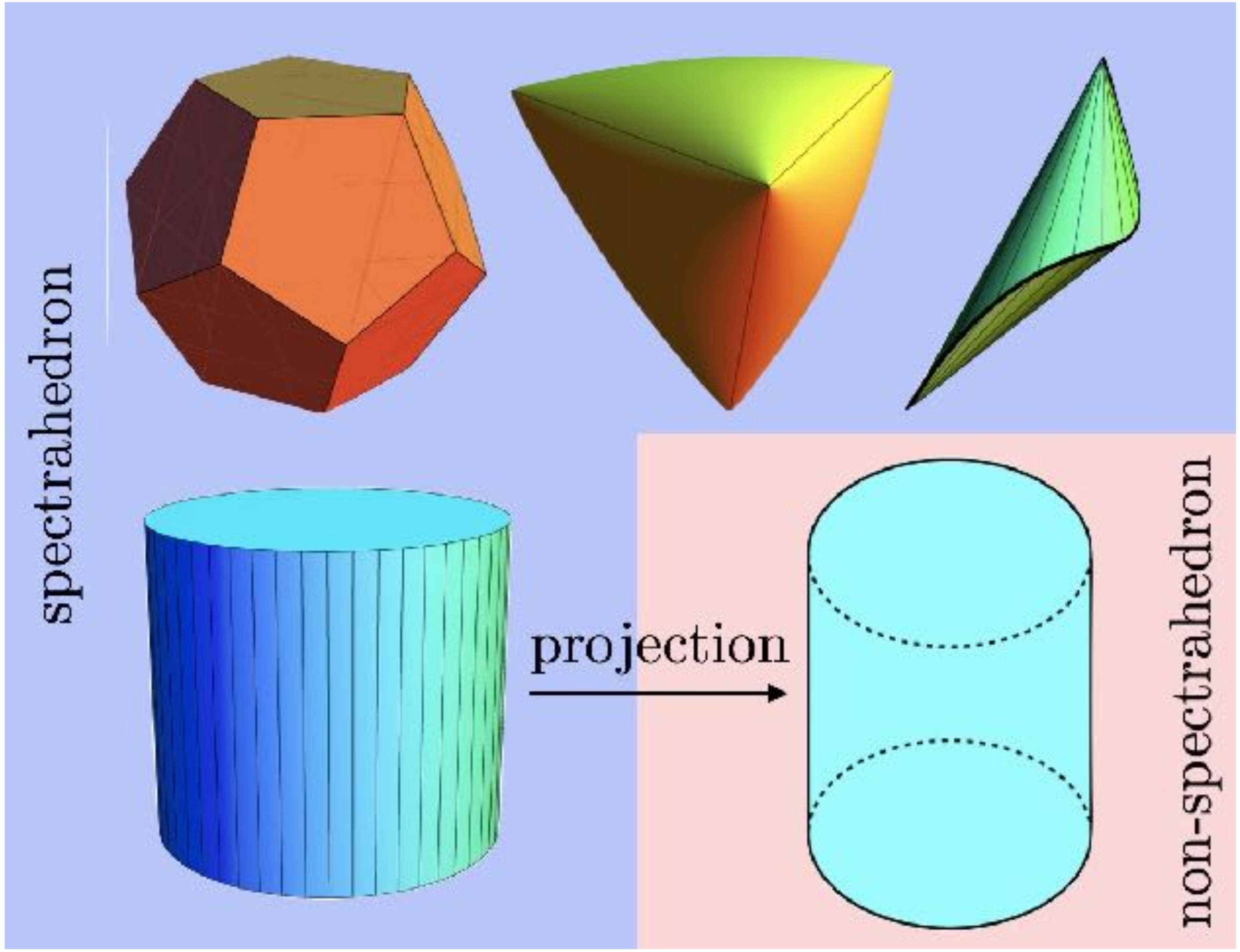
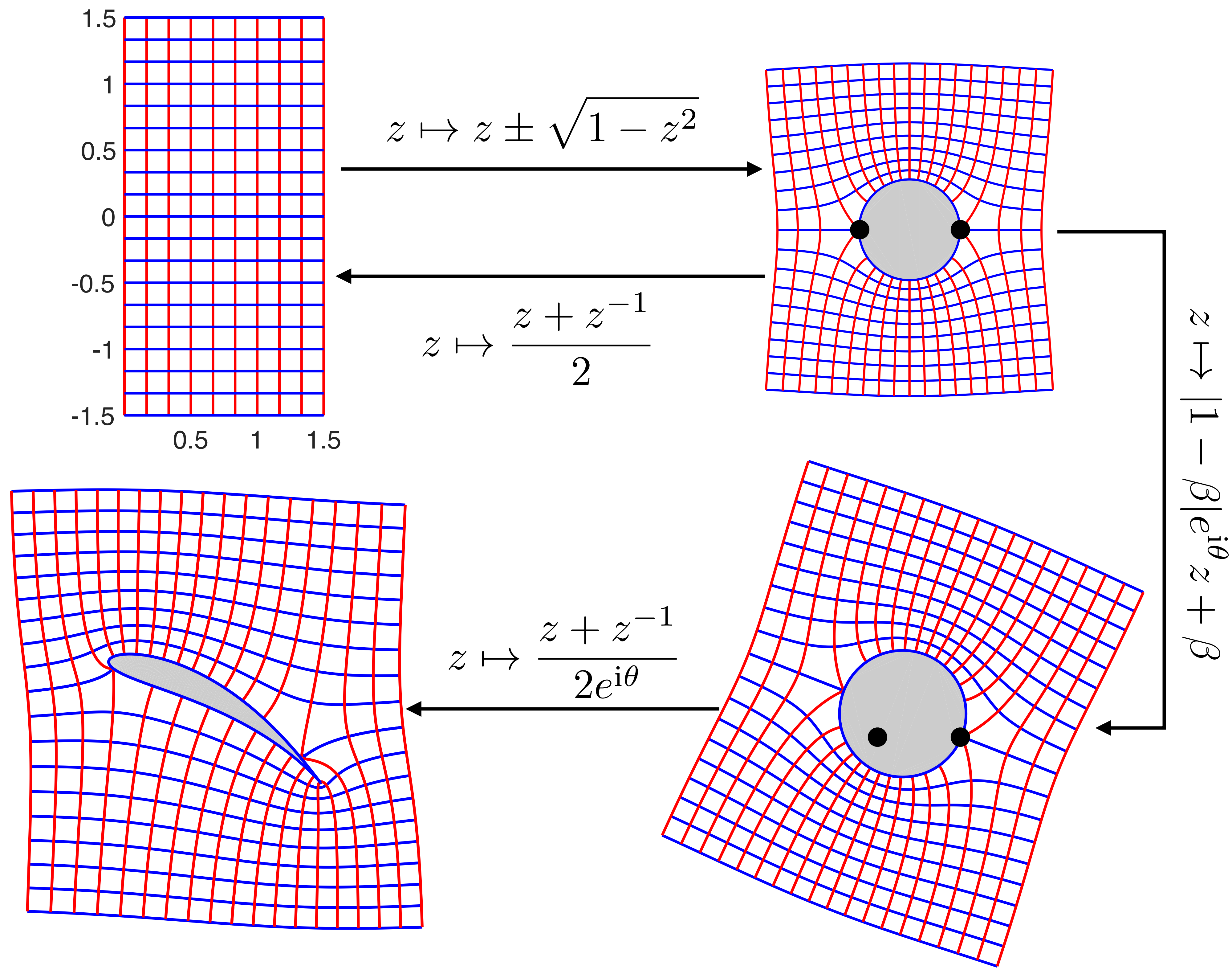
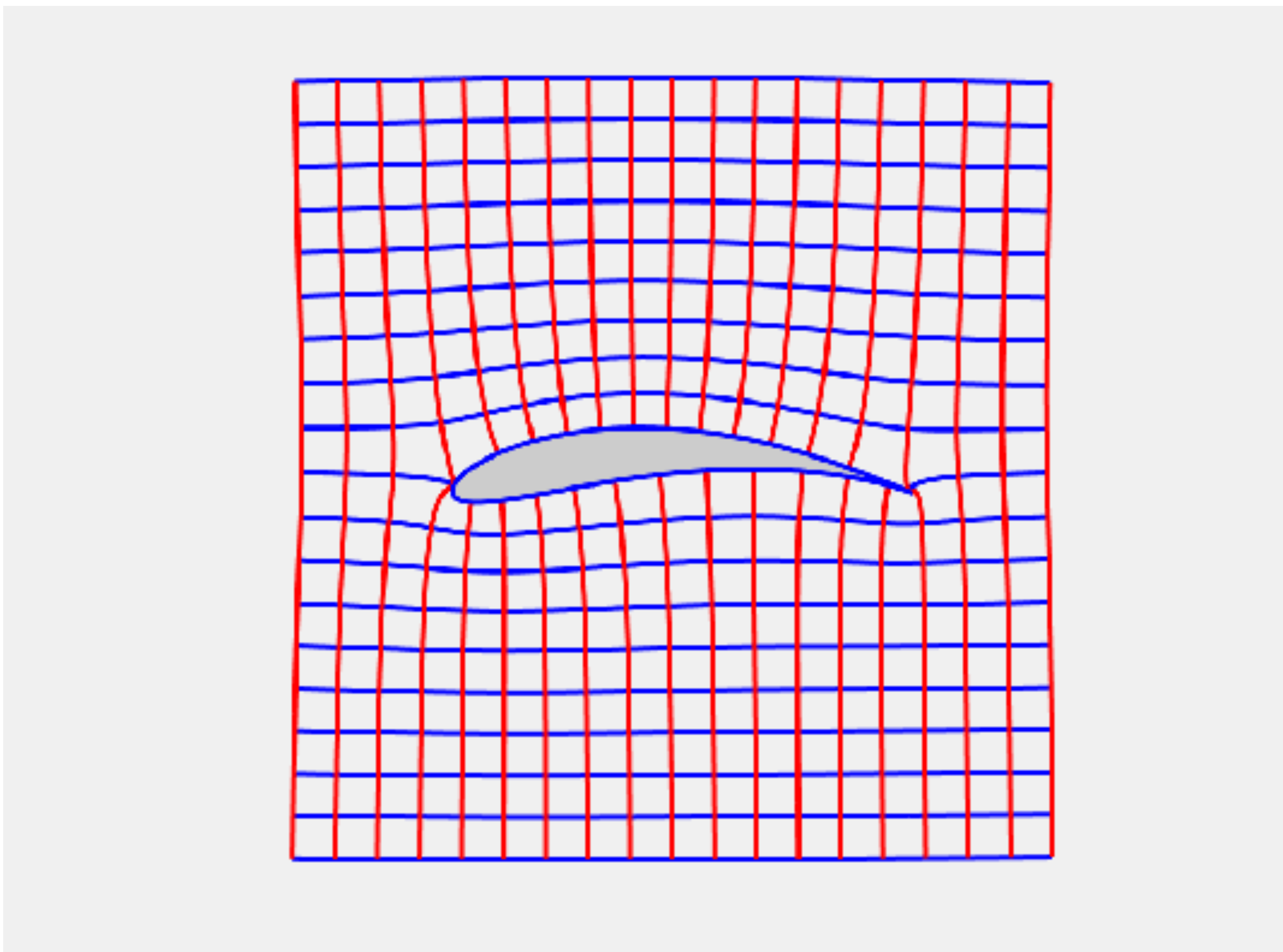
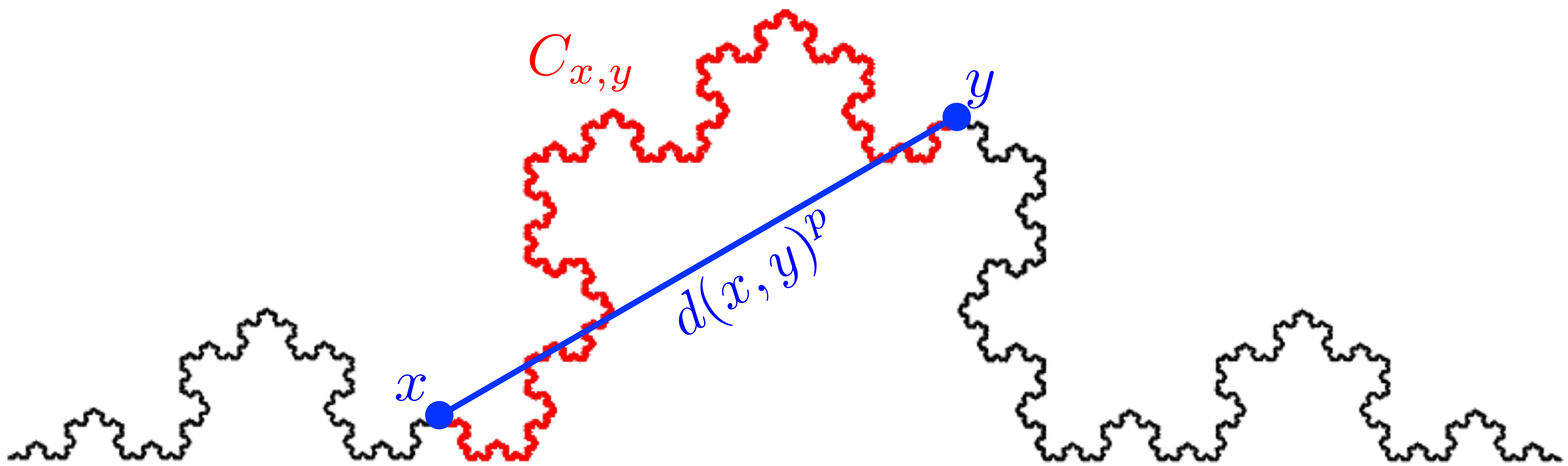
Geometry


spectrahedron
projection
non-spectrahedron

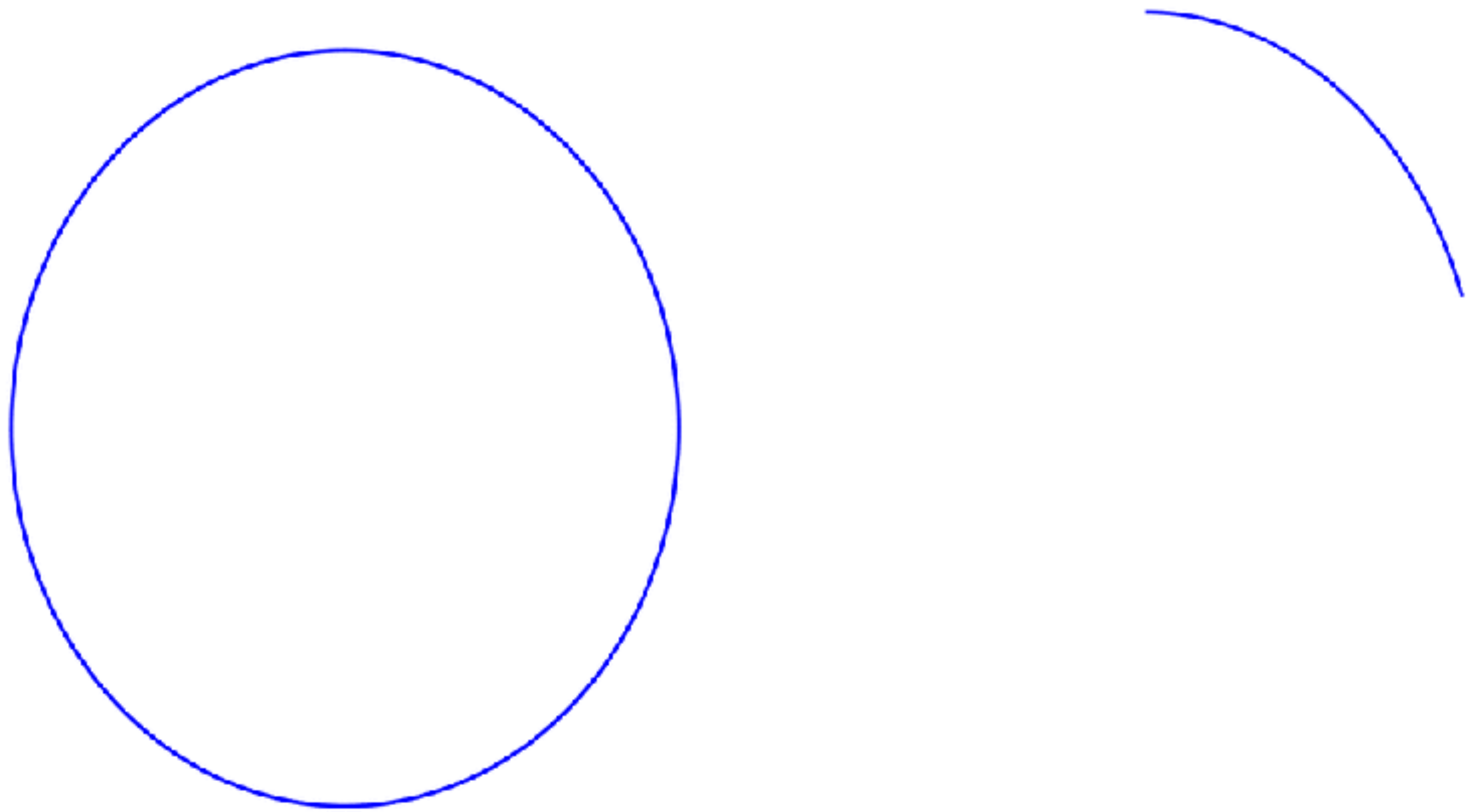
Joukowski Conformal Mapping







$$d(x,y) = \mathrm{Hausdorff}_p(C_{x,y})^{\frac{1}{p}} \qquad p = \frac{\log(4)}{\log(3)}$$

