Errata

(Mathematical Introduction to Data Science by Sven A. Wegner)

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- Page 8, Line −4:
$$\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})=\sum_{i=1}^{n}x_iy_i-\overline{y}\sum_{i=1}^{n}x_i-\overline{x}\sum_{i=1}^{n}y_i+n\overline{x}\,\overline{y}$$
- Page 9, Line −3:
$$0=\sum_{i=1}^{n}(ax_i+b-y_i)=a\sum_{i=1}^{n}x_i+nb-\sum_{i=1}^{n}y_i=an\overline{x}+nb-n\overline{y},$$
- Page 13, Line 10: …constant random variable av.
- Page 13, Line −5: since $\overline{x^{(n)2}}=\operatorname{var}(x^{(n)})+\overline{x^{(n)}}^{2}$ as the sum …
- Page 15, Line −9: $\cdots+\frac{2}{n^2}\sum_{i<j}x_ix_j\operatorname{E}(\mathcal{E}_i)\operatorname{E}\mathcal{E}_j)$
- Page 16, Line 10: …, i.e., $f^*(x)=\langle a^*,x\rangle+b^*$ …
- Page 14, Line −2: If the latter is the case, then $\operatorname{sign}(r_{xy})=\operatorname{sign}(\langle u,v\rangle)=\cdots$
- Page 17, Line 13: …we calculate (with just for now $\phi(\tilde{a})=\langle\tilde{a},X^{\mathrm{T}}X\tilde{a}\rangle$):
- Page 23, Line 9: In the picture it must be the z-axis.
- Page 23, Line −3: …and $(w,b)=(w_1,\dots,w_d,b)$ for …
- Page 24, Line 16: $\operatorname{P}\big[Y_i\cancel{(f)}=y_i \text{ for all } i\big]=\cdots$
- Page 24, Line 18: $L\colon\big\{f\colon\mathbb{R}^d\to(0,1)\,|\,f \text{ logistic function}\big\}\to\mathbb{R}$
- Page 26, Line 13: $h'(t)=\operatorname{sig}(t)\cancel{+1}$
- Page 27, Line 18: …the data *overlaps* if for every $w\in\mathbb{R}^{d+1}\backslash\{0\}$ there exists …
- Page 27, Line 22: …, the rounded ~~logisitie~~ logistic regressor, …
- Page 27, Line −10: if $\operatorname{sig}(\langle w,\widehat{x}\rangle)\geqslant 1/2$
- Page 37, Line 1:

  function K-NN CLASSIFIER(D,k,x)
      $D'\leftarrow D,\;\; A\leftarrow\emptyset$
      for $j\leftarrow 1$ to k **do**
          $z^*\leftarrow\operatorname{argmin}_{z\in D'}\rho(x,\pi_1(z))$
          $A\leftarrow A\cup\{z^*\},\;D'\leftarrow D'\setminus\{z^*\}$
      for y in Y **do**
          $N(y)\leftarrow\#\{a\in A\,|\,\pi_2(a)=y\}$
      $\ell\leftarrow\operatorname{argmax}_{y\in Y}N(y)$
      return ℓ

  Here, $\pi_2(x,y)=y$ denotes the projection onto the second entry of $(x,y)\in D$ ~~and y^* is the label of z^*~~.
- Page 39, Line 9: …*and* $k<n$.
- Page 39, Line 10: *The calculation of the k-neareast neighbors of x can be implemented such that at most $(n\cdot d\cdot k)$-many mulitplikations have to be carried out.*
- Page 39, Line 14: In the Euklidean metric, it requires $(d-1)$-many multiplications to compute one distance if we omit the root, which we can do as it does not change the argmin. This leads to
$$(d-1)\cdot(n+(n-1)+\cdots+(n-k+1))\;\leqslant\;\cancel{C}\,d\cdot k\cdot n$$
multiplikationen ~~with a suitable $C\in\mathbb{N}$~~.
- Page 40, Line 3: …, we choose k-nearest neighbors $x_1,\dots,x_k$ of x and denote their labels by $y_1,\dots,y_k$.

- Page 40, Line 9: $f\colon X \to Y,\ f(x) = \dfrac{\sum_{i=1}^{k} w(x_i, x) \cdot y_i}{\sum_{i=1}^{k} w(x_i, x)}$
- Page 41, Line 5: $\tilde{x}^{(i)} = \left(a + \dfrac{(x_1^{(i)} - \min_{j=1,\dots,n} x_1^{(j)})(b-a)}{\max_{j=1,\dots,n} x_1^{(j)} - \min_{j=1,\dots,n} x_1^{(j)}}, \dots \right)$
- Page 41, Line 7: $\tilde{x}^{(i)} = \left(\dfrac{x_1^{(i)} - \overline{x_1^{(\cdot)}}}{\sigma_1}, \dots \right)$
- Page 42, Line 14: $\rho(x^{(1)}, x^{(4)}) = 3.681$
- Page 46, Line 1: We discuss some of these methods in Exercise 3.10.
- Page 45, Line 28: We thus see that texts no. 1 and text no. 2 are significantly more cosine similar than text no. 1 and text no. 3 or text no. 2 and text no. 3.
- Page 48, Line 26: The cosine distance, on the other hand, may appear here more natural, as the scalar product increases if the frequency of the fixed word increases in the second text.
- Page 52, Line 11: For finite subsets $A, B \subseteq X$ we define ...
- Page 53, Line 20:

