Parallel Optimization in Machine Learning

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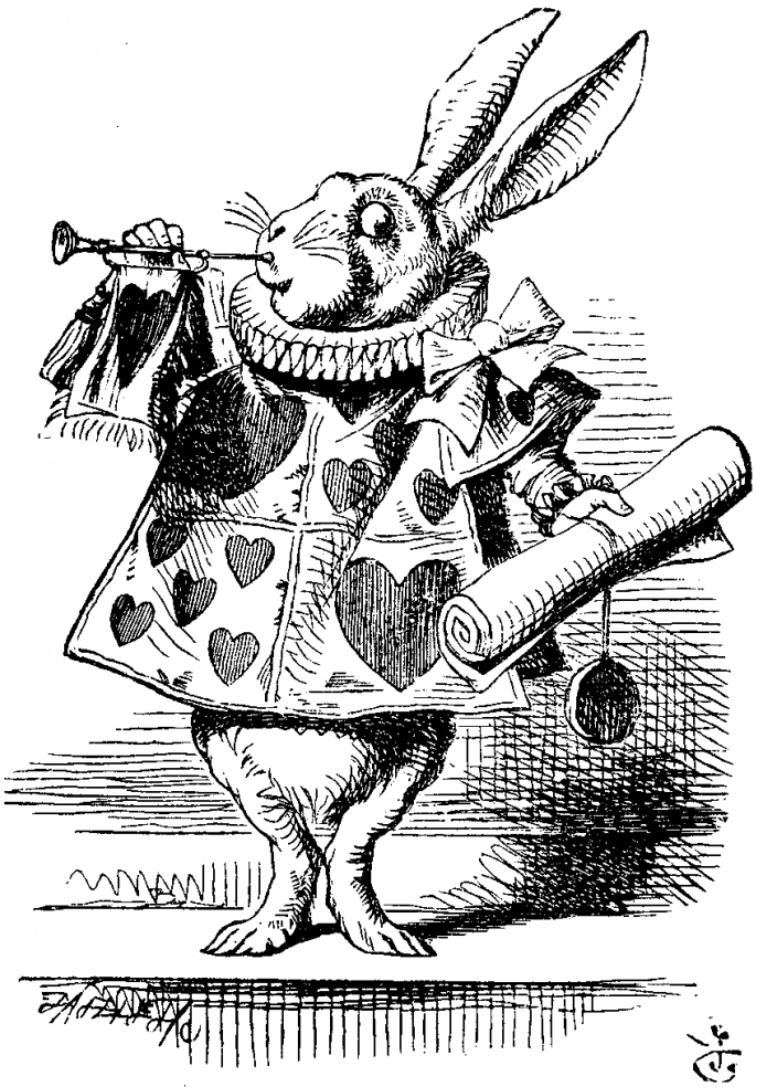
About me



- Engineer (2010-2012), Inria Saclay (scikit-learn kickstart).
- PhD (2012-2015, Inria Saclay)
- Postdoc (2015-2016), Dauphine–ENS–Inria Paris.
- Postdoc (2017-present), UC Berkeley - ETH Zurich (Marie-Curie fellowship, European Commission)

Hacker at heart … trapped in a researcher's body.

Motivation

Computer add in 1993

Computer add in 2006

What has changed?

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2006 = no longer mentions to speed of processors.

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Primary feature: number of cores.

Moore's law

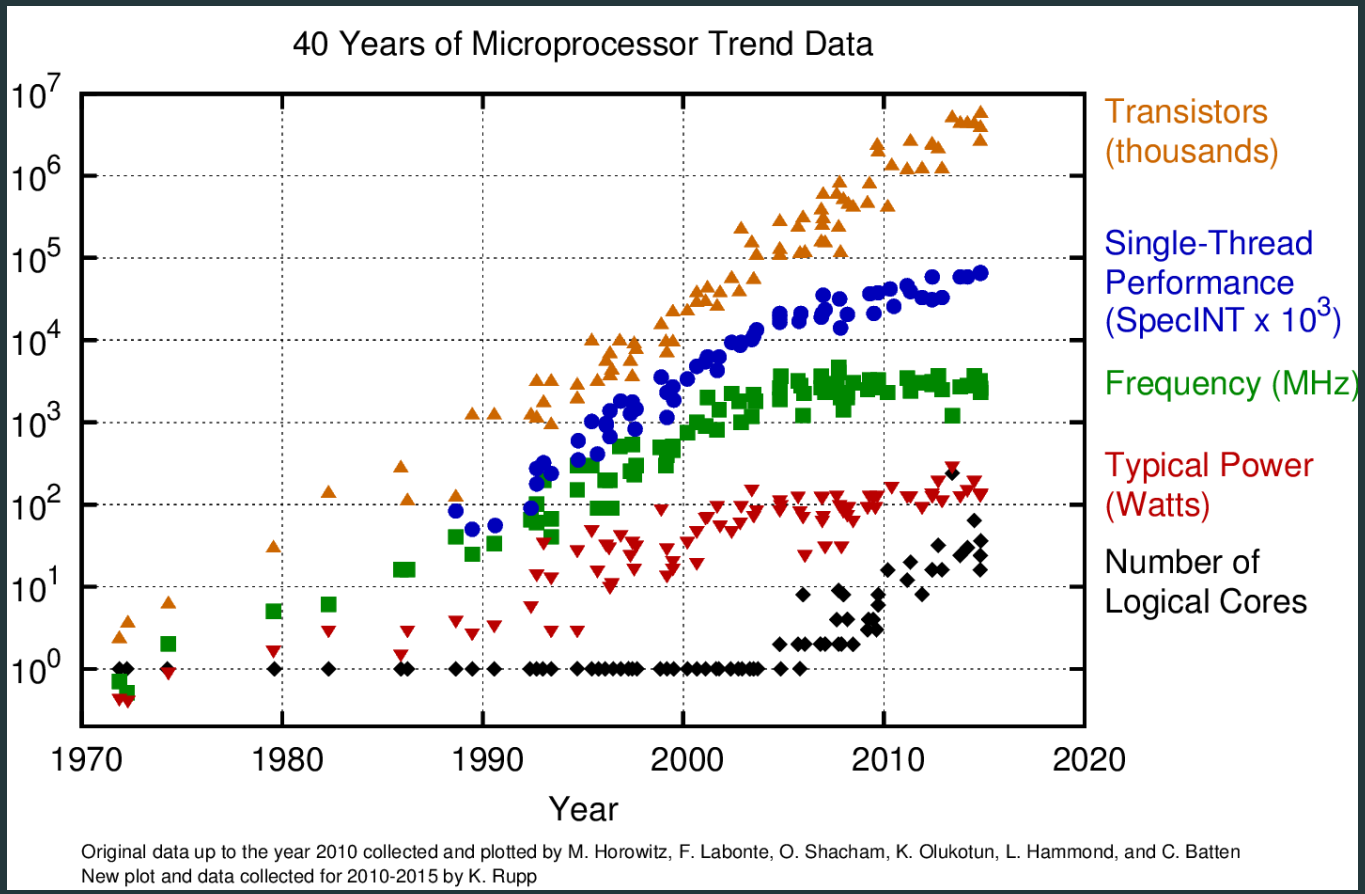
> *The complexity for minimum component costs has increased at a rate of roughly a factor of two per year. Certainly over the short term this rate can be expected to continue*

Gordon Moore (Intel), 1965

> *OK, maybe a factor of two every two years.*

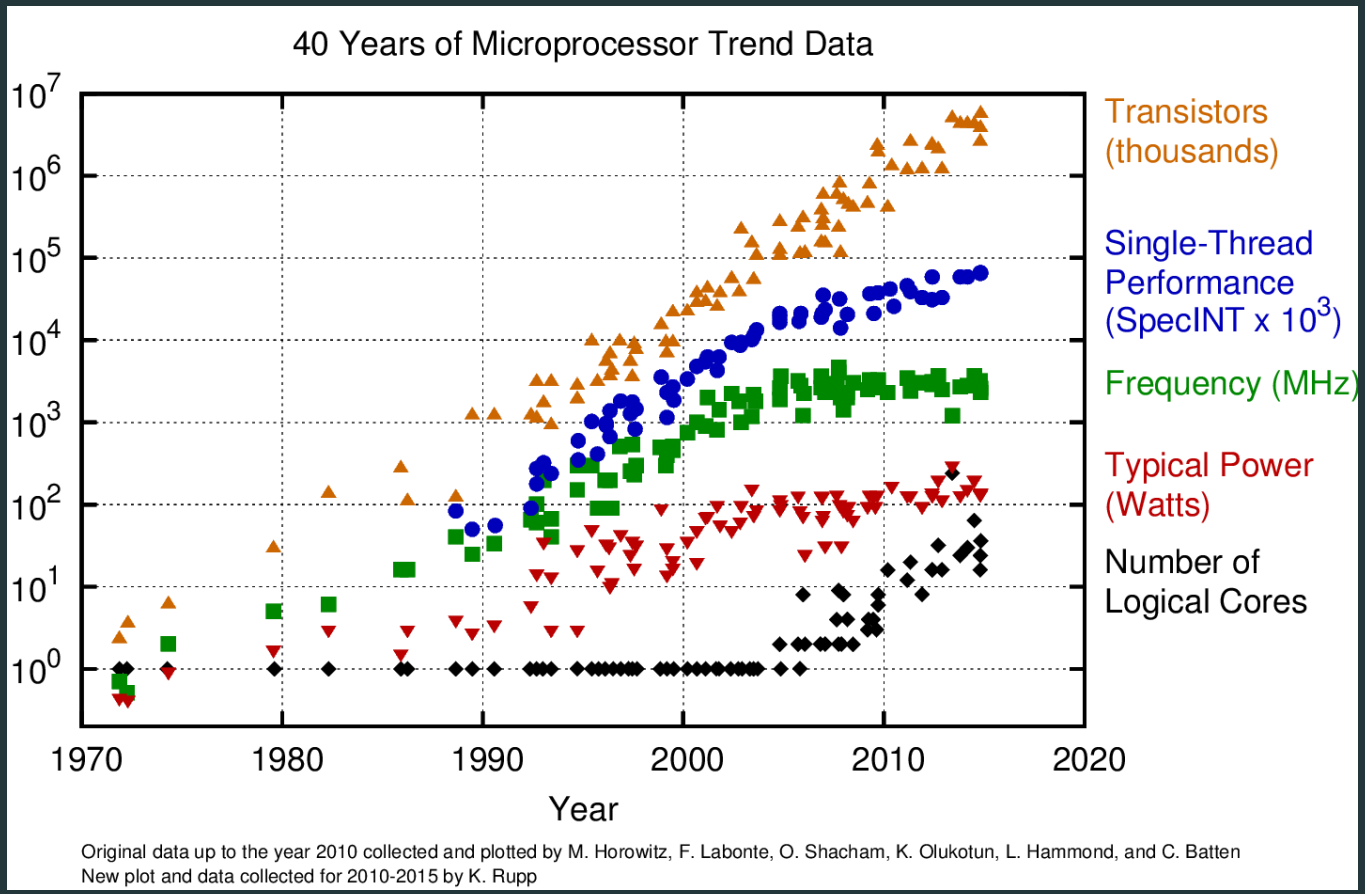
Gordon Moore (Intel), 1975 [paraphrased]