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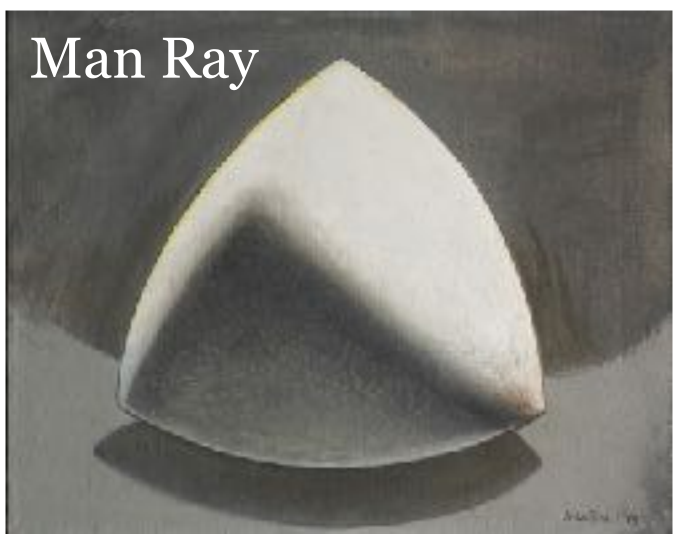
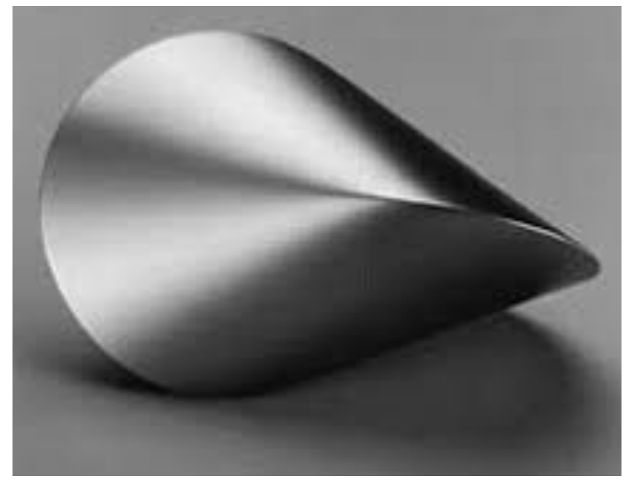
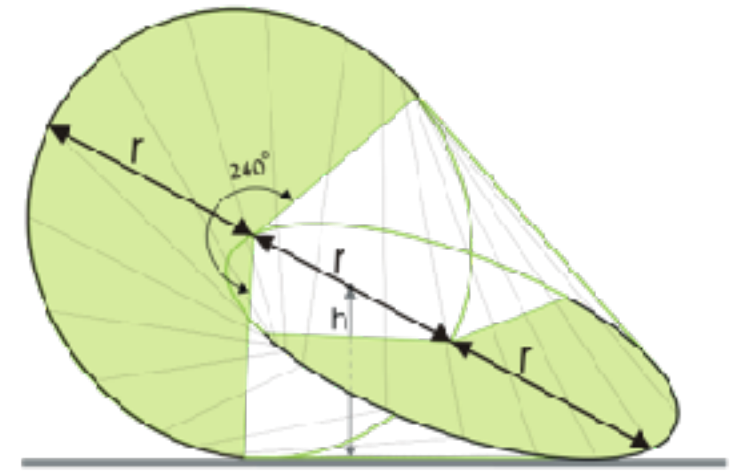
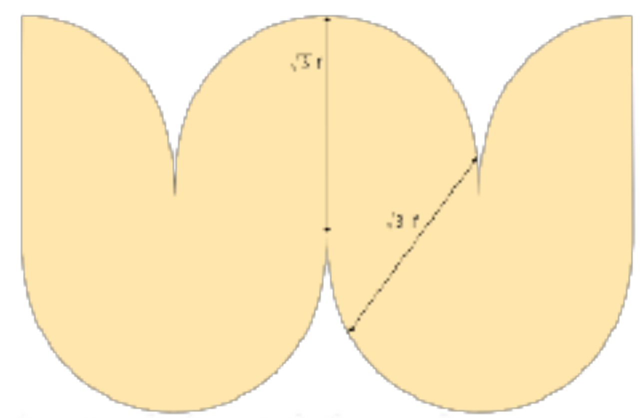
2016


Man Ray







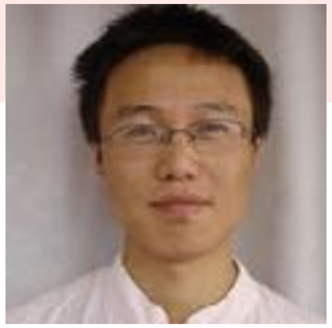
$$
\begin{aligned}
&4x^2+4y^2+4x^3+4xy^2+4xz^2+\\
&-7x^4-18x^2y^2-11y^4-6x^2z^2-10y^2z^2+z^4+\\
&-8x^5-8xy^4-6x^3y^2-48x^3z^2-52xy^2z^2-8xz^4+\\
&2x^6+22x^2y^4+14x^4y^2+10y^6-46x^4z^2-46x^2y^2z^2-50x^2z^4-12y^2z^4-2z^6+\\
&4x^7+12x^3y^4+12x^5y^2-4xy^6-12x^5z^2+12x^3y^2z^2+24xy^4z^2-36x^3z^4+\\
&x^8-6x^4y^4-8x^2y^6-3y^8+6x^4y^2z^2+12x^2y^4z^2+6y^6z^2-6x^4z^4+12x^2y^2z^4-9x^4z^4-20x^2z^6+6y^2z^6-3z^8=0
\end{aligned}
$$

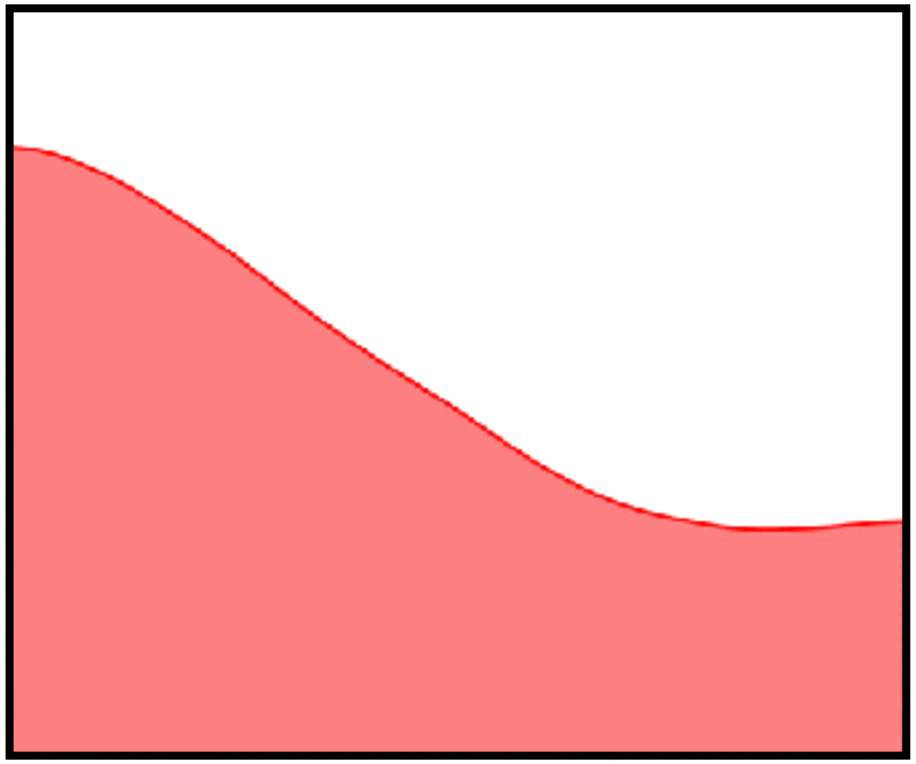
Weiestrass function: $f_{a,b}(x) \stackrel{\text{def.}}{=} \sum_{n=0}^{+\infty} a^n \cos(b^n \pi x)$

Its graph: $F_{a,b} \stackrel{\text{def.}}{=} \left\{(x, f(x)) \in \mathbb{R}^2 \; ; \; x \in \mathbb{R}\right\}$

Theorem: [W. Shen, 2016]

$$\dim_{\mathrm{H}}(F_{a,b}) = 2 + \frac{\log(a)}{\log(b)}$$





$b = 2 \qquad 0.2 \leqslant a \leqslant 0.8$

The Weierstrass function is continuous if a<1 but nowhere differentiable if ab>1.
The Hausdorff dimension of its graph was conjectured by Mandelbrot in 1977
and proved by Shen in 2016.

https://arxiv.org/abs/1505.03986

Lemma 3.3. *Under the above circumstances, the following holds:*

(1) *For each $1 \le i < j \le e(1)$, we have*

$$|\sin(2\pi x_i) - \sin(2\pi x_j)| \ge \frac{2\theta_1(b,\gamma)}{b}, \tag{3.6}$$

where

$$\theta_1(b,\gamma) = \sqrt{\max\left(0, \left(b\sin\frac{\pi}{b}\right)^2 - \frac{4\gamma^2 b^2}{(b-\gamma)^2}\right)}. \tag{3.7}$$

(2) *If $k_i = k$ or $k_j = k$, then*

$$|\sin(2\pi x_i) - \sin(2\pi x_j)| \ge \frac{2\theta_0(b,\gamma)}{b}, \tag{3.8}$$

where

$$\theta_0(b,\gamma) = \sqrt{\max\left(0, \left(b\sin\frac{\pi}{b}\right)^2 - \frac{\gamma^2 b^2}{(b-\gamma)^2}\right)}. \tag{3.9}$$

(3) *If $k_i - k_j \not\equiv \pm 1 \mod b$, then*

$$|\sin(2\pi x_i) - \sin(2\pi x_j)| \ge \frac{2\theta_2(b,\gamma)}{b}, \tag{3.10}$$

where

$$\theta_2(b,\gamma) = \sqrt{\max\left(0, \left(b\sin\frac{2\pi}{b}\right)^2 - \frac{4\gamma^2 b^2}{(b-\gamma)^2}\right)}. \tag{3.11}$$

Proof. For each $1 \le i < j \le e(1)$, we have

(3.12)
$$|\cos(2\pi x_i)-\cos(2\pi x_j)|^2+|\sin(2\pi x_i)-\sin(2\pi x_j)|^2 = 4\sin^2(\pi(x_i-x_j)) \ge 4\sin^2\frac{\pi}{b}.$$

If $k_i = k$ or $k_j = k$ then by (3.4), the inequality (3.8) follows.

In general, from (3.4), we obtain

$$|\cos(2\pi x_i) - \cos(2\pi x_j)| \le \frac{4\gamma}{b-\gamma}, \tag{3.13}$$

which, together with (3.12), implies (3.6).

If $k_i - k_j \not\equiv \pm 1 \mod b$, then

$$|\cos(2\pi x_i) - \cos(2\pi x_j)|^2 + |\sin(2\pi x_i) - \sin(2\pi x_j)|^2 \ge 4\sin^2\frac{2\pi}{b}, \tag{3.14}$$

which, together with (3.13), implies (3.10). □

Consequently,

$$e(1) - 2 = \frac{y^{e(1)-1} - y^1}{2\pi/b} < \frac{2\arcsin(0.8\gamma)}{2\pi/b} \le 0.4\gamma b, \tag{3.22}$$

where we used $\arcsin t \le \pi t/2$ for each $t \in [0,1]$. If $2 + 0.4\gamma b \le \gamma b$, then we are done. So assume the contrary. Then $\gamma b < 10/3$ and hence $e(1) - 2 < 4/3$. Therefore $e(1) \le 3$. If $\gamma > 1/2$, then $e(1) < \gamma b$ holds. So assume $\gamma \le 1/2$. Then

$$\frac{\arcsin(0.8\gamma)}{0.8\gamma} \le \frac{\arcsin(0.5)}{0.5} = \frac{\pi}{3},$$

and hence (3.22) improves to the following $e(1) - 2 < 4\gamma b/15$. If $2 + 4\gamma b/15 \le \gamma b$ then we are done. So assume $2 + 4\gamma b/15 > \gamma b$. Then $\gamma b < 30/11$ and hence $e(1) - 2 < 4\gamma b/15 < 1$. It follows that $e(1) = 1$ or 2. If $\gamma b > 2$ then $e(1) < \gamma b$. So assume $\gamma b \le 2$. To complete the proof we need to show $e(1) = 1$. By (3.5), it suffices to show

$$\left(\frac{2\gamma}{b-\gamma}\right)^2 + \left(\frac{2\gamma\Delta_{b,\gamma}}{1-\gamma^2}\right)^2 < 4\sin^2\frac{\pi}{b}.$$

Since $\gamma b \le 2$, we are reduced to show

$$\frac{16}{(b^2-2)^2} + \frac{16}{(b-2)^2}\left(\frac{\Delta_{b,2/b}}{1+2/b}\right)^2 < 4\sin^2\frac{\pi}{b}. \tag{3.23}$$

In the case $b = 6$, by (3.1), $\Delta_{6,1/3} \le \max(0.99 + 1/3, 1 + 0.972/3) = 1.324$, then an easy numerical calculation shows that the left hand side of (3.23) is less than the right hand side which is equal to 1. Assume now $b \ge 7$. Using $\Delta_{b,2/b} \le 1 + 2/b$, we are further reduced to show

$$\frac{4b^2}{(b^2-2)^2} + \frac{4b^2}{(b-2)^2} < b^2\sin^2\frac{\pi}{b}. \tag{3.24}$$

Note that the left hand side is decreasing in b and the right hand side is increasing in b. Thus it suffices to verify this inequality in the case $b = 7$, which is an easy exercise. □

3.2. The case $b = 5$. We use $\sin(\pi/5) = \sqrt{10 - 2\sqrt{5}}/4$. By (3.6), for each $1 \le i < e(1)$, since $\gamma < 1$,

$$|\sin(2\pi x_i) - \sin(2\pi x_{i+1})| \ge \sqrt{4\sin^2\frac{\pi}{5} - (4/4)^2} = (\sqrt{5} - 1)/2 > 0.6.$$

Moreover, by (3.8) if either $k_i = k$ or $k_{i+1} = k$, then

$$|\sin(2\pi x_i) - \sin(2\pi x_{i+1})| \ge \sqrt{4\sin^2\frac{\pi}{5} - (2/4)^2} = \sqrt{9 - 2\sqrt{5}}/2 > 1.$$

Thus

$$2 \ge |\sin(2\pi x_{e(1)}) - \sin(2\pi x_1)| > 1 + 0.6(e(1) - 2),$$

$$|\sin(2\pi x_i) - \sin(2\pi x_{i+1})| \ge \sqrt{4\sin^2\frac{\pi}{5} - (3/5)^2} = \sqrt{(4.64 - \sqrt{5})/2} > 1.$$

Thus $2 \ge 1.1 + (e(1) - 2)$ which implies $e(1) \le 2$.

3.3. The case $b = 4$. We use $\sin(\pi/4) = \sqrt{2}/2$. By (3.6), for each $1 \le i < e(1)$, since $\gamma < 1$,

$$|\sin(2\pi x_i) - \sin(2\pi x_{i+1})| \ge \sqrt{4\sin^2\frac{\pi}{4} - (4/3)^2} = \frac{\sqrt{2}}{3}.$$

Moreover, by (3.8), if $k_i = k$ or $k_{i+1} = k$, then

$$|\sin(2\pi x_i) - \sin(2\pi x_{i+1})| \ge \sqrt{4\sin^2\frac{\pi}{4} - (2/3)^2} = \frac{\sqrt{14}}{3}.$$

Thus

$$2 \ge |\sin(2\pi x_{e(1)}) - \sin(2\pi x_1)| \ge \frac{\sqrt{14}}{3} + (e(1) - 2)\frac{\sqrt{2}}{3},$$

which implies $e(1) \le 3$. Therefore, either $e(1) < \gamma b$ or $\gamma \le 3/4$. Assume the latter. Then by (3.6), for each $1 \le i < e(1)$ we have

$$|\sin(2\pi x_i) - \sin(2\pi x_{i+1})| \ge \sqrt{4\sin^2\frac{\pi}{4} - 1} = 1.$$

Thus $2 \ge \sqrt{14}/3 + (e(1) - 2)$, which implies $e(1) \le 2$.

3.4. The case $b = 3$. We use $\sin(\pi/3) = \sqrt{3}/2$. We claim that for each $x \in [0,1]$, $E(1,x) \ne \{0,1,2\}^2$, so that by Lemma 2.7, $\sigma(1) \le \sqrt{2} + 1$. Otherwise, there exists $x \in [0,1]$ such that $E(1,x) = \{0,1,2\}^2$. Using the notation introduced above, for any $1 \le i < j \le 3$, as in (3.8), we have

$$|\sin(2\pi x_i) - \sin(2\pi x_j)| \ge \sqrt{4\sin^2\frac{\pi}{3} - \frac{4}{4}} = \sqrt{2},$$

which contradicts the fact

$$2 \ge \sin(2\pi x_3) - \sin(2\pi x_1) = |\sin(2\pi x_3) - \sin(2\pi x_2)| + |\sin(2\pi x_2) - \sin(2\pi x_1)|.$$

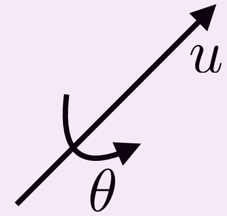
Assume $\sigma(1) \ge \gamma b$. Then $\gamma < (1 + \sqrt{2})/3 < 0.81$. Keep the notation x_i, $e(1)$ as above. By (3.6), for each $1 \le i < e(1)$ we have

$$|\sin(2\pi x_i) - \sin(2\pi x_{i+1})| \ge \sqrt{4\sin^2\frac{\pi}{3} - \frac{16\gamma^2}{(3-\gamma)^2}} > 0.9.$$

By (3.8), if $k_i = k$ or $k_{i+1} = k$, then $|\sin(2\pi x_i) - \sin(2\pi x_{i+1})| \ge \sqrt{2}$. Thus $2 \ge \sqrt{2} + (e(1) - 2) \cdot 0.9$ which implies that $e(1) \le 2$.

