Errata

(Mathematical Introduction to Data Science by Sven A. Wegner)

August 24, 2024

- Page 8, Line -4:
$$\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y}) = \sum_{i=1}^{n} x_iy_i - \overline{y}\sum_{i=1}^{n} x_i - \overline{x}\sum_{i=1}^{n} y_i + n\overline{x}\,\overline{y}$$
- Page 9, Line -3:
$$0=\sum_{i=1}^{n}(ax_i+b-y_i) = a\sum_{i=1}^{n}x_i + nb - \sum_{i=1}^{n} y_i = an\overline{x}+nb-n\overline{y},$$
- Page 13, Line 10: ...constant random variable av.
- Page 15, Line -9: $\cdots + \frac{2}{n^2}\sum_{i<j} x_ix_j \operatorname{E}(\mathcal{E}_i)\operatorname{E}\mathcal{E}_j)$
- Page 15, Line -5: since $\overline{x^{(n)2}} = \operatorname{var}(x^{(n)}) + \overline{x^{(n)}}^2$ as the sum ...
- Page 14, Line -2: If the latter is the case, then $\operatorname{sign}(r_{xy}) = \operatorname{sign}(\langle u, v\rangle) = \cdots$
- Page 39, Line 10: *The calculation of the k-neareast neighbors of x can be implemented such that at most $(n\cdot d\cdot k)$-many mulitplikations have to be carried out.*
- Page 39, Line 14: In the Euklidean metric, it requires $(d-1)$-many multiplications to compute one distance if we omit the root, which we can do as it does not change the argmin. This leads to
$$(d-1)\cdot(n+(n-1)+\cdots+(n-k+1)) \leqslant \cancel{C}\, d\cdot k\cdot n$$
multiplikationen ~~with a suitable $C\in\mathbb{N}$~~.
- Page 40, Line 3: ..., we choose k-nearest neighbors $x_1,\ldots,x_k$ of x and denote their labels by $y_1,\ldots,y_k$.
- Page 40, Line 9: $f\colon X\to Y,\; f(x) = \dfrac{\sum_{i=1}^{k} w(x_i,x)\cdot y_i}{\sum_{i=1}^{k} w(x_i,x)}$
- Page 41, Line 5: $\tilde{x}^{(i)} = \Big(a + \dfrac{(x_1^{(i)} - \min_{j=1,\ldots,n} x_1^{(j)})(b-a)}{\max_{j=1,\ldots,n} x_1^{(j)} - \min_{j=1,\ldots,n} x_1^{(j)}}, \ldots\Big)$
- Page 41, Line 7: $\tilde{x}^{(i)} = \Big(\dfrac{x_1^{(i)} - \overline{x_1^{(\cdot)}}}{\sigma_1}, \ldots\Big)$
- Page 42, Line 14: $\rho(x^{(1)}, x^{(4)}) = 3.681$
- Page 46, Line 1: We discuss some of these methods in Exercise 3.10.
- Page 45, Line 28: We thus see that texts no. 1 and text no. 2 are significantly more cosine similar than text no. 1 and text no. 3 or text no. 2 and text no. 3.
- Page 48, Line 26: The cosine distance, on the other hand, may appear hear more natural, as the scalar product increases if the frequency of the fixed word increases in the second text.
- Page 52, Line 11: For finite subsets $A, B\subseteq X$ we define ...

- Page 53, Line 20:

function LINKAGE-BASED CLUSTERING (X, ρ, D, δ)
  $k \leftarrow \#D$
  for $i \leftarrow 1$ to k **do**
    $C_i \leftarrow \{x_i\}$
  while $\min_{i \neq j} \rho(C_i, C_j) \leqslant \delta$ and $k \geqslant 2$ **do**
    $m \leftarrow 0$
    $(i^*, j^*) \leftarrow \operatorname{argmin}_{i \neq j} \rho(C_i, C_j)$
    for $\ell \leftarrow 1$ to $k-1$ **do**
      if $\ell = \min(i^*, j^*)$ **then**
        $C_\ell \leftarrow C_{i^*} \cup C_{j^*}$
      if $\ell = \max(i^*, j^*)$ **then**
        $m \leftarrow 1$
        $C_\ell \leftarrow C_{\ell+m}$
      else
        $C_\ell \leftarrow C_{\ell+m}$
    $k \leftarrow k - 1$
  return $C_1, \ldots, C_k$

- Page 54, Line −6: $K\colon \mathcal{C}_k \to \mathbb{R}$
- Page 56, Line −16: *The following pseudocode approximates a minimizer of the k-means cost function.*
- Page 56, Pseudocode:

1: **function** K-MEANS (D, k, X, ρ)
2:   $\mu_1, \ldots, \mu_k \leftarrow$ pairwise different points from X
3:   **for** $i \leftarrow 1$ to k **do**
4:     $C_i \leftarrow \{x \in D \mid i \in \operatorname{argmin}_{j=1,\ldots,k} \rho(x, \mu_j)\}$
5:   $U \leftarrow$ True
6:   **while** $U =$ True **do**
7:     $U \leftarrow$ False
8:     **for** $i \leftarrow 1$ to k **do**
9:       $\mu_i \leftarrow \mu(C_i)$
10:     **for** $i \leftarrow 1$ to k **do**
11:       $C_i' \leftarrow \{x \in D \mid i \in \operatorname{argmin}_{j=1,\ldots,k} \rho(x, \mu_j)\}$
12:       **if** $C_i' \neq C_i$
13:         $C_i \leftarrow C_i'$
14:         $U \leftarrow$True
15:   **return** $C_1, \ldots, C_k$

In the lines 4 and 9 of the pseudocode we pick as a single i in the case that the armin is not unique.