40 years of CPU trends



- Speed of CPUs has stagnated since 2005.

40 years of CPU trends



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- Multi-core architectures are here to stay.

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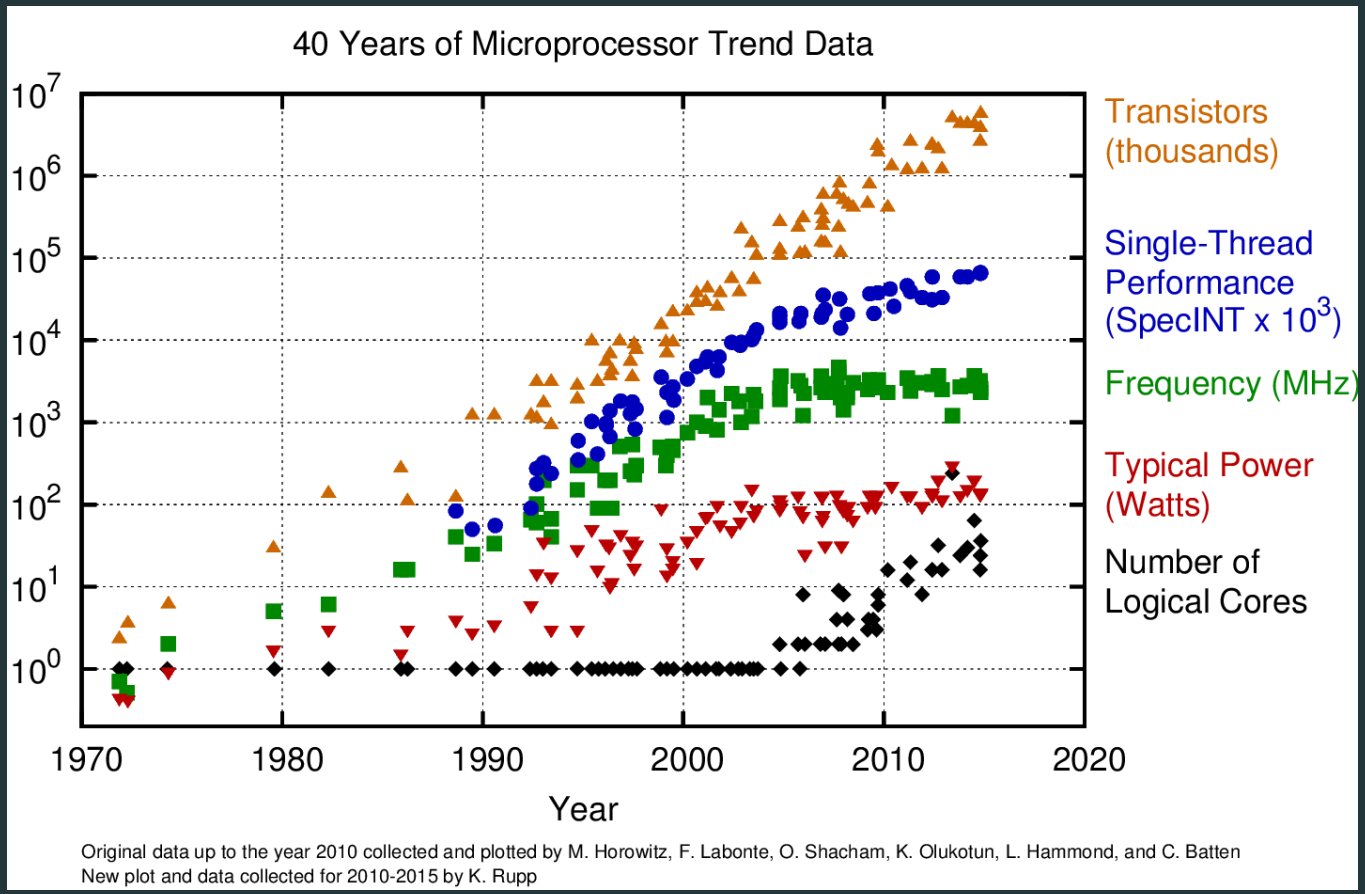

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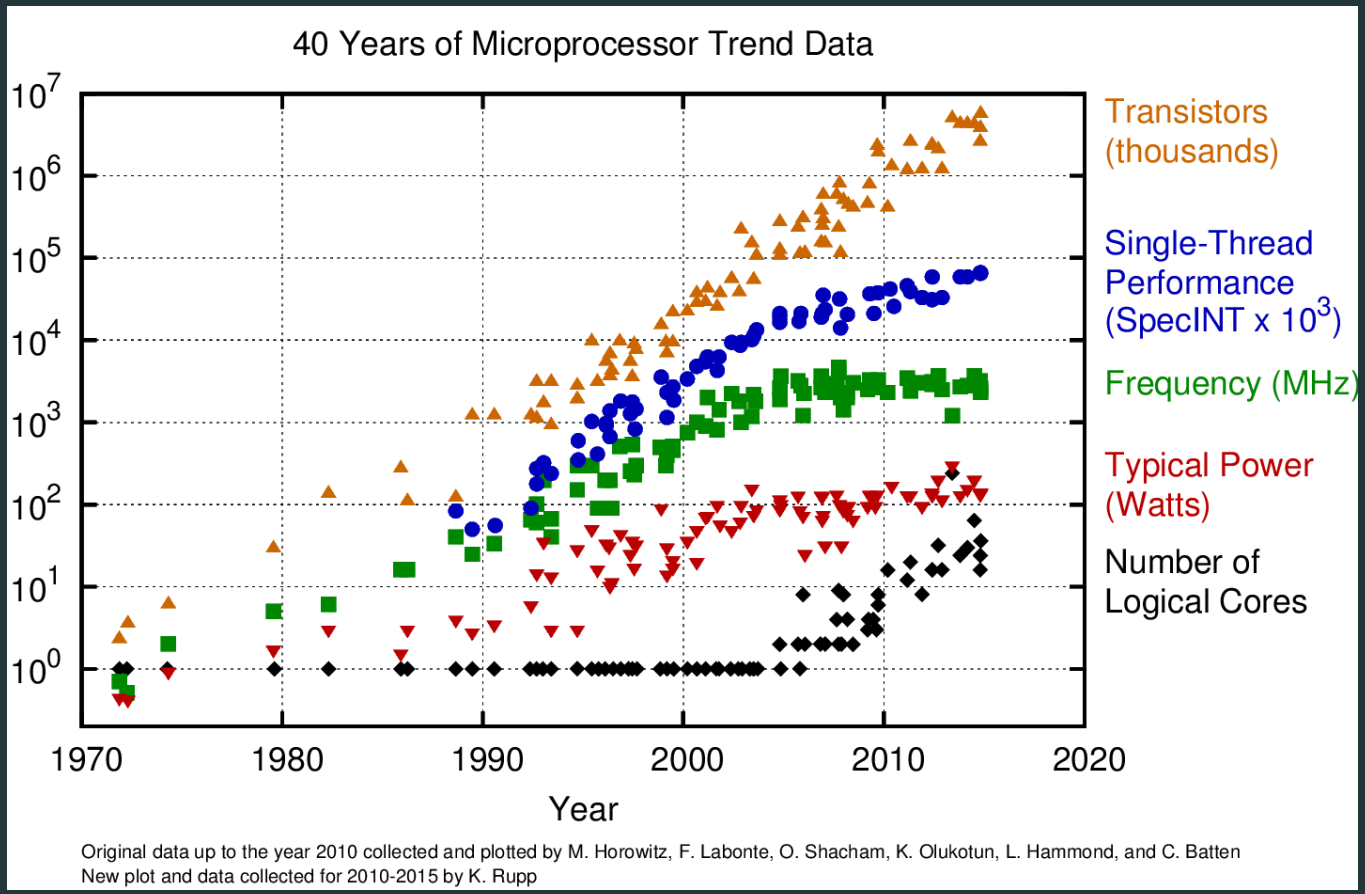

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Parallel algorithms needed to take advantage of modern CPUs.