Rotation $R_{u,\theta}$:

axis u, angle θ



$$\begin{pmatrix} q_\mathrm{r}^2+q_\mathrm{i}^2-q_\mathrm{j}^2-q_\mathrm{k}^2 & 2q_\mathrm{i}q_\mathrm{j}-2q_\mathrm{r}q_\mathrm{k} & 2q_\mathrm{i}q_\mathrm{k}+2q_\mathrm{r}q_\mathrm{j} \\ 2q_\mathrm{i}q_\mathrm{j}+2q_\mathrm{r}q_\mathrm{k} & q_\mathrm{r}^2-q_\mathrm{i}^2+q_\mathrm{j}^2-q_\mathrm{k}^2 & 2q_\mathrm{j}q_\mathrm{k}-2q_\mathrm{r}q_\mathrm{i} \\ 2q_\mathrm{i}q_\mathrm{k}-2q_\mathrm{r}q_\mathrm{j} & 2q_\mathrm{j}q_\mathrm{k}+2q_\mathrm{r}q_\mathrm{i} & q_\mathrm{r}^2-q_\mathrm{i}^2-q_\mathrm{j}^2+q_\mathrm{k}^2 \end{pmatrix}$$

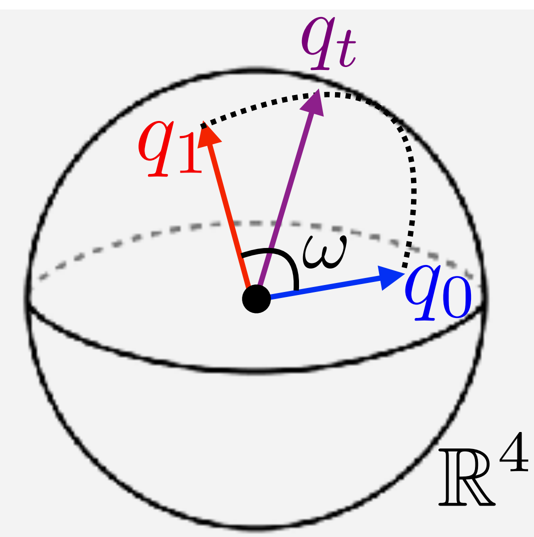
Unit quaternion:

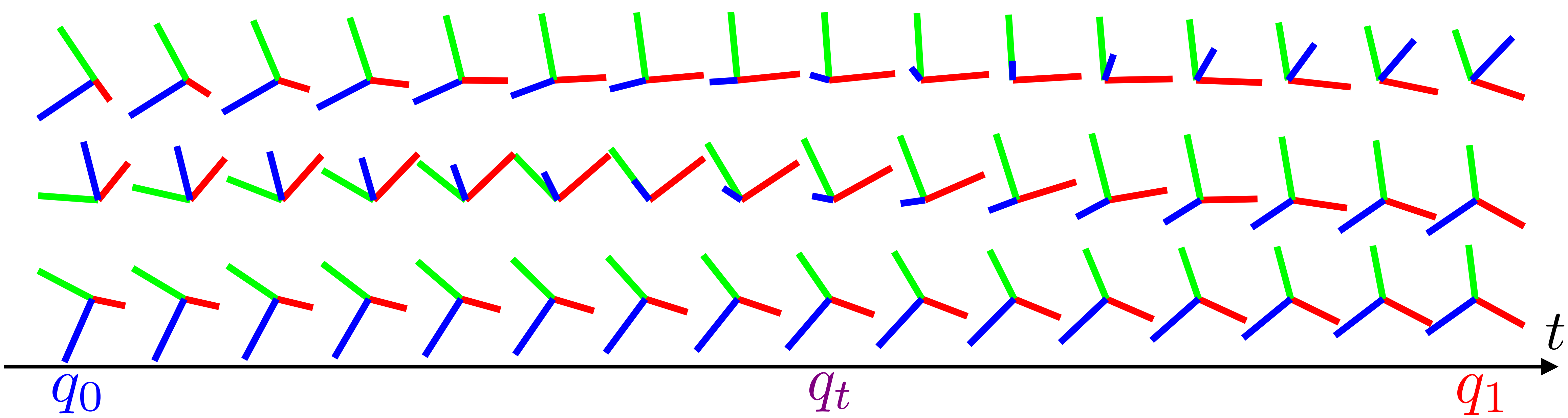
$$q = \cos(\theta/2) + (u_x\mathrm{i} + u_y\mathrm{j} + u_z\mathrm{k})\sin(\theta/2).$$

$$q = q_\mathrm{r} + q_\mathrm{i}\mathrm{i} + q_\mathrm{j}\mathrm{j} + q_\mathrm{k}\mathrm{k}$$

Spherical Linear Interpolation (SLERP):

$$q_t \overset{\text{def.}}{=} \frac{\sin((1-t)\omega)q_0 + \sin(t\omega)q_1}{\sin(\omega)}$$



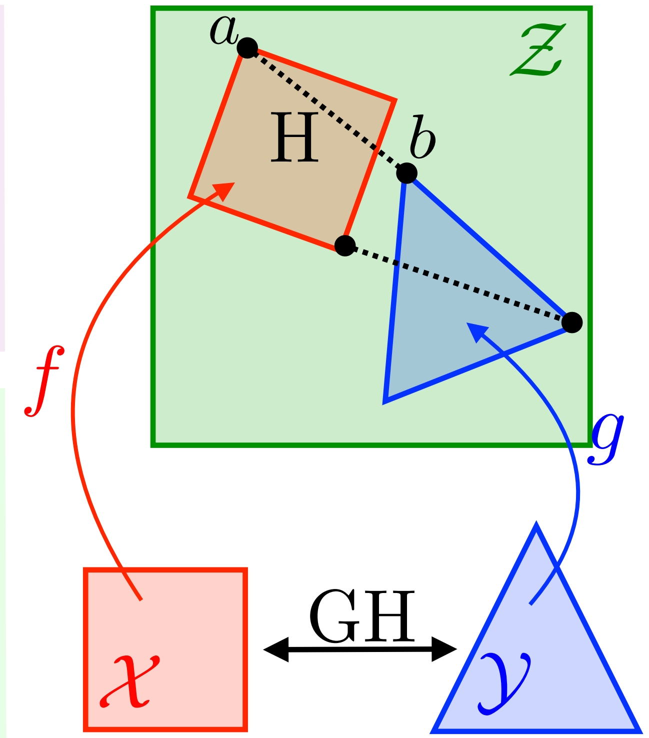


Hausdorff distance on $(\mathcal{Z}, d_\mathcal{Z})$:

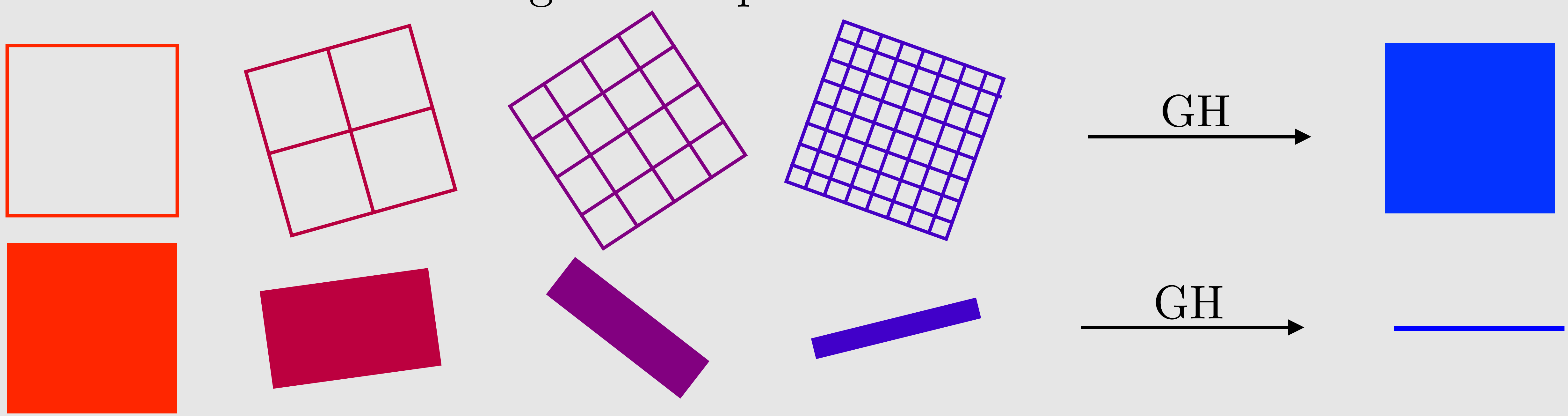
$$d_{\mathrm{H}}^{\mathcal{Z}}(A, B) \stackrel{\text{def.}}{=} \max\left(\sup_{a \in A} \inf_{b \in B} d_\mathcal{Z}(a, b), \sup_{b \in B} \inf_{a \in A} d_\mathcal{Z}(a, b), \right)$$

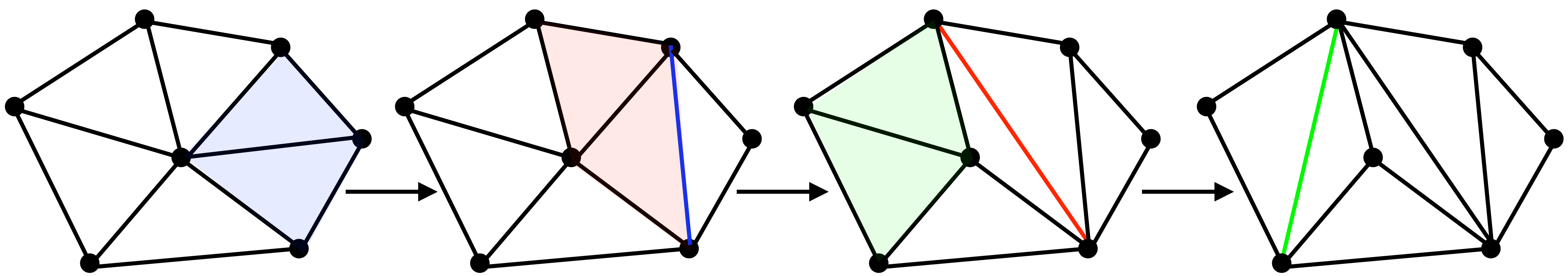
Gromov-Hausdorff distance between metric spaces:

$$d_{\mathrm{GH}}(\mathcal{X}, \mathcal{Y}) \stackrel{\text{def.}}{=} \inf_{\mathcal{Z}, f, g} \left\{ d_{\mathrm{H}}^{\mathcal{Z}}(f(\mathcal{X}), g(\mathcal{Y})) \; ; \; \begin{array}{l} f : \mathcal{X} \xrightarrow{\text{isom}} \mathcal{Z} \\ g : \mathcal{Y} \xrightarrow{\text{isom}} \mathcal{Z} \end{array} \right\}$$



Gromov-Hausdorff convergence of spaces:

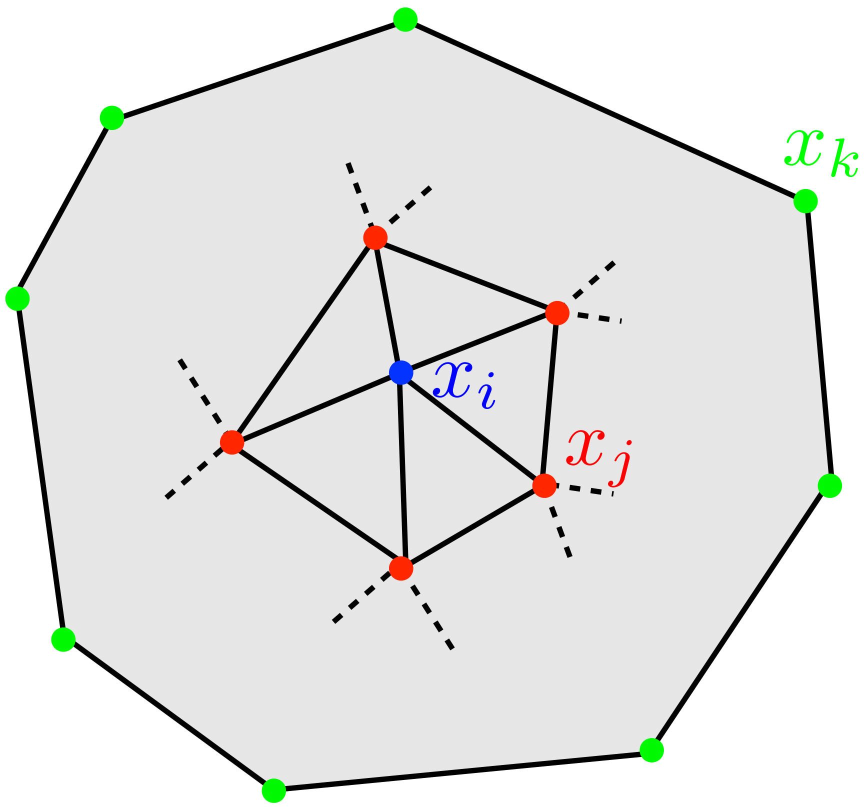




Theorem: Any two planar triangulations of n vertices are connected by $O(n^k)$ edge flips.

[Klaus Wagner 1936] $k \leqslant 2$.

[Hideo Komuro 1997] $k = 1$.



$$\forall x_i \in \mathrm{Int}(D),\ x_i = \frac{\sum_{j\sim i} x_j}{|\{j\ ;\ j\sim i\}|}$$

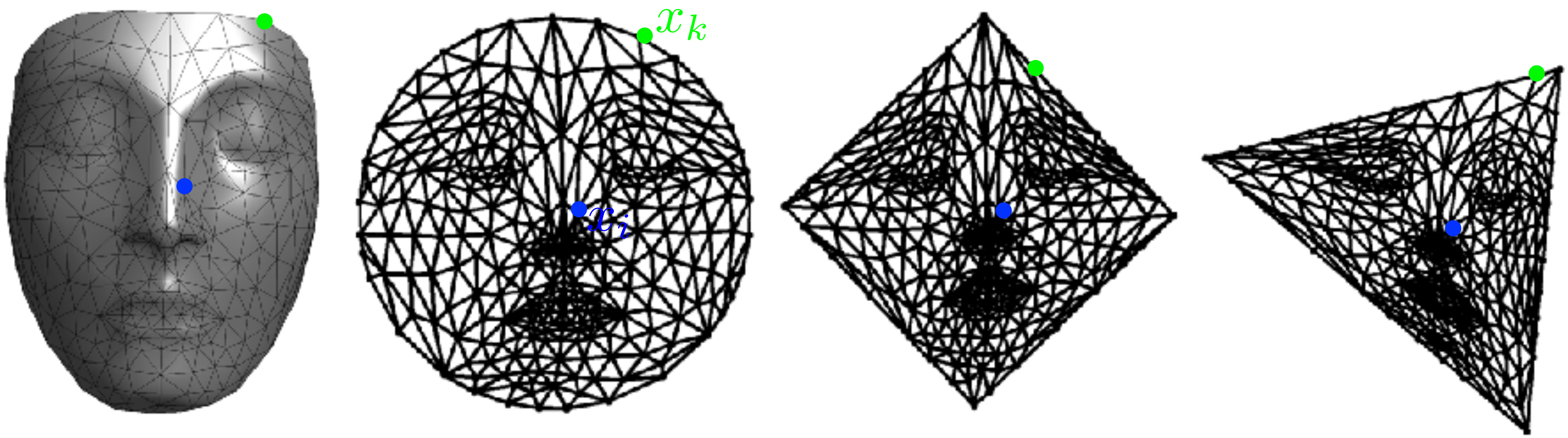
$$\forall x_k \in \mathrm{Bound}(D),\ x_k \leftarrow \text{fixed}$$

Theorem: [William Tutte, 1968]

if $\{i \sim j\}_{i,j}$ is a planar graph,

and B is convex

$i \mapsto x_i \in \mathbb{R}^2$ is a valid embedding.

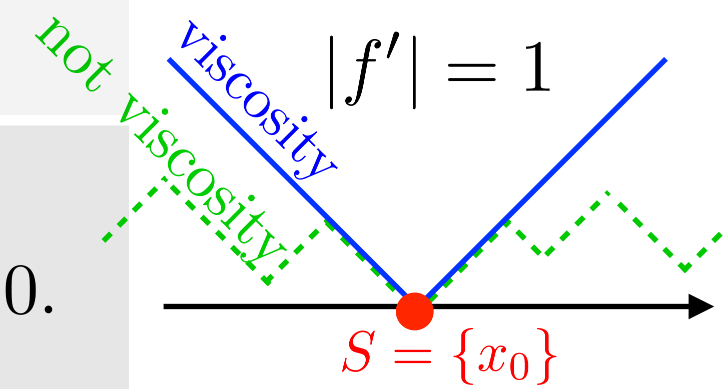


Distance to a set: $\quad f_S(x) \overset{\text{def.}}{=} \min_{y \in S} \|x - y\|$

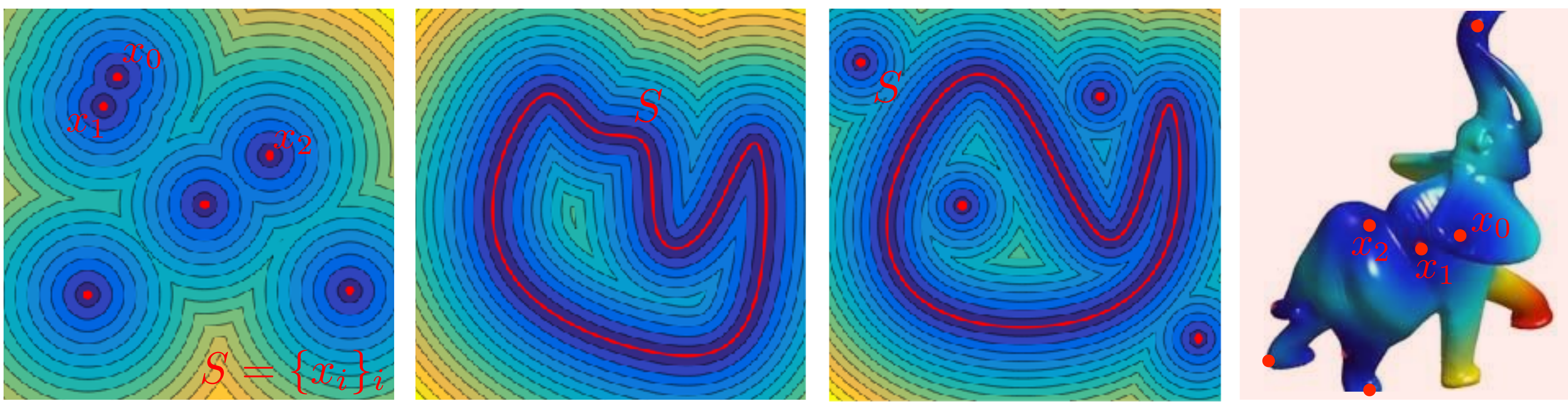
Theorem: f_S is the unique viscosity solution of

$$\forall\, x \notin S,\ \|\nabla f(x)\| = 1 \quad \text{and} \quad \forall\, x_0 \in S,\ f(x_0) = 0.$$

[Crandall, Ishii, Lions]

