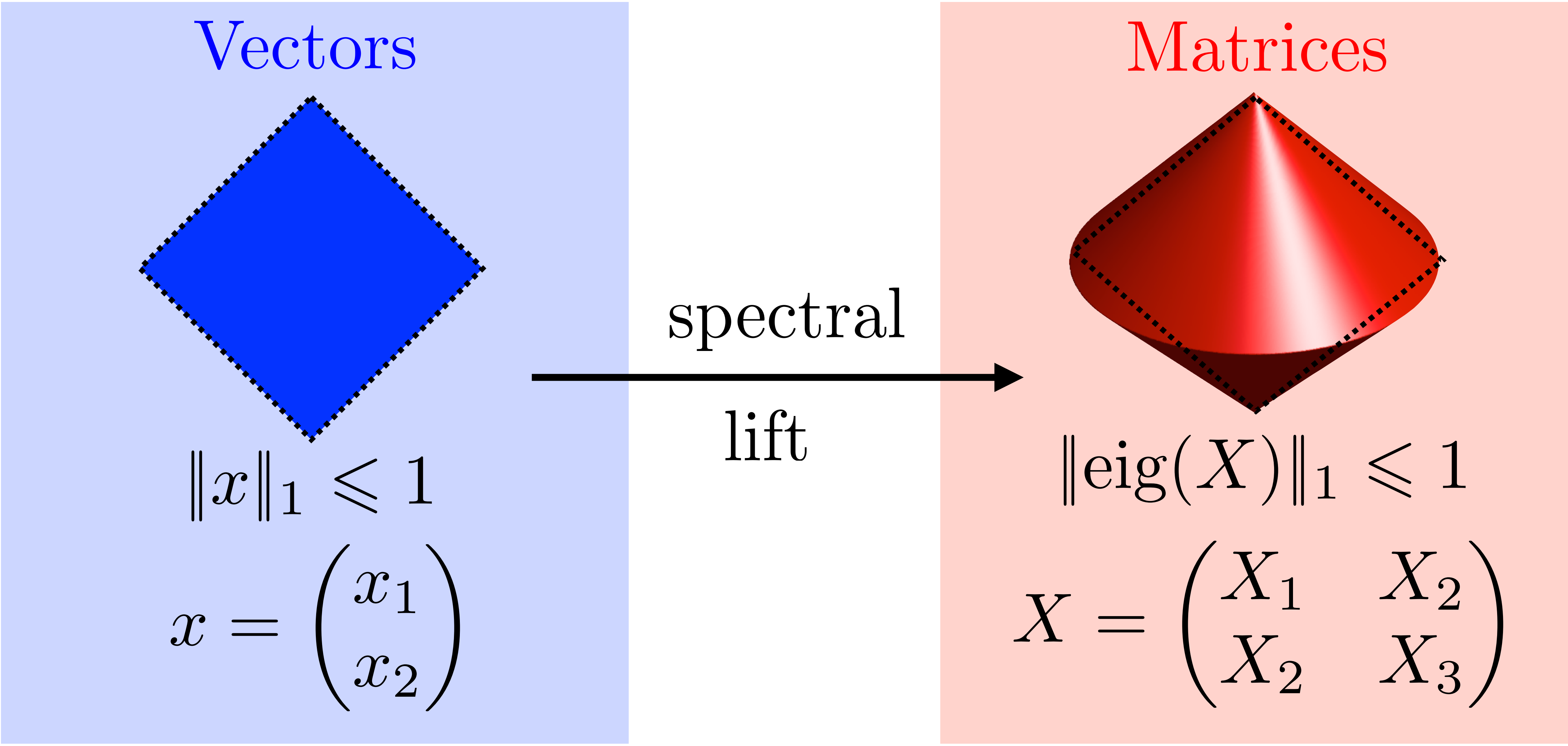


Vectors
$\|x\|_1 \leqslant 1$
$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$
spectral
lift
Matrices
$\|\mathrm{eig}(X)\|_1 \leqslant 1$
$X = \begin{pmatrix} X_1 & X_2 \\ X_2 & X_3 \end{pmatrix}$

$$f^*(\omega) \overset{\text{def.}}{=} \sup_x \langle x,\, \omega\rangle - f(x)$$

$$(f \diamond g)(x) \overset{\text{def.}}{=} \inf_y f(y) + g(x-y)$$

Theorem: $(f \diamond g)^* = f^* + g*$

$$\hat{f}(\omega) \overset{\text{def.}}{=} \int f(x) e^{-\mathrm{i}\langle \omega,\, x\rangle} \mathrm{d}x$$

$$(f \star g)(x) \overset{\text{def.}}{=} \int f(y) g(x-y) \mathrm{d}y$$

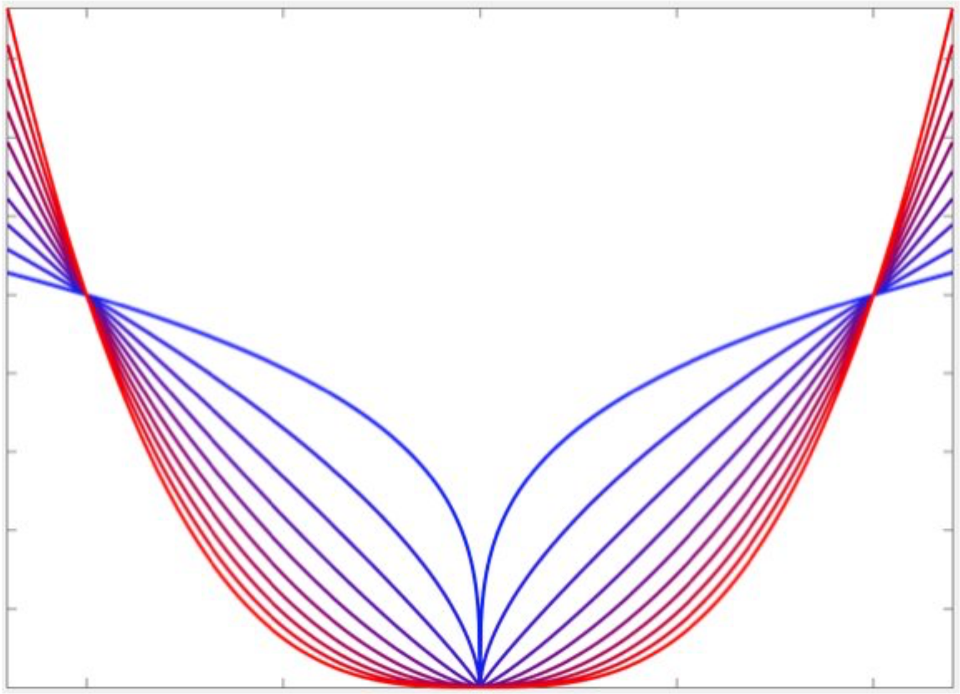
Theorem: $\widehat{f \star g} = \hat{f} \cdot \hat{g}$

Kurdyka-Łojasiewicz:

(at minimum $f(0) = 0$)

$$\exists \varphi, \|\nabla(\varphi \circ f)\| \geqslant 1$$

$$\iff \quad \exists (K, \tau), \|\nabla f(x)\| \geqslant K |f(x)|^\tau$$



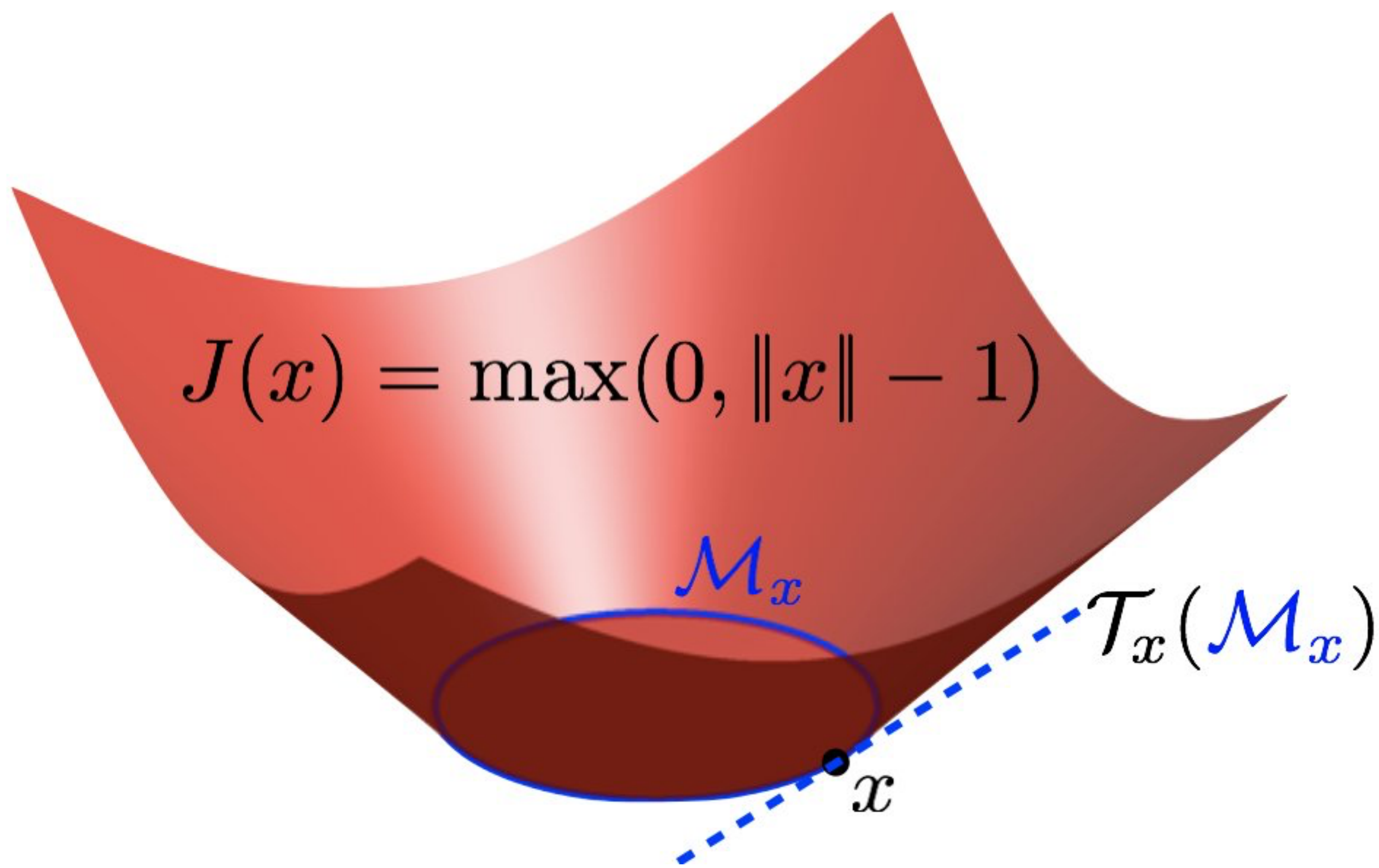
$J : \mathbb{R}^N \to \mathbb{R}$ is partly smooth at x for a manifold $\mathcal{M}_x$