Parallel optimization

Parallel algorithms can be divided into two large categories: **synchronous** and **asynchronous**.

Image credits: (Peng et al. 2016)

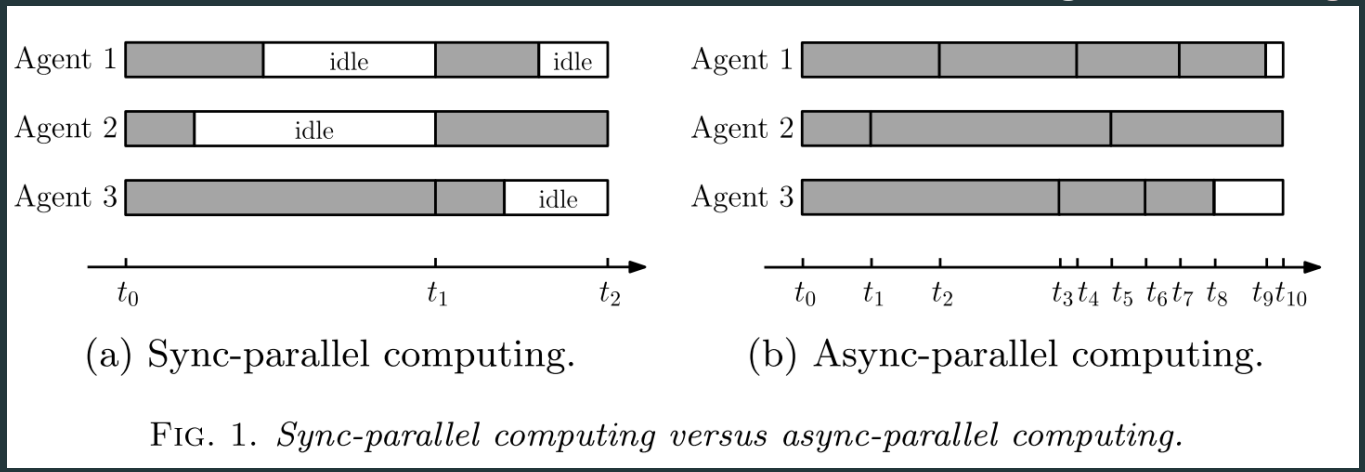


FIG. 1. *Sync-parallel computing versus async-parallel computing.*

Synchronous methods

✔ Easy to implement (i.e., developed software packages).

✔ Well understood.

✖ Limited speedup due to synchronization costs..

Asynchronous methods

✔ Faster, typically larger speedups.

✖ Not well understood, large gap between theory and practice.

✖ No mature software solutions.

Outline

Synchronous methods

- Synchronous (stochastic) gradient descent.

Asynchronous methods

- Asynchronous SGD (Hogwild) (Niu et al. 2011)
- Asynchronous variance-reduced stochastic methods (Leblond, P., and Lacoste-Julien 2017), (Pedregosa, Leblond, and Lacoste-Julien 2017).
- Analysis of asynchronous methods.
- Codes and implementation aspects.

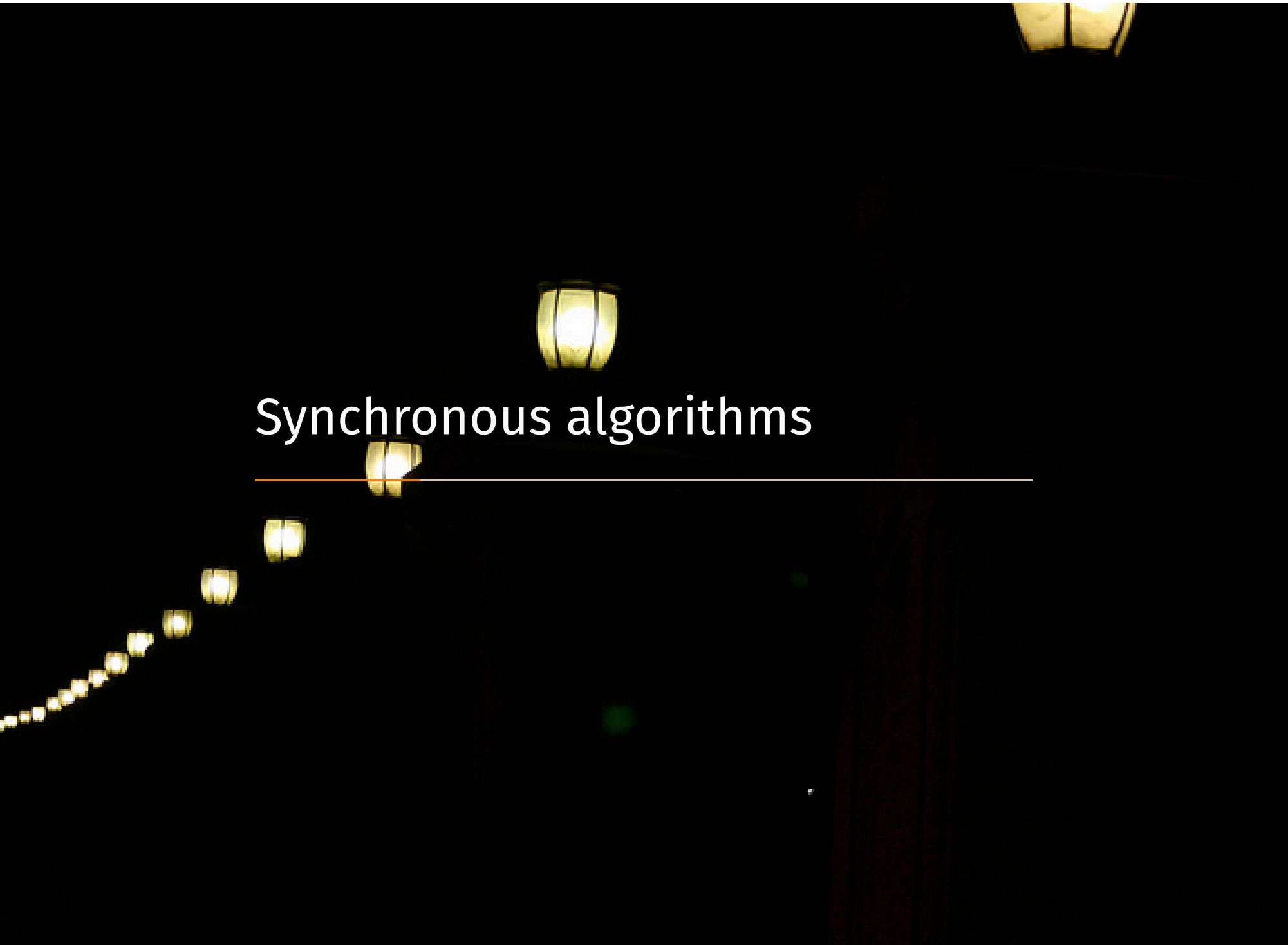
Leaving out many parallel synchronous methods: ADMM (Glowinski and Marroco 1975), CoCoA (Jaggi et al. 2014), DANE (Shamir, Srebro, and Zhang 2014), to name a few.

Outline

Most of the following is joint work with Rémi Leblond and Simon Lacoste-Julien

Rémi Leblond

Simon Lacoste–Julien



Synchronous algorithms

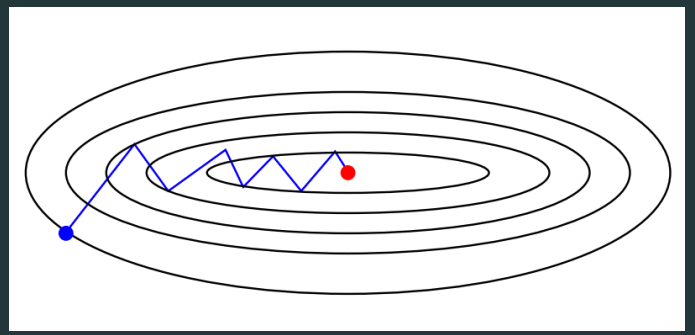
Optimization for machine learning

Large part of problems in machine learning can be framed as optimization problems of the form

$$\underset{x}{\text{minimize}}\, f(x) \stackrel{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n} f_i(x)$$

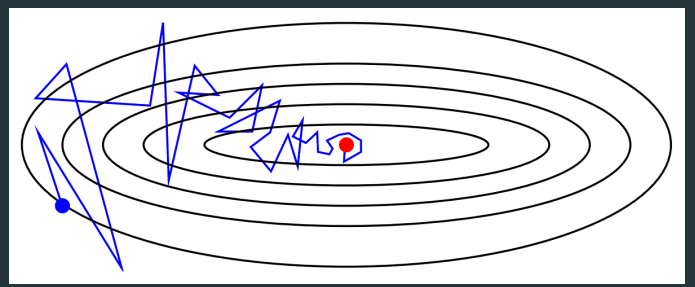
Gradient descent (Cauchy 1847). Descend along steepest direction $(-\nabla f(x))$

$$x^+ = x - \gamma \nabla f(x)$$



Stochastic gradient descent (SGD) (Robbins and Monro 1951). Select a random index i and descent along $\nabla f_i(x)$:

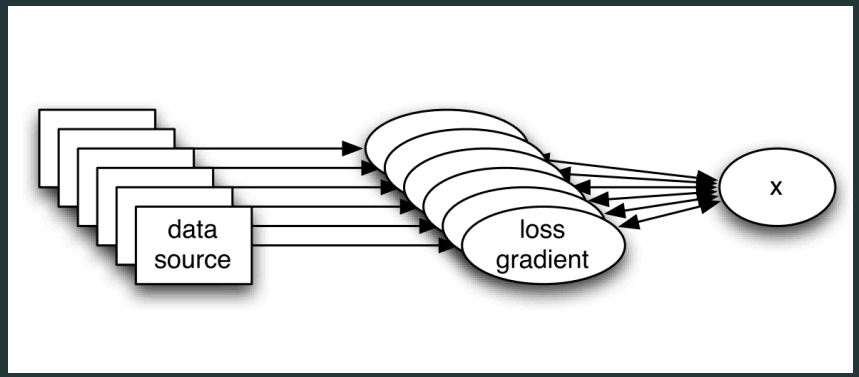
$$x^+ = x - \gamma \nabla f_i(x)$$



images source: Francis Bach

Parallel synchronous gradient descent

Computation of gradient is distributed among k workers



- Workers can be: different computers, CPUs or GPUs
- Popular frameworks: Spark, Tensorflow, PyTorch, neHadoop.