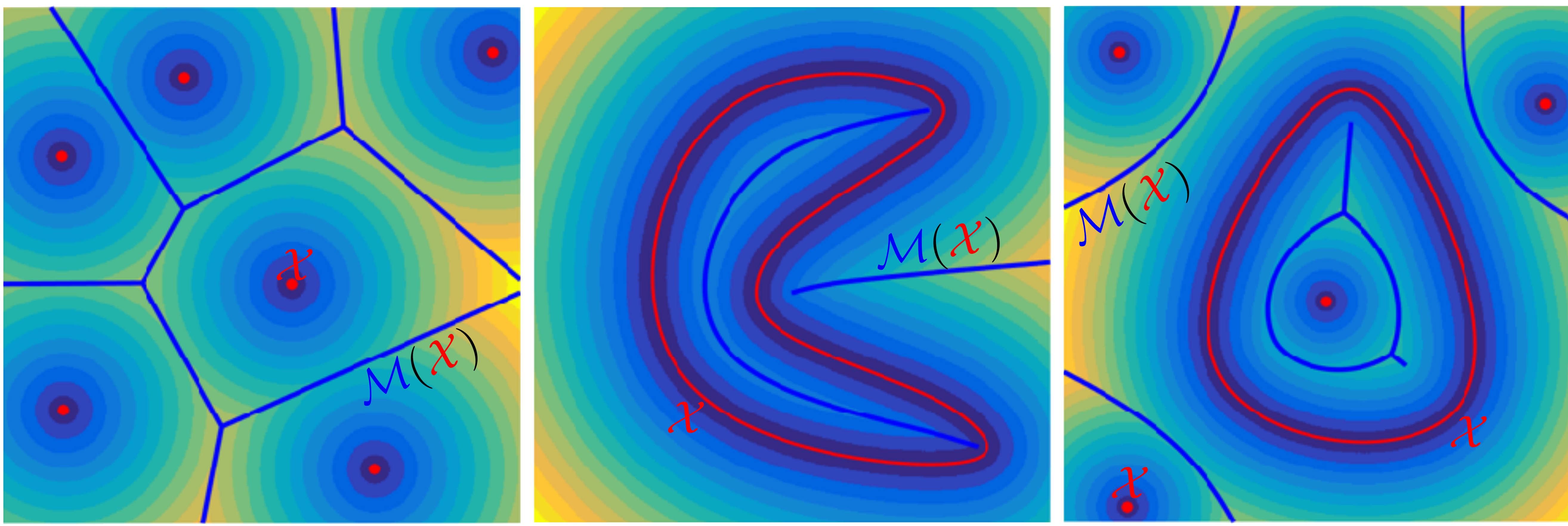


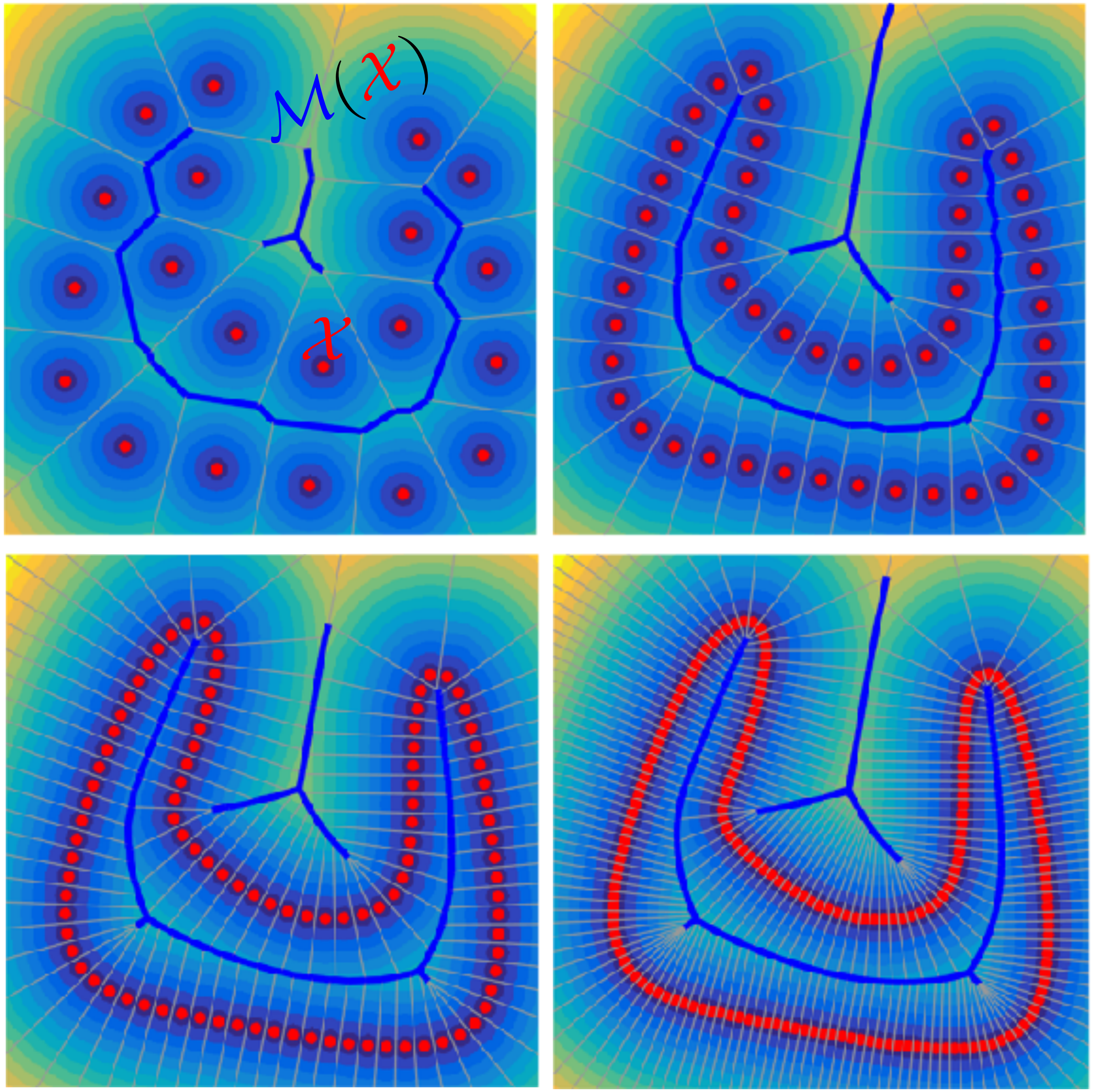
Generalization to Riemannian manifold (f_S =geodesic distance).

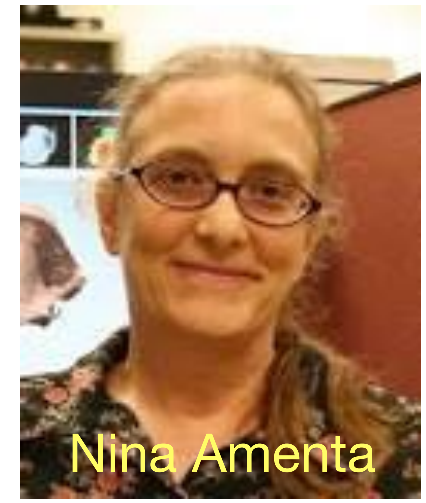


Distance function: $d_{\mathcal{X}}(y) \stackrel{\text{def.}}{=} \min_{x \in \mathcal{X}} \|x - y\|$

Medial axis: $\mathcal{M}(\mathcal{X}) \stackrel{\text{def.}}{=} \{y \,;\, \exists x \neq x' \in \mathcal{X}, d_{\mathcal{X}}(y) = \|x - y\| = \|x' - y\|\}$



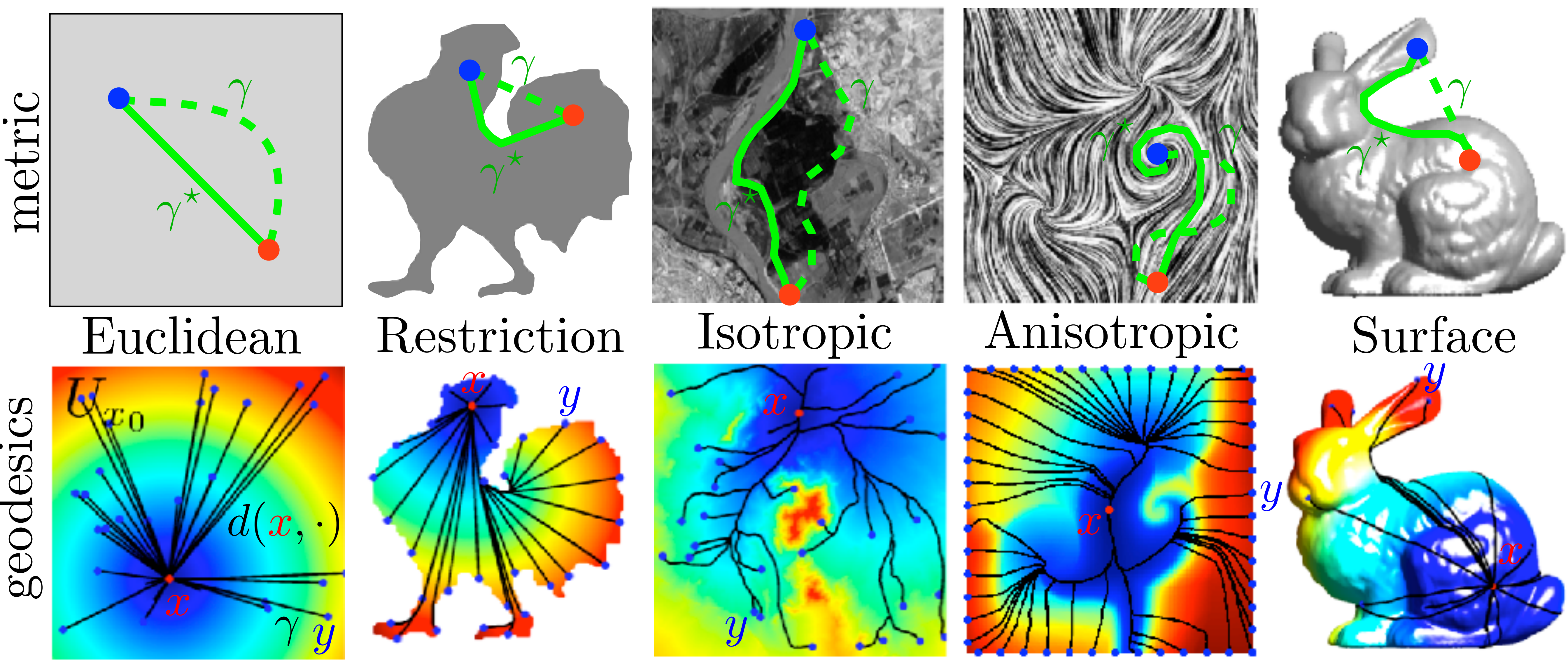

M(X)
X


Nina Amenta

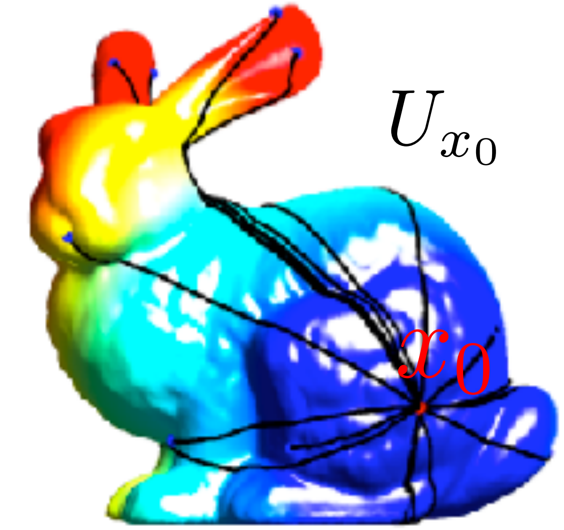

Tamal Dey

Geodesic distance: $d(x, y) = \min_{\gamma : x \to y} \text{Length}(\gamma)$

Geodesic curve: $\gamma^\star$ such that $\text{Length}(\gamma^\star) = d(x, y)$.



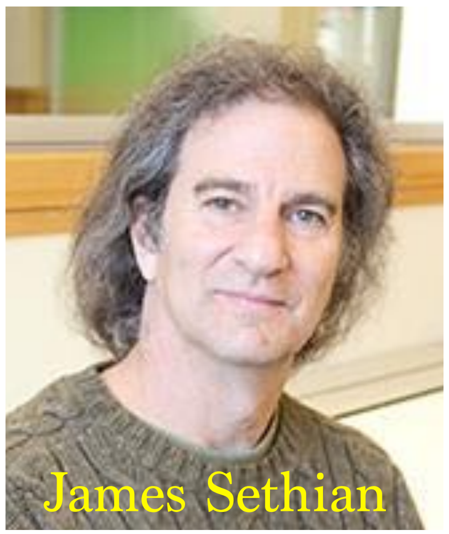
Distance map to a point: $U_{x_0}(x) = d_{\mathcal{M}}(x_0, x)$.

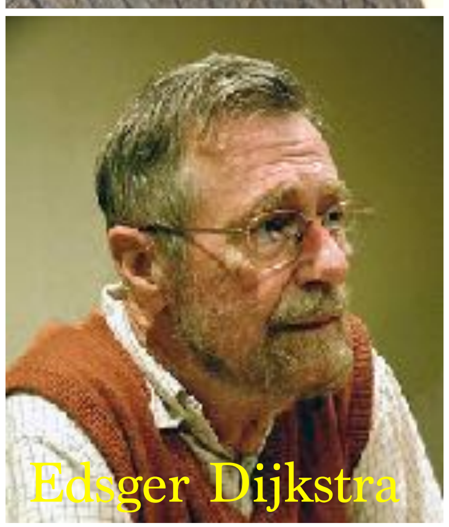


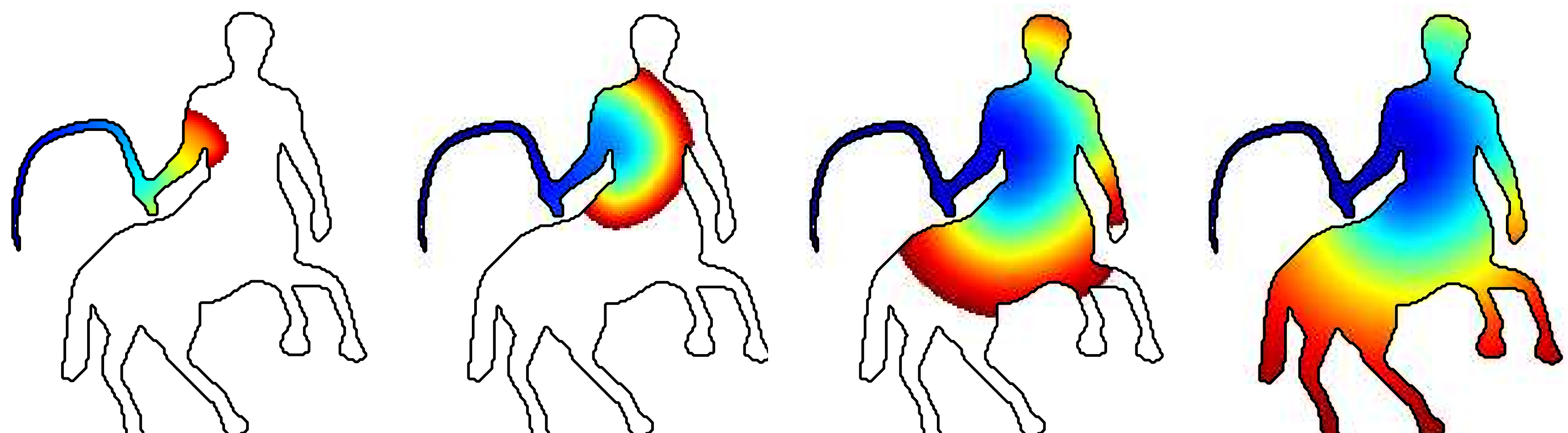
Non-linear PDE: (viscosity)

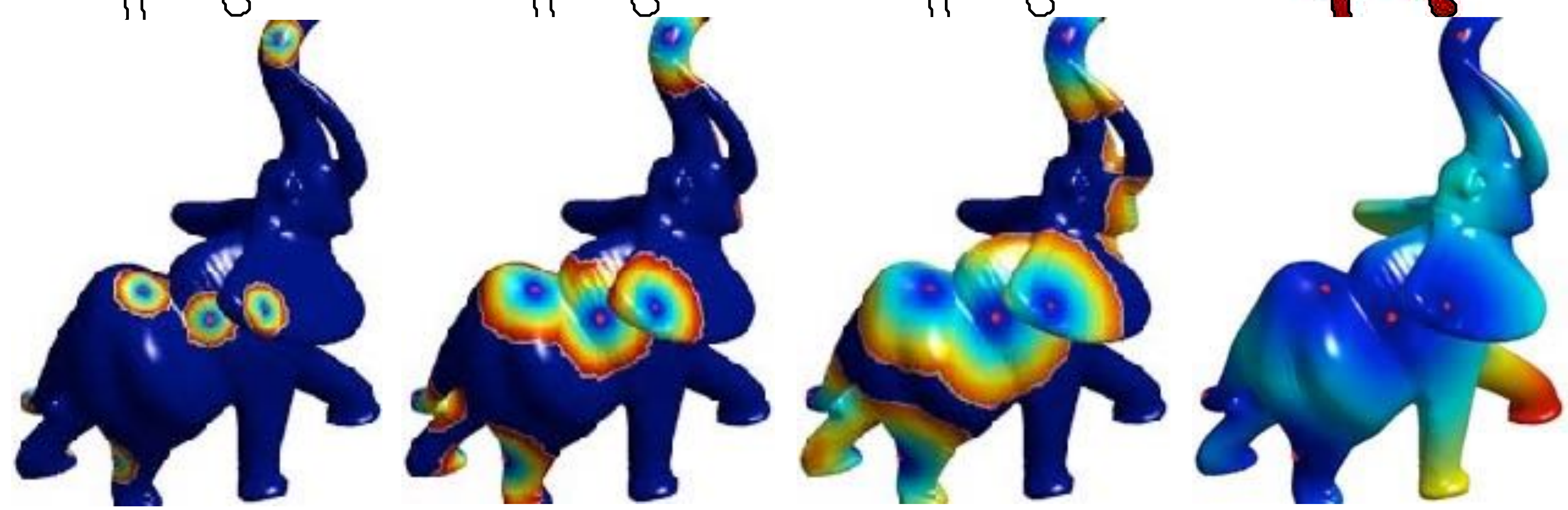
$$\begin{cases} \|\nabla U_{x_0}(x)\| = W(x) \\ U_{x_0}(x_0) = 0, \end{cases}$$

Fast Marching: front propagation in $O(N \log(N))$ operations.





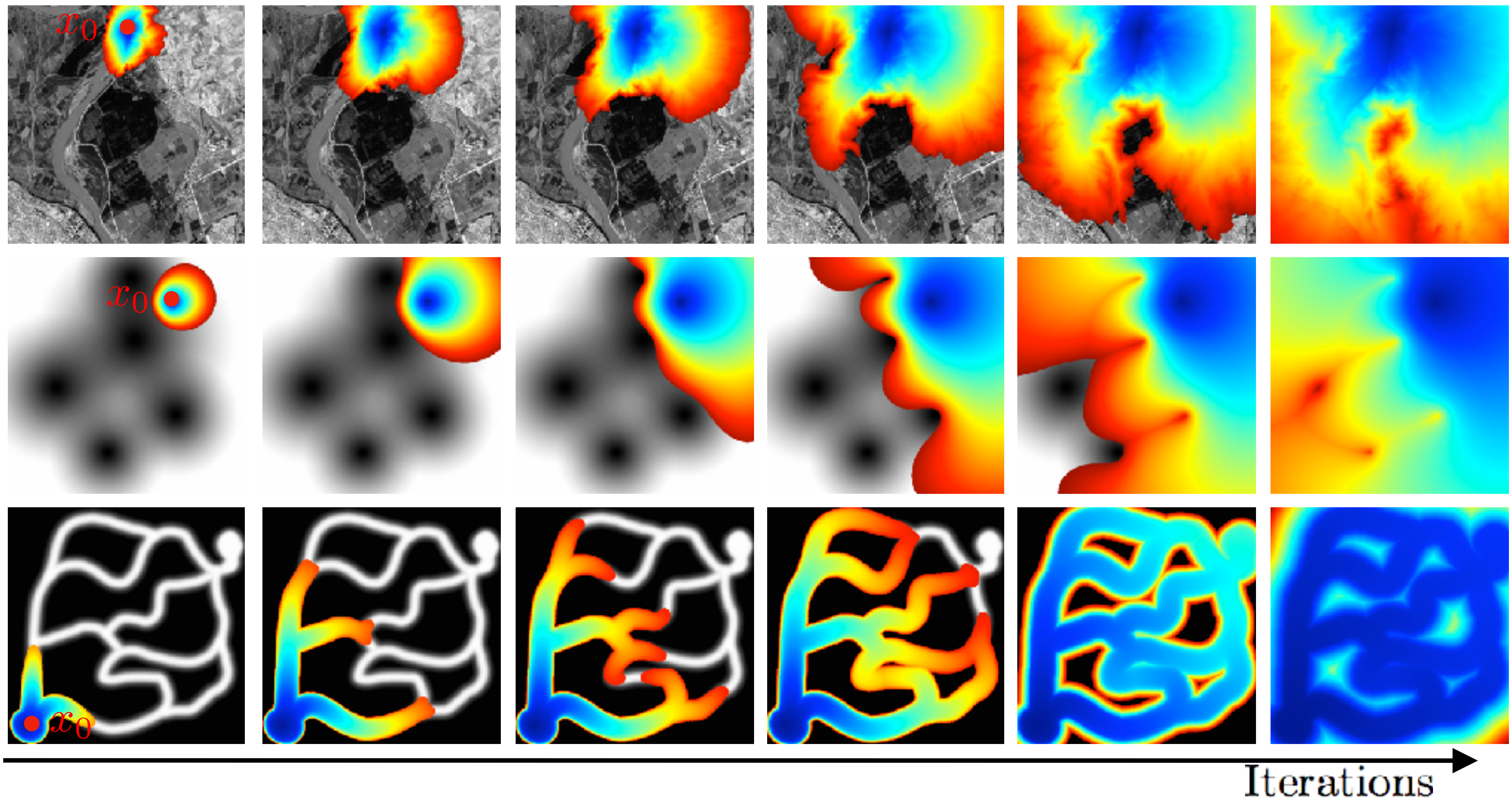




Length of a curve γ: $L(\gamma) \stackrel{\text{def.}}{=} \int_0^1 W(\gamma(t))\|\gamma'(t)\|\mathrm{d}t$

Geodesic distance to x_0: $d_{x_0}(x) \stackrel{\text{def.}}{=} \min_{\gamma} \{L(\gamma) \; ; \; \gamma(0) = x_0, \gamma(1) = x\}$

Eikonale equation: $\|\nabla d_{x_0}(x)\| = W(x)$



Init: $D^{(0)}_{i,j} \begin{cases} \in \mathbb{R} \text{ if } i \sim j, \\ = +\infty \text{ otherwise.} \end{cases}$

For $k = 1, \ldots, N$
  For $i = 1, \ldots, N$
    For $j = 1, \ldots, N$

$$D^{(k)}_{i,j} \overset{\text{def.}}{=} \min(D^{(k-1)}_{i,j}, D^{(k-1)}_{i,k} + D^{(k-1)}_{k,j})$$

Theorem: $D^{(N)}_{i,j} = \text{ShortestDist}(i,j)$

Complexity N^3.