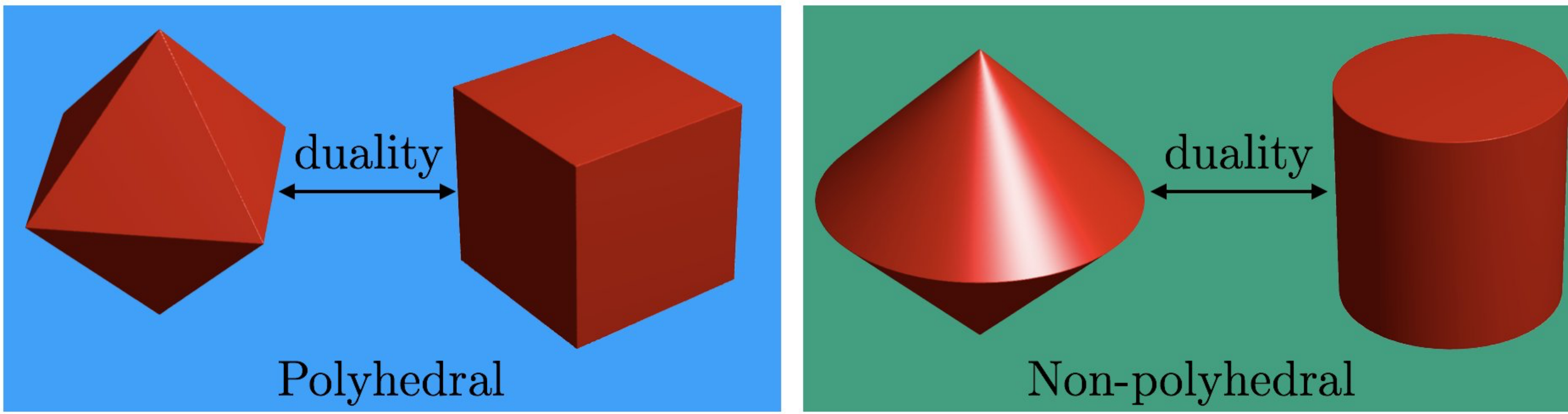
(i) J is C^2 along $\mathcal{M}_x$ around x ;

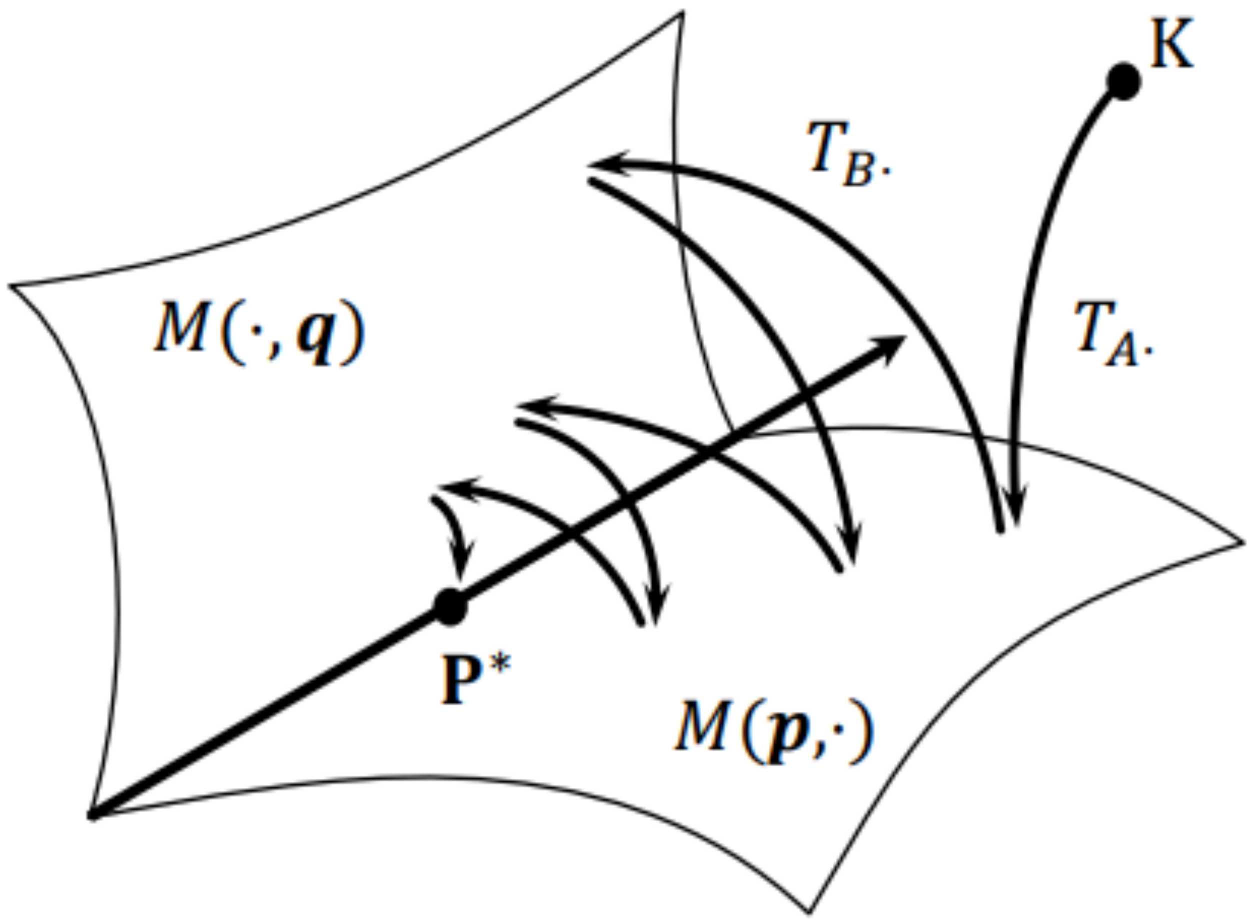
(ii) $\forall\, h \in \mathcal{T}_x(\mathcal{M}_x)^\perp$, $t \mapsto J(x+th)$ non-smooth at $t=0$.

(iii) ∂J is continuous on $\mathcal{M}_x$ around x.

[Lewis 2003]




duality
Polyhedral
duality
Non-polyhedral



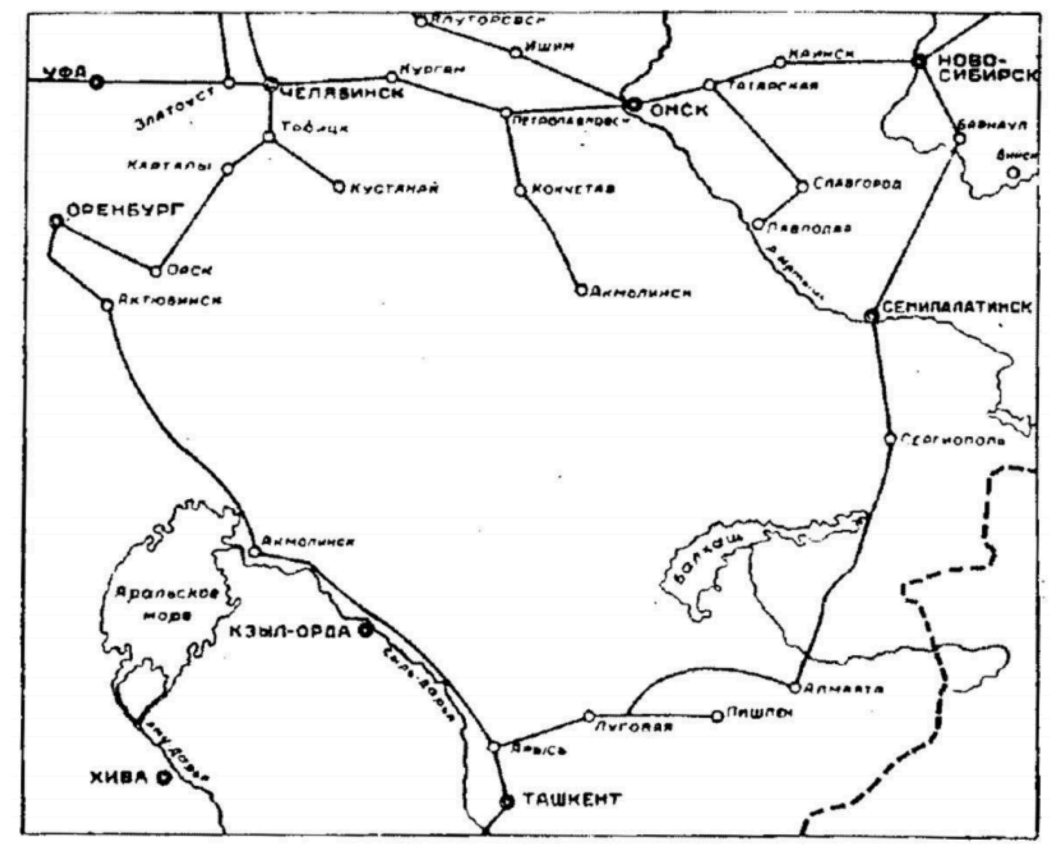
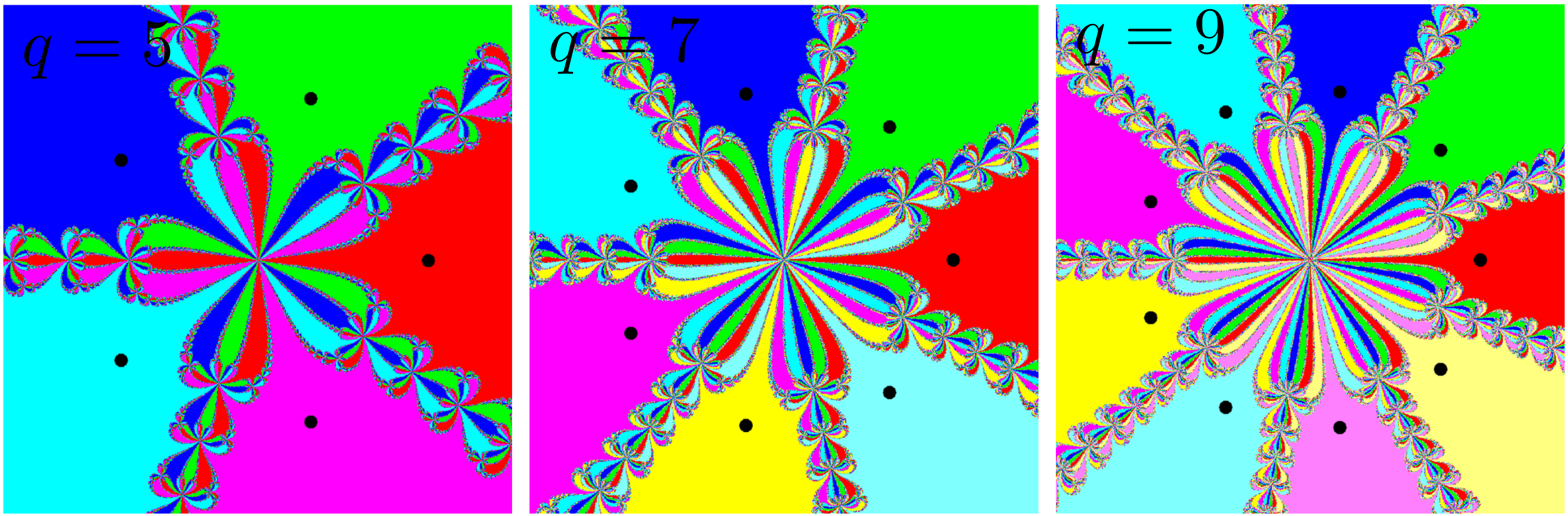