function LINKAGE-BASED CLUSTERING (X, ρ, D, δ)
    $k \leftarrow \#D$
    for $i \leftarrow 1$ to k **do**
        $C_i \leftarrow \{x_i\}$
    while $\min_{i \neq j} \rho(C_i, C_j) \leqslant \delta$ and $k \geqslant 2$ **do**
        $m \leftarrow 0$
        $(i^*, j^*) \leftarrow \operatorname{argmin}_{i \neq j} \rho(C_i, C_j)$
        for $\ell \leftarrow 1$ to $k-1$ **do**
            if $\ell = \min(i^*, j^*)$ **then**
                $C_\ell \leftarrow C_{i^*} \cup C_{j^*}$
            if $\ell = \max(i^*, j^*)$ **then**
                $m \leftarrow 1$
                $C_\ell \leftarrow C_{\ell+m}$
            else
                $C_\ell \leftarrow C_{\ell+m}$
        $k \leftarrow k - 1$
    return $C_1, \dots, C_k$

- Page 54, Line −6: $K\colon \mathfrak{C}_k \to \mathbb{R}$
- Page 56, Line −16: *The following pseudocode approximates a minimizer of the k-means cost function.*
- Page 56, Pseudocode:

function K-MEANS (D, k, X, ρ)
    $\mu_1, \dots, \mu_k \leftarrow$ pairwise different points from X
    for $i \leftarrow 1$ to k **do**
        $C_i \leftarrow \{x \in D \mid i \in \operatorname{argmin}_{j=1,\dots,k} \rho(x, \mu_j)\}$
    $U \leftarrow$ True
    while $U =$ True **do**
        $U \leftarrow$ False
        for $i \leftarrow 1$ to k **do**
            $\mu_i \leftarrow \mu(C_i)$
        for $i \leftarrow 1$ to k **do**
            $C_i' \leftarrow \{x \in D \mid i \in \operatorname{argmin}_{j=1,\dots,k} \rho(x, \mu_j)\}$
            if $C_i' \neq C_i$
                $C_i \leftarrow C_i'$
                $U \leftarrow$ True
    return $C_1, \dots, C_k$

In the lines 4 and 9 of the pseudocode we pick as a single i in the case that the armin is not unique.

- Page 57, Line 7:

$$\mu(A) \in \underset{\mu \in A}{\operatorname{argmin}} \sum_{x \in A} \rho(x,\mu)^2, \quad \text{respectively} \quad \mu(A) \in \underset{\mu \in X}{\operatorname{argmin}} \sum_{x \in A} \rho(x,\mu).$$

- Page 57, Line -7: For $j \geqslant 1$ denote by $(C_1^{(j)}, \dots, C_k^{(j)})$ that clustering which the algorithm produces in the j-th round. For $j \geqslant 2$ we have

$$K(C_1^{(j)}, \dots, C_k^{(j)}) \;=\; \min_{\mu_1,\dots,\mu_k \in X} \sum_{i=1}^{k} \sum_{x \in C_i^{(j)}} \rho(x,\mu_i)^2$$

- Page 58, Line 3: …line 11 of Algorithm 4.9 …
- Page 58, Line 7: There, Picture 1b corresponds to the penultimate line in the estimate and Picture 2a to the line above that.
- Page 58, Line 10: …as we have just moved point x_3 from cluster C_2 in Figure 1b to cluster C_1 in Figure 2a, …
- Page 59, Line -4: We assume that we start with the initial values $\mu_1 = 2$ and …
- Page 62, Line 11: $A = (a_{ij})_{i,j=1,\dots,n}$
- Page 62, Line 13: $L = (\ell_{ij})_{i,j=1,\dots,n}$
- Page 63, Line -2: …In Example 5.7, $\lambda_2 \neq 0$ and there are no clusters (or, depending on how one prefers to see it, one single cluster), in Example 5.8 …
- Page 67, Line 8: For the other direction let $\{v_1, \dots, v_n\}$ be a basis consisting of eigenvectors corresponding to the λ_i and let $U \subseteq \mathbb{R}^n$ be a subspace with $\dim U = n-k+1$. By construction $U \cap \operatorname{span}\{v_1,\dots,v_k\} \neq \{0\}$ and we can select $0 \neq x = \alpha_1 v_1 + \cdots + \alpha_k v_k \in U$. Then it follows

$$\frac{\langle x, Mx\rangle}{\langle x,x\rangle} = \frac{\sum_{i=1}^{k} \lambda_i \alpha_i^2}{\sum_{i=1}^{k} \alpha_i^2} \leqslant \frac{\lambda_k \sum_{i=1}^{k} \alpha_i^2}{\sum_{i=1}^{k} \alpha_i^2} = \lambda_k,$$

since the λ_i's are increasing. With this we get $\min_{0 \neq x \in U} \frac{\langle x,Mx\rangle}{\langle x,x\rangle} \leqslant \lambda_k$ which then leads to

$$\max_{\substack{U \subseteq \mathbb{R}^n \\ \dim U = n-k+1}} \; \min_{\substack{x \in U \\ x \neq 0}} \frac{\langle x, Mx\rangle}{\langle x,x\rangle} \leqslant \lambda_k.$$