Parallel synchronous gradient descent

1. Choose $n_1, \dots n_k$ that sum to n.
2. Distribute computation of $\nabla f(x)$ among k nodes

$$\nabla f(x) = \frac{1}{n} \sum_{i=1} \nabla f_i(x)$$

$$= \frac{1}{k} (\underbrace{\frac{1}{n_1} \sum_{i=1}^{n_1} \nabla f_i(x)}_{\text{done by worker 1}} + \dots + \underbrace{\frac{1}{n_1} \sum_{i=n_{k-1}}^{n_k} \nabla f_i(x)}_{\text{done by worker } k})$$

3. Perform the gradient descent update by a master node

$$x^+ = x - \gamma \nabla f(x)$$

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$$x^+ = x - \gamma \nabla f(x)$$

✔ Trivial parallelization, same analysis as gradient descent.

✖ Synchronization step every iteration (3.).

Parallel synchronous SGD

Can also be extended to stochastic gradient descent.

1. Select k samples $i_0, \ldots, i_k$ uniformly at random.
2. Compute in parallel ∇f_{i_t} on worker t
3. Perform the (mini-batch) stochastic gradient descent update

$$x^+ = x - \gamma \frac{1}{k} \sum_{t=1}^{k} \nabla f_{i_t}(x)$$

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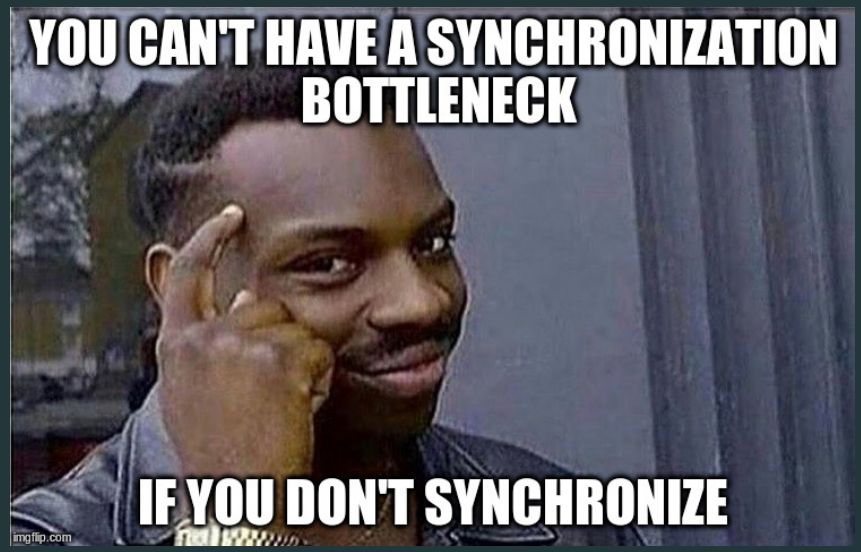
Asynchronous algorithms

Asynchronous SGD

Synchronization is the bottleneck.

What if we just ignore it?



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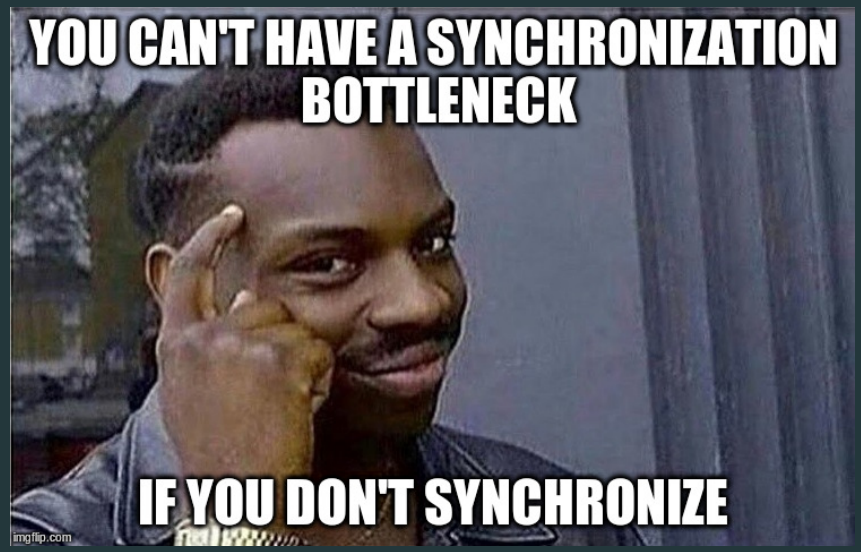

Hogwild (Niu et al. 2011): each core runs SGD in parallel, without synchronization, and updates the same vector of coefficients.

In theory: convergence under very strong assumptions.

In practice: just works.

Hogwild in more detail

Each core follows the same procedure

1. Read the information from shared memory $\hat{x}$.
2. Sample $i \in \{1, \ldots, n\}$ uniformly at random.
3. Compute partial gradient $\nabla f_i(\hat{x})$.
4. Write the SGD update to shared memory $x = x - \gamma \nabla f_i(\hat{x})$.

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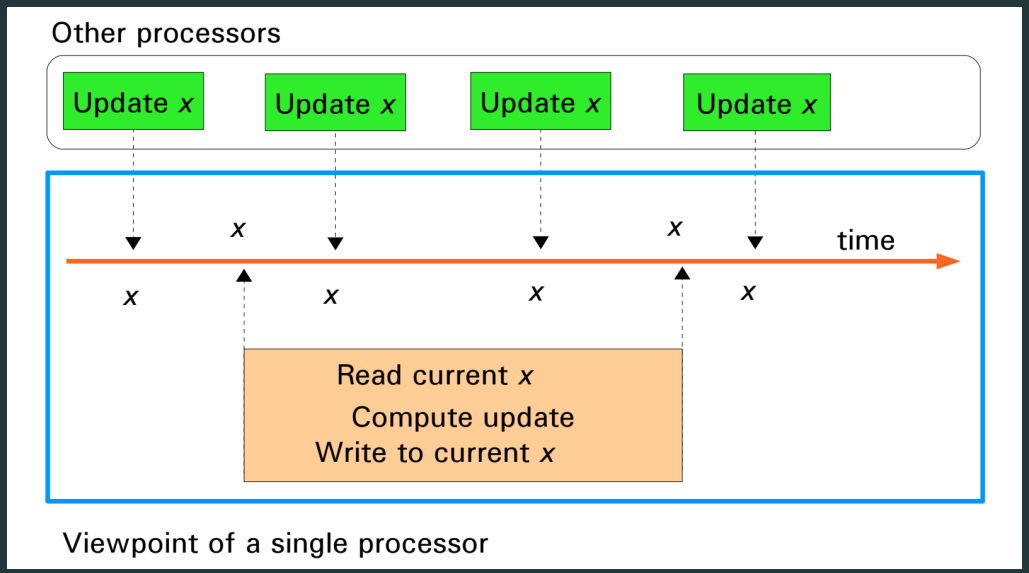
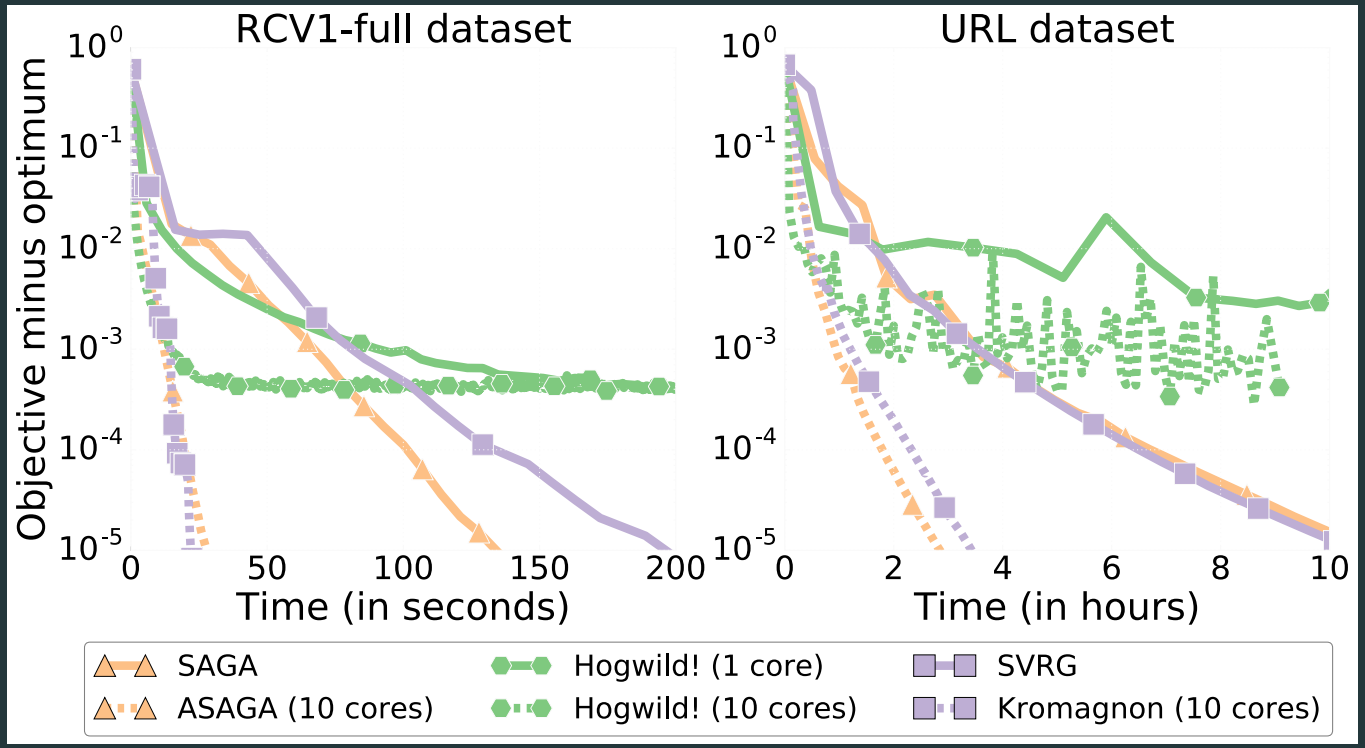