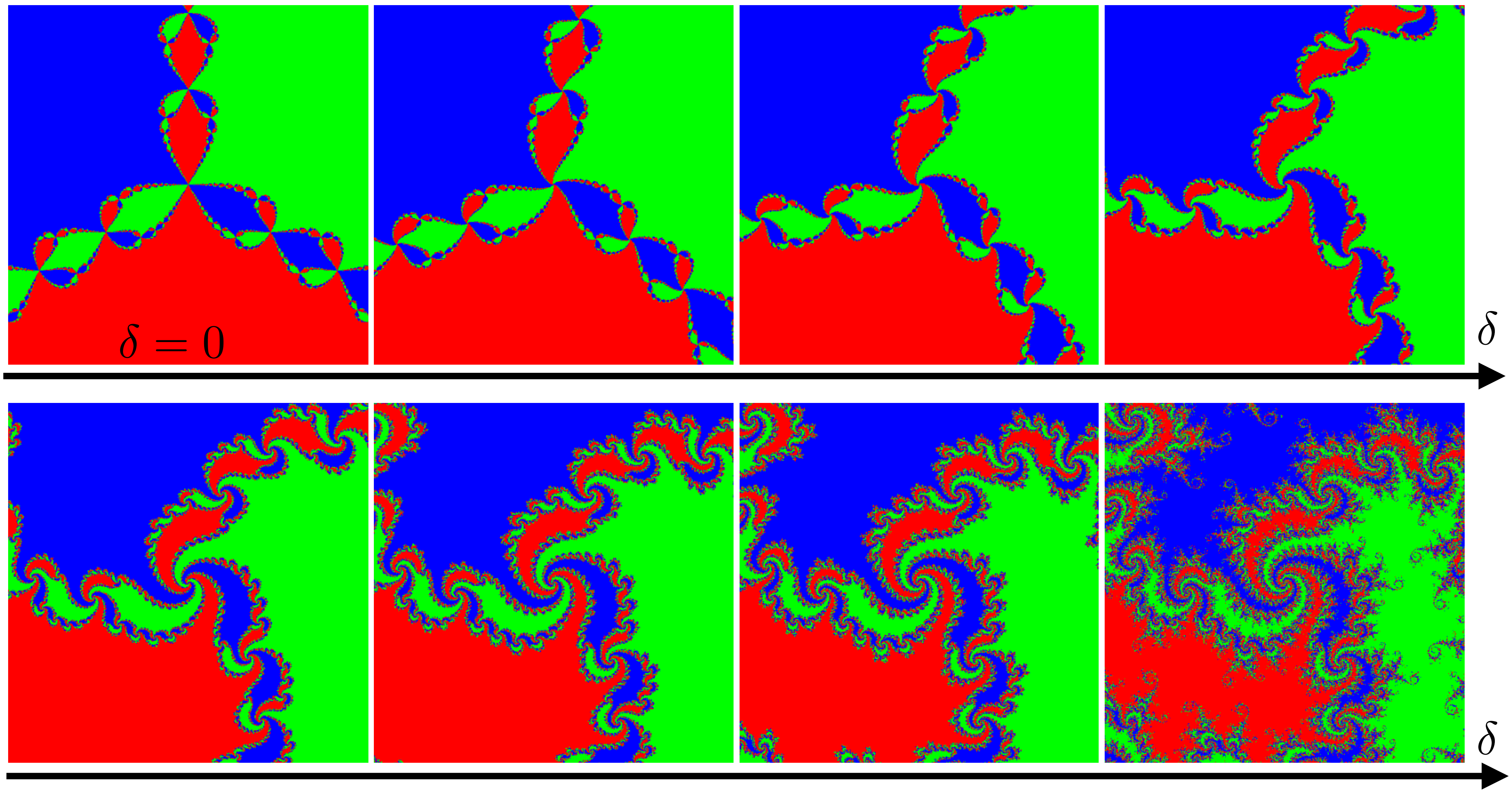
Figure 1: Figure from Tolstoĭ [1930] to illustrate a negative cycle

Newton method: $z_{k+1} = z_k - \frac{f(z_k)}{f'(z_k)}$



Attraction bassins for $f(z) = z^q - 1$

"Twisted" Newton: $z_{k+1} = z_k - (1 + \delta e^{\mathrm{i}\theta}) \frac{f(z_k)}{f'(z_k)}$ $\quad f(z) = z^3 - 1$

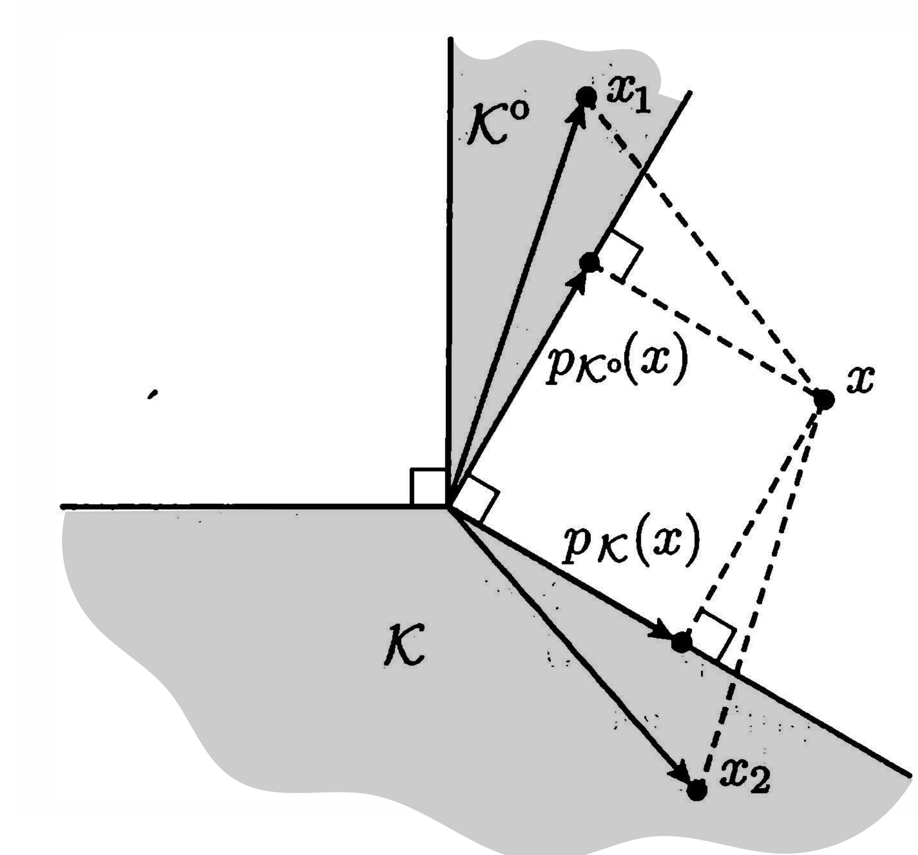


Polar cone:

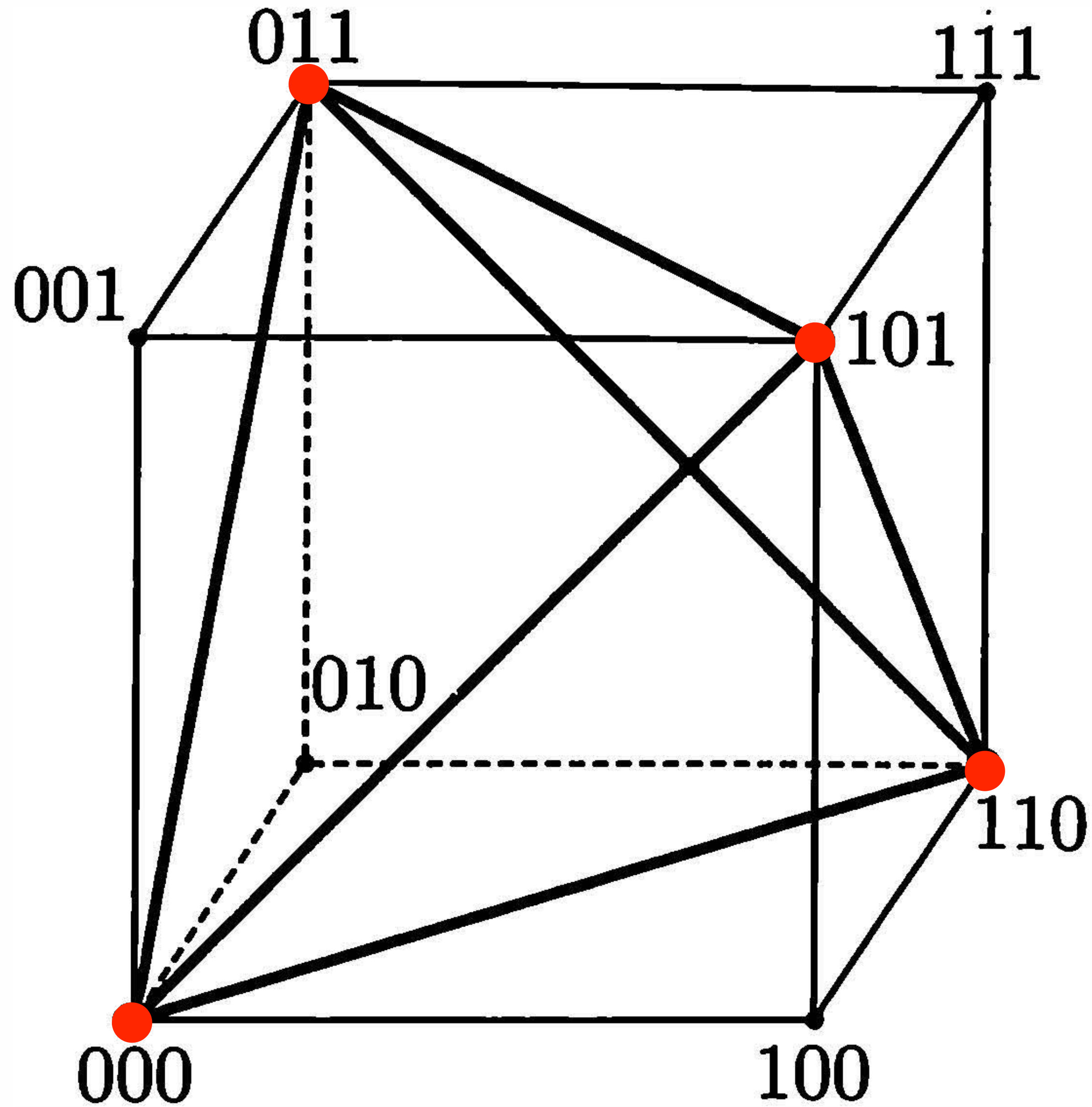
$$\mathcal{K}^\circ \overset{\text{def.}}{=} \{x \; ; \; \forall y \in \mathcal{K}, \langle x,\, y\rangle \leqslant 0\}$$

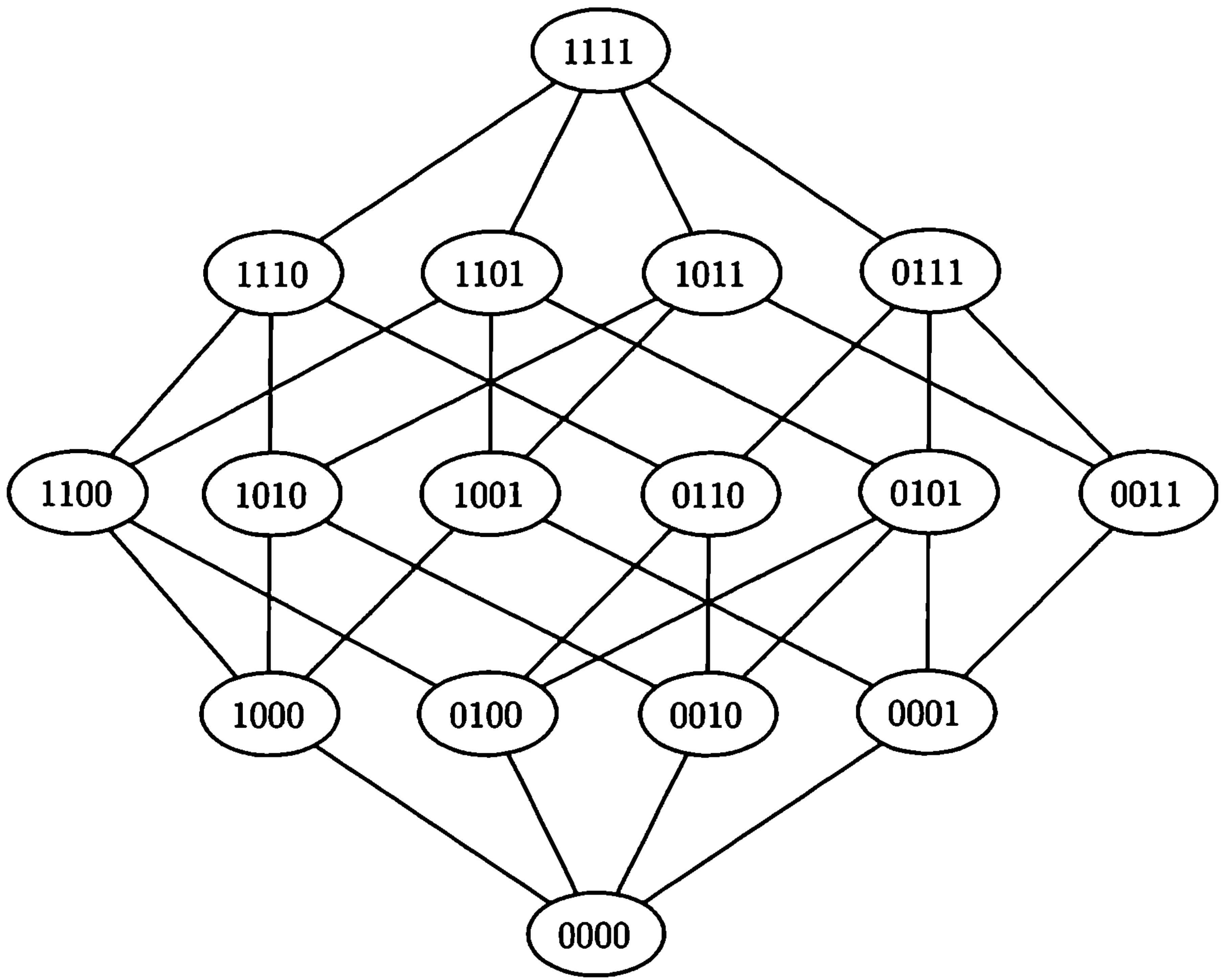
Theorem: (Moreau's decomposition)

$$x = \mathrm{Proj}_{\mathcal{K}}(x) +^{\perp} \mathrm{Proj}_{\mathcal{K}^\circ}(x)$$