- Page 67, Line -4: …and any corresponding eigenvector v_2 of norm 1:
- Page 68, Line 14: Let $G = (V,E)$ be a graph with $\deg(v) > 0$ for all $v \in V$.
- Page 70, Line 10:

$$\lambda_2(\mathcal{L}) = \min_{\substack{x \neq 0 \\ \langle Dx, \mathbb{1}\rangle = 0}} \frac{\sum_{\{i,j\} \in E} (x_i - x_j)^2}{\sum_{i=1}^{n} x_i^2 d_i}.$$

- Page 70, Line 16:

$$\mathcal{L} D^{1/2}\mathbb{1} = D^{-1/2} L D^{-1/2} D^{1/2}\mathbb{1} = D^{-1/2} L \mathbb{1} \underset{\substack{\uparrow \\ \text{Proposition} \\ 5.6}}{=} D^{-1/2} 0 \mathbb{1} = 0 D^{1/2}\mathbb{1}.$$

- Page 70, Line 18:

$$\begin{aligned}
\lambda_2(\mathcal{L}) &\overset{\substack{\text{Theorem} \\ 5.13}}{=} \min_{\substack{x \neq 0 \\ \langle x, D^{1/2}\mathbb{1}\rangle = 0}} \frac{\langle x, \mathcal{L}x\rangle}{\langle x,x\rangle} \overset{(*)}{=} \min_{\substack{y \neq 0 \\ \langle D^{1/2}y, D^{1/2}\mathbb{1}\rangle = 0}} \frac{\langle D^{1/2}y, \mathcal{L} D^{1/2}y\rangle}{\langle D^{1/2}y, D^{1/2}y\rangle} \\
&= \min_{\substack{y \neq 0 \\ \langle y, D\mathbb{1}\rangle = 0}} \frac{\langle y, Ly\rangle}{\langle y, Dy\rangle} = \min_{\substack{y \neq 0 \\ \langle Dy, \mathbb{1}\rangle = 0}} \frac{\sum_{\{i,j\} \in E} (y_i - y_j)^2}{\sum_{i=1}^{n} y_i^2 d_i}
\end{aligned}$$

- Page 71, Line 14: *(ii)* $\min(\operatorname{vol} S_k, \operatorname{vol} S_k^c) = \operatorname{vol} S_k^c$ *and* $\operatorname{vol} S_k^c - \operatorname{vol} S_{k+1}^c = d_{k+1}$ *hold whenever* $r \leqslant k \leqslant n-1$.
- Page 71, Line 17:

$$\operatorname{vol} S_k^c - \operatorname{vol} S_{k+1}^c = \sum_{i=k+1}^{n} d_i - \sum_{i=k+2}^{n} d_i = d_{k+1}$$

- Page 72, Line 9:

$$\langle Dx, \mathbb{1}\rangle = \sum_{i=1}^{n} d_i x_i = \dots$$

- Page 72, Line −1 (and Page 73, Line 1):

$$\begin{aligned} \dots &= \operatorname{vol} S - \operatorname{vol} S \cdot \tfrac{\operatorname{vol} S}{\operatorname{vol} S + \operatorname{vol} S^c} \\ &\geqslant \operatorname{vol} S - \operatorname{vol} S \cdot \tfrac{\operatorname{vol} S}{2 \operatorname{vol} S}, \end{aligned}$$

- Page 73, Line 9: ... and our goal in the following will be to show $\lambda_2 \geqslant \alpha^2/2$, ...
- Page 73, Line 12 (Equation (5.2)):

$$\cdots \text{ and } \langle Dx, \mathbb{1}\rangle = \sum_{i=1}^{n} d_i x_i = 0$$

- Page 73, Line 21:

$$\begin{bmatrix} x_1 - x_r \\ \vdots \\ x_{r-1} - x_r \\ 0 \\ x_{r+1} - x_r \\ \vdots \\ x_n - x_r \end{bmatrix} = \begin{bmatrix} x_1 - x_r \\ \vdots \\ x_{r-1} - x_r \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ x_r - x_{r+1} \\ \vdots \\ x_r - x_n \end{bmatrix} =: p - n.$$

- Page 74, Line −3 (until top of page 75):

$$\begin{aligned} \lambda_2 &= \frac{\sum_{\{i,j\}\in E} (x_i - x_j)^2}{\sum_{i=1}^n x_i^2 d_i} \\ &\underset{\substack{\uparrow \\ (5.3)}}{\geqslant} \frac{\sum_{\{i,j\}\in E} \left((p_i - p_j)^2 + (n_i - n_j)^2\right)}{\sum_{i=1}^n (p_i^2 + n_i^2) d_i} \\ &\overset{(5.4)}{=} \frac{\sum_{\{i,j\}\in E} (p_i - p_j)^2 + \sum_{\{i,j\}\in E} (n_i - n_j)^2}{\sum_{i=1}^n p_i^2 d_i + \sum_{i=1}^n n_i^2 d_i} \\ &\underset{\substack{\uparrow \\ (5.5)}}{\geqslant} \min\left(\frac{\sum_{\{i,j\}\in E} (p_i - p_j)^2}{\sum_{i=1}^n p_i^2 d_i}, \frac{\sum_{\{i,j\}\in E} (n_i - n_j)^2}{\sum_{i=1}^n n_i^2 d_i}\right) \\ &= \min\left(\frac{\sum_{\{i,j\}\in E} (p_i - p_j)^2}{\sum_{i=1}^n p_i^2 d_i} \cdot \frac{\sum_{\{i,j\}\in E} (p_i + p_j)^2}{\sum_{\{i,j\}\in E} (p_i + p_j)^2}, \dots\right) \\ &=: \min\left(\frac{Z}{N}, \dots\right). \end{aligned}$$

