Saint Mary's University

Department of Mathematics
& Computing Science

MATH 4442 – Real Analysis II

Winter 2022

Instructor: Dr. Manuela Girotti

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Please put "MATH 4442" in the subject line, use the *plain text format*, and make sure that you are clearly identified (first and last names). I do not answer anonymous email. I do not check emails during evenings or weekends. I usually answer during the first business day after receiving an email.

Lectures: Synchronous.

Mondays and Wednesdays, 10:00am–11:15am (Halifax time)

Loyola Academic room 280

Office hours: TBD

Overview: The course is a continuation of MATH 3441 - Real Analysis I where we focused on the basic theory of metric spaces and their properties (separability, completeness and compactness). In this course we will start by focussing on the space of continuous functions on (usually compact) metric spaces. We will then dedicate most of the course on **Measure Theory and Lebesgue Integration Theory**.

Prerequisites: MATH 3441 (Real Analysis I). Good knowledge of MATH 2310 (Intro to Analysis) and MATH 2311 (Multivariable calculus).

Textbooks: Pre-started class notes from Real Analysis I (Lectures 23–28) and Measure Theory class notes (posted on Brightspace).
These notes are for your use only - please, do not share them with anyone!

Other useful resources:

- H. L. Royden and P.M. Fitzpatrick, *Real Analysis*, 4th ed, Pearson, 2010.
- W. Rudin, *Real and Complex Analysis*, Series in Higher Mathematics, McGraw-Hill, 1976.

A diary of the lectures will be regularly kept on the Brightspace calendar with the sections covered in each class.

Forum: Discussions about class lectures, homework exercises, information about exams, etc. will happen on the forum set up on Brightspace (under the voice Discussion). You are highly encouraged to use this tool to ask questions to me or to your other peers and share your understanding!

Evaluations: The course mark will be calculated as follows:

45% assignments,
5% *active* in-class participation,
20% midterms,
30% final exam.

Note that there is no “100% final exam” option in this course. The term work contributes 70% to the final grade. Therefore, active participation in classes and continuous work on the course material during the semester is essential for success in this course.
The final score will be out of 100 and the breakdown of the grades is the following:

Grade	F	D	C	C+	B-	B	B+	A-	A	A+
Percentage	0-50	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90	90-100

Homework: You will be required to hand in about **5 assignments** along the semester. The assignments will be posted on the Brightspace website with due dates and they reflect the content of the course.

Discussions and work group are highly encouraged!

The assignments will be posted on Brightspace (and on Crowdmark) and they reflect the content of the course.
No late assignments will be accepted.
To submit your assignment you can either

- write it on paper (filling the empty spaces provided on the assignment) and scan it;
- type it on the computer by using LateX, Overleaf or other softwares that support Mathematics symbols;
- write it on your tablet using a handwriting app.

You will then need to *upload your homework on Crowdmark.*

Midterm exams: There will be **two midterm exams**. They will be held during class hours (10am-11:15am) in the usual classroom. The dates and content of the midterms will be communicated at least 10 days in advance.

Final exam: The final examination will be a 3-hour **take-home exam**, to be taken during a continuous 3 hour period.
The final exam due date will be scheduled by the Registrar for some time during the exam period in April.
The final exam will cover material from the entire course and it will be open book: you can use all class material (class notes, homework problems, solutions). It is forbidden to use any other material, to look up solutions online, and to discuss with other peers.

Make-ups: Alternate arrangements will be discussed only in case a valid medical excuse is provided in a timely fashion and no later then 24 hours after the exam. No special arrangements to accommodate travel that coincides with midterms or the final will be made.

Expectations: All individuals participating in courses are expected to be professional and constructive throughout the course, including in their communications.

Academic Integrity: This course will adhere to the SMU Academic Integrity Policy as found on the Academic Integrity and Student Responsibility page.

