MATH 250 – Foundations of Mathematics (Spring 2024)

(as of February 21, 2024)

Land Acknowledgement

Emory University acknowledges the Muscogee (Creek) people who lived, worked, produced knowledge on, and nurtured the land where Emory's Oxford and Atlanta campuses are now located. In 1821, fifteen years before Emory's founding, the Muscogee were forced to relinquish this land. We recognize the sustained oppression, land dispossession, and involuntary removals of the Muscogee and Cherokee peoples from Georgia and the Southeast. Emory seeks to honor the Muscogee Nation and other Indigenous caretakers of this land by humbly seeking knowledge of their histories and committing to respectful stewardship of the land.

Instructor: Dr. Manuela Girotti

Office: Math & Science Center, room N416

Email: manuela.girotti@emory.edu

Please put "MATH 250" in the subject line, use the *plain text format*, and make sure that you are clearly identified (first and last names). I do not answer anonymous email. I make every effort to answer by the first business day after receiving an email.

Lectures: Synchronous, in person.

Tuesdays and Thursdays, 2:30pm–3:45pm (Eastern time)
Math & Science Center, room W307C

Office hours: – *Ask-Me-Anything* hours –

Tuesdays and Thursdays 12:30pm–1:30pm and 3:45pm–4:45pm
or by appointment

Prerequisites: Calculus I and II (MATH 111 and MATH 112 or equivalent).

Overview:

This course is a first exploration into the world of Mathematics in its abstract form. It will give an introduction to proofs in advanced mathematics and is intended as a transition to upper division courses including complex variables (MATH 318), abstract vector spaces (MATH 321), number theory (MATH 328), partial differential equations (MATH 351), or real analysis (MATH 411).

The main topics will be the following:

- Logic: statements, negation, converse, etc.

- Mathematical prose and rigor: how to write mathematics correctly and in complete sentences.

- Techniques of proof: proof by contradiction, induction, proof by cases.

- Sets, relations, functions: the building blocks of mathematics.

- Some additional topics such as

   - Cardinality: different sizes of infinity

   - Numbers: what they are, divisibility of integers (Euclid algorithm, Diophantine equation), transcendental numbers vs. algebraic numbers

A diary of the lectures will be regularly kept on the Canvas calendar with the material covered in each class. Please, refer to that when preparing for the final exam or in case of missed class, because that will be the official and ultimate syllabus for the class.

Learning outcomes:

At the end of the semester students will have learned

- basic elements of logic

- how to write simple mathematical proofs and sentences with standard symbols ($\exists$, $\in$, $\subseteq$, etc.) and specific techniques: induction, contradiction, etc.

- the concept of "infinity" in the mathematical sense and the different sizes of infinity

- fundamental properties of numbers: divisibility of integers, difference between transcendental vs. algebraic number (i.e., the difference between π and $2^{\frac{1}{2}}$), etc.

Textbook: Robert J. Bond, William J. Keane, *An introduction to abstract mathematics*, Waveland Press, 2007 (ISBN 978-1-57766-539-7).

Evaluations: The course mark will be calculated as follows:

10% quizzes,

10% class participation,

30% homework,

20% midterm exam,

30% final exam.

Final letter grades are assigned according to the ECAS Catalog:

Grade	F	D	D+	C-	C	C+	B-	B	B+	A-	A
Percentage	0-62	63-66	67-69	70-72	73-76	77-79	80-82	83-86	87-89	90-92	93-100

Quizzes: There will be a short quiz almost every Thursday, available on Gradescope. It will be available until the next Tuesday and it will consists of a few short and/or multiple choice questions about what we just saw in class during the week.

The quizzes are intended to encourage the class to regularly review the material, such as personal notes and/or the textbook, and to give feedback (both to you the student, and to me the teacher) about your progress.

Forum: Discussions about class lectures, homework exercises, information about exams, etc. will happen on the forum set up on Canvas (under the voice Discussions). You are highly encouraged to use this tool to ask questions to me or to your other peers and share your understanding!

Homework: You will be required to hand in about 12 assignments along the semester (approximately one every week).

The assignment should be produced using LaTeX. A template for the submission will be provided for each assignment on Canvas and on Gradescope. You can use the following online tools for typing on it:

- https://www.overleaf.com/edu/emory
- https://latexbase.com/

You will then need to upload your homework on Gradescope.

No late assignments will be accepted.

A few things to remember:

- <u>Assignments are very important!</u> Taking them seriously and doing them well is by far the best way to learn.

  Make sure that you start working on them well before the deadline. One day will usually not be enough. You should at least read the questions on the day the assignment is posted. Before you attempt a question write out relevant definitions and results.

  Do not hand in your rough work. Always use full sentences and provide justification for your answers, or you will receive no partial credit.