- Page 75, Line 8:

$$N = \sum_{i=1}^{n} p_i^2 d_i \cdot \sum_{\{i,j\}\in E} (p_i + p_j)^2 \underset{\substack{\uparrow \\ (5.5)}}{\geqslant} \sum_{i=1}^{n} p_i^2 d_i \cdot \sum_{\{i,j\}\in E} 2(p_i^2 + p_j^2)$$

- Page 76, Line 3:

$$\cdots \underset{\substack{\uparrow \\ \text{telescopic} \\ \text{sum}}}{=} \left(\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \mathbb{1}_E(i,j) \sum_{k=i}^{j-1} \left(p_k^2 - p_{k+1}^2\right) \right)^2$$

- Page 79, Line 13: Finally, we want to note that Theorem 5.21 together with Remark 5.17(i) provides an upper bound for the eigenvalue $\lambda_2(\mathcal{L})$.
- Page 82, Line −5: *Let a dataset* $D=\{x_1,\dots,x_n\}\subseteq\mathbb{R}^d$ *be given, …*
- Page 83, Line 3:
$$\sum_{j=1}^{k}\|Xv_j\|^2 \geqslant \sum_{j=1}^{k}\|Xw_j\|^2$$
- Page 83, Line −8:
$$\{v_1,\dots,v_k\} \in \underset{\substack{\{\tilde v_1,\dots,\tilde v_k\}\subseteq\mathbb{R}^d\\ \text{orthonormal}\\ \text{system}}}{\operatorname{argmax}} \sum_{j=1}^{n}\|X\tilde v_j\|^2.$$
- Page 84, Line 10: Let $D=\{x_1,\dots,x_n\}\subseteq\mathbb{R}^d$ be given …
- Page 84, Line 13: for $j=1,\dots,k$, where …
- Page 85, Line −1: $\langle w_{k+1},v_i\rangle=\langle w_{k+1},\underbrace{v_i-\pi_W(v_i)}_{\perp W}\rangle+\langle w_{k+1},\underbrace{\pi_W(v_i)}_{\in U}\rangle=0+0=0$
- Page 86, Line 11: Let $D=\{x_1,\dots,x_n\}\subseteq\mathbb{R}^d$ be given…
- Page 87, Line −9: Determine, by using Theorem 6.3 and a Python package for maximizations under nonlinear constraints, a 1-best fitting subspace for …
- Page 86, Line 16: …hold if $r=\dim\operatorname{span}D>0$. If $r=0$, then all σ_k vanish.
- Page 86, Line 19: For $k\geqslant r+1$ we apply the equation in the proof of Lemma 6.2 twice, on the one hand for V_k with ONB $\{v_1,\dots,v_k\}$, and on the other for V_{k-1} with ONB $\{v_1,\dots,v_{k-1}\}$. Then, after rearranging, we obtain
$$\sigma_k^2=\|Xv_k\|^2=\sum_{i=1}^{n}\operatorname{dist}(x_i,V_{k-1})^2-\sum_{i=1}^{n}\operatorname{dist}(x_i,V_k)^2.$$
As $V_r\subseteq V_{k-1}\subseteq V_k$ holds, Lemma 6.4 implies that both terms on the right hand side are zero. Thus, $\sigma_{r+1},\dots,\sigma_d=0$.
- Page 90, Line 8: $\|u\|^2=\|Av\|^2=\langle Av,Av\rangle=\cdots$
- Page 94, Line 6: …, it may be easier to start with this matrix and proceed analologuously (notice that one then gets the u_i's first and has to compute the v_i's by applying A^{T}).
- Page 94, Line −9: $V\in\mathbb{R}^{d\times d}$
- Page 94, Line −7: Then precisely r-many of the σ_i are nonzero and these are the singular values of A …
- Page 96, Line 10: …and let $\{v_1,\dots,v_r\}$ be some arbitrary orthonormal system …
- Page 96, Line 14: By construction, the $v_1,\dots,v_r$ that we defined in the theorem form an orthonormal system.
- Page 96, Line 20:
$$\|Av\|^2=\langle v,A^{\mathrm{T}}Av\rangle=\langle v,B\operatorname{diag}(\cdots)B^{\mathrm{T}}v\rangle=\langle B^{\mathrm{T}}v,\operatorname{diag}(\cdots)B^{\mathrm{T}}v\rangle=\sum_{j=1}^{r}\lambda_j^{\cancel{2}}(B^{\mathrm{T}}v)_j^2$$
In the rest of the proof everywhere the squares at the eigenvalues λ_j have to be removed!
- Page 98, Line 4: …v_i is therefore an eigenvector of $A^{\mathrm{T}}A$ corresponding to σ_i^2 for $i=1,\dots,k$.
- Page 99, Line −3: ~~Multiplication with V^{T} from the left in~~
- Page 99, Line 12: $v=\begin{bmatrix}0\\ \vdots\\ 0\\ \sigma_{k+1}\\ w_{k+1}\end{bmatrix}\Bigg/\left\|\begin{bmatrix}0\\ \vdots\\ 0\\ \sigma_{k+1}\\ w_{k+1}\end{bmatrix}\right\|$
- Page 99, Line 14: …, we want to discuss how the singular value decomposition ~~indeed~~ generalizes the orthogonal diagonalization theorem.