Hogwild is fast

Hogwild can be very fast. But its still SGD...



- With constant step size, bounces around the optimum.
- With decreasing step size, slow convergence.
- There are better alternatives (Emilie already mentioned some)


Looking for excitement? ...
analyze asynchronous methods!

Analysis of asynchronous methods

Simple things become counter-intuitive, e.g, how to **name** the iterates?

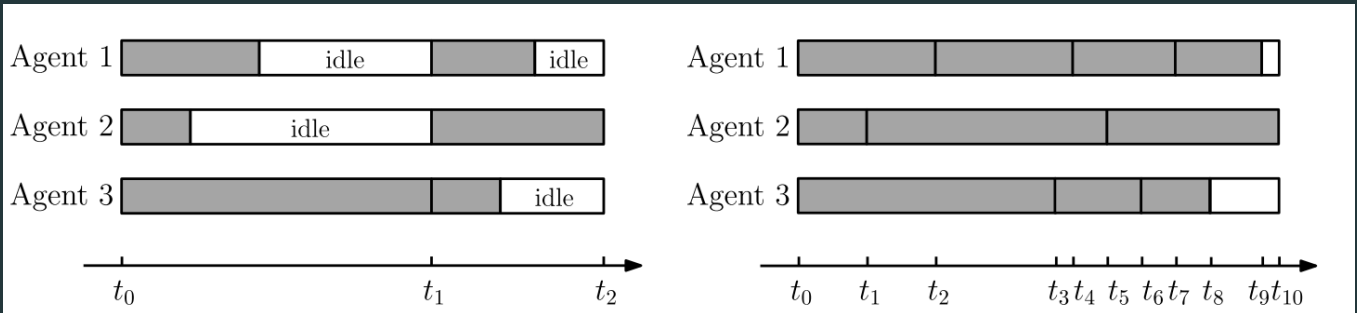


FIG. 1. *Sync-parallel computing versus async-parallel computing.*

⚠ Iterates will change depending on the speed of processors

Naming scheme in Hogwild

Simple, intuitive and wrong

Each time a core has finished writing to shared memory, increment iteration counter.

$\iff \hat{x}_t = (t+1)$-th succesfull update.

Value of $\hat{x}_t$ and i_t are not determined until the iteration has finished.

$\implies \hat{x}_t$ and i_t are not necessarily independent.

Unbiased gradient estimate

SGD-like algorithms crucially rely on the unbiased property $\mathbb{E}_i[\nabla f_i(x)] = \nabla f(x)$.

For synchronous algorithms, follows from the uniform sampling of i

$$\mathbb{E}_i[\nabla f_i(x)] = \sum_{i=1}^{n} \text{Proba}(\text{selecting } i) \nabla f_i(x)$$

$$\overset{\text{uniform sampling}}{=} \sum_{i=1}^{n} \frac{1}{n} \nabla f_i(x) = \nabla f(x)$$

A problematic example

Selecting i uniformly at random is not enough.

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Illustration: problem with two samples and two cores $f = \frac{1}{2}(f_1 + f_2)$.
Computing ∇f_1 is much expensive than ∇f_2.

A problematic example

Selecting i uniformly at random is not enough.

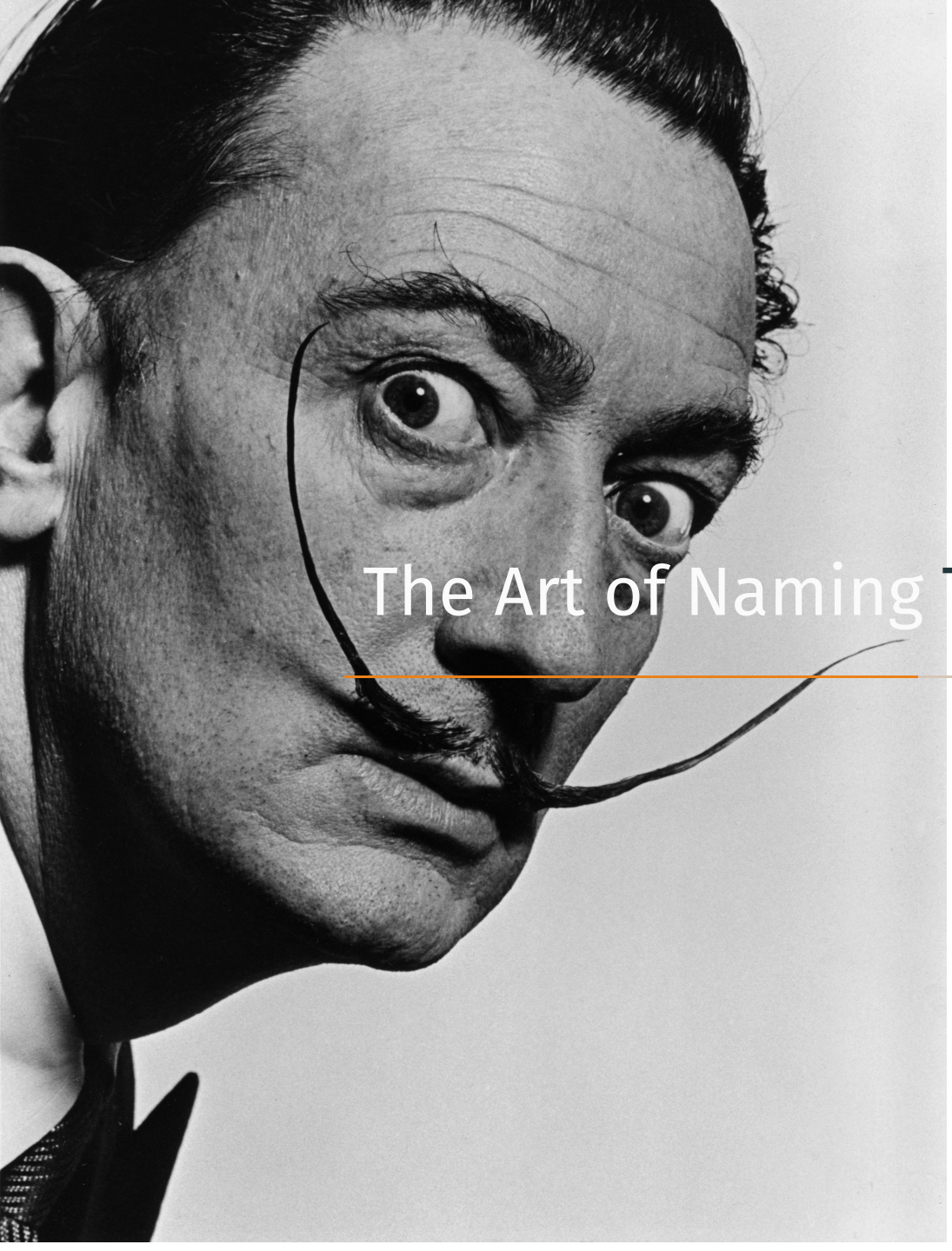
Illustration: problem with two samples and two cores $f = \frac{1}{2}(f_1 + f_2)$. Computing ∇f_1 is much expensive than ∇f_2.

Start at x_0. Because of the random sampling there are 4 possible scenarios:

1. Core 1 selects f_1, Core 2 selects f_1 $\implies$ $x_1 = x_0 - \nabla f_1(x)$
2. Core 1 selects f_1, Core 2 selects f_2 $\implies$ $x_1 = x_0 - \nabla f_2(x)$
3. Core 1 selects f_2, Core 2 selects f_1 $\implies$ $x_1 = x_0 - \nabla f_2(x)$
4. Core 1 selects f_2, Core 2 selects f_2 $\implies$ $x_1 = x_0 - \nabla f_2(x)$

So we have

$$\begin{aligned} \mathbb{E}_i \left[\nabla f_i\right] &= \frac{1}{4} f_1 + \frac{3}{4} f_2 \\ &\neq \frac{1}{2} f_1 + \frac{1}{2} f_2 \text{ !!} \end{aligned}$$



The Art of Naming Things

A new labeling scheme

New way to name iterates.

