56.  $\left(-\frac{3}{8}, \frac{1}{4}\right), \left(\frac{1}{2}, -\frac{1}{6}\right)$   
$$20x + 42y = 3$$

Find the slope of each line described. (Examples 6 and 7)

57. A line parallel to  $y = 0.5x - 9$  0.5

58. A line parallel to  $3x - 9y = 4$  1/3

59. A line perpendicular to  $3y - 3x = 7$  -1

60. A line perpendicular to  $2x - 3y = 8$  -3/2

61. A line perpendicular to the line  $x = 4$  0

62. A line parallel to  $y = 5$  0

Write an equation in standard form using only integers for each of the lines described. In each case make a sketch. (Examples 6 and 7)

63. The line with slope 2, going through (1, -2)  $2x - y = 4$

64. The line with slope -3, going through (-3, 4)  $3x + y = -5$

65. The line through (1, 4), parallel to  $y = -3x$   $3x + y = 7$

66. The line through (-2, 3) parallel to  $y = \frac{1}{2}x + 6$   $x - 2y = -8$

67. The line parallel to  $5x - 7y = 35$  and containing (6, 1)  
$$5x - 7y = 23$$

68. The line parallel to  $4x + 9y = 5$  and containing (-4, 2)  
$$4x + 9y = 2$$

69. The line perpendicular to  $y = \frac{2}{3}x + 5$  and containing (2, -3)  
$$3x + 2y = 0$$

70. The line perpendicular to  $y = 9x + 5$  and containing (5, 4)  
$$x + 9y = 41$$

71. The line perpendicular to  $x - 2y = 3$  and containing (-3, 1)  
$$2x + y = -5$$

72. The line perpendicular to  $3x - y = 9$  and containing (0, 0)  
$$x + 3y = 0$$

73. The line perpendicular to  $x = 4$  and containing (2, 5)  $y = 5$

74. The line perpendicular to  $y = 9$  and containing (-1, 3)  
$$x = -1$$

Find the value of  $a$  in each case. (Examples 1-7)

75. The line through (-2, 3) and (8, 5) is perpendicular to  
 $y = ax + 2$ . -5

76. The line through (3, 4) and (7,  $a$ ) has slope 2/3. 20/3

77. The line through (-2,  $a$ ) and ( $a$ , 3) has slope -1/2. 8

78. The line through (-1,  $a$ ) and (3, -4) is parallel to  $y = ax$ .  
-4/5

Either prove or disprove each statement. Use a graph only as a guide. Your proof should rely on algebraic calculations. (Example 8)

79. The points (-1, 2), (2, -1), (3, 3), and (-2, -2) are the vertices of a parallelogram. T

80. The points (-1, 1), (-2, -5), (2, -4), and (3, 2) are the vertices of a parallelogram. T

81. The points (-5, -1), (-3, -4), (3, 0), and (1, 3) are the vertices of a rectangle. T

82. The points (-5, -1), (1, -4), (4, 2), and (-1, 5) are the vertices of a square. F

83. The points (-5, 1), (-2, -3), and (4, 2) are the vertices of a right triangle. F

84. The points (-4, -3), (1, -2), (2, 3), and (-3, 2) are the vertices of a rhombus. T

 Use a graphing calculator to solve each problem.

85. Graph  $y_1 = (x - 5)/3$  and  $y_2 = x - 0.67(x + 4.2)$ . Do the lines appear to be parallel? Are the lines parallel? Yes, no

86. Graph  $y_1 = 99x$  and  $y_2 = -x/99$ . Do the lines appear to be perpendicular? Should they appear perpendicular? Yes, Yes

87. Graph  $y = (x^3 - 8)/(x^2 + 2x + 4)$ . Use TRACE to examine points on the graph. Write a linear function for the graph. Explain why this function is linear.

$$y = x - 2, x^3 - 8 = (x - 2)(x^2 + 2x + 4)$$

88. Graph  $y = (x^3 + 2x^2 - 5x - 6)/(x^2 + x - 6)$ . Use TRACE to examine points on the graph. Write a linear function for the graph. Factor  $x^3 + 2x^2 - 5x - 6$  completely.  
$$y = x + 1, (x + 1)(x + 3)(x - 2)$$

Solve each problem. (Examples 9 and 10)

89. Celsius to Fahrenheit Formula Fahrenheit temperature  $F$  is a linear function of Celsius temperature  $C$ . The ordered pair (0, 32) is an ordered pair of this function because  $0^\circ\text{C}$  is equivalent to  $32^\circ\text{F}$ , the freezing point of water. The ordered pair (100, 212) is also an ordered pair of this function because  $100^\circ\text{C}$  is equivalent to  $212^\circ\text{F}$ , the boiling point of water. Use

the two given points and the point-slope formula to write  $F$  as a function of  $C$ . Find the Fahrenheit temperature of an oven at  $150^{\circ}\text{C}$ .  $F = \frac{9}{5}C + 32$ ,  $302^{\circ}\text{F}$

