1. *Warm Up.* Towers of Hanoi: Suppose we have three pegs in a row. Start with n disks of decreasing size on the left-most peg. We want to move all the disks to the right-most peg while following these rules:

   (a) Only one disk can be moved at a time;

   (b) Each move takes the top disk from one stack and moves it to the top of another;

   (c) No disk can be put on a smaller disk.

   Find the smallest number of moves to achieve that!

2. **Prove that:** The sum of the first n natural numbers is $\dfrac{n(n+1)}{2}$.

   (a) Check that the statement is true for $n = 1, 2, 3, 4, 5$.

   (b) What is the open statement "$P(n)$"?

   $P(n) =$

   (c) What is the statement "$P(1)$"? Why is $P(1)$ true?

   $P(1) =$

   (d) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that $P(k-1) \Rightarrow P(k)$).

   Assume that

   We want to show that

   (Inductive step)

3. **Prove that:** For every positive integer n,

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

   (a) What is the open sentence "$P(n)$"?

   $P(n) =$

   (b) What is the statement "$P(1)$"? Why is $P(1)$ true?

   $P(1) =$

   (c) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that $P(k-1) \Rightarrow P(k)$).

   Assume that

   We want to show that

   (Inductive step)

4. Prove that if $n \in \mathbb{Z}$ and $n \geq 4$, then $n! > 2^n$.

5. Prove that if $n \in \mathbb{Z}$ and $n \geq 1$, then $6 \mid 7^n - 1$.

6. Recall that the *Fibonacci numbers* are described as follows:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55. \ldots$$

$$F_1 = 1$$

$$F_2 = 1$$

$$F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 3$$

   Prove that the Fibonacci sequence satisfies:

$$F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{2n}$$

   for all n.