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"After read" labeling (Leblond, P., and Lacoste-Julien 2017). Increment counter each time we *read* the vector of coefficients from shared memory.

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✔ No dependency between i_t and the cost of computing ∇f_{i_t}.

✔ Full analysis of Hogwild and other asynchronous methods in *"Improved parallel stochastic optimization analysis for incremental methods", Leblond, P., and Lacoste-Julien* (submitted).

Analysis

Analysis inspired by the perturbed iterate framework of (Mania et al. 2017). Distinguishes two quantities:

- $\hat{x}_t = (t+1)$-th fully completed read from shared memory.
- $x_{t+1} \stackrel{\text{def}}{=} x_t - \gamma g(\hat{x}_t)$ "virtual" iterate, its a definition and does not need to exist in memory (for SGD $g = \nabla f_i$).

Analysis is carried out in x_{t+1}. Finally, relate x_t to $\hat{x}_t$.

Asynchronous SAGA

The SAGA algorithm

Setting:

$$\underset{x}{\text{minimize}} \frac{1}{n} \sum_{i=1}^{n} f_i(x)$$

The **SAGA** algorithm (Defazio, Bach, and Lacoste-Julien 2014).

Select $i \in \{1, \ldots, n\}$ and compute (x^+, α^+) as

$$x^+ = x - \gamma(\nabla f_i(x) - \alpha_i + \overline{\alpha}) \,;\; \alpha_i^+ = \nabla f_i(x)$$

- Like SGD, update is unbiased, i.e., $\mathbb{E}_i[\nabla f_i(x) - \alpha_i + \overline{\alpha})] = \nabla f(x)$.
- Unlike SGD, because of memory terms α, variance $\to 0$.
- Unlike SGD, converges with fixed step size $(1/3L)$

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Super easy to use in scikit-learn

```
from sklearn.linear_model import LogisticRegression
clf = LogisticRegression(solver='saga')
clf.fit(X, y)
```

Sparse SAGA

Need for a sparse variant of SAGA

- A large part of large scale datasets are sparse.
- For sparse datasets and generalized linear models (e.g., least squares, logistic regression, etc.), partial gradients ∇f_i are sparse too.
- Asynchronous algorithms work best when updates are sparse.

SAGA update is inefficient

$$x^+ = x - \gamma(\underbrace{\nabla f_i(x)}_{\text{sparse}} - \underbrace{\alpha_i}_{\text{sparse}} + \underbrace{\overline{\alpha}}_{\text{dense!}}) \,;\; \alpha_i^+ = \nabla f_i(x)$$

[many tricks are used in in scikit-learn that we cannot use in asynchronous version]

Sparse SAGA

Sparse variant of SAGA. Relies on

- Diagonal matrix $P_i =$ projection onto the support of ∇f_i
- Diagonal matrix D defined as $D_{j,j} = 1/\text{number of times } \nabla_j f_i \text{ is nonzero.}$

Sparse SAGA algorithm (Leblond, P., and Lacoste-Julien 2017)

$$x^+ = x - \gamma(\nabla f_i(x) - \alpha_i + P_i D\overline{\alpha})\,;\ \alpha_i^+ = \nabla f_i(x)$$

- All operations are sparse, cost per iteration is $\mathcal{O}(\text{nonzeros in } \nabla f_i)$.
- Same convergence properties than SAGA, but with cheaper iterations in presence of sparsity.
- Crucial insight: $\mathbb{E}_i[P_i D] = I$.

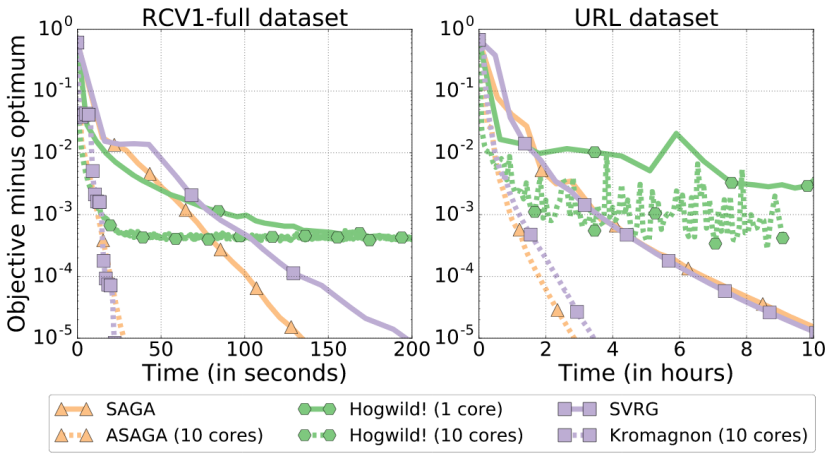
Asynchronous SAGA (ASAGA)

- Each core runs an instance of Sparse SAGA.
- Updates the same vector of coefficients $\boldsymbol{\alpha}, \overline{\boldsymbol{\alpha}}$.

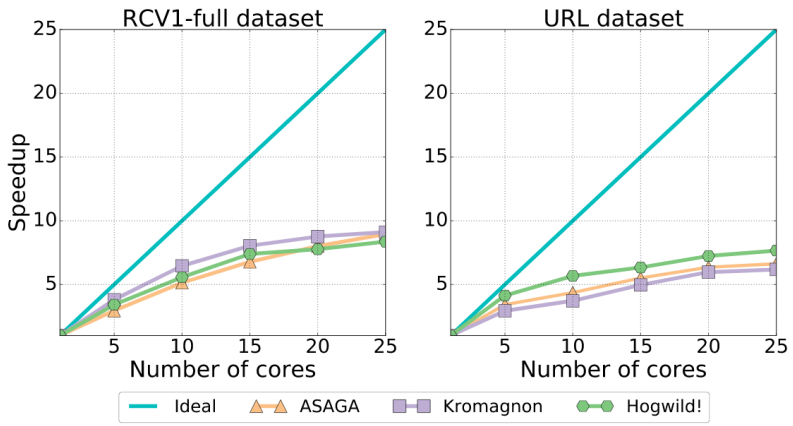
Theory: Under standard assumptions (bounded dalays), same convergence rate than sequential version.

$\Longrightarrow$ theoretical linear speedup with respect to number of cores.

Experiments



(a) Suboptimality as a function of time.



(b) Speedup as a function of the number of cores

- Improved convergence of variance-reduced methods wrt SGD.
- Significant improvement between 1 and 10 cores.
- Speedup is significant, but far from ideal.

Extension to non-smooth optimization

Composite objective

Previous methods assume objective function is smooth.

Cannot be applied to Lasso, Group Lasso, box constraints, etc.

Objective: minimize composite objective function:

$$\underset{x}{\text{minimize}}\ f(x) + h(x)\ , \quad \text{with } f(x) = \frac{1}{n}\sum_{i=1}^{n} f_i(x)$$

where f_i is smooth and h is a block-separable (i.e., $h(x) = \sum_B h([x]_B)$) convex function for which we have access to its proximal operator. (for simplicity we will assume this is the ℓ_1 norm)

(Prox)SAGA

The ProxSAGA update is inefficient

$$x^+ = \underbrace{\mathrm{prox}_{\gamma h}}_{\text{dense!}}(x - \gamma(\underbrace{\nabla f_i(x)}_{\text{sparse}} - \underbrace{\alpha_i}_{\text{sparse}} + \underbrace{\overline{\alpha}}_{\text{dense!}})); \; \alpha_i^+ = \nabla f_i(x)$$

$\Longrightarrow$ a sparse variant is needed as a prerequisite for a practical parallel method.

Sparse Proximal SAGA

Sparse Proximal SAGA. (Pedregosa, Leblond, and Lacoste-Julien 2017)
Extension of Sparse SAGA to composite optimization problems

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$$v_i = \nabla f_i(x) - \alpha_i + DP_i\overline{\alpha}\,;$$

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Where P_i, D are as in Sparse SAGA and $\varphi_i \overset{\text{def}}{=} \sum_j^d (P_i D)_{i,i} |x_j|$.

φ_i has two key properties: *i*) support of φ_i = support of ∇f_i (sparse updates) and *ii*) $\mathbb{E}_i[\varphi_i] = \|x\|_1$ (unbiasedness)

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φ_i has two key properties: *i*) support of φ_i = support of ∇f_i (sparse updates) and *ii*) $\mathbb{E}_i[\varphi_i] = \|x\|_1$ (unbiasedness)

Convergence: same linear convergence rate as SAGA, with cheaper updates in presence of sparsity.

Proximal Asynchronous SAGA (ProxASAGA)

Each core runs Sparse Proximal SAGA asynchronously without locks and updates x, α and $\overline{\alpha}$ in shared memory.