- <u>Discussions and work group are highly encouraged!</u>

  However,

  (a) **Acknowledge** all people you worked with or got help from (e.g., I worked with John Smith on Problems 2, 3, 5, and got additional help from Jane Doe on Problems 1, 3 and 4.).
  You must also properly acknowledge all other sources you got help from (e.g., textbooks and online sources, like Mathoverflow). If you find a solution somewhere online make sure that the acknowledgement is very precise; in particular this means that you must provide the exact web address for each problem (or part of a problem).

  (b) **Write** out the solutions in your own words and on your own (if you work in groups you are not allowed to produce template from which you all copy). Do not just copy from a book or an online source.

  (c) Do not submit anything that you do not **understand**! I reserve the right to quiz you on any part of the assignment you submit.

Exams: There will be one Midterm and the Final Exam.

The midterm exam will be written in class, during scheduled times. It will tentatively take place during Week 8. The exact date and contents of the midterm will be communicated at least 10 days in advance.
Date for the final exam will be scheduled by the university registrar. The final exam will cover material from the entire course.

Notes, books, or electronic devices may not be used when taking the exams.

Missed tests: Make-up midterm exams will not be given. If you miss the midterm exam due to unavoidable, compelling, and well-documented circumstances, the exam will be excused and your other exams will be weighted more heavily. Make-up final exams will be given only in extreme situations, with justification verified by the Office of Undergraduate Education (OUE).

Extra help: Do not hesitate to come to my office during office hours or by appointment to discuss a homework problem or any aspect of the course.

You may also want to check the Academic Success Program for resources to succeed in the course!

Academic Integrity: This course will adhere to the Emory University Academic Honor Code http://catalog.college.emory.edu/policies/honor-code.html

Students are expected to do their own work during tests and exams. The following activities, although not exhaustive, are examples of activities that are prohibited:

- Copying from another student;

- Allowing another student to copy from you;

- Using unauthorized aids, including: sheets, cell phones and calculators, during test or exam;

- Getting aid from or giving aid to another student during tests and exams;

- Having another student write for you or writing for another student.

Offenders will be subject to discipline. In particular, cheating may be reported to both the student's college and the Honor Council.

An incident of academic dishonesty can have extremely negative consequences: it could delay or bar a student from graduating or even affect job opportunities.

This course is a precious opportunity for you to learn something new and valuable. It's an investment on your future. Failing to acquire it will sadly be your loss.

Accommodation: Emory University is committed to providing reasonable accommodations for all persons with disabilities. Students with disabilities who need accommodations shall contact the Department of Accessibility Services to learn more about the registration process and steps for requesting accommodations. Students who have accommodations in place are encouraged to coordinate with the instructor during the first week of the semester to communicate your specific needs for the course.

(Tentative) course calendar:

Week	Topic	Important dates
1 (Jan 15th)	Welcome! 1.1 Statements, Quantifiers, Negations, pt 1.	Jan 15th – *Martin Luther King Day*
2 (Jan 22nd)	1.1 Statements, Quantifiers, Negations, pt 2. 1.2 Conjunctions, Disjunctions, and Truth Tables.	
3 (Jan 29th)	1.3 Implications and their Negations. 1.4 Contrapositive, Converse, and Contradictions.	Jan 30th – course add/drop/swap deadline
4 (Feb 5th)	More on Contradictions (notes) 2.1 Sets, Subsets, Equality of Sets, Complements.	
5 (Feb 12th)	2.2 Unions and Intersections, DeMorgan's Laws. 2.2 Cartesian Products. More on sets (notes)	
6 (Feb 19th)	2.3 Power Sets, Indexed Sets, Partitions. Partitions of integers (notes) 2.3 Pigeonhole Principle.	
7 (Feb 26th)	5.1 Well-Ordering Principle. 5.2 Induction. More on Induction (notes)	Mar 1st – partial withdrawal deadline
8 (Mar 4th)	4.2 Relations. Equivalence Relations. 4.2 Equivalence Classes. Partial and Linear Ordering.	
9 (Mar 11th)		*** *Spring break* ***
10 (Mar 18th)	3.1 Functions and their Images. Midterm.	
11 (Mar 25th)	3.1 Inverse Images. 3.2 Surjective and Injective Functions.	
12 (Apr 1st)	3.3 Composition of Functions and Invertibility.	
13 (Apr 8th)	5.3 Division Algorithm and Greatest Common Divisors. Diophantine Equations (notes)	
14 (Apr 15th)	5.4 Primes and Unique Factorization. 6.1 Countable Sets.	
15 (Apr 22nd)	6.2 Uncountable Sets. Cantor's Theorem. Continuity and Epsilon/Delta. (notes) Review & Conclusions.	

Disclaimer: this syllabus is subject to change and revision, as needed, to meet the learning goal of the course. Necessary revisions will be announced in class and course materials will be updated.