90. *Cost of Business Cards* Speedy Printing charges \$23 for 200 deluxe business cards and \$35 for 500 deluxe business cards. Given that the cost is a linear function of the number of cards printed, find a formula for that function and find the cost of 700 business cards.  $C = 0.04n + 15$ , \$43

91. *Volume Discount* Mona Kalini gives a walking tour of Honolulu to one person for \$49. To increase her business, she advertised at the National Orthodontist Convention that she would lower the price by \$1 per person for each additional person. Write the cost per person  $c$  as a function of the number of people  $n$  on the tour. How much does she make for a tour with 40 people?  $c = 50 - n$ , \$400

92. *Ticket Pricing* At \$10 per ticket, Willie Williams and the Wranglers will fill all 8000 seats in the Assembly Center. The manager knows that for every \$1 increase in the price, 500 tickets will go unsold. Write the number of tickets sold,  $n$ , as a function of the ticket price,  $p$ . How much money will be taken in if the tickets are \$20 each?  $n = -500p + 13,000$ , \$60,000

93. *Lindbergh's Air Speed* Charles Lindbergh estimated that at the start of his historic flight the practical economical air speed was 95 mph and at 4000 statute miles from the starting point it was 75 mph ([www.charleslindbergh.com](http://www.charleslindbergh.com)). Assume that the practical economical air speed  $S$  is a linear function of the distance  $D$  from the starting point as shown in the accompanying figure. Find a formula for that function.

$$S = -0.005D + 95$$

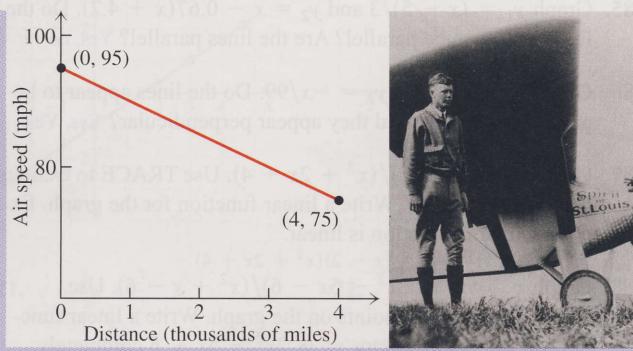


Figure for Exercise 93

94. *Speed Over Newfoundland* In Lindbergh's flying log he recorded his air speed over Newfoundland as 98 mph, 1100 miles into his flight. According to the formula from Exercise 93, what should have been his air speed over Newfoundland? 89.5 mph

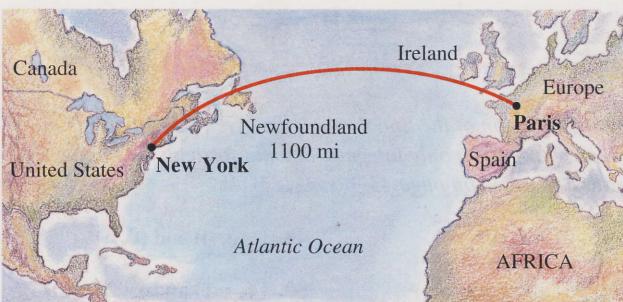


Figure for Exercise 94

95. *Computers and Printers* An office manager will spend a total of \$60,000 on computers at \$2000 each and printers at \$1500 each. Write the number of computers purchased as a function of the number of printers purchased. Find and interpret the slope.  $c = -\frac{3}{4}p + 30$ ,  $-\frac{3}{4}$ , If  $p$  increases by 4 then  $c$  decreases by 3.

96. *Carpenters and Helpers* Because a job was finished early, a contractor will distribute a total of \$2400 in bonuses to 9 carpenters and 3 helpers. The carpenters all get the same amount and the helpers all get the same amount. Write the amount of a helper's bonus as a function of the amount of a carpenter's bonus. Find and interpret the slope.

$$h = -3c + 800$$

$-3$ , If  $c$  increases by \$1, then  $h$  decreases by \$3.

### For Writing/Discussion

97. *Equal Slopes* Show that if  $y = mx + b_1$  and  $y = mx + b_2$  are equations of lines with equal slopes, but  $b_1 \neq b_2$ , then they have no point in common.

98. *Unequal Slopes* Show that the lines  $y = m_1x + b_1$  and  $y = m_2x + b_2$  intersect at a point with  $x$ -coordinate  $(b_2 - b_1)/(m_1 - m_2)$  provided  $m_1 \neq m_2$ . Explain how this exercise and the previous exercise prove the theorem that two nonvertical lines are parallel if and only if they have equal slopes.

99. *Summing Angles* Consider the angles shown in the accompanying figure. Show that the degree measure of angle  $A$  plus the degree measure of angle  $B$  is equal to the degree measure of angle  $C$ .

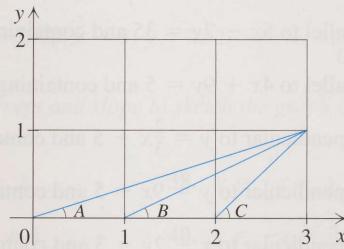


Figure for Exercise 99