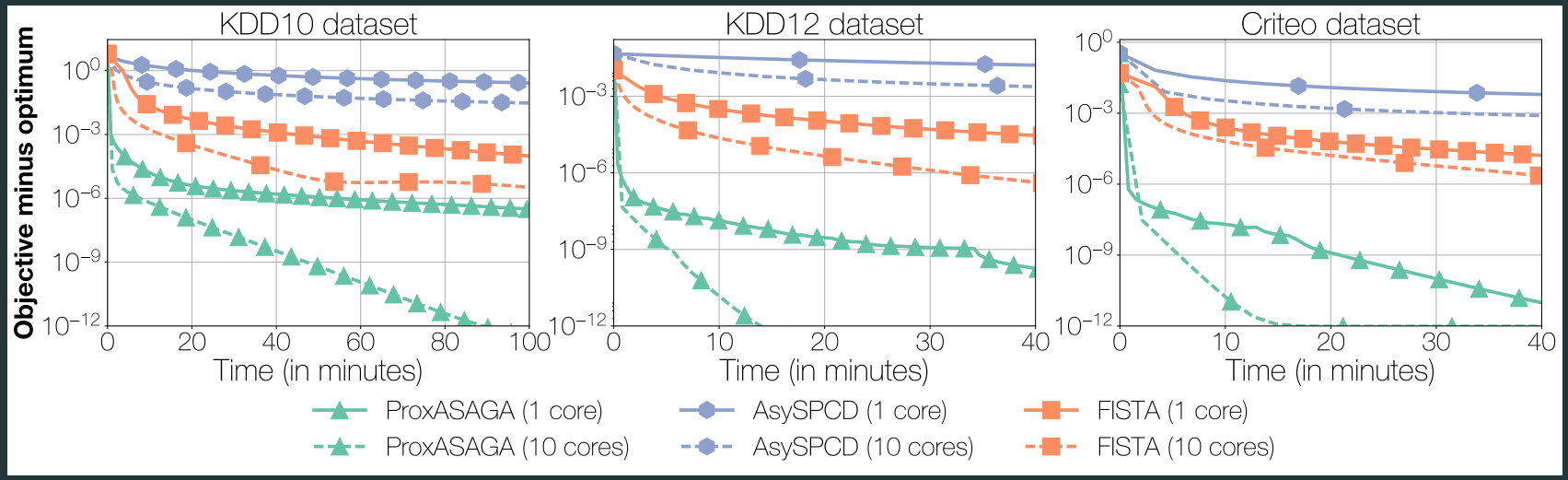
🔀 All read/write operations to shared memory are *inconsistent*, i.e., no performance destroying vector-level locks while reading/writing.

Convergence: under sparsity assumptions, ProxASAGA converges with the same rate as the sequential algorithm $\implies$ theoretical linear speedup with respect to the number of cores.

Empirical results

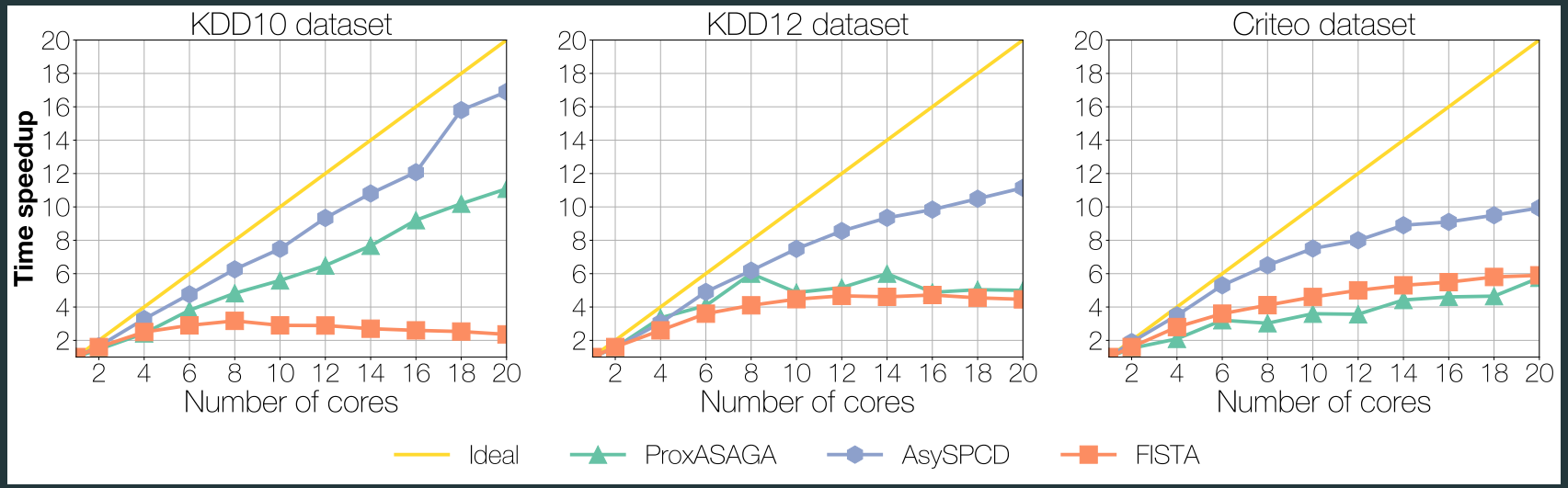
ProxASAGA vs competing methods on 3 large-scale datasets, ℓ_1-regularized logistic regression

Dataset	n	p	density	L	Δ
KDD 2010	19,264,097	1,163,024	10^{-6}	28.12	0.15
KDD 2012	149,639,105	54,686,452	2×10^{-7}	1.25	0.85
Criteo	45,840,617	1,000,000	4×10^{-5}	1.25	0.89



Empirical results - Speedup

$$\text{Speedup} = \frac{\text{Time to } 10^{-10} \text{ suboptimality on one core}}{\text{Time to same suboptimality on } k \text{ cores}}$$



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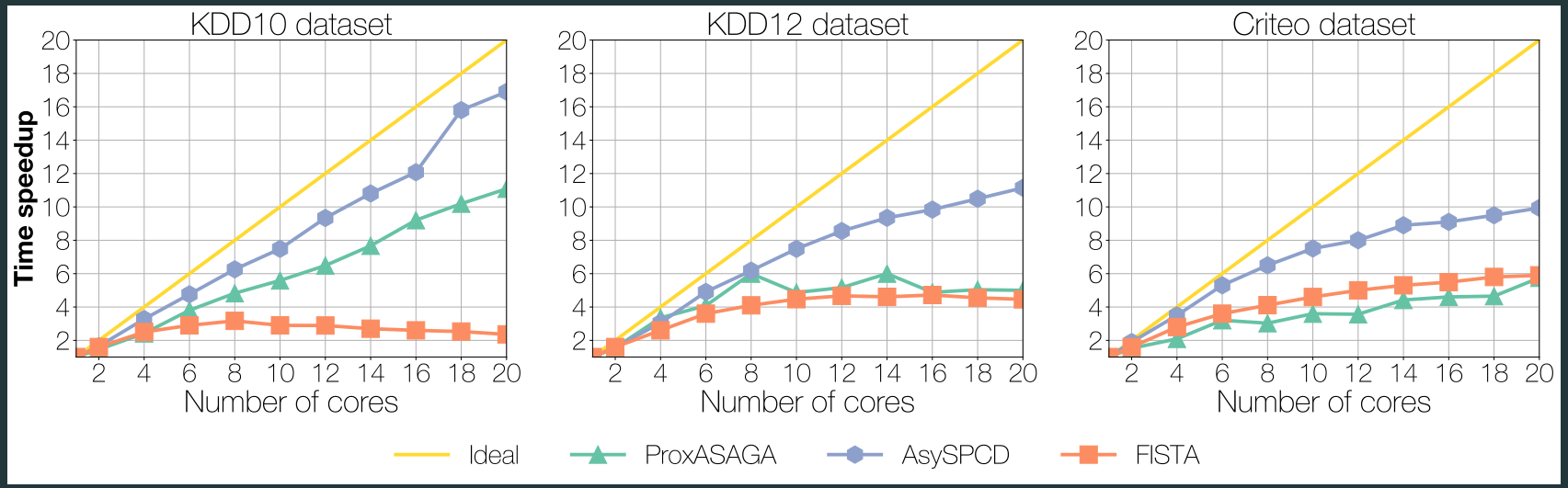

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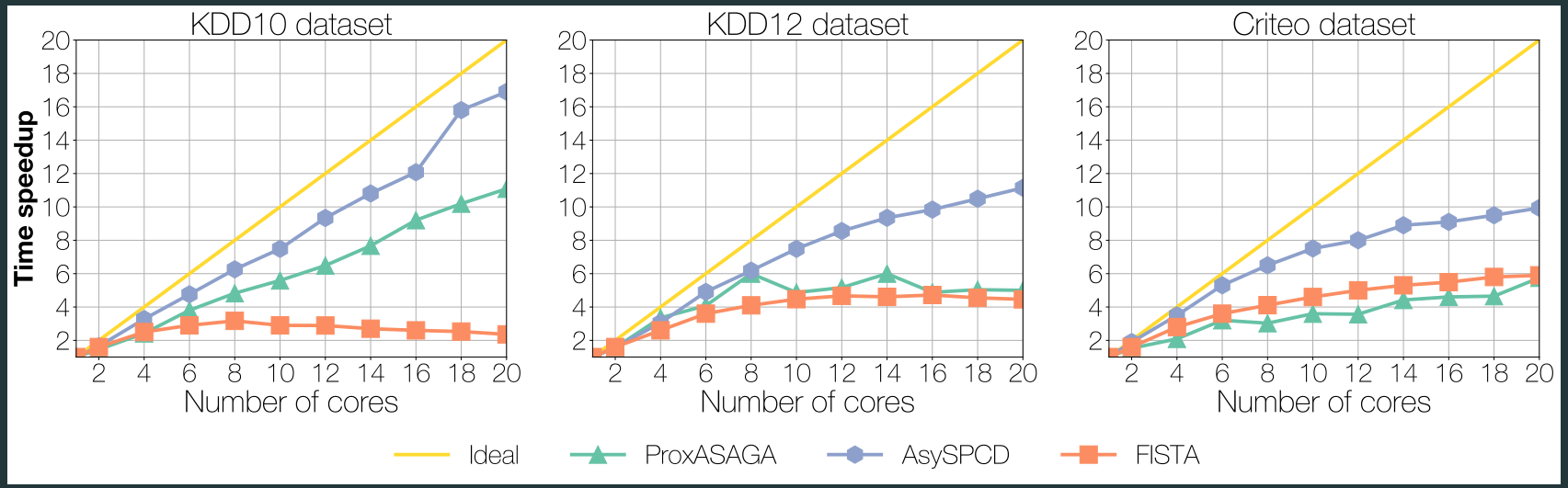


- ProxASAGA achieves speedups between 6x and 12x on a 20 cores architecture.
- As predicted by theory, there is a high correlation between degree of sparsity and speedup.

