Pension system of Smurfland

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In the happy village of Smurfland, the following pension system has been put into place: every Smurf starts working for the local Smurf-company when he (or she) turns 25 years of age.

They all start as junior employees, with a possibility to be promoted to executive employees starting their second year of employment.

At the end of each year both junior and executive employees devote a quote of their salary to the central Smurf-bank so that the total amount could be equally distributed to all the retired Smurfs in the village. If that year there are no retired Smurfs, then the amount of Smurf-dollars will be simply stored in the bank for the year to come to be possibly distributed to the new retired Smurfs, if any.

The retirement age is mandatory at 70 years old, with the possibility to retire earlier starting at 55 years old.

It is known that Smurfs live a very long life! And they're still quite energetic at the age of their retirement. Therefore, Papa Smurf is now wondering whether it would be more profitable to increase the mandatory retirement age at 75 years old, with earlier retirement option starting at 60 years old.

Asking Mathe-Smurf to give a prediction of the Smurf pensions within the next 100 years starting at the current year, here is what he got: Figure 1 and Figure 2.

Numerical simulation. Graph 1 shows the amount of Smurf-dollars that each retired Smurf is receiving each year. Graph 2 shows the interval of confidence on how much Smurf-dollars each retired Smurf is likely to receive.

Both graphs have been produced using a MATLAB code where

- the number of new employees each year is a Poisson with parameter $\lambda = 11$;
- the probability that a junior Smurf gets promoted is a Bernoullib $B(1, \theta)$ with $\theta = 0.01$;
- unfortunately, some deaths among the older Smurfs needs to be taken into account: the number is a binomial $B(n, p)$ where n is the number of retired Smurfs at the year under consideration, and $p = 0.1$;
- the probability that a Smurf decides to retire when he/she reaches the retirement window is a Bernoulli $B(1, q)$ with $q = 0.6$;