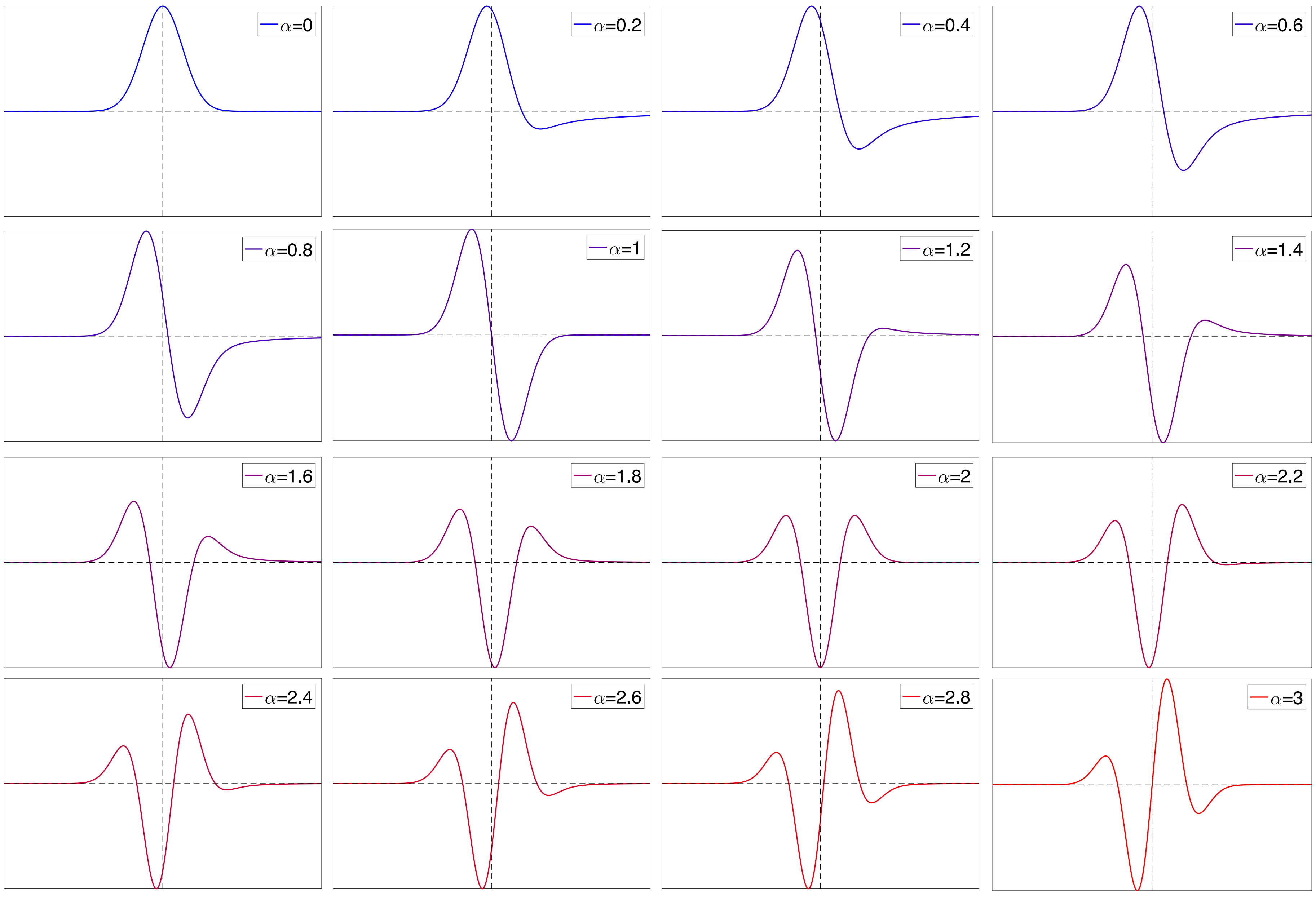
Parity bits

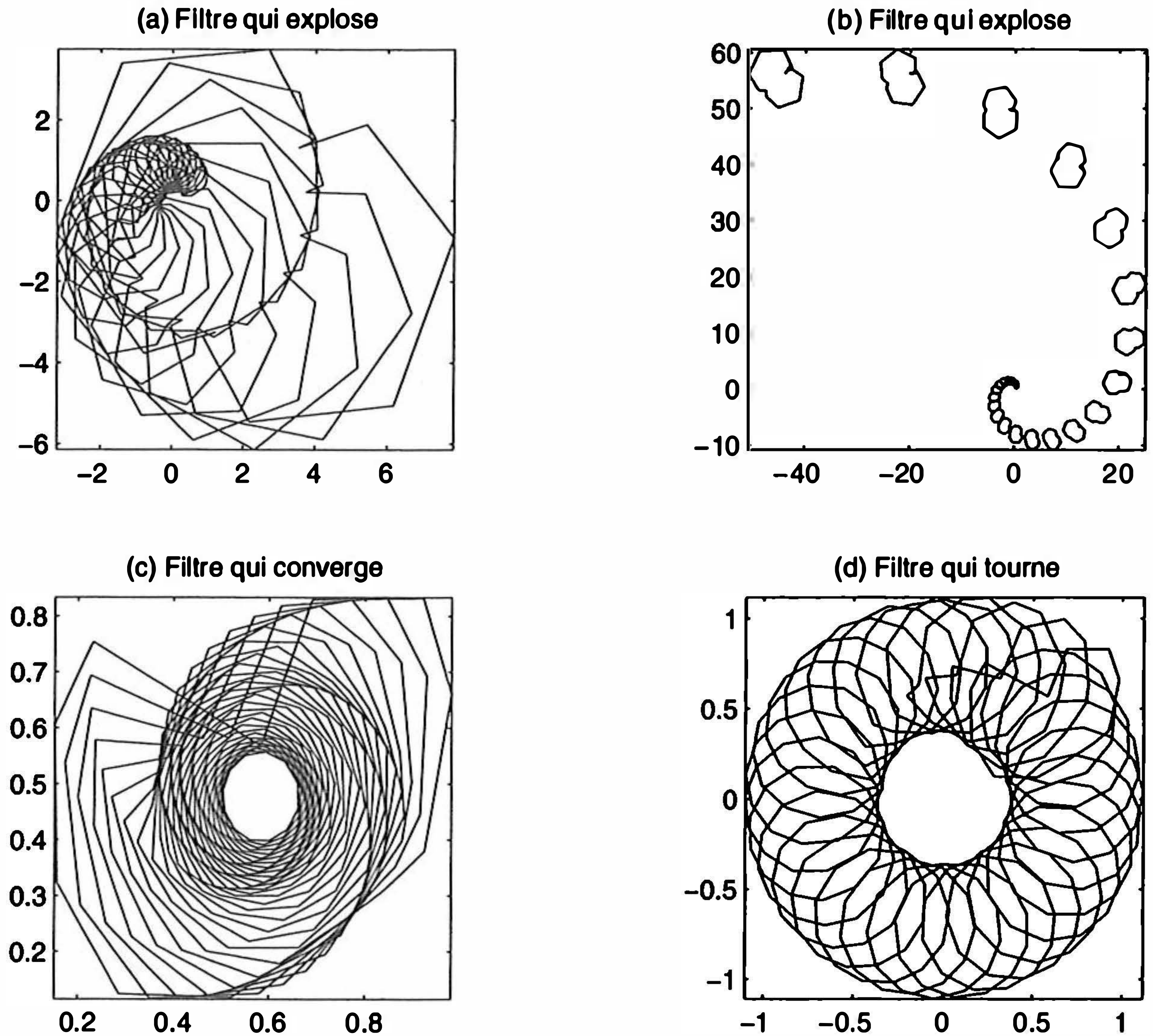



1111
1110
1101
1011
0111
1100
1010
1001
0110
0101
0011
1000
0100
0010
0001
0000

Fourier transform: $\mathcal{F}(f)(\omega) \stackrel{\text{def.}}{=} \int_{\mathbb{R}} f(x) e^{-\mathrm{i}\omega x} \mathrm{d}x$

Fractional derivative: $\mathcal{F}(f^{(\alpha)}) \stackrel{\text{def.}}{=} (\mathrm{i}\omega)^{\alpha} \mathcal{F}(f)(\omega)$



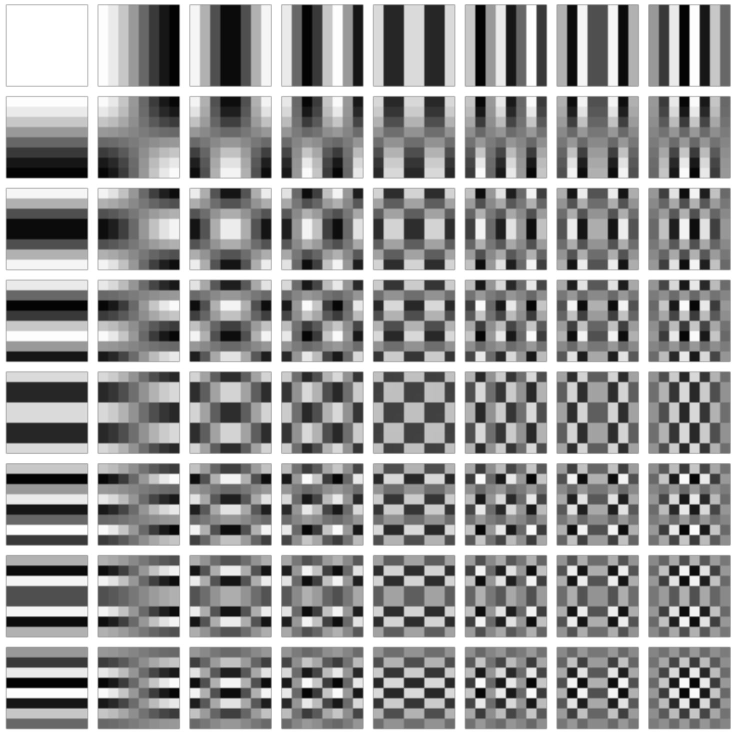

(a) Filtre qui explose
2
0
−2
−4
−6
−2
0
2
4
6
(b) Filtre qui explose
60
50
40
30
20
10
0
−10
−40
−20
0
20
(c) Filtre qui converge
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.2
0.4
0.6
0.8
(d) Filtre qui tourne
1
0.5
0
−0.5
−1
−1
−0.5
0
0.5
1

DCT-1: $\cos jk \frac{\pi}{N-1}$ (divide by $\sqrt{2}$ when j or k is 0 or $N-1$)

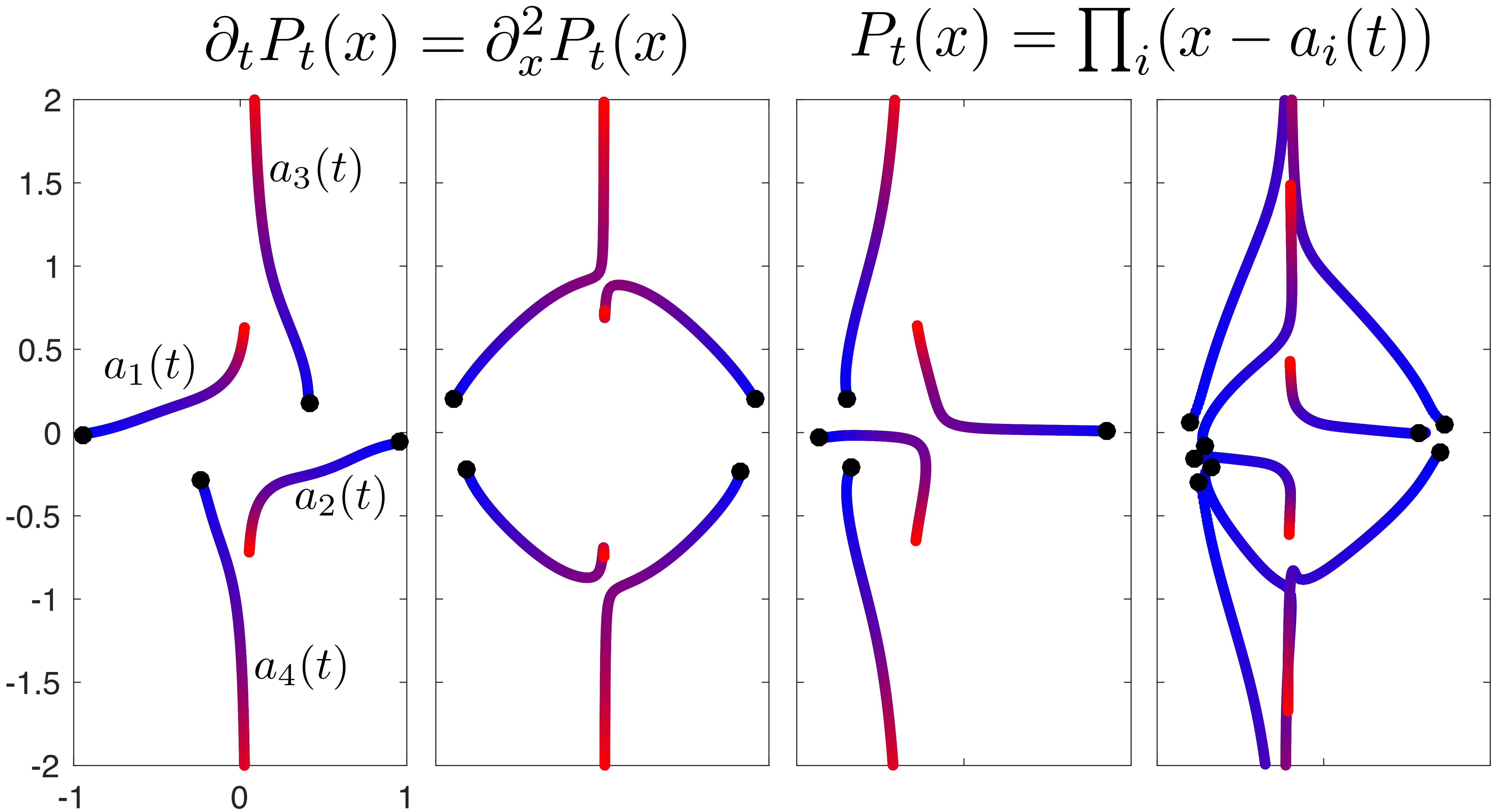
DCT-2: $\cos\left(j+\frac{1}{2}\right) k \frac{\pi}{N}$ (divide by $\sqrt{2}$ when $k=0$)

