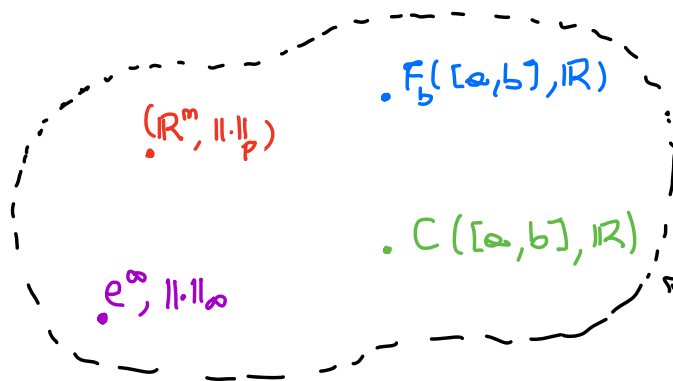# Why metric spaces???

Meet M. Fréchet (1878-1973)

Ph. D. thesis 1906

many spaces (Ascoli, Arzelá, Volterra,...)
each with its own definition of CONVERGENCE

$(\mathbb{R}^m, \|\cdot\|_p)$

$. F_b([a,b], \mathbb{R})$

$. C([a,b], \mathbb{R})$

$\ell^\infty, \|\cdot\|_\infty$

these are actually
all the same type
of space!
METRIC SPACES
(Hausdorff, 1914)

Fréchet came up with the definition of   (abstract!)   (Pythagoras, Thales,...)

DISTANCE

$d: X \times X \to \mathbb{R}$ with properties $\Delta 1, \Delta 2, \Delta 3, \Delta 4$

consequences: 1) concept of BALLS (open/closed)

simplify our life! → 2) (unified) definition of

CONVERGENCE

→ Topological Sp.
(Hausdorff)

are metric spaces really all equals?

They have different properties depending on

* SEPARABILITY

* COMPLETENESS

* COMPACTNESS