- Page 57, Line 7:

$$\mu(A) \in \operatorname*{argmin}_{\mu \in A} \sum_{x \in A} \rho(x, \mu)^2, \quad \text{respectively} \quad \mu(A) \in \operatorname*{argmin}_{\mu \in X} \sum_{x \in A} \rho(x, \mu).$$

- Page 57, Line −7: *Proof.* For $j \geqslant 1$ denote by $(C_1^{(j)}, \ldots, C_k^{(j)})$ that clustering which the algorithm produces in the j-th round.
- Page 57, Line −4: $K(C_1^{(j)}, \ldots, C_k^{(j)}) \;=\; \min_{\mu_1, \ldots, \mu_k \in X} \sum_{i=1}^{k} \sum_{x \in C_i^{(j)}} \rho(x, \mu_i)^2$
- Page 58, Line 3: …line 11 of Algorithm 4.9 …
- Page 58, Line 7: There, Picture 1b corresponds to the penultimate line in the estimate and Picture 2a to the line above that.
- Page 58, Line 10: …as we have just moved point x_3 from cluster C_2 in Figure 1b to cluster C_1 in Figure 2a, …

- Page 59, Line -4: We assume that we start with the initial values $\mu_1 = 2$ and ...
- Page 63, Line -2: ... In Example 5.7, $\lambda_2 \neq 0$ and there are no clusters (or, depending on how one prefers to see it, one single cluster), in Example 5.8 ...
- Page 67, Line 8: For the other direction let $\{v_1, \dots, v_n\}$ be a basis consisting of eigenvectors corresponding to the λ_i and let $U \subseteq \mathbb{R}^n$ be a subspace with $\dim U = n-k+1$. By construction $U \cap \operatorname{span}\{v_1, \dots, v_k\} \neq \{0\}$ and we can select $0 \neq x = \alpha_1 v_1 + \cdots + \alpha_k v_k \in U$. Then it follows

$$\frac{\langle x, Mx\rangle}{\langle x, x\rangle} = \frac{\sum_{i=1}^{k} \lambda_i \alpha_i^2}{\sum_{i=1}^{k} \alpha_i^2} \leqslant \frac{\lambda_k \sum_{i=1}^{k} \alpha_i^2}{\sum_{i=1}^{k} \alpha_i^2} = \lambda_k,$$

since the λ_i's are increasing. With this we get $\min_{0\neq x \in U} \frac{\langle x, Mx\rangle}{\langle x, x\rangle} \leqslant \lambda_k$ which then leads to

$$\max_{\substack{U \subseteq \mathbb{R}^n \\ \dim U = n-k+1}} \min_{\substack{x \in U \\ x \neq 0}} \frac{\langle x, Mx\rangle}{\langle x, x\rangle} \leqslant \lambda_k.$$

- Page 68, Line 14: Let $G = (V, E)$ be a graph with $\deg(v) > 0$ for all $v \in V$.
- Page 70, Line 10:

$$\lambda_2(\mathcal{L}) = \min_{\substack{x \neq 0 \\ \langle Dx, \mathbb{1}\rangle = 0}} \frac{\sum_{\{i,j\}\in E} (x_i - x_j)^2}{\sum_{i=1}^{n} x_i^2 d_i}.$$

- Page 70, Line 16:

$$\mathcal{L} D^{1/2}\mathbb{1} = D^{-1/2} L D^{-1/2} D^{1/2} \mathbb{1} = D^{-1/2} L \mathbb{1} \underset{\substack{\uparrow \\ \text{Proposition} \\ 5.6}}{=} D^{-1/2} 0 \mathbb{1} = 0 D^{1/2} \mathbb{1}.$$