- Page 99, Line 16: **Proposition 7.14.** Let $A \in \mathbb{R}^{n\times n}$ be a symmetric and positive semidefinite matrix, ...

- Page 99, Line −3: Proposition 7.14 thus yields exactly an orthogonl diagonalization $V^{\mathrm{T}}AV = \operatorname{diag}(\sigma_1, \ldots, \sigma_r, 0, \ldots, 0)$; but only under the additional assumption of A being positive semidefinite.

  Corollary 5.22. Let $G = (V, E)$ be a graph with $\deg(i) > 0$ for all $i \in V$. Then for the second smallest eigenvalue λ_2 of the normalized Laplace matrix of G the estimate $\lambda_2 \leqslant 2$ holds. □

- Page 102, Line 17: *Proof.* Let $B \in \mathbb{R}^{n\times d}$ be arbitrary. Put

$$V_k := \operatorname{span}\{v_1, \ldots, v_k\} \;\text{ and }\; W := \operatorname{span}\{b_1, \ldots, b_n\} \;\cancel{\text{and } \check{V} := \operatorname{span}\{\check{a}_1, \ldots, \check{a}_n\},}$$

  where $b_i \in \mathbb{R}^d$ is the vector that has as entries those of the i-th row of B. According to Theorem 7.12, V_k is a k-best fitting subspace for $\{a_1, \ldots, a_n\}$. ~~We claim that $\check{V} = V_k$. By Theorem 7.17(ii), we have $\check{a}_i \in V_k$ for $i = 1, \ldots, n$ and thus $\check{V} \subseteq V_k$. The equality then follows from $\dim \check{V} = \operatorname{rk} \check{A} = k = \dim V_k$.~~ Consequently ~~we obtain for each $i = 1, \ldots, n$ the estimates~~

$$\begin{aligned}\|A - B\|_{\mathrm{F}}^2 &= \sum_{i=1}^n \|a_i - b_i\|^2 \underset{\substack{\uparrow\\ b_i \in W}}{\geqslant} \sum_{i=1}^n \|a_i - \pi_W(a_i)\|^2 \underset{\substack{\uparrow\\ V_k\ k\text{-best}\\ \text{fitting for}\\ a_1,\ldots,a_n}}{\geqslant} \sum_{i=1}^n \|a_i - \pi_{V_k}(a_i)\|^2\\ &\underset{\substack{\uparrow\\ \text{Thm}\\ 7.17(\text{ii})}}{=} \sum_{i=1}^n \|a_i - \check{a}_i\|^2 = \|A - \check{A}\|_{\mathrm{F}}^2\end{aligned}$$

  which concludes the proof. □

- Page 105, Line −1 and Page 106, Line 1:

$$A = \underbrace{\begin{bmatrix} 0.07 & 0.29 & 0.32 & 0.51 & 0.66 & 0.18 & -0.23\\ 0.13 & -0.02 & -0.01 & -0.79 & 0.59 & -0.02 & -0.06\\ 0.68 & -0.11 & -0.05 & -0.05 & -0.24 & 0.56 & -0.35\\ 0.15 & 0.59 & 0.65 & -0.25 & -0.33 & -0.09 & 0.11\\ 0.41 & -0.07 & -0.03 & 0.10 & -0.02 & -0.78 & -0.43\\ 0.07 & 0.73 & -0.67 & 0.00 & -0.00 & 0.00 & 0.00\\ 0.55 & -0.09 & -0.04 & 0.17 & 0.17 & -0.11 & 0.78 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} 12.4 & 0.0 & 0.0 & 0.0 & 0.0\\ 0.0 & 9.5 & 0.0 & 0.0 & 0.0\\ 0.0 & 0.0 & 1.3 & 0.0 & 0.0\\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0\\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0\\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0\\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} 0.56 & 0.09 & 0.56 & 0.09 & 0.59\\ -0.12 & 0.69 & -0.12 & 0.69 & 0.02\\ -0.40 & -0.09 & -0.40 & -0.09 & 0.80\\ \cancel{0.41} & \cancel{0.09} & \cancel{0.40} & \cancel{0.09} & \cancel{-0.80}\\ 0.51 & 0.48 & -0.51 & -0.48 & -0.00\\ 0.48 & -0.51 & -0.48 & 0.51 & -0.00 \end{bmatrix}}_{V^{\mathrm{T}}}$$

$$= \begin{bmatrix} 0.07 & 0.29 & 0.32\\ 0.13 & -0.02 & -0.01\\ 0.68 & -0.11 & -0.05\\ 0.15 & 0.59 & 0.65\\ 0.41 & -0.07 & -0.03\\ 0.07 & 0.73 & -0.67\\ 0.55 & -0.09 & -0.04 \end{bmatrix} \begin{bmatrix} 12.4 & & \\ & 9.5 & \\ & & 1.3 \end{bmatrix} \begin{bmatrix} 0.56 & 0.09 & 0.56 & 0.09 & 0.59\\ -0.12 & 0.69 & -0.12 & 0.69 & 0.02\\ -0.40 & -0.09 & -0.40 & -0.09 & 0.80 \end{bmatrix}.$$