$(\min,+)$ matrix multiplication:

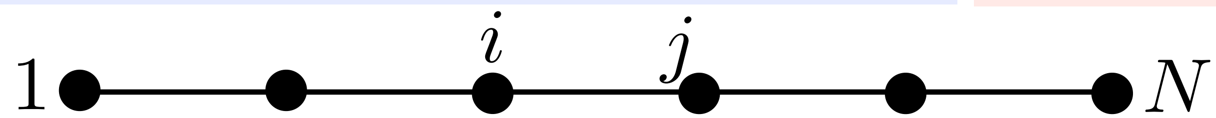
$$(A \star B)_{i,j} \overset{\text{def.}}{=} \min_k A_{i,k} + B_{k,j}$$

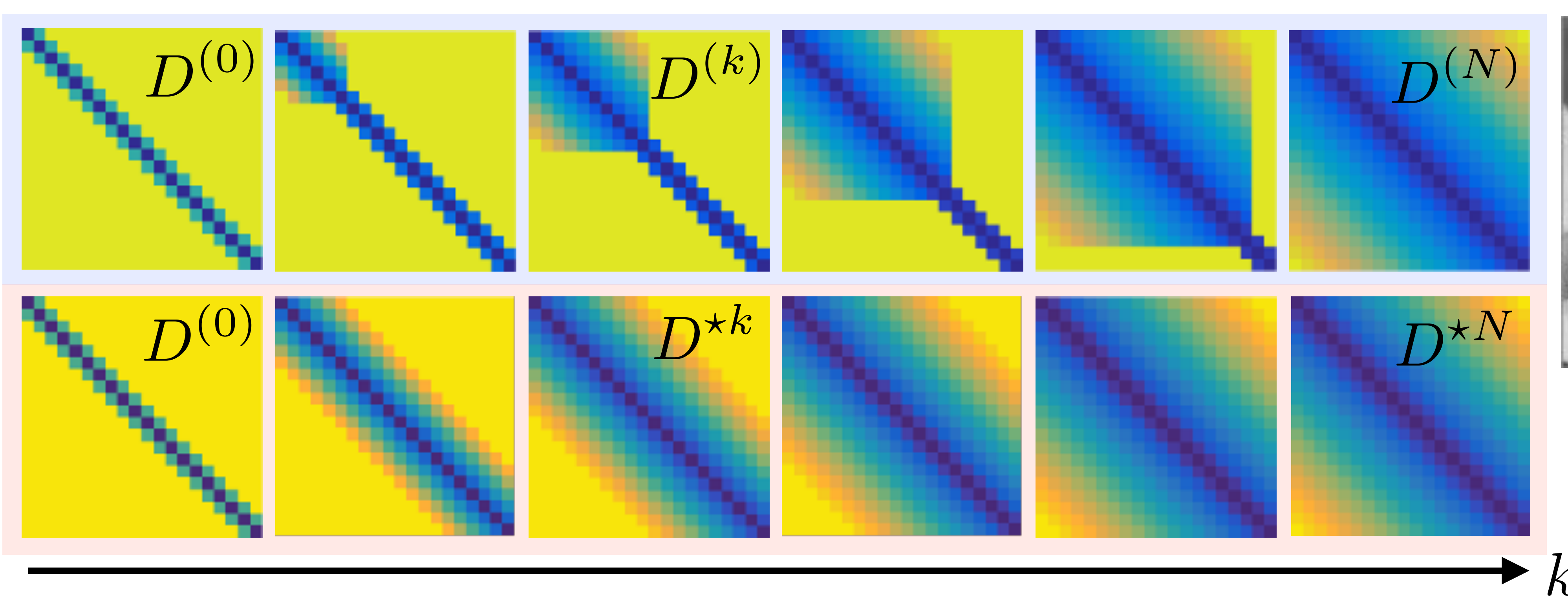
$$D^{\star k} \overset{\text{def.}}{=} \underbrace{D^{(0)} \star \ldots \star D^{(0)}}_{k \text{ times}}$$

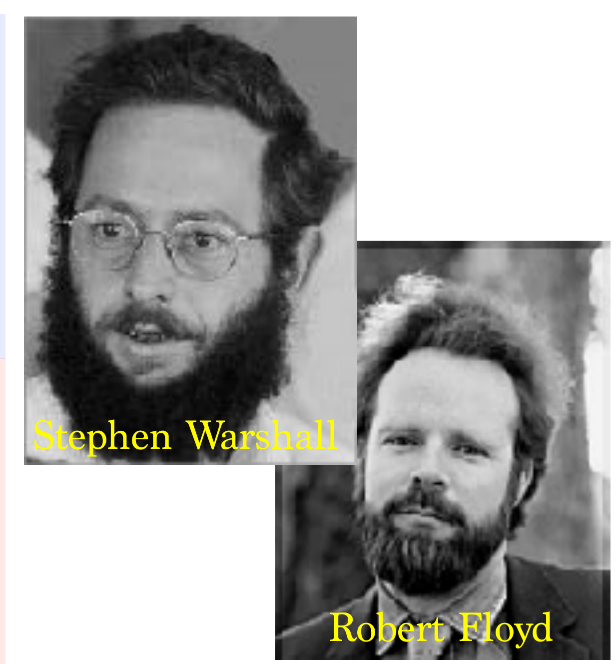
Theorem:
$D^{\star k}_{i,j}$ distance btw (i,j)
with paths of length $\leqslant k$

$$\Longrightarrow \quad D^{\star N} = D^{(N)}$$

Example:



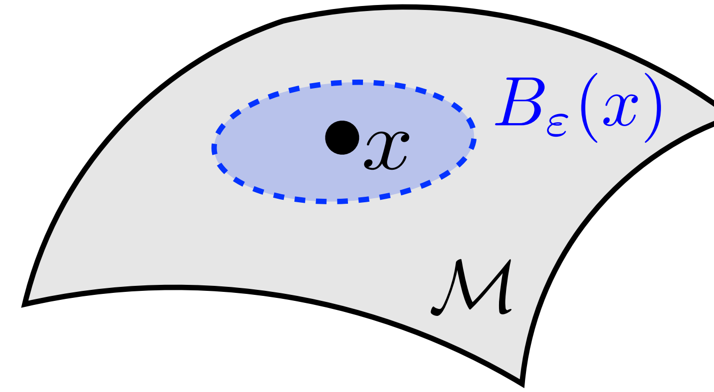




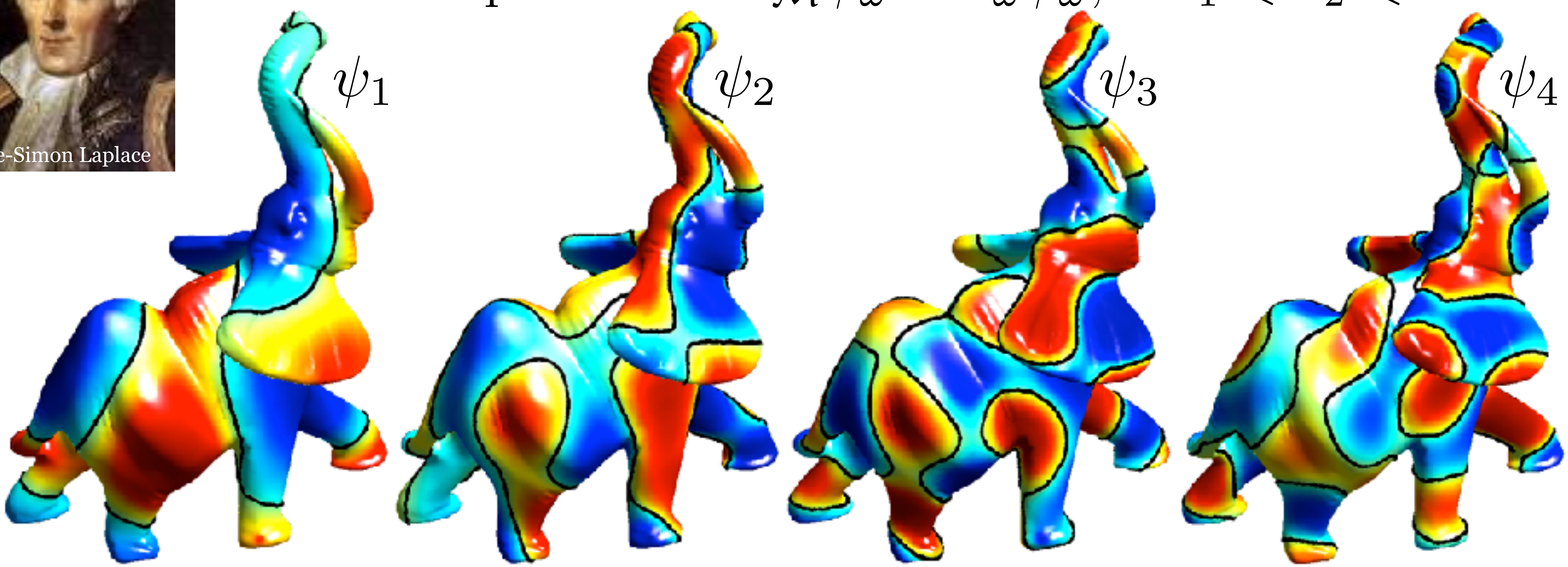
$\mathcal{M}$ compact manifold

Laplace-Beltrami operator:

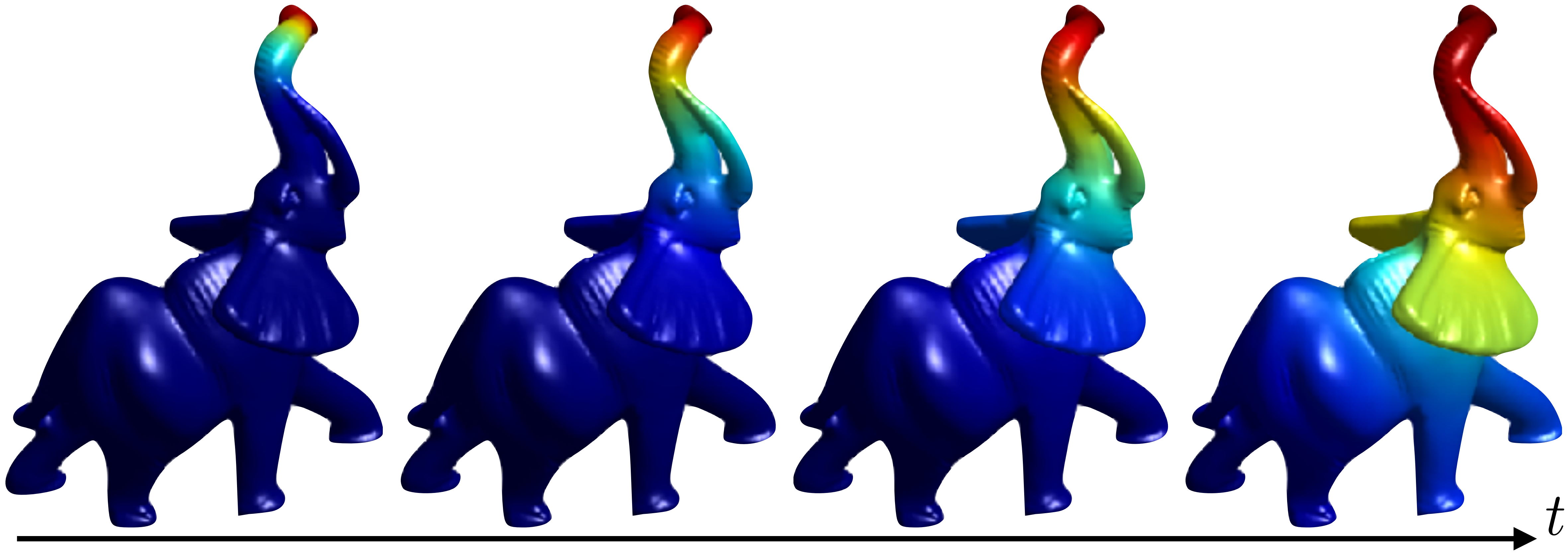
$$(\Delta_{\mathcal{M}} f)(x) = \lim_{\varepsilon \to 0} f(x) - \frac{1}{|B_\varepsilon(x)|} \int_{B_\varepsilon(x)} f(x) \mathrm{d}x$$



Discrete spectrum: $\Delta_{\mathcal{M}} \psi_\omega = \lambda_\omega \psi_\omega, \quad \lambda_1 \leqslant \lambda_2 \leqslant \ldots$



Heat equation on a manifold $\mathcal{M}$: $\partial_t u_t = \Delta_{\mathcal{M}} u_t, \quad u_0(x, \cdot) = \delta_x$

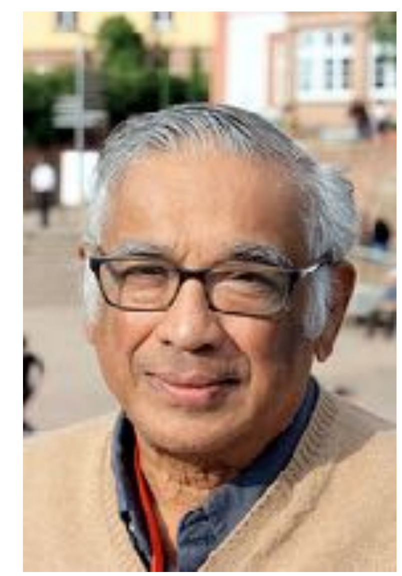


$$\text{On } \mathcal{M} = \mathbb{R}^d: \quad u_t(y) = \frac{1}{\sqrt{4\pi t}} e^{-\frac{\|x-y\|^2}{4t}}$$

Theorem:
[Srinivasa Varadhan, 1967]

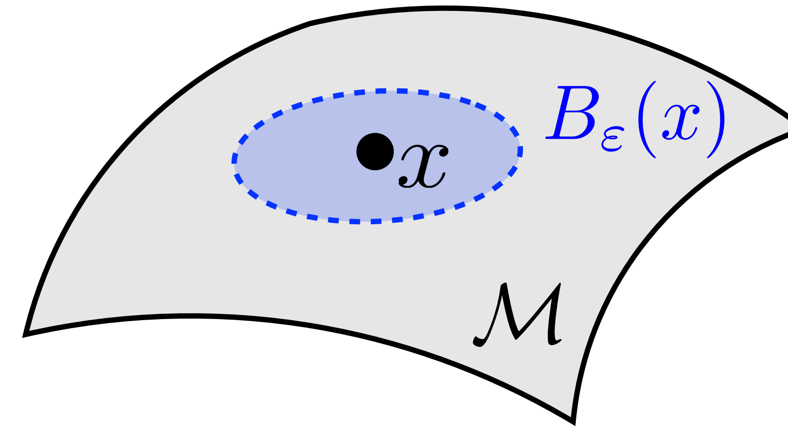
$$-4t \log(u_t(y)) \xrightarrow{t \to 0} d_{\mathcal{M}}(x, y)^2$$

(geodesic distance)



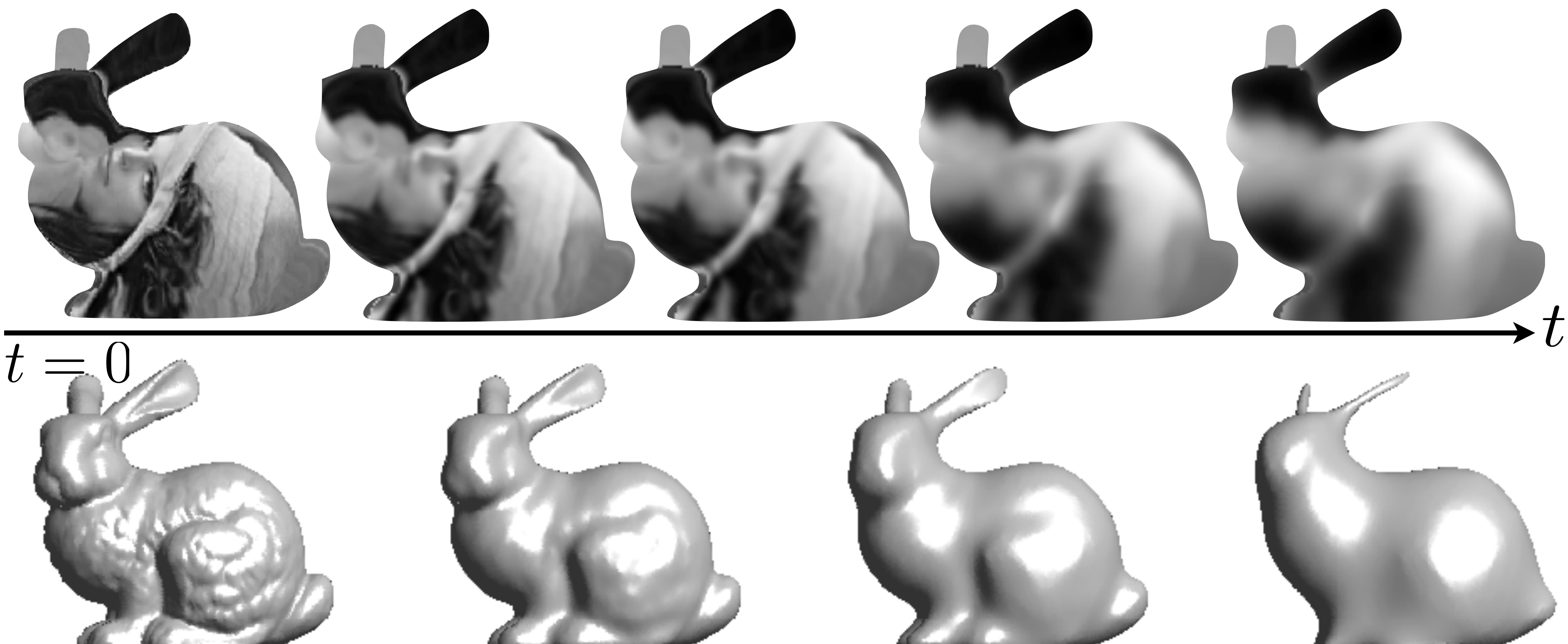
Laplace-Beltrami operator:

$$(\Delta_{\mathcal{M}} f)(x) = \lim_{\varepsilon \to 0} f(x) - \frac{1}{|B_\varepsilon(x)|} \int_{B_\varepsilon(x)} f(x) \mathrm{d}x$$