Codes

Code is in github: `https://github.com/fabianp/ProxASAGA`.

Computational code is C++ (use of atomic type) but wrapped in Python.

A very efficient implementation of SAGA can be found in the scikit-learn and lightning (`https://github.com/scikit-learn-contrib/lightning`) libraries.

References

- Cauchy, Augustin (1847). “Méthode générale pour la résolution des systemes d’équations simultanées”. In: *Comp. Rend. Sci. Paris*.
- Defazio, Aaron, Francis Bach, and Simon Lacoste-Julien (2014). “SAGA: A fast incremental gradient method with support for non-strongly convex composite objectives”. In: *Advances in Neural Information Processing Systems*.
- Glowinski, Roland and A Marroco (1975). “Sur l’approximation, par éléments finis d’ordre un, et la résolution, par pénalisation-dualité d’une classe de problèmes de Dirichlet non linéaires”. In: *Revue française d’automatique, informatique, recherche opérationnelle. Analyse numérique*.
- Jaggi, Martin et al. (2014). “Communication-Efficient Distributed Dual Coordinate Ascent”. In: *Advances in Neural Information Processing Systems 27*.
- Leblond, Rémi, Fabian P., and Simon Lacoste-Julien (2017). “ASAGA: asynchronous parallel SAGA”. In: *Proceedings of the 20th International Conference on Artificial Intelligence and Statistics (AISTATS 2017)*.
- Mania, Horia et al. (2017). “Perturbed iterate analysis for asynchronous stochastic optimization”. In: *SIAM Journal on Optimization*.
- Niu, Feng et al. (2011). “Hogwild: A lock-free approach to parallelizing stochastic gradient descent”. In: *Advances in Neural Information Processing Systems*.


Pedregosa, Fabian, Rémi Leblond, and Simon Lacoste-Julien (2017). “Breaking the Nonsmooth Barrier: A Scalable Parallel Method for Composite Optimization”. In: *Advances in Neural Information Processing Systems 30*.


Peng, Zhimin et al. (2016). “ARock: an algorithmic framework for asynchronous parallel coordinate updates”. In: *SIAM Journal on Scientific Computing*.


Robbins, Herbert and Sutton Monro (1951). “A Stochastic Approximation Method”. In: *Ann. Math. Statist.*


Shamir, Ohad, Nati Srebro, and Tong Zhang (2014). “Communication-efficient distributed optimization using an approximate newton-type method”. In: *International conference on machine learning*.

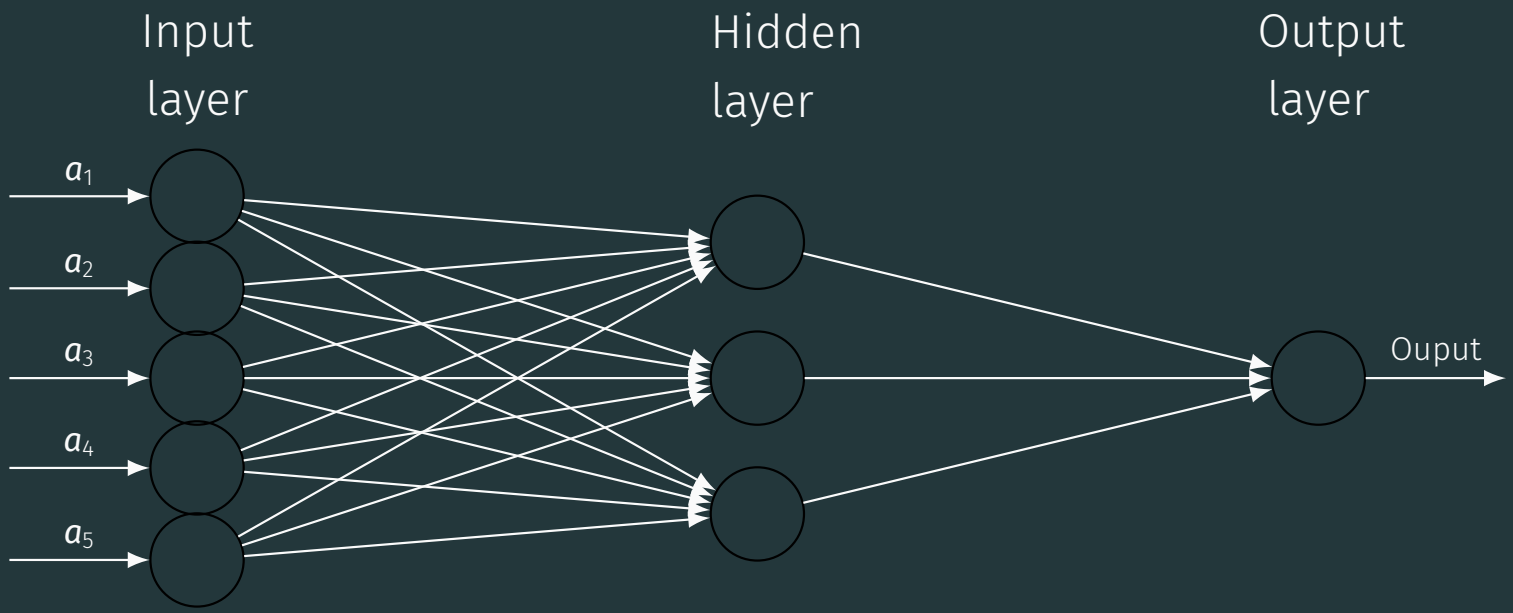
Supervised Machine Learning

Data: n observations $(a_i, b_i) \in \mathbb{R}^p \times \mathbb{R}$

Prediction function: $h(a, x) \in \mathbb{R}$

Motivating examples:

- Linear prediction: $h(a, x) = x^T a$
- Neural networks: $h(a, x) = x_m^T \sigma(x_{m-1} \sigma(\cdots x_2^T \sigma(x_1^T a))$



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Minimize some distance (e.g., quadratic) between the prediction

$$\underset{x}{\text{minimize}} \frac{1}{n} \sum_{i=1}^{n} \ell(b_i, h(a_i, x)) \overset{\text{notation}}{=} \frac{1}{n} \sum_{i=1}^{n} f_i(x)$$

where popular examples of ℓ are

- Squared loss, $\ell(b_i, h(a_i, x)) \overset{\text{def}}{=} (b_i - h(a_i, x))^2$
- Logistic (softmax), $\ell(b_i, h(a_i, x)) \overset{\text{def}}{=} \log(1 + \exp(-b_i h(a_i, x)))$

Sparse Proximal SAGA

For step size $\gamma = \frac{1}{5L}$ and f μ-strongly convex ($\mu > 0$), Sparse Proximal SAGA converges geometrically in expectation. At iteration t we have

$$\mathbb{E}\|x_t - x^*\|^2 \leq (1 - \tfrac{1}{5}\min\{\tfrac{1}{n}, \tfrac{1}{\kappa}\})^t C_0 \ ,$$

with $C_0 = \|x_0 - x^*\|^2 + \frac{1}{5L^2}\sum_{i=1}^n \|\alpha_i^0 - \nabla f_i(x^*)\|^2$ and $\kappa = \frac{L}{\mu}$ (condition number).

Implications

- Same convergence rate than SAGA with cheaper updates.
  - In the "big data regime" ($n \geq \kappa$): rate in $\mathcal{O}(1/n)$.
  - In the "ill-conditioned regime" ($n \leq \kappa$): rate in $\mathcal{O}(1/\kappa)$.
- Adaptivity to strong convexity, i.e., no need to know strong convexity parameter to obtain linear convergence.

Convergence ProxASAGA

Suppose $\tau \leq \frac{1}{10\sqrt{\Delta}}$. Then:

- If $\kappa \geq n$, then with step size $\gamma = \frac{1}{36L}$, ProxASAGA converges geometrically with rate factor $\Omega(\frac{1}{\kappa})$.
- If $\kappa < n$, then using the step size $\gamma = \frac{1}{36n\mu}$, ProxASAGA converges geometrically with rate factor $\Omega(\frac{1}{n})$.

In both cases, the convergence rate is the same as Sparse Proximal SAGA $\implies$ ProxASAGA is **linearly faster** up to constant factor. In both cases the **step size does not depend on** τ.

If $\tau \leq 6\kappa$, a universal step size of $\Theta(\frac{1}{L})$ achieves a similar rate than Sparse Proximal SAGA, making it adaptive to local strong convexity (knowledge of κ not required).