DCT-3: $\cos j\left(k+\frac{1}{2}\right) \frac{\pi}{N}$ (divide by $\sqrt{2}$ when $j=0$)

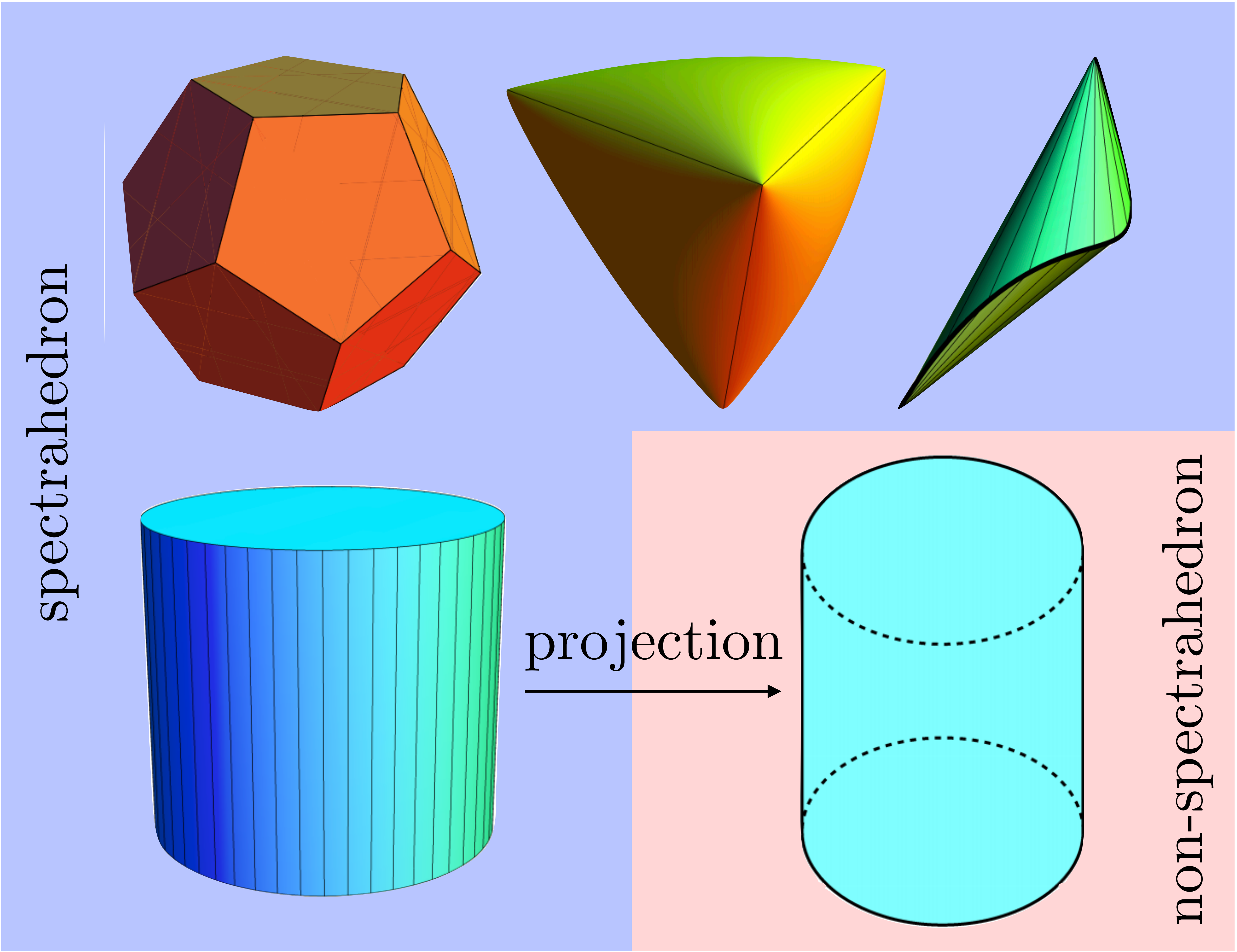
DCT-4: $\cos\left(j+\frac{1}{2}\right)\left(k+\frac{1}{2}\right) \frac{\pi}{N}$

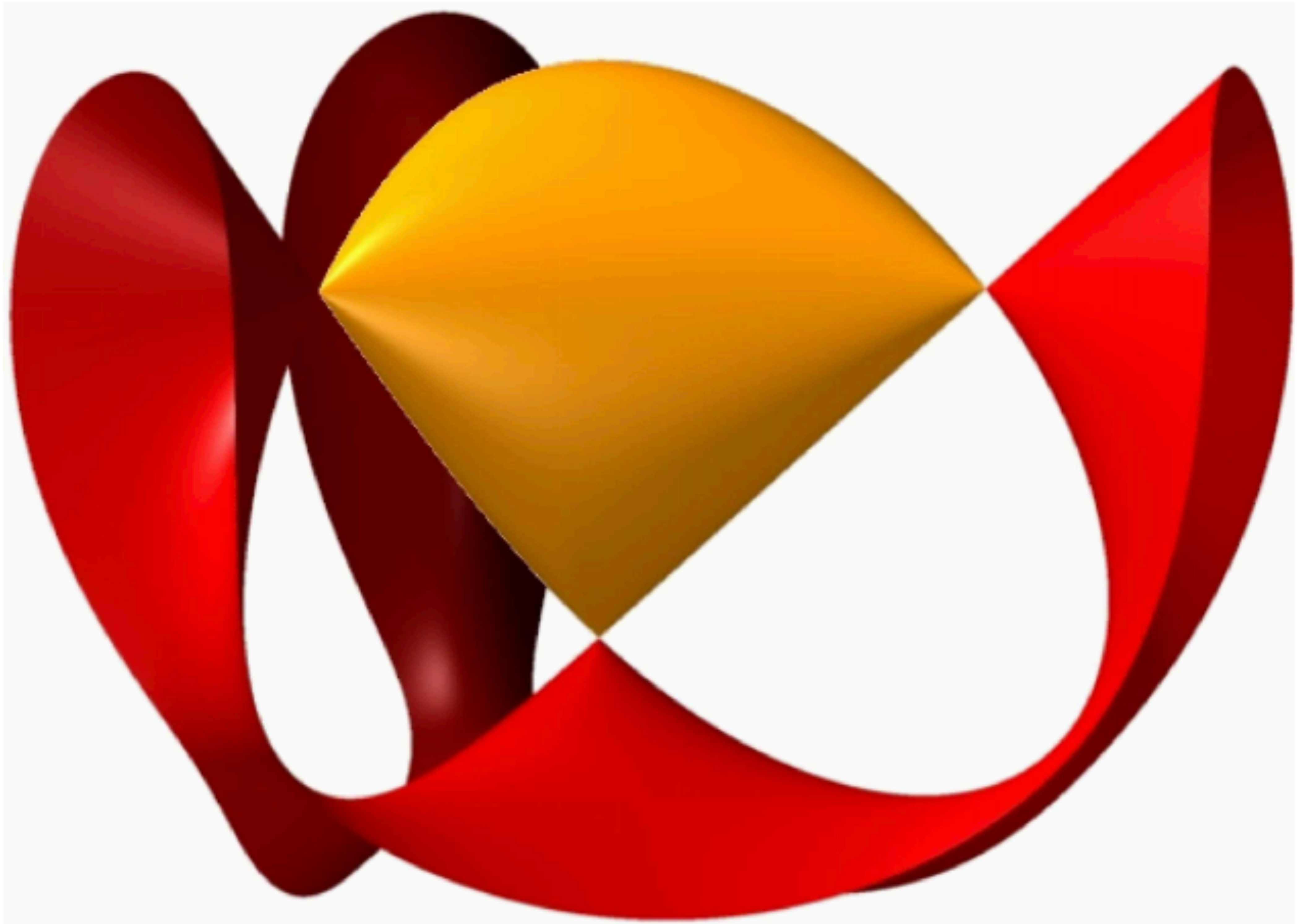


The discrete case has a new level of variety and complexity, often appearing in the boundary conditions [G. Strang - SIAM review, 1999]

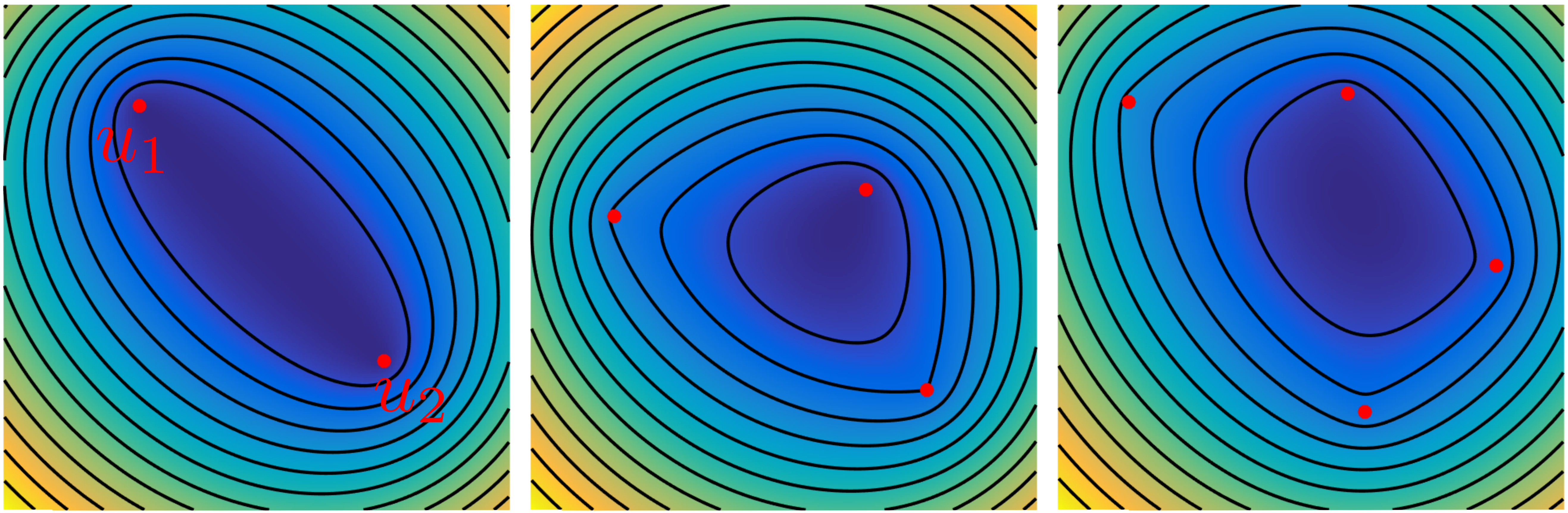


$\partial_t P_t(x) = \partial_x^2 P_t(x)$
$P_t(x) = \prod_i (x - a_i(t))$
$a_1(t)$
$a_2(t)$
$a_3(t)$
$a_4(t)$
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
0
1