- Page 106, Line 6:

$$\check{A} = \begin{bmatrix} 0.15 & 1.97 & 0.15 & 1.97 & 0.56\\ 0.92 & 0.01 & 0.92 & 0.01 & 0.94\\ 4.84 & 0.03 & 4.84 & 0.03 & 4.95\\ 0.36 & 4.03 & 0.36 & 4.03 & 1.20\\ 2.92 & -0.00 & 2.92 & -0.00 & 2.98\\ -0.34 & 4.86 & -0.34 & 4.86 & 0.65\\ 3.92 & 0.02 & 3.92 & 0.02 & 4.00 \end{bmatrix} = \begin{bmatrix} 0.07 & 0.29\\ 0.13 & -0.02\\ 0.68 & -0.11\\ 0.15 & 0.59\\ 0.41 & -0.07\\ 0.07 & 0.73\\ 0.55 & -0.09 \end{bmatrix} \begin{bmatrix} 12.4 & \\ & 9.5 \end{bmatrix} \begin{bmatrix} 0.56 & 0.09 & 0.56 & 0.09 & 0.59\\ -0.12 & 0.69 & -0.12 & 0.69 & 0.02 \end{bmatrix}$$

- Page 107, Line 15:

$$= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0.07 & 0.29 & \cdots & -0.23\\ 0.13 & -0.02 & & -0.06\\ 0.68 & -0.11 & & -0.35\\ 0.15 & 0.59 & & 0.11\\ 0.41 & -0.07 & & -0.43\\ 0.07 & 0.73 & & 0.00\\ 0.55 & -0.09 & \cdots & 0.78 \end{bmatrix} \begin{bmatrix} 12.4 & & & \\ & 9.5 & & \\ & & 1.3 & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} 0.56 & 0.09 & 0.56 & 0.09 & 0.59\\ -0.12 & 0.69 & -0.12 & 0.69 & 0.02\\ \vdots & & & & \vdots\\ 0.48 & -0.51 & -0.48 & 0.51 & 0.00 \end{bmatrix} \begin{bmatrix} 0\\ \vdots\\ 0\\ 1\\ 0 \end{bmatrix}$$

- Page 107, Line −2:

$$u_2 = \quad 0.29 \cdot \text{Abbie} - 0.02 \cdot \text{Bailey} + \cdots - 0.09 \cdot \text{Gladys},$$

- Page 109, Line 10:

$$\breve{A} = \begin{array}{r} \text{Abbie} \\ \text{Bailey} \\ \text{Catherine} \\ \text{Darlene} \\ \\ \text{Elena} \\ \text{Fatima} \\ \text{Gladys} \end{array} \begin{bmatrix} 0.07 & 0.29 \\ 0.13 & -0.02 \\ 0.68 & -0.11 \\ 0.15 & 0.59 \\ 0.41 & -0.07 \\ 0.07 & 0.73 \\ 0.55 & -0.09 \end{bmatrix} \begin{bmatrix} 12.4 & \\ & 9.5 \end{bmatrix} \begin{array}{c} \begin{array}{ccccc} \text{Alien} & \text{Casablanca} & \text{Star Wars} & \text{Titanic} & \text{The Matrix} \end{array} \\ \begin{bmatrix} 0.56 & 0.09 & 0.56 & 0.09 & 0.59 \\ -0.12 & 0.69 & -0.12 & 0.69 & 0.02 \end{bmatrix} \end{array}$$

- Page 109, Line -3: ... and $\check{\mathcal{V}} = \{v_1, v_2\}$.
- Page 119, Line -7 ..., we assume that the distribution has mean zero and variance 1.
- Page 120, Line 1 in eq. (8.2): $V(\|X\|^2) = V(X_1^2) + \cdots + V(X_d^2)$.
- Page 121, Line 4: ..., we get $\mathrm{E}(S_d) = 0$
- Page 121, Line 13: $= 2\sqrt{d}\,\mathrm{E}(\|X\| - \sqrt{d}\,)$
- Page 121, Line -2: **Theorem 8.2.** Let $X, Y \sim \mathcal{N}(0, 1, \mathbb{R}^d)$ be independent. Then *(i)* $\forall\, d \in \mathbb{N}\colon |\mathrm{E}(\|X - Y\| - \sqrt{2d})| \leqslant \sqrt{2/d}$,
- Page 122, Line 1: *(ii)* $\forall\, d \in \mathbb{N}\colon\ \mathrm{V}(\|X - Y\|) \leqslant 4$.
- Page 122, Line 3 in eq. (8.2): $d \cdot (\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} x^4 \exp(-x^2/2)\,\mathrm{d}x - \mathrm{V}(X_1)^2))$
- Page 125, Line -8: *(i)* $\forall\, d \in \mathbb{N}\colon |\mathrm{E}(\|X - Y\| - \sqrt{2d})| \leqslant \sqrt{2/d}$,
- Page 125, Line -7: *(ii)* $\forall\, d \in \mathbb{N}\colon\ \mathrm{V}(\|X - Y\|) \leqslant 4$.
- Page 125, Line -6: Firstly, calculate $\mathrm{V}((X_i - Y_i)^2) = 8$ ...
- Page 125, Line -5: Then infer $\mathrm{V}((X_i - Y_i)^2) = 8d$ ...
- Page 140, Line -7: *Proof.* We put $Y_1 + \cdots + Y_d$ and ...
- Page 140, last expression in eq. (10.1): $= \mathrm{e}^{-ta} \prod_{i=1}^{d} \mathrm{E}(\mathrm{e}^{tY_i})$
- Page 143, Lines 2 – 6:

$$\begin{aligned} \mathrm{P}\big[\,\big|\|X\| - \sqrt{d}\,\big| > \varepsilon\big] &\leqslant \mathrm{P}\big[\,\big|\|X\| - \sqrt{d}\big| \cdot (\|X\| + \sqrt{d}) \geqslant \varepsilon \cdot \sqrt{d}\,\big] \\ &= \mathrm{P}\big[\,|X_1^2 + \cdots + X_d^2 - d| \geqslant \varepsilon\sqrt{d}\,\big] \\ &= \mathrm{P}\big[\,|(X_1^2 - 1) + \cdots + (X_d^2 - 1)| \geqslant \varepsilon\sqrt{d}\,\big] \\ &= \mathrm{P}\big[\,\big|\tfrac{X_1^2-1}{2} + \cdots + \tfrac{X_d^2-1}{2}\big| \geqslant \tfrac{\varepsilon\sqrt{d}}{2}\,\big] \\ &= \mathrm{P}\big[\,|Y_1 + \cdots + Y_d| \geqslant a\,\big], \end{aligned}$$

- Page 143, Line 10: ① The Y_i are ~~pairwise~~ independent according to Fact 10.1(i).
- Page 152, Line -11:

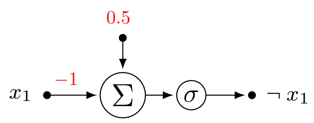
$$\mathrm{P}\left[\,\left|\left\langle \frac{u_i}{\|u_i\|}, \frac{u_j}{\|u_j\|} \right\rangle\right| \leqslant \varepsilon\,\right] \geqslant 1 - \frac{2/\varepsilon + 7}{\sqrt{d}}.$$

- Page 155, Line 5: ... (i) The dimension d of the ambient space ...
- Page 155, Line -3ff:

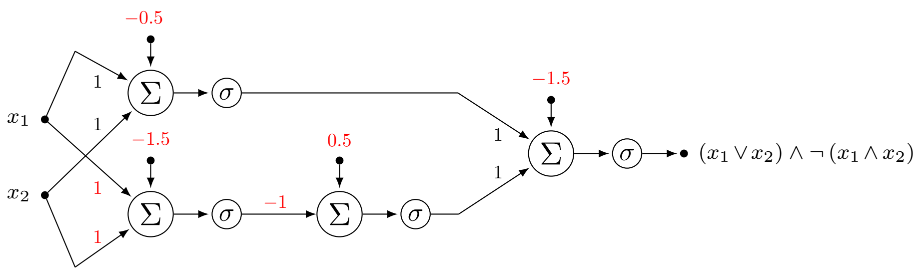
$$\mathrm{P}[\,\cdots] = 1 - \mathrm{P}\big[\,\exists\, i \neq j\colon \|T_U x_i - T_U x_j\| \notin \big((1-\varepsilon)\|x_i - x_j\|, (1+\varepsilon)\|x_i - x_j\|\big)\big]$$

$$
\begin{aligned}
&\geqslant 1-\sum_{i\neq j}(1-P_{ij}) \geqslant 1-\sum_{i\neq j}2\,\mathrm{e}^{-k\varepsilon^2/16} \geqslant 1-\frac{n^2}{2}\cdot 2\,\mathrm{e}^{-k\varepsilon^2/16}\\
&\geqslant 1-n^2\,\mathrm{e}^{-(\frac{48}{\varepsilon^2}\log n)\,\varepsilon^2/16} = 1-n^2\,\mathrm{e}^{\log(n^{-3})}\\
&= 1-\frac{1}{n}
\end{aligned}
$$

- Page 169, Line -1: $\sqrt{(\Delta(d))^2+2d}-\sqrt{2d}\to\infty$
- Page 170, Line -1: $\lim_{d\to\infty}\sqrt{(\Delta(d))^2+2d}-\sqrt{2d}=\infty$
- Page 171, Line 4: $\sqrt{(\Delta(d))^2+2d}-\sqrt{2d}\geqslant\sqrt{R\sqrt{d}+2d}-\sqrt{2d}\xrightarrow{d\to\infty}\frac{R}{2\sqrt{2}}$
- Page 171, Line 5: Multiplication with $\sqrt{(\Delta(d))^2+2d}+\sqrt{2d}\,\ldots$
- Page 173, Line 6: $\cdots\geqslant R\cdot\big(\sqrt{(\Delta(d))^2+2d}-\sqrt{2d}\,\big)\geqslant\cdots$
- Page 184, Line -6: $y_i\langle w^{(j+1)},\widehat{x}_i\rangle = y_i\langle w^{(j)}+y_i\widehat{x}_i,\widehat{x}_i\rangle = \underbrace{y_i\langle w^{(j)},\widehat{x}_i\rangle}_{\substack{\text{erroneously}\\ \text{less than zero}}} + \underbrace{y_i^2\langle\widehat{x}_i,\widehat{x}_i\rangle}_{\substack{\text{addition of a}\\ \text{positive cor-}\\ \text{rection term}}}.$
- Page 185, Line 18: Since D is linearly separable, there exists some $w\in\mathbb{R}^{d+1}\backslash\{0\}$, such that all points will be classified correctly. ~~This vector w cannot be the zero vector, because then $h\equiv 0$ and all data points would be classified incorrectly.~~
- Page 185, Line -4: $\langle w^*,w^{(j+1)}\rangle\geqslant\langle w^*,w^{(0)}\rangle+(j+1)\cdot\gamma=(j+1)\cdot\gamma$
- Page 186, Line 3: $\|w^{(j+1)}\|^2\leqslant\|w^{(0)}\|^2+(j+1)\cdot R^2=(j+1)\cdot R^2$.
- Page 186, Line 3: $(j+1)\cdot\gamma\langle w^*,w^{(j+1)}\rangle\leqslant\|w^*\|^{\cancel{2}}\|w^{(j+1)}\|^{\cancel{2}}=\|w^{(j+1)}\|^{\cancel{2}}+\sqrt{j+1}\cdot R$
- Page 186, Line 1: under the underbrace: $\leqslant 0$, as ...
- Page 187, Line -8: Assume that the perceptron algorithm outputs the generalized weight vector ...
- Page 190, Line 2: and ~~let~~ $b\in\mathbb{R}$
- Page 227, Line 11:



- Page 227, Line -3:



- Page 229, Line 13: Alternatively, with sigmoid activation, ...
- Page 229, Line -4: $N=\begin{bmatrix}a_{11}&\cdots&a_{1\ell}\\ \vdots&&\vdots\\ a_{m1}&\cdots&a_{m\ell}\end{bmatrix}\begin{bmatrix}n_1\\ \vdots\\ n_\ell\end{bmatrix}$
- Page 229, Line -3: ...and a matrix $(a_{ij})_{i,j}\in\mathbb{R}^{m\times\ell}$ ...
- Page 237, Line 5: $\forall\, w\in\mathbb{R}^{\cancel{n}},\ b\in\mathbb{R}\colon\ \displaystyle\int_\Omega\sigma(wx+b)+\ell\,\mathrm{d}\mu(x)=0$
- Page 237, Line -7: $\forall\, w\in\mathbb{R}^{\cancel{n}},\ b\in\mathbb{R}\colon\ \displaystyle\int_\Omega\sigma(wx+b)\,\mathrm{d}\mu(x)=0$
- Page 242, Line -4: Since Ω is compact there exists $c_0\geqslant 0$ such that $a_1f_1+\cdots+a_if_i+c_0\geqslant 0$ on Ω for all $i=1,\ldots,\ell$. We construct ...
- Page 282, Line 6: (ii) For $A,\,B\in\Sigma$ with $\mathrm{P}(B)\neq 0$, $\mathrm{P}(A|B):=\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$ ...

- Page 283, Line 19:

$$\rho(x) = \tfrac{1}{(2\pi\sigma^2)^{d/2}}\,\mathrm{e}^{-\frac{\|x-\mu\|^2}{2\sigma^2}} \quad \text{respectively} \quad \rho(x) = \tfrac{1}{\lambda^d(B)} \cdot \mathbb{1}_B(x),$$

- Page 286, Line −5: For $A = A_1 \times \cdots \times A_d \subseteq \mathbb{R}^d$ with $A_i \in \mathcal{B}^{\cancel{d}}$ we calculate
- Page 288, Line 2:

$$\begin{aligned}
(\rho_1 * \rho_2)(s) &= \frac{1}{2\pi\sqrt{ab}} \int_{\mathbb{R}} \exp\big(-\frac{(s-t)^2}{2a}\big)\exp\big(-\tfrac{t^2}{2b}\big)\,\mathrm{d}t \\
&= \frac{1}{2\pi\sqrt{ab}} \int_{\mathbb{R}} \exp\big(-\frac{b(s^2-2st+t^2)+at^2}{2ab}\big)\,\mathrm{d}t \\
&= \frac{1}{2\pi\sqrt{ab}} \int_{\mathbb{R}} \exp\big(-\frac{t^2(b+a)-2stb+bs^2}{2ab}\big)\,\mathrm{d}t \\
&= \frac{1}{2\pi\sqrt{ab}} \int_{\mathbb{R}} \exp\big(-\frac{t^2(b+a)/c-2stb/c+bs^2/c}{2ab/c}\big)\,\mathrm{d}t \\
&= \frac{1}{\sqrt{2\pi c}\sqrt{2\pi(ab/c)}} \int_{\mathbb{R}} \exp\big(-\frac{(t-(bs)/c)^2-(sb/c)^2+s^2(b/c)}{2ab/c}\big)\,\mathrm{d}t \\
&= \frac{1}{\sqrt{2\pi c}} \exp\big(+\frac{(sb/c)^2-s^2(b/c)}{2ab/c}\big)\frac{1}{\sqrt{2\pi(ab/c)}} \int_{\mathbb{R}} \exp\big(-\frac{(t-(bs)/c)^2}{2ab/c}\big)\,\mathrm{d}t \\
&= \frac{1}{\sqrt{2\pi c}} \exp\big(+\frac{(sb/c)^2c^2-s^2(b/c)c^2}{2abc}\big) \\
&= \frac{1}{\sqrt{2\pi c}} \exp\big(+\frac{s^2(b^2-bc)}{2abc}\big) \\
&= \frac{1}{\sqrt{2\pi c}} \exp\big(-\frac{s^2}{2c}\big),
\end{aligned}$$