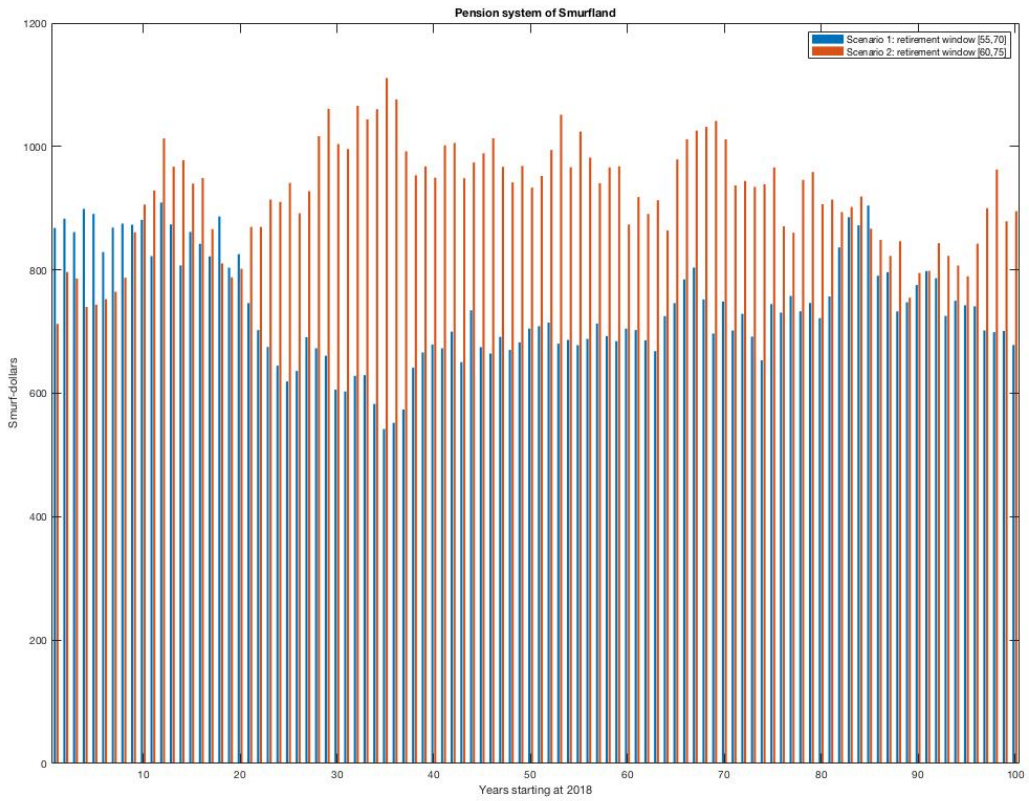

Figure 1: Pension system of Smurfland.

- the interval of confidence in the second graph has a quantile t_α with $\alpha = 0.2$ based on a Student's t-distribution and it's calculated over $N = 30$ simulations;

- the initial settings are: 80 junior Smurfs, 20 executive Smurfs, 20 retired Smurfs.

Ongoing and future developments I'm working on:

- developing an interactive version/app for the web;
- the parameters for the random variables should be dictated by some smart Machine Learning algorithm, once I'll collect enough history data from the book of the Smurfs.

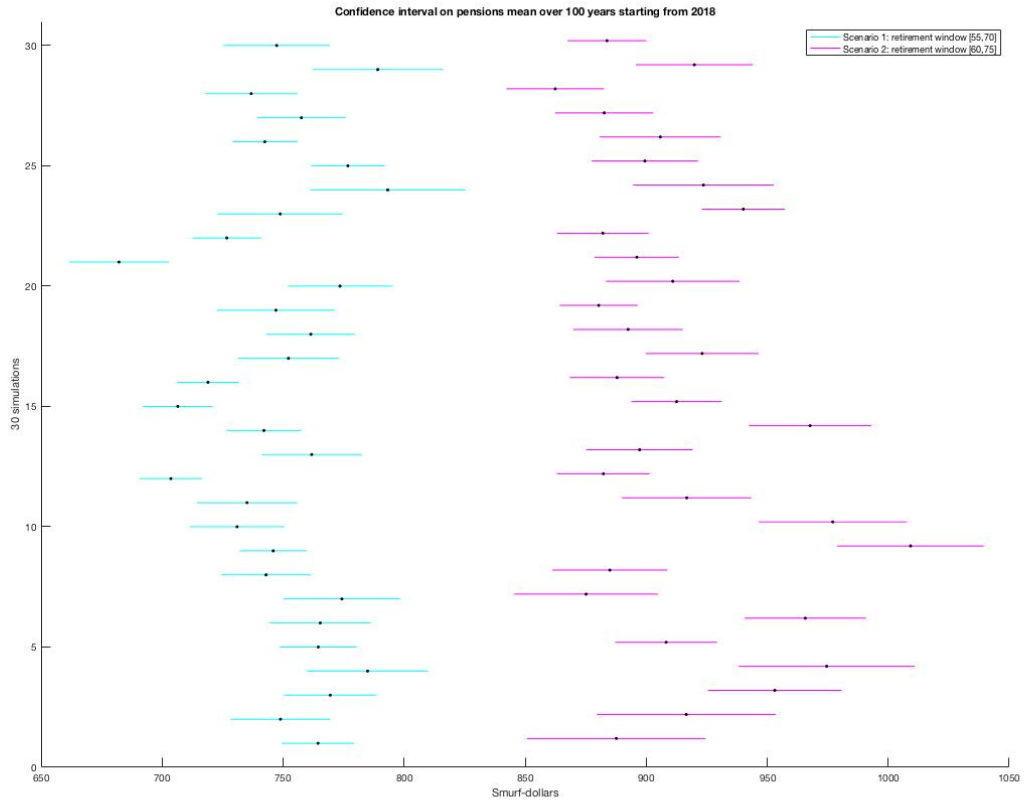

Figure 2: Intervals of confidence.