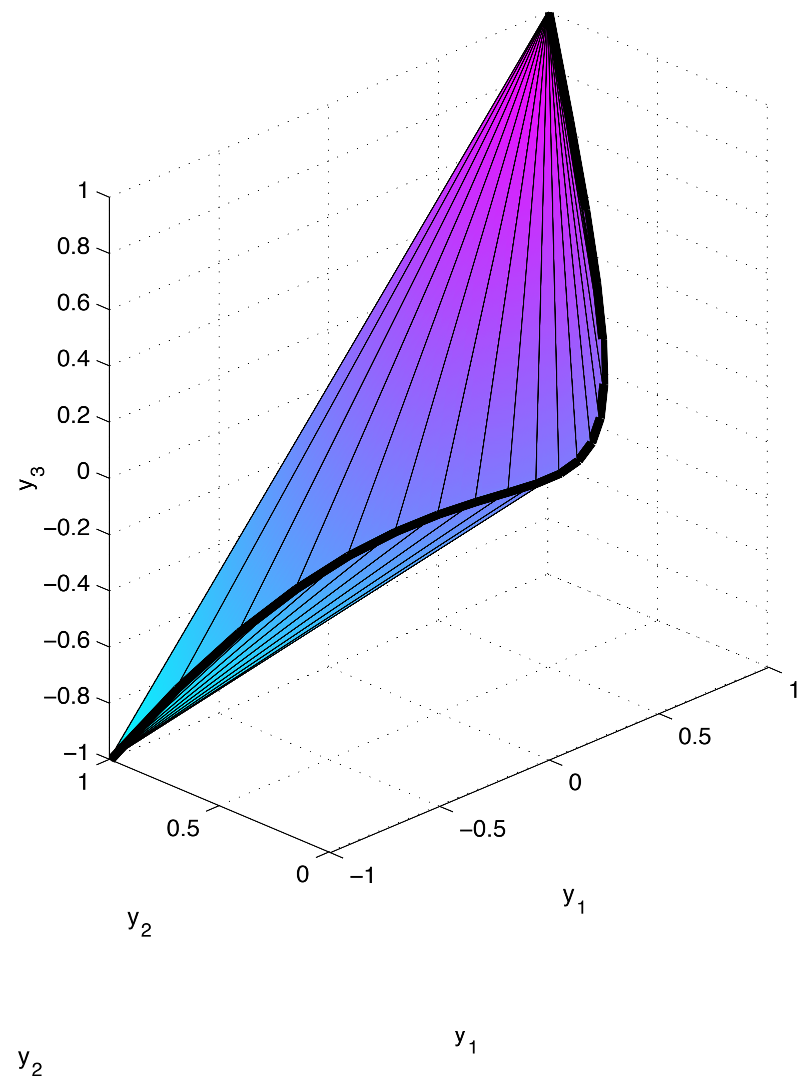

spectrahedron
projection
non-spectrahedron



$$\left\{ (x,y) \in \mathbb{R}^2 \; ; \; \sum_{i=1}^{n} \sqrt{(x-u_k)^2 + (y-v_k)^2} \leqslant t \right\}$$



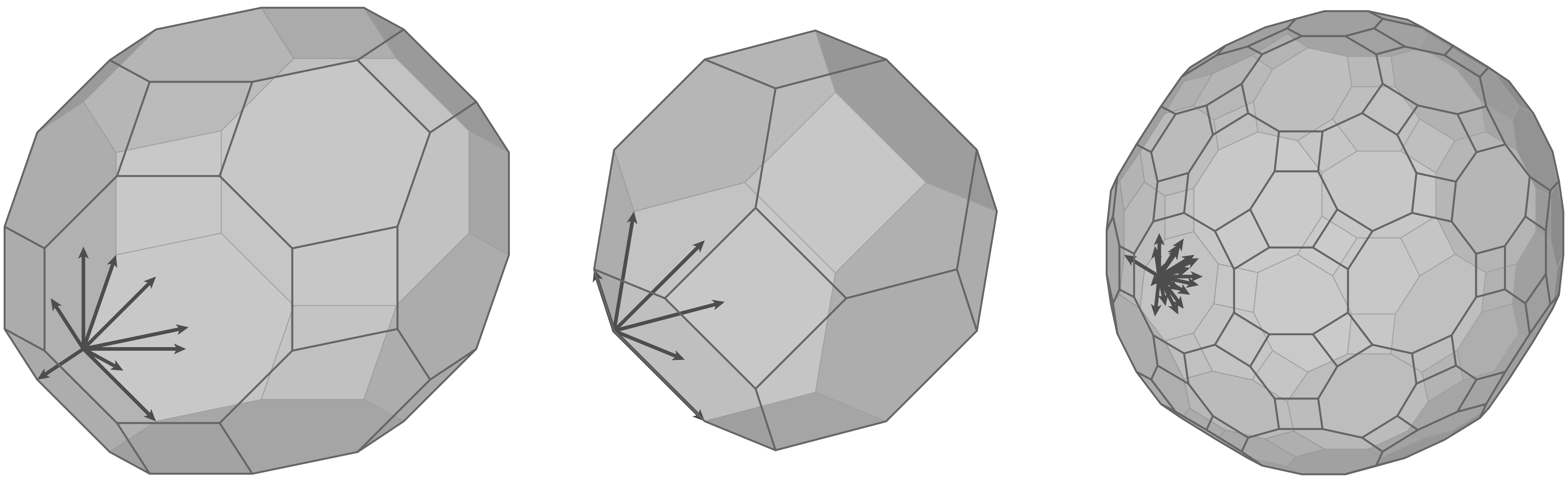
Input : $\left\{(t, t^2, t^3) \in \mathbb{R}^3 : -1 \leq t \leq 1\right\}$



The convex hull of the moment curve is a spectrahedron.

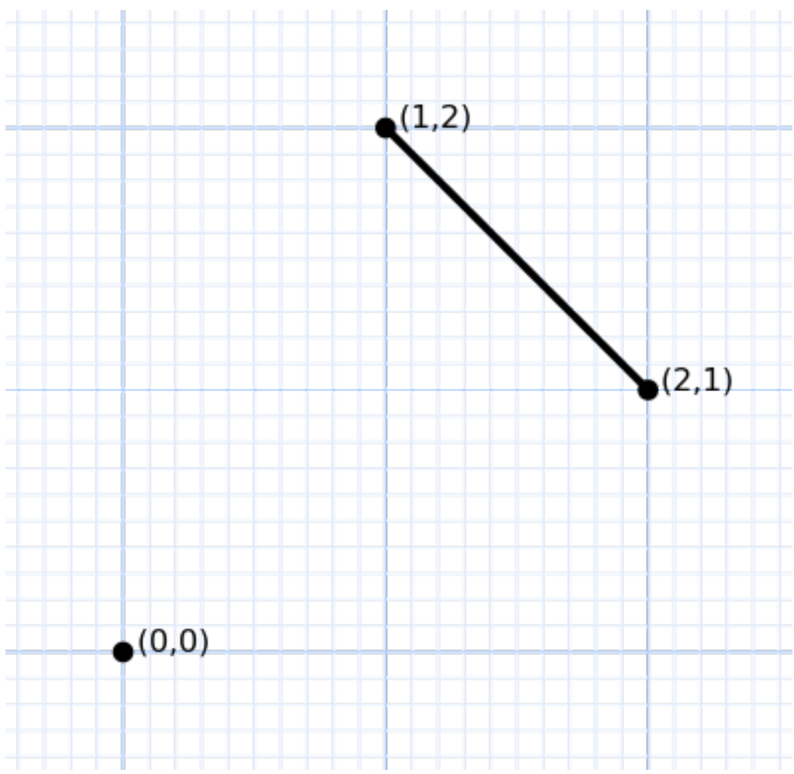
Output : $$\begin{pmatrix} 1 & x \\ x & y \end{pmatrix} \pm \begin{pmatrix} x & y \\ y & z \end{pmatrix} \succeq 0$$

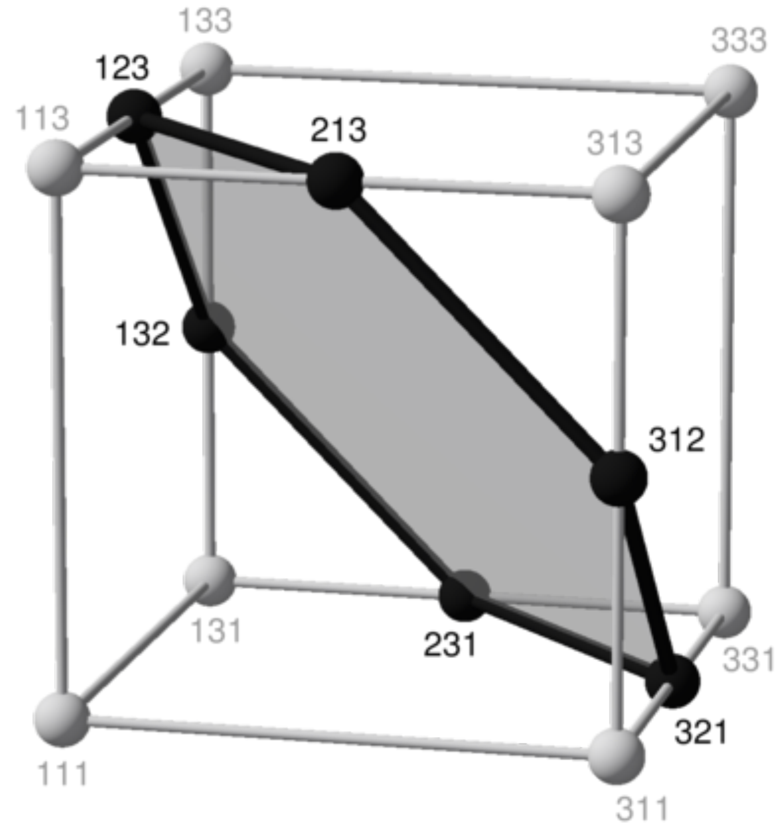
Zonohedra are images of l^inf ball by linear map / symmetric polyhedra are images of the l1 ball