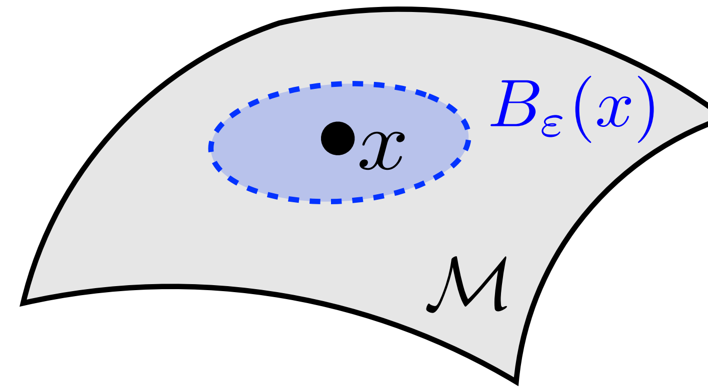
Wave equation on manifold $\mathcal{M}$:

$$\frac{\partial f}{\partial t} = \Delta_{\mathcal{M}} f \qquad \text{and} \quad f_{t=0} = f_0$$



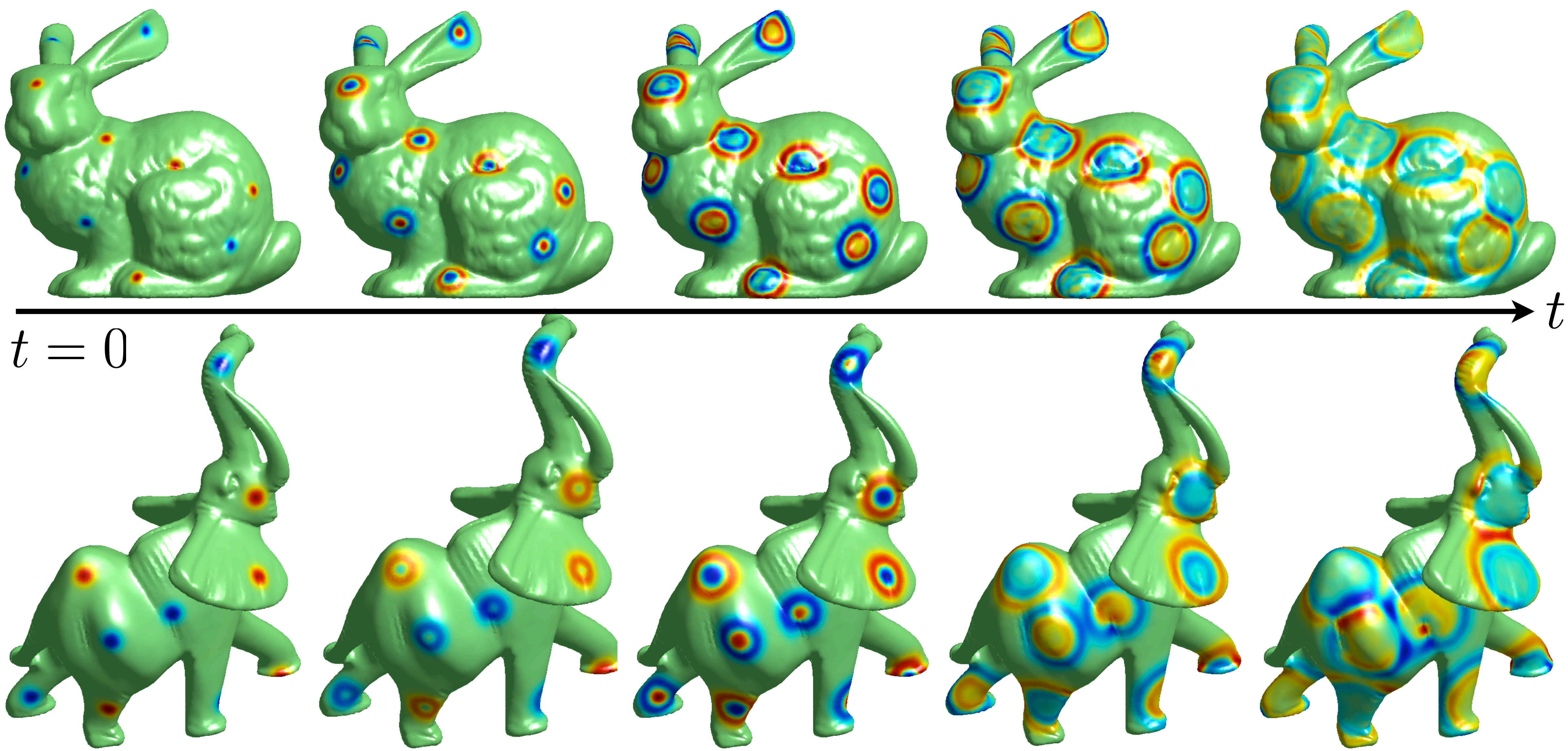
Laplace-Beltrami operator:

$$(\Delta_{\mathcal{M}} f)(x) = \lim_{\varepsilon \to 0} f(x) - \frac{1}{|B_\varepsilon(x)|} \int_{B_\varepsilon(x)} f(x) \mathrm{d}x$$



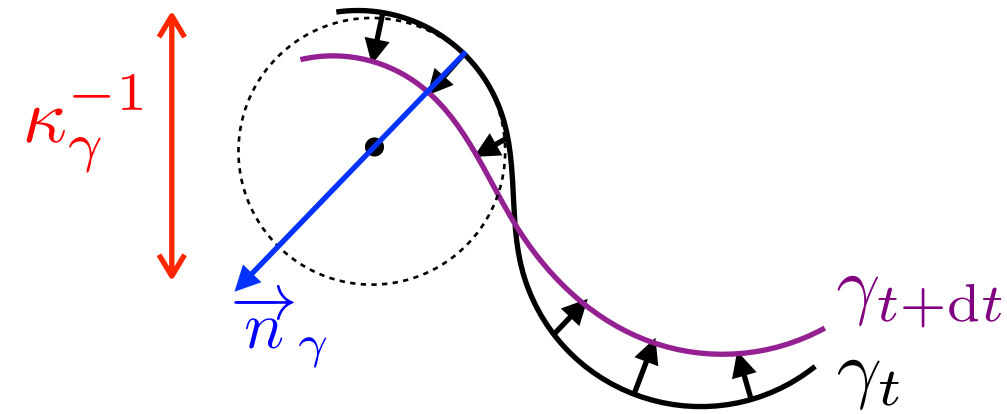
Wave equation on manifold $\mathcal{M}$:

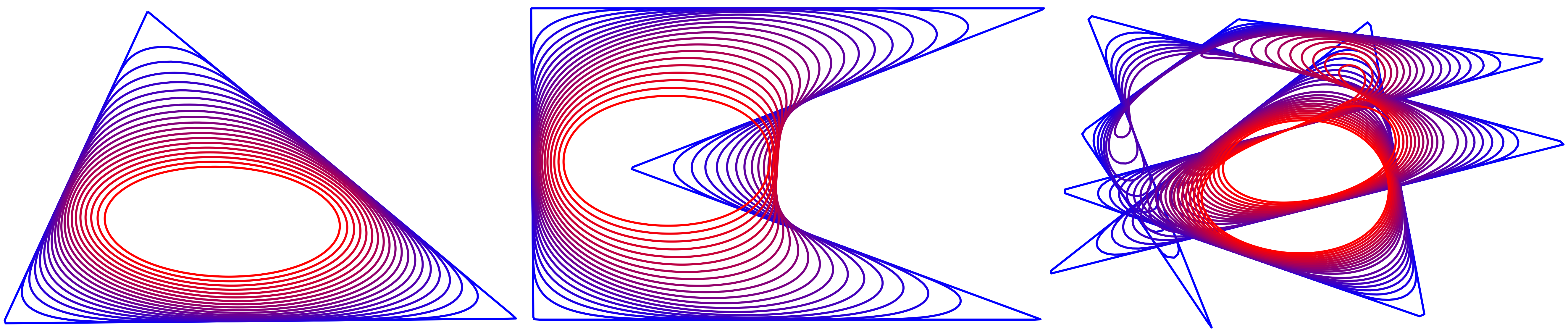
$$\frac{\partial^2 f}{\partial t^2} = \Delta_{\mathcal{M}} f \quad \text{and} \quad \left\{ \begin{array}{l} f_{t=0} = f_0 \\ f'_{t=0} = g_0 \end{array} \right.$$



Mean curvature motion:

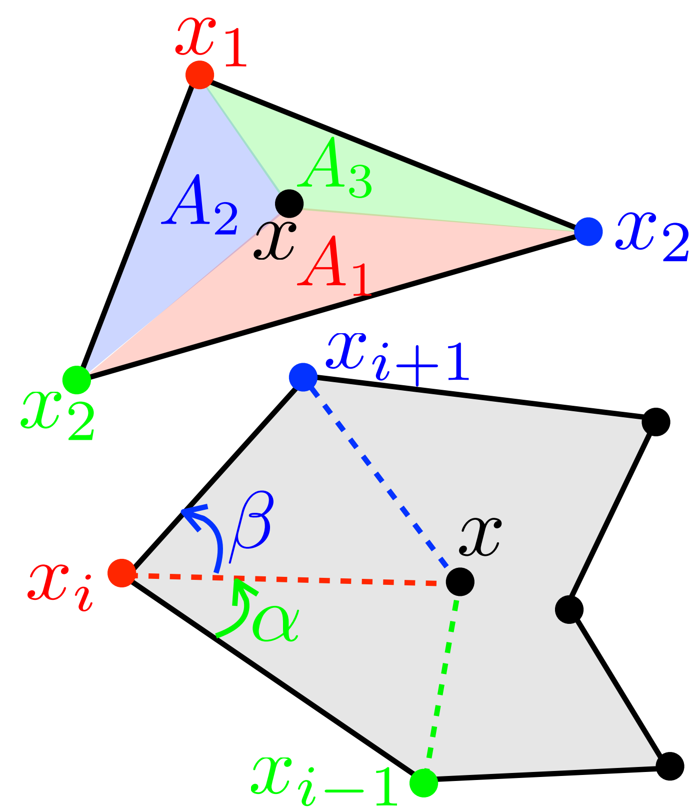
$$\frac{\partial \gamma}{\partial t} = \kappa_\gamma \overrightarrow{n}_\gamma = \begin{array}{c}\text{normal}\\ \times \\ \text{curvature}\end{array}$$





Barycentric coordinates: $x = \frac{\sum_i \lambda_i(x) x_i}{\sum_j \lambda_j(x)}$,

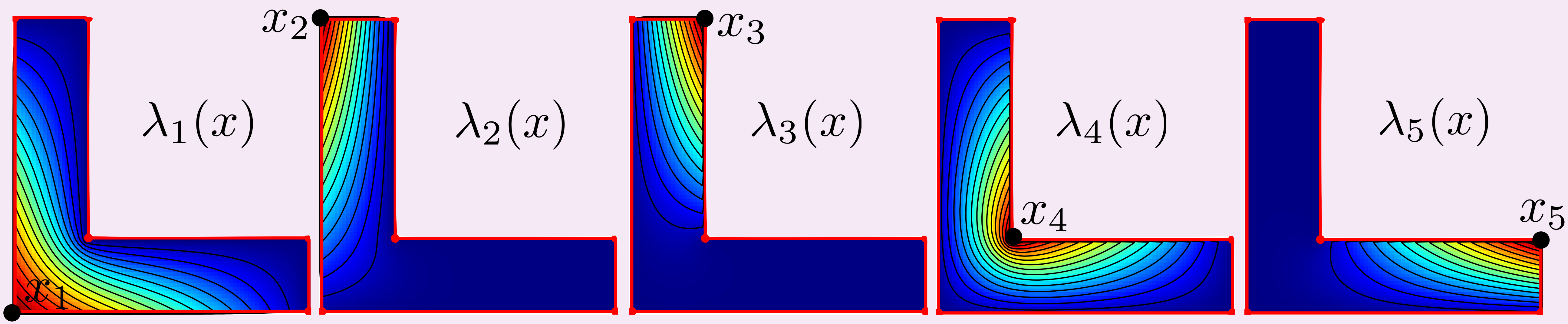
For 3 points: unique, $\lambda_i = \mathrm{Area}(A_i)$.



For $\geqslant 3$ points: mean-value coordinates:

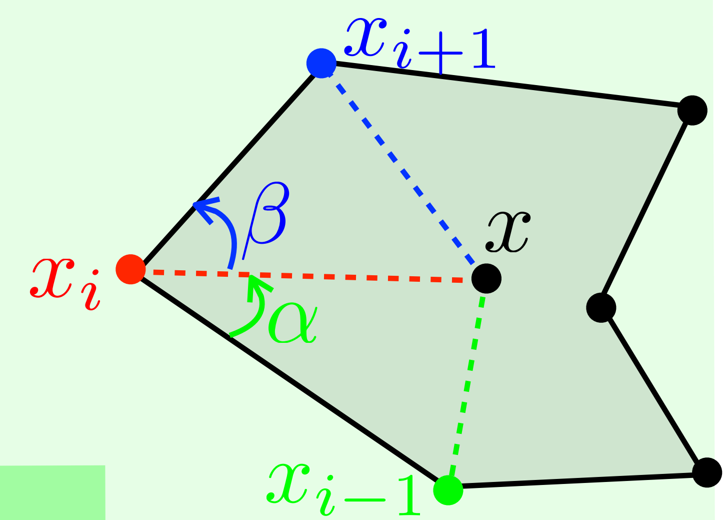
$$\lambda_i(x) \overset{\text{def.}}{=} \frac{\tan(\alpha/2) + \tan(\beta/2)}{\|x - x_i\|}$$

[Michael Floater, 2003]



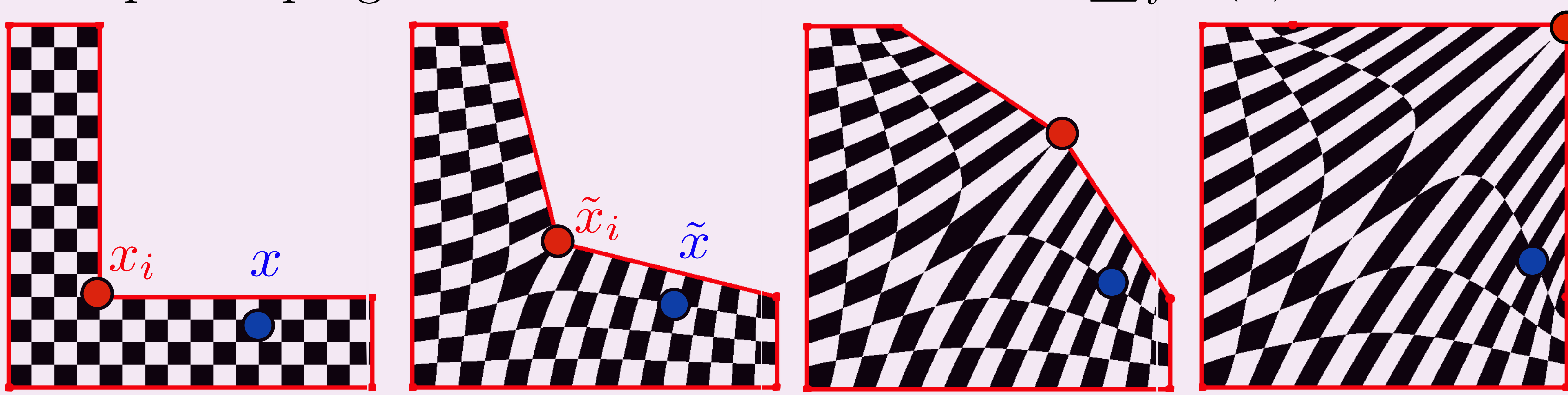
Mean-value coordinates: $\bar{\lambda}_i = \dfrac{\lambda_i}{\sum_j \lambda_j}$

$$\lambda_i(x) \overset{\text{def.}}{=} \frac{\tan(\alpha/2) + \tan(\beta/2)}{\|x - x_i\|}$$



Proposition: One has $\lambda_i \geqslant 0$ and $x = \sum_i \bar{\lambda}_i(x) x_i$

Shape warping: $x_i \mapsto \tilde{x}_i$ $\quad x \mapsto \tilde{x} \overset{\text{def.}}{=} \sum_i \bar{\lambda}_i(x) \tilde{x}_i$

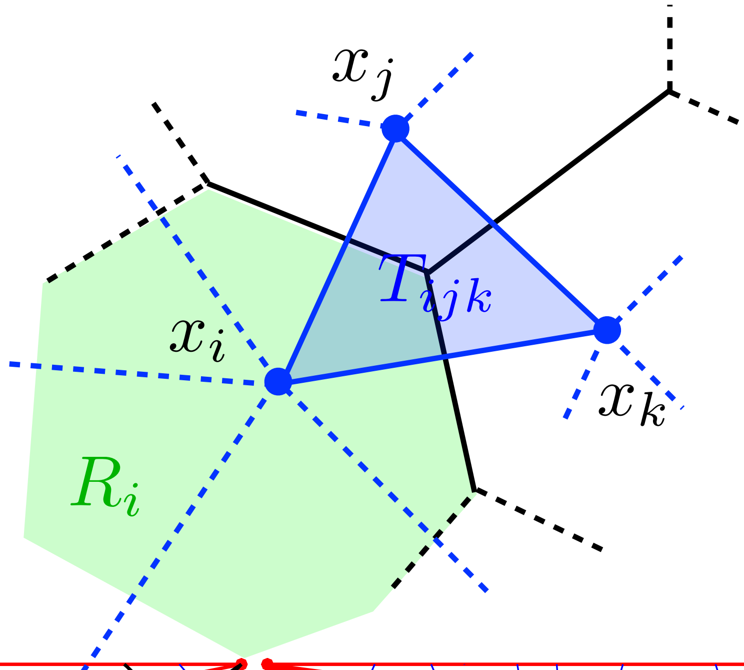


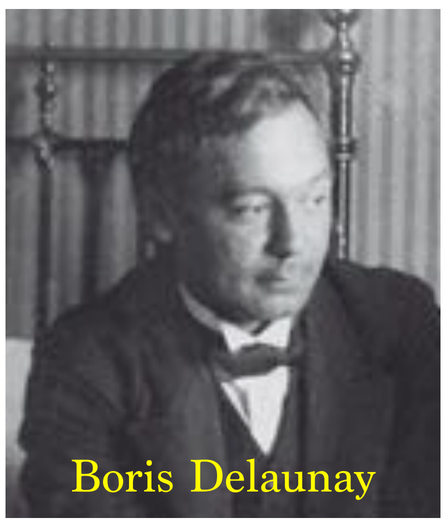
Voronoi diagram:

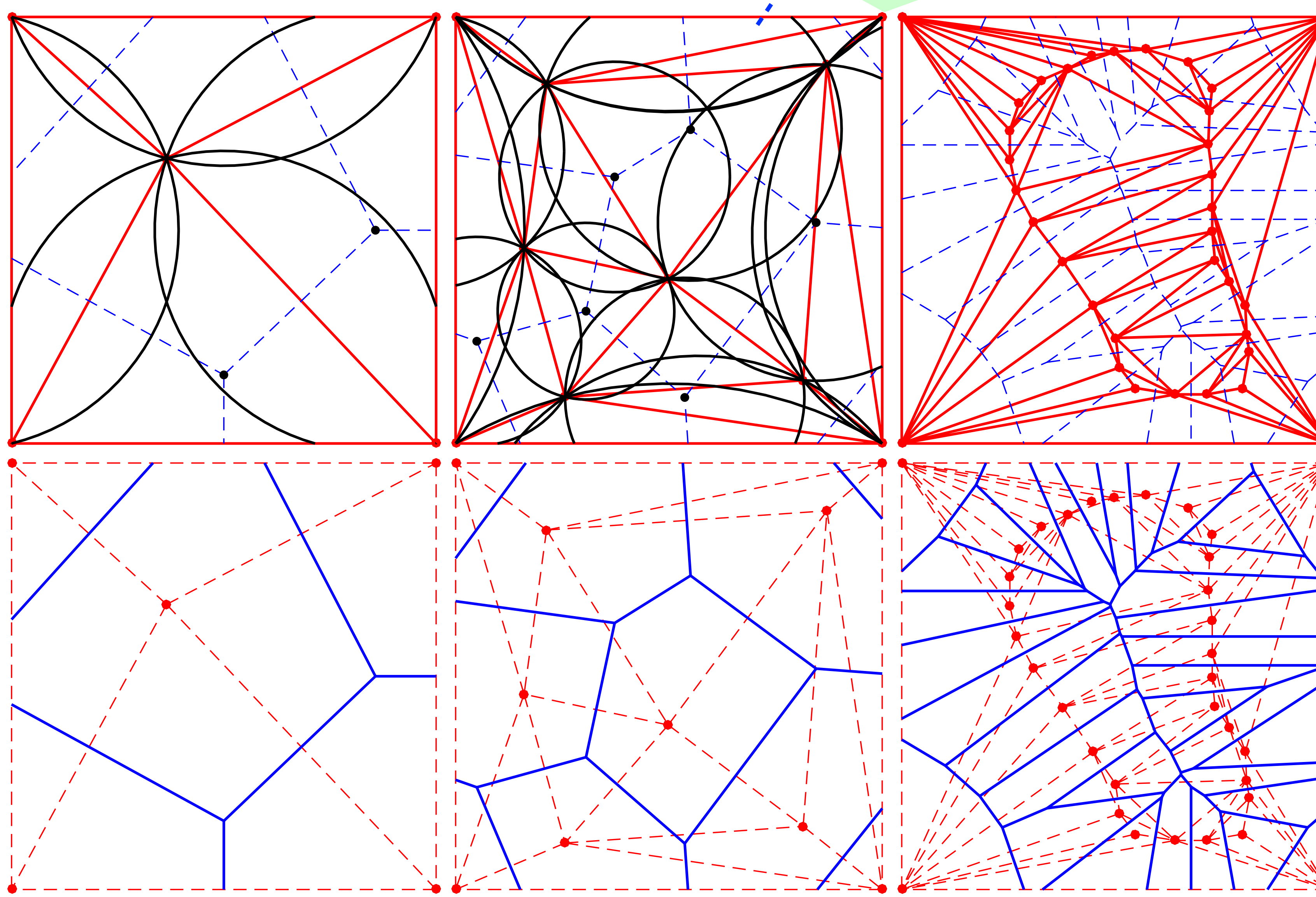
$$R_i \overset{\text{def.}}{=} \{x \;;\; \forall j \neq i, \|x_j - x\| \geqslant \|x_i - x\|\}$$

Delaunay triangulation: T_{ijk}

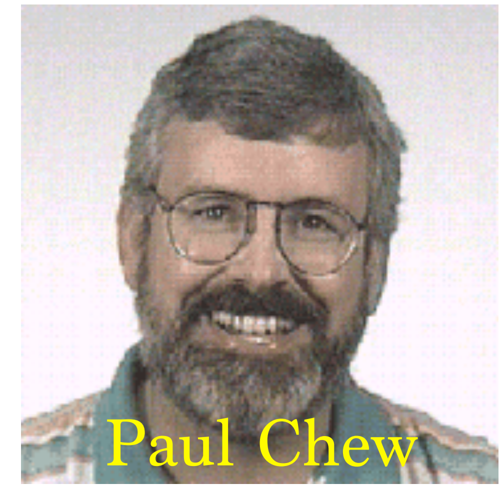
$$(i \sim j) \Leftrightarrow (R_i \cap R_j \neq \emptyset)$$

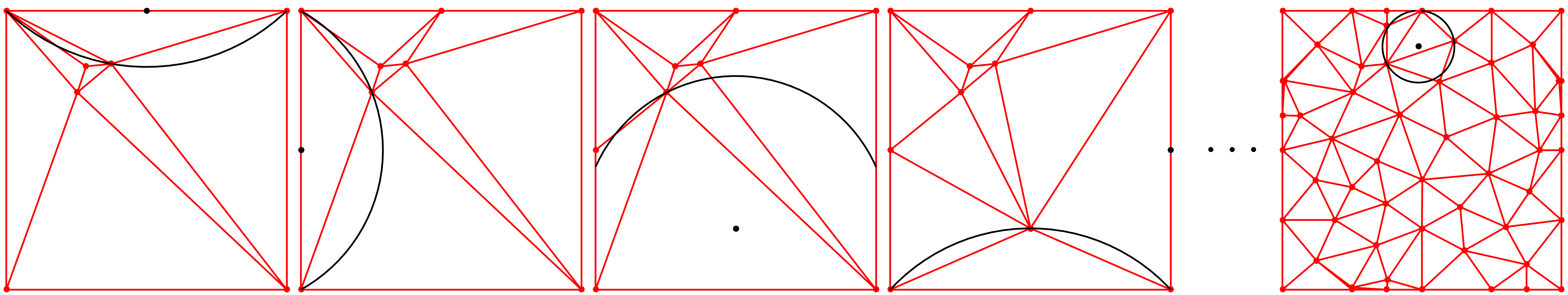


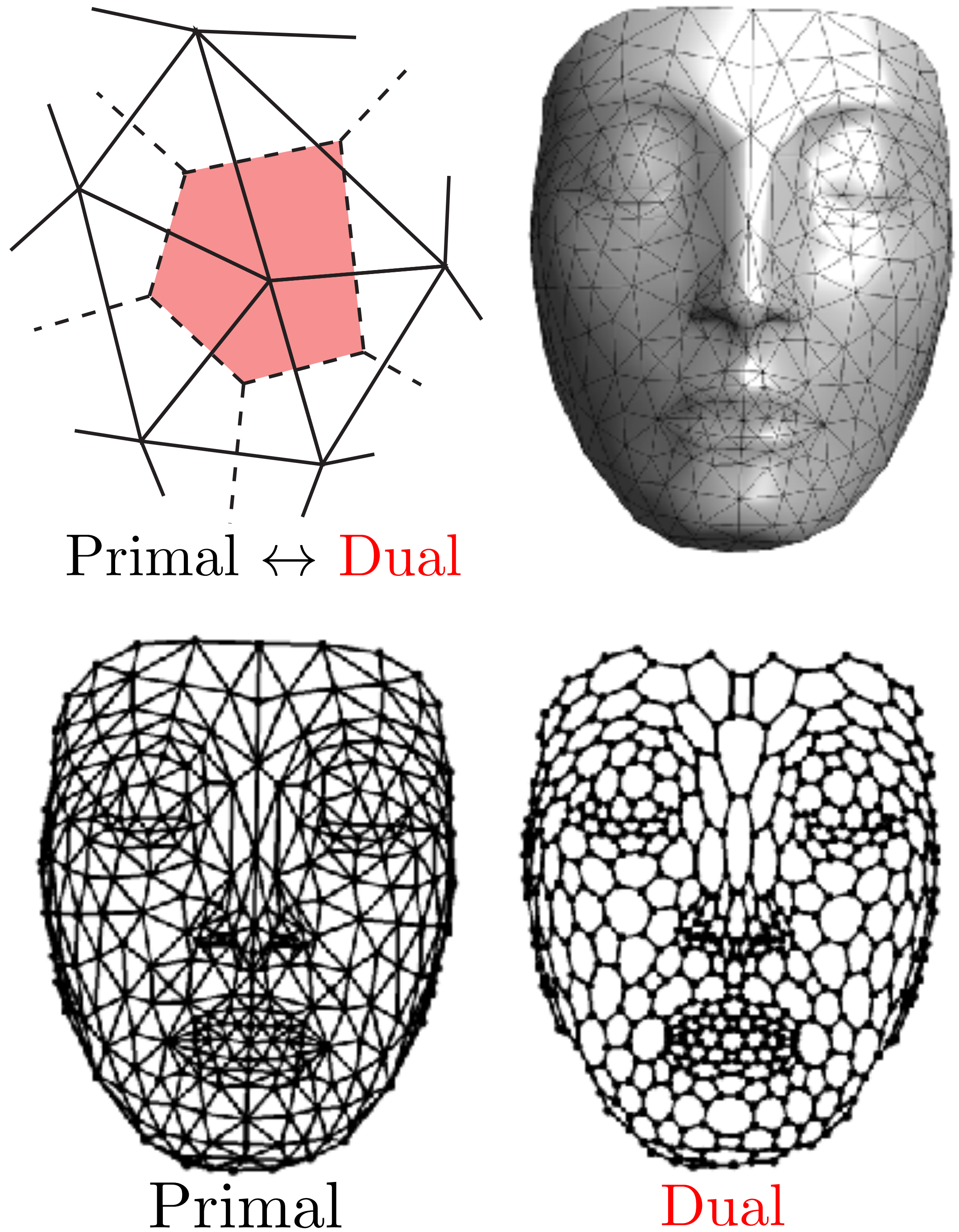


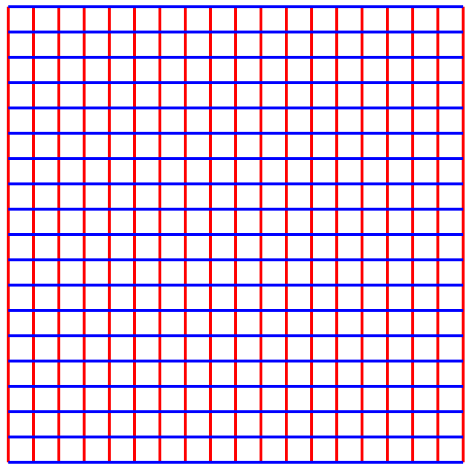


$$(T_i)_i \overset{\text{def.}}{=} \text{Delaunay}((x_k)_k)$$
$$r_i = \text{CircumCircleRadius}(T_i)$$
$$i^\star = \text{argmax}_i r_i$$
$$(x_k)_k \leftarrow (x_k)_k \cup \text{CircumCenter}(T_{i^\star})$$