Students are expected to do their own work during tests and exams. The following activities, although not exhaustive, are examples of activities that are prohibited:

- Copying from another student;
- Allowing another student to copy from you;
- Using unauthorized aids, including: sheets, cell phones and calculators, during test or exam;
- Getting aid from or giving aid to another student during tests and exams;
- Having another student write for you or writing for another student.

Offenders are subject to discipline. Students are urged to read the Academic Integrity Handbook.

An incident of academic dishonesty can have extremely negative consequences: it could delay or bar a student from graduating. A note on a transcript referring to academic dishonesty could very well bar a student from graduate school or affect job opportunities.

This course is a precious opportunity for you to learn something new and valuable. It's an investment on your future. Failing to acquire it will sadly be your loss.

Intellectual property: Content belonging to the instructor and shared in online courses, including, but not limited to, online lectures, course notes, quizzes, assignments, and video recordings of classes remain the intellectual property of the faculty member. It may not be distributed, published or broadcast, in whole or in part, without the express permission of the faculty member.

Students are also forbidden to use their own means of recording any elements of an online class or lecture without express permission of the instructor. Any unauthorized sharing of course content may constitute a breach of the Academic Regulations.

Disabilities: Saint Mary's University is committed to providing reasonable accommodations for all persons with disabilities. Students with disabilities who need accommodations shall first contact the Fred Smithers Centre before requesting accommodations for this class.

Students who need accommodations in this course must contact the instructor in a timely manner (at least one week before examinations) to discuss needed accommodations.

Territorial Acknowledgement

Saint Mary's University acknowledges that the university is located in Mi'kma'ki, the ancestral and unceded territory of the Mi'kmaq People.

This territory is covered by the "Treaties of Peace and Friendship" which Mi'kmaq, Wəlastəkwiyik (Maliseet), and Passamaquoddy Peoples first signed with the British Crown in 1726.

The treaties did not deal with surrender of lands and resources but in fact recognized Mi'kmaq and Wəlastəkwiyik (Maliseet) title and established the rules for what was to be an ongoing relationship between nations.

(Tentative) course calendar:

Week	Topic	Important dates
1 (Jan 10th)	Welcome! Pointwise and uniform convergence for a sequence of functions. The Banach space $C_b(X, \mathbb{R})$.	Jan 14th – course registration deadline
2 (Jan 17th)	Equicontinuity and total boundedness in $C(X, \mathbb{R})$. Algebras of functions and the Stone–Weierstrass theorem.	Jan 18th – course drop deadline
3 (Jan 24th)	Intro. An analyst's motivation for the Lebesgue' integral. The topology induced by a metric.	
4 (Jan 31th)	Borel sets and functions. Borel σ-algebra on $\mathbb{R}^n$ and second-countability.	
5 (Feb 7th)	Borel σ-algebra on the extended real line $\mathbb{R} \cup \{-\infty, +\infty\}$ and limits. Measures.	
6 (Feb 14th)	$\limsup$ of a sequence of sets. Integration of positive functions.	
7 (Feb 21st)	*** *Winter break* ***	Feb 21st – *Heritage Day*
8 (Feb 28th)	Integration of real and complex functions. Increasing functions as distribution functions of measures on $\mathbb{R}$.	
9 (Mar 7th)	Outer measure and Carathéodory's Theorem. Borel measures on $\mathbb{R}$.	
10 (Mar 14th)	Comparison with the Riemann integrals and with improper integrals. Cantor set and Cantor function.	Mar 17th – course withdrawal deadline
11 (Mar 21st)	The monotone class theorem. Regularity of Lebesgues–Stieltjes measures.	
12 (Mar 28th)	Product measures. Lebesgue measure on $\mathbb{R}^n$. Tonelli's and Fubini's Theorems.	
13 (Apr 4th)	Applications of Tonelli's and Fubini's theorems. Review & Conclusions	

Disclaimer: the instructor reserves the right to make changes to the course outline and course content should this be necessary for academic or other reasons. Changes will also be posted on Brightspace and promptly communicated. Every effort will be made to minimize such changes.