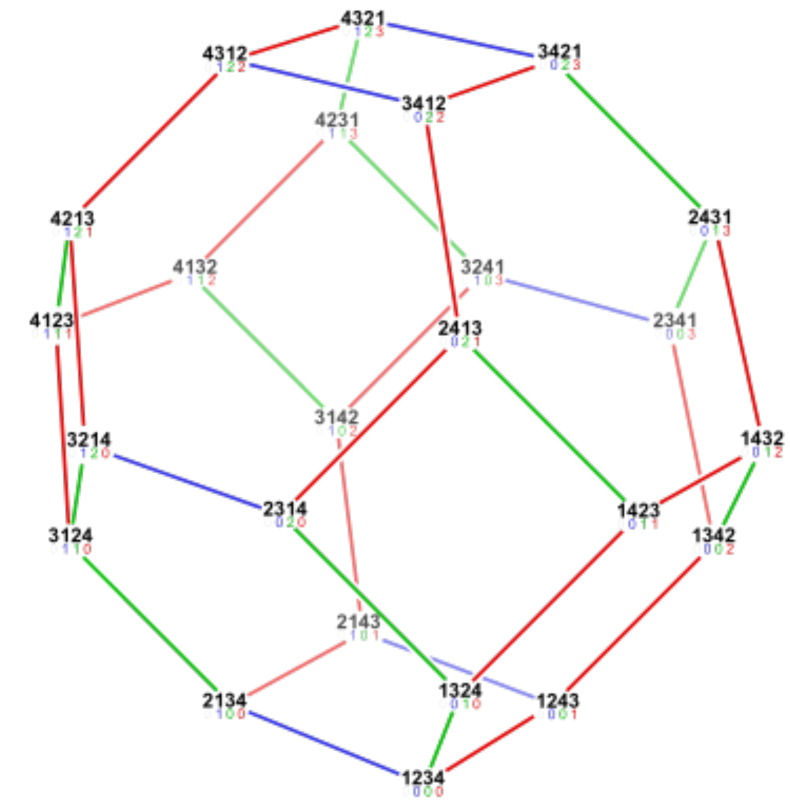


zonoids are limits of Zonohedra

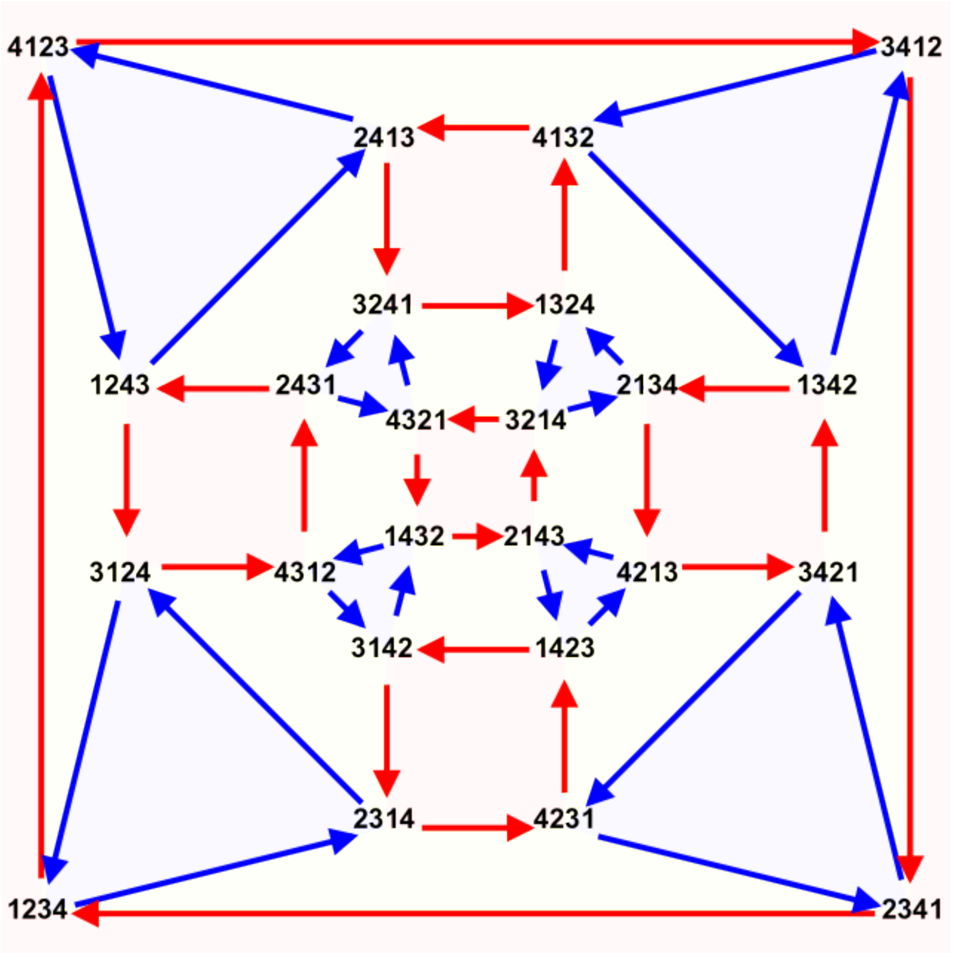
permutahedron

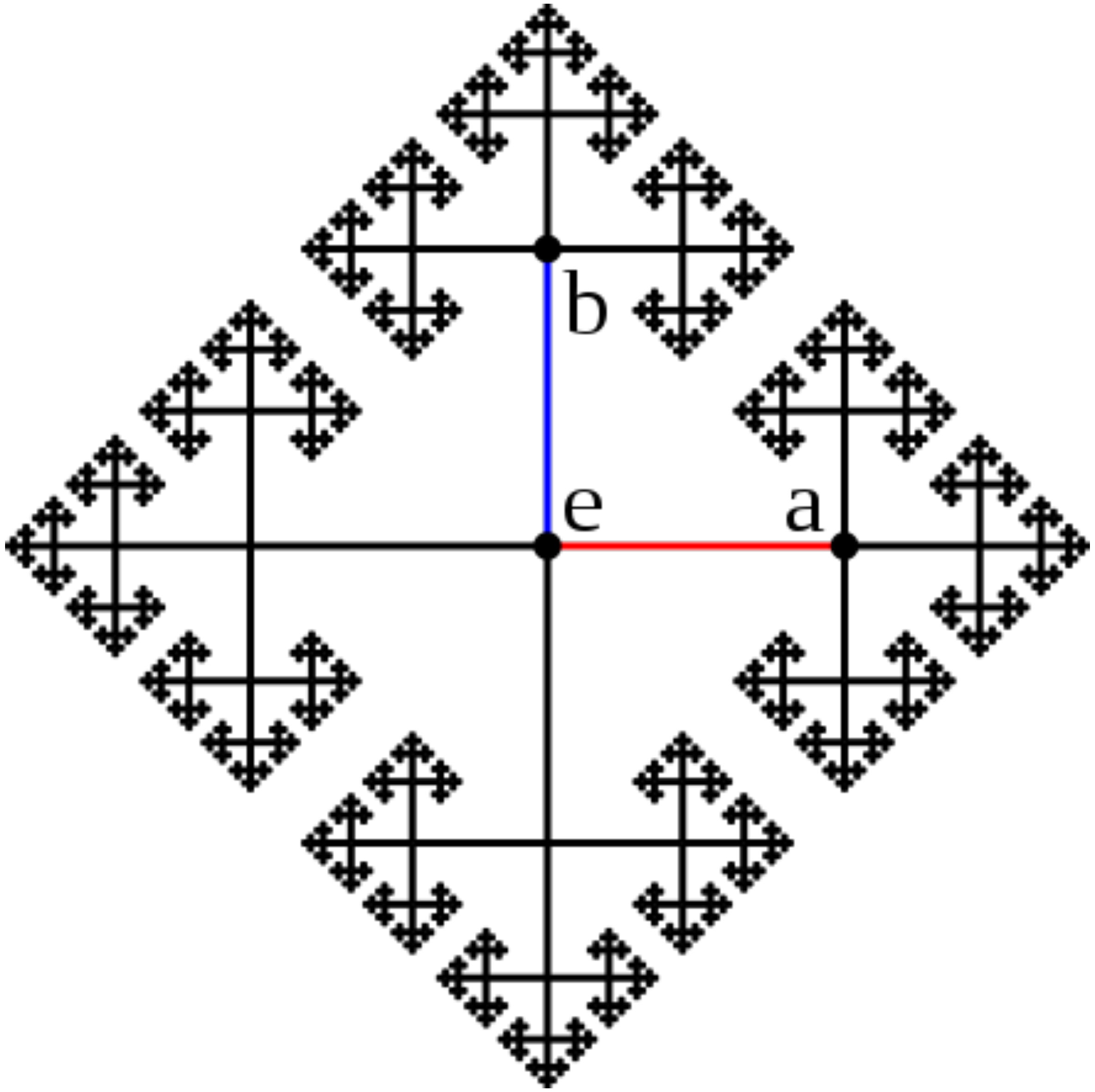






Cayley graph of S_4





- Conform application / fluids flows

- SDP=Lorentz for 2x2

- The only « Fourier modes » on permutation groups are signature and identity.

- Positive polynomia on R are exactly sums of squares

- Eigenvectors of the FFT / 4