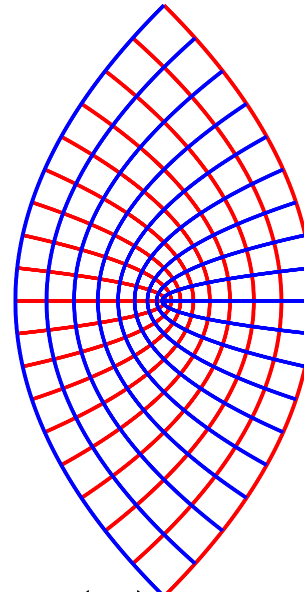




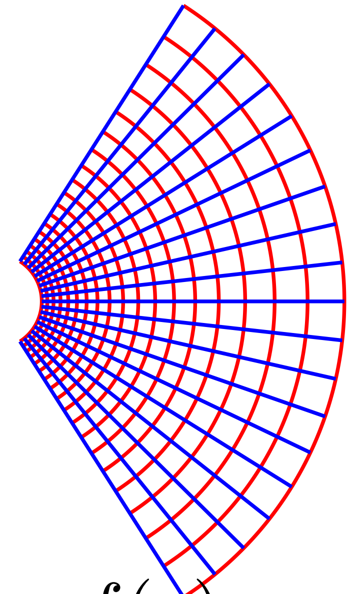

Primal ↔ Dual
Primal
Dual



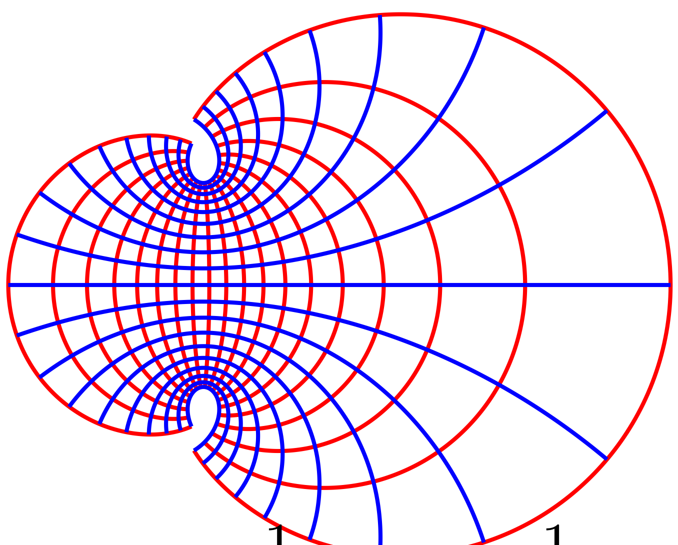
$f(z) = z$



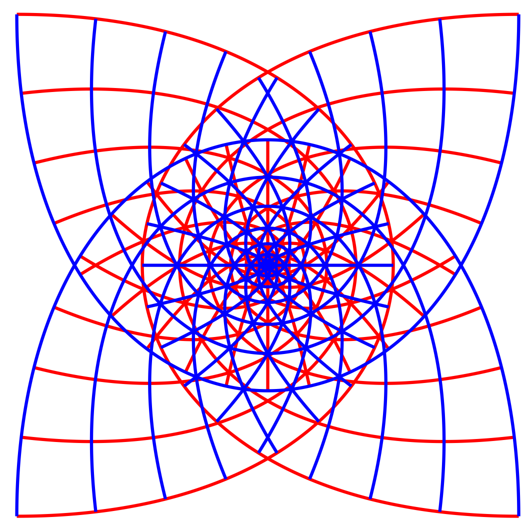
$f(z) = z^2$



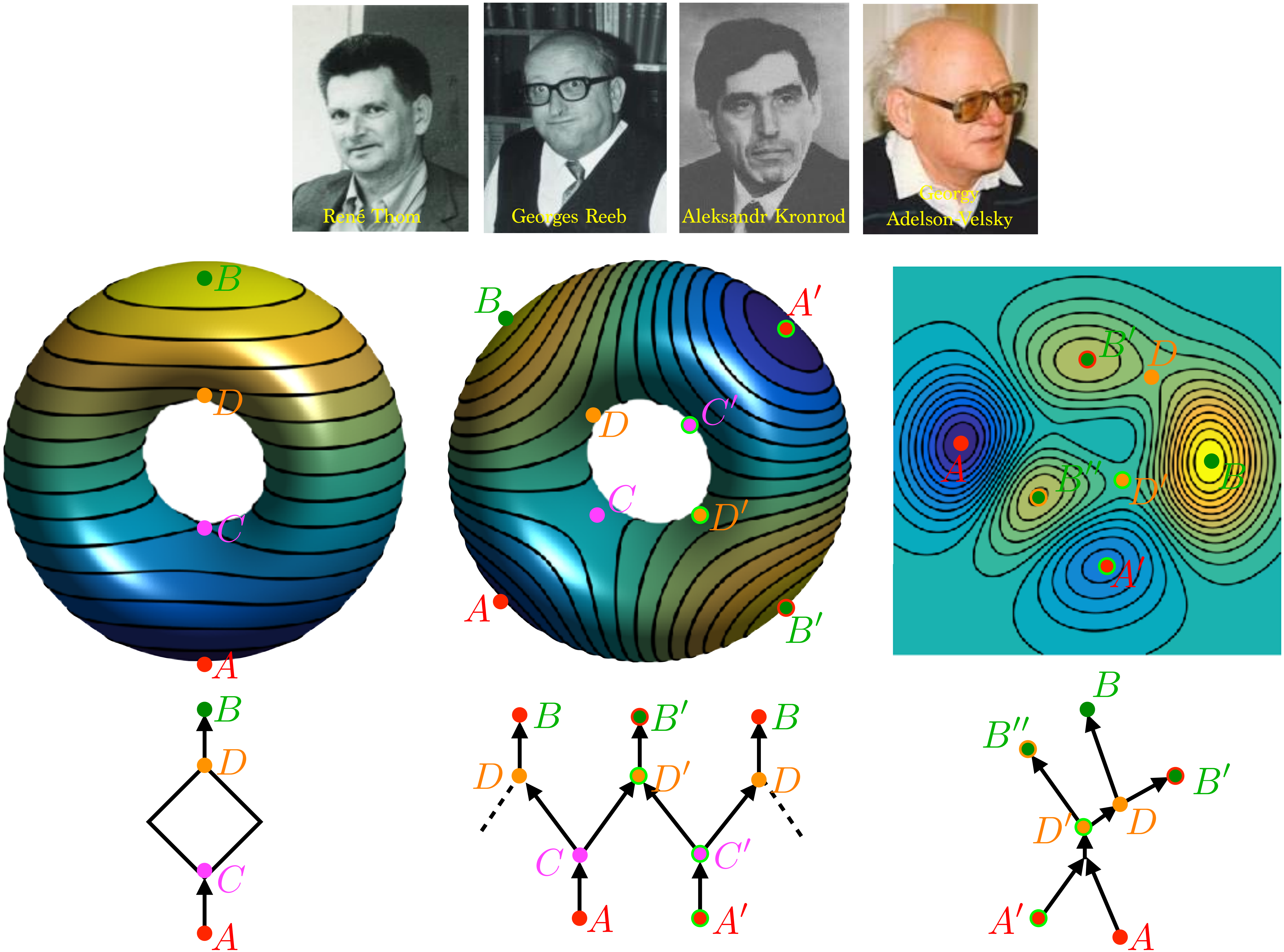
$f(z) = e^z$



$$f(z) = \frac{1}{z + 1.3} + \frac{1}{z - 1.6}$$

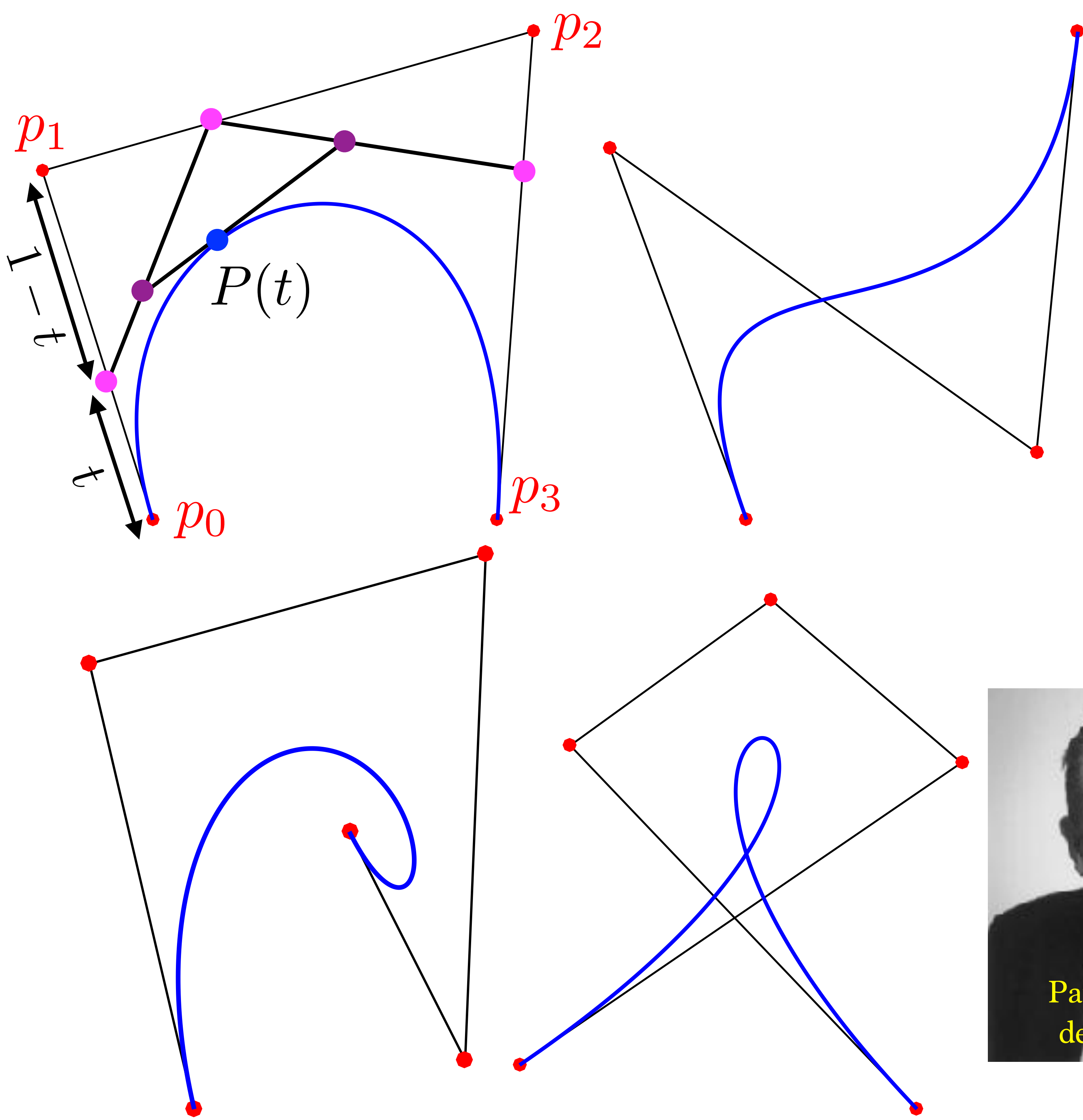


$f(z) = z^3$


René Thom
Georges Reeb
Aleksandr Kronrod
Georgy
Adelson-Velsky
A
B
C
D
A'
B'
B''
C'
D'

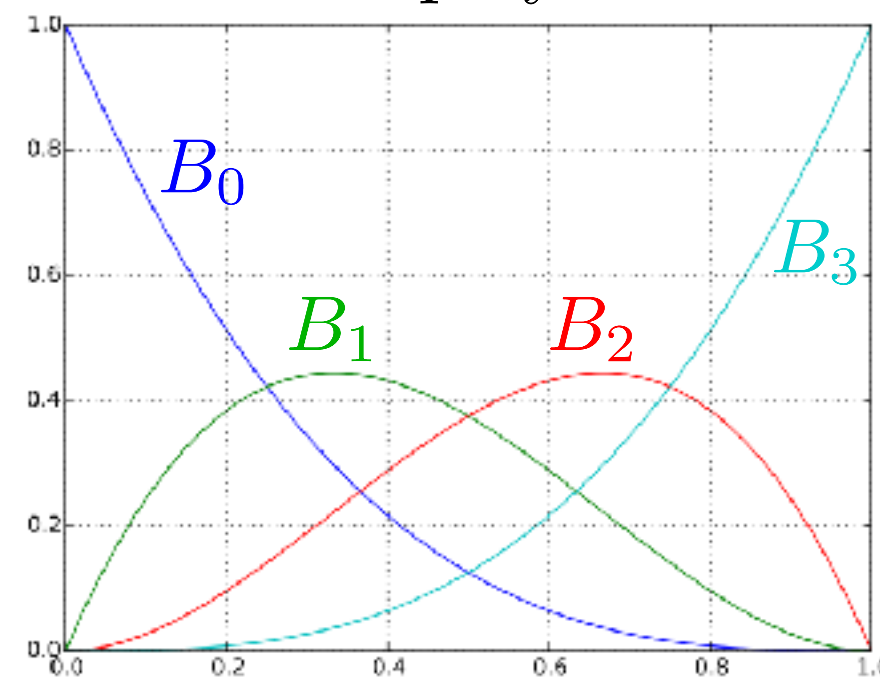
Geodesic Voronoi diagram: $R_i \overset{\text{def.}}{=} \{x \,;\, \forall j \neq i, d(x_j, x) \geqslant d(x_i, x)\}$

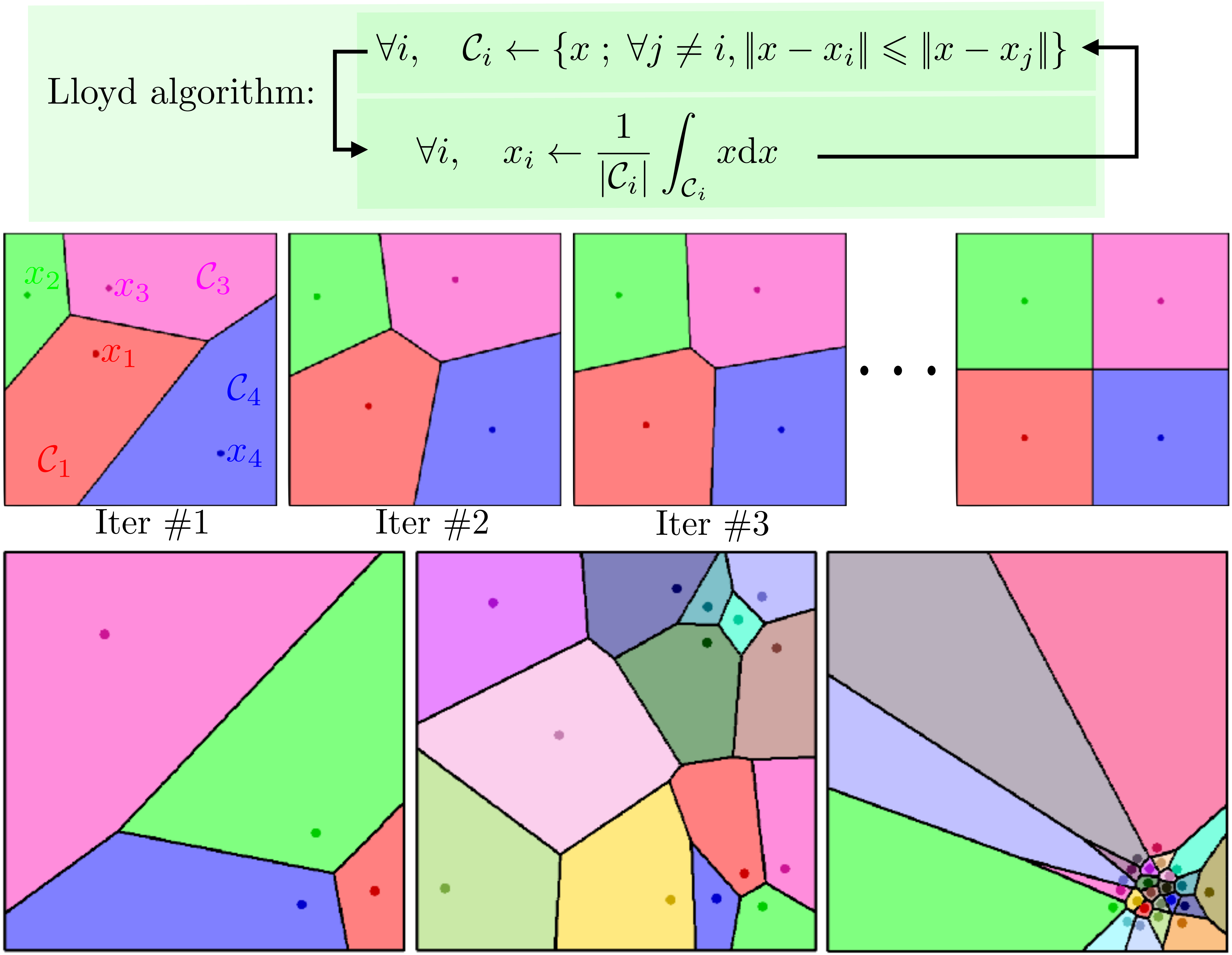
Geodesic Delaunay triangulation: $(i \sim j) \Leftrightarrow (R_i \cap R_j \neq \emptyset)$



$$P(t) = \sum_i P_i B_i(t)$$

Bernstein polynomials:

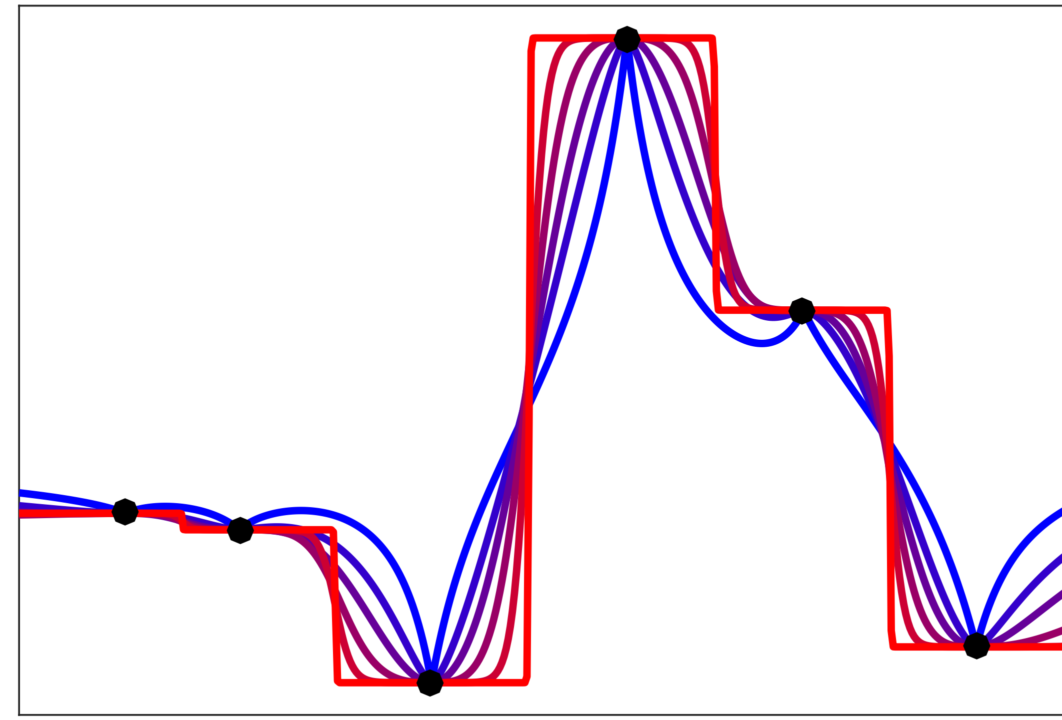


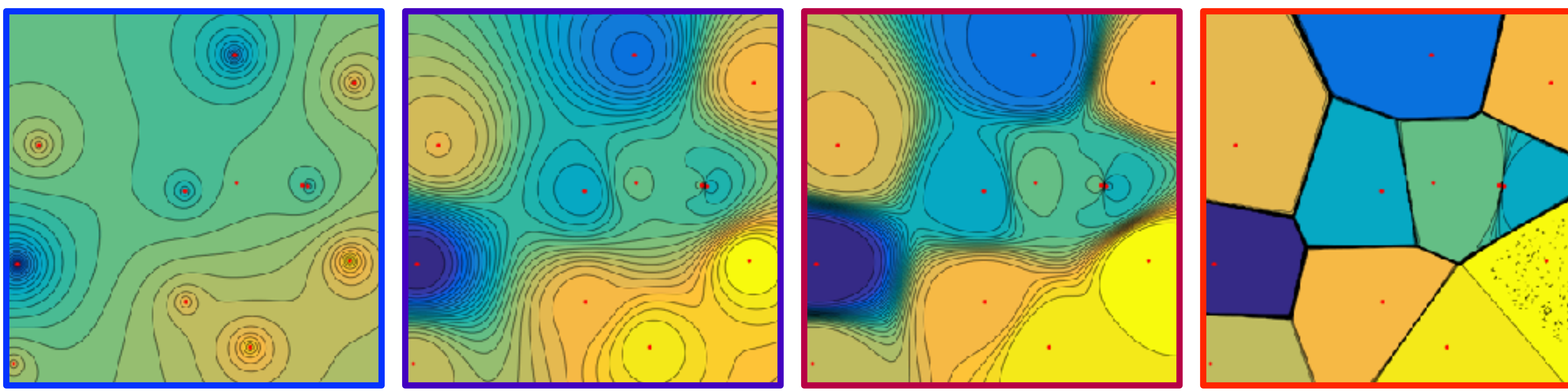


Lloyd algorithm:
$\forall i, \quad \mathcal{C}_i \leftarrow \{x \; ; \; \forall j \neq i, \|x - x_i\| \leqslant \|x - x_j\|\}$
$\forall i, \quad x_i \leftarrow \frac{1}{|\mathcal{C}_i|} \int_{\mathcal{C}_i} x \mathrm{d}x$
x_2
x_3
$\mathcal{C}_3$
x_1
$\mathcal{C}_4$
$\mathcal{C}_1$
x_4
Iter #1
Iter #2
Iter #3

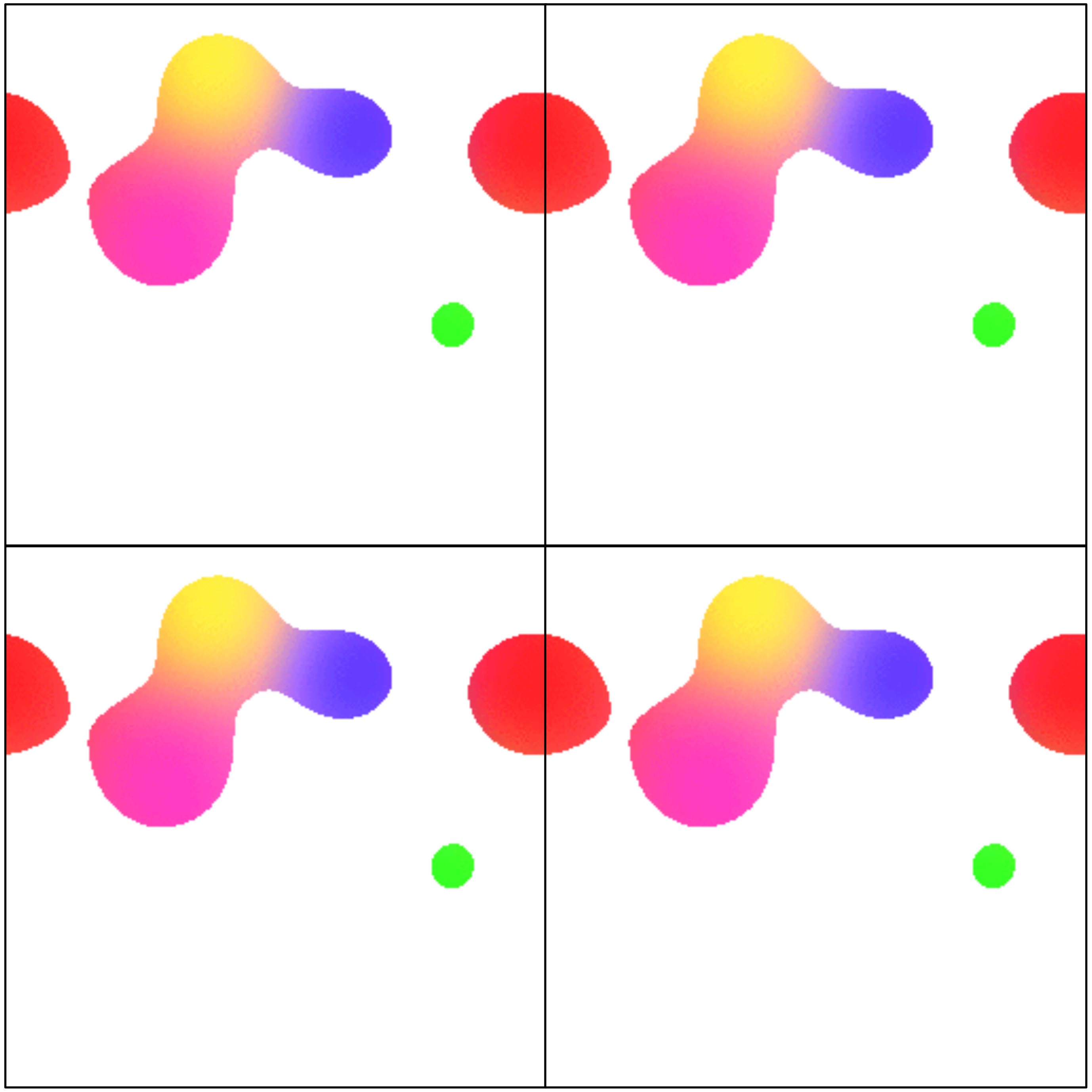
Shepard interpolation:

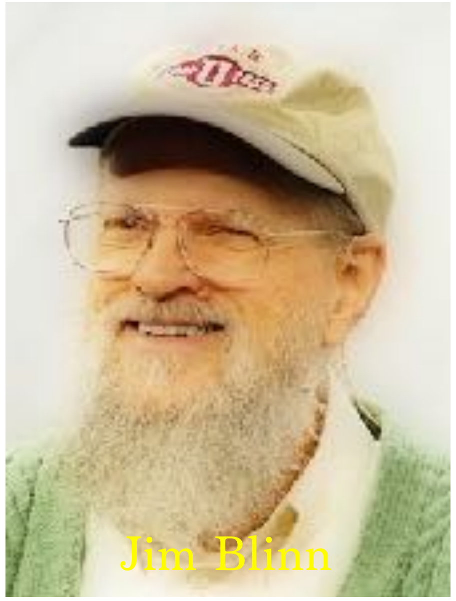
$$f(x) = \frac{\sum_i \|x - x_i\|^{-q} f(x_i)}{\sum_i \|x - x_i\|^{-q}}$$





$q = 1$ $\quad$ $q = 3$ $\quad$ $q = 6$ $\quad$ $q = 100$





Balls centers x_i.

Level set t.

$$\sum_i \frac{1}{\|x - x_i\|^2} \leqslant t$$