- Page 70, Line 18:

$$\begin{aligned} \lambda_2(\mathcal{L}) &\overset{\substack{\text{Theorem} \\ 5.13}}{=} \min_{\substack{x \neq 0 \\ \langle x, D^{1/2}\mathbb{1}\rangle = 0}} \frac{\langle x, \mathcal{L}x\rangle}{\langle x, x\rangle} \overset{(*)}{=} \min_{\substack{y \neq 0 \\ \langle D^{1/2}y, D^{1/2}\mathbb{1}\rangle = 0}} \frac{\langle D^{1/2}y, \mathcal{L} D^{1/2} y\rangle}{\langle D^{1/2} y, D^{1/2} y\rangle} \\ &= \min_{\substack{y \neq 0 \\ \langle y, D\mathbb{1}\rangle = 0}} \frac{\langle y, Ly\rangle}{\langle y, Dy\rangle} = \min_{\substack{y \neq 0 \\ \langle Dy, \mathbb{1}\rangle = 0}} \frac{\sum_{\{i,j\}\in E} (y_i - y_j)^2}{\sum_{i=1}^{n} y_i^2 d_i} \end{aligned}$$

- Page 71, Line 14: *(ii)* $\min(\operatorname{vol} S_k, \operatorname{vol} S_k^{\mathrm{c}}) = \operatorname{vol} S_k^{\mathrm{c}}$ *and* $\operatorname{vol} S_k^{\mathrm{c}} - \operatorname{vol} S_{k+1}^{\mathrm{c}} = d_{k+1}$ *hold whenever* $r \leqslant k \leqslant n-1$.
- Page 71, Line 17:

$$\operatorname{vol} S_k^{\mathrm{c}} - \operatorname{vol} S_{k+1}^{\mathrm{c}} = \sum_{i=k+1}^{n} d_i - \sum_{i=k+2}^{n} d_i = d_{k+1}$$

- Page 72, Line 9:

$$\langle Dx, \mathbb{1}\rangle = \sum_{i=1}^{n} d_i x_i = \dots$$

- Page 72, Line -1 (and Page 73, Line 1):

$$\begin{aligned} \cdots &= \operatorname{vol} S - \operatorname{vol} S \cdot \tfrac{\operatorname{vol} S}{\operatorname{vol} S + \operatorname{vol} S^{\mathrm{c}}} \\ &\geqslant \operatorname{vol} S - \operatorname{vol} S \cdot \tfrac{\operatorname{vol} S}{2 \operatorname{vol} S}, \end{aligned}$$

- Page 73, Line 9: ... and our goal in the following will be to show $\lambda_2 \geqslant \alpha^2/2$, ...
- Page 73, Line 12 (Equation (5.2)):

$$\cdots \text{ and } \langle Dx, \mathbb{1}\rangle = \sum_{i=1}^{n} d_i x_i = 0$$

- Page 73, Line 21:

$$\begin{bmatrix} x_1 - x_r \\ \vdots \\ x_{r-1} - x_r \\ 0 \\ x_{r+1} - x_r \\ \vdots \\ x_n - x_r \end{bmatrix} = \begin{bmatrix} x_1 - x_r \\ \vdots \\ x_{r-1} - x_r \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ x_r - x_{r+1} \\ \vdots \\ x_r - x_n \end{bmatrix} =: p - n.$$

- Page 74, Line -3 (until top of page 75):

$$\begin{aligned}
\lambda_2 &= \frac{\sum_{\{i,j\}\in E}(x_i - x_j)^2}{\sum_{i=1}^n x_i^2 d_i} \\
&\underset{\substack{\uparrow \\ (5.3) \\ (5.4)}}{\geqslant} \frac{\sum_{\{i,j\}\in E}\left((p_i - p_j)^2 + (n_i - n_j)^2\right)}{\sum_{i=1}^n (p_i^2 + n_i^2) d_i} \\
&= \frac{\sum_{\{i,j\}\in E}(p_i - p_j)^2 + \sum_{\{i,j\}\in E}(n_i - n_j)^2}{\sum_{i=1}^n p_i^2 d_i + \sum_{i=1}^n n_i^2 d_i} \\
&\underset{\substack{\uparrow \\ (5.5)}}{\geqslant} \min\left(\frac{\sum_{\{i,j\}\in E}(p_i - p_j)^2}{\sum_{i=1}^n p_i^2 d_i}, \frac{\sum_{\{i,j\}\in E}(n_i - n_j)^2}{\sum_{i=1}^n n_i^2 d_i}\right) \\
&= \min\left(\frac{\sum_{\{i,j\}\in E}(p_i - p_j)^2}{\sum_{i=1}^n p_i^2 d_i} \cdot \frac{\sum_{\{i,j\}\in E}(p_i + p_j)^2}{\sum_{\{i,j\}\in E}(p_i + p_j)^2}, \dots\right) \\
&=: \min\left(\frac{Z}{N}, \dots\right).
\end{aligned}$$

- Page 75, Line 8:

$$N = \sum_{i=1}^n p_i^2 d_i \cdot \sum_{\{i,j\}\in E} (p_i + p_j)^2 \underset{\substack{\uparrow \\ (5.5)}}{\geqslant} \sum_{i=1}^n p_i^2 d_i \cdot \sum_{\{i,j\}\in E} 2(p_i^2 + p_j^2)$$

- Page 76, Line 3:

$$\cdots \underset{\substack{\uparrow \\ \text{telescopic} \\ \text{sum}}}{=} \left(\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \mathbb{1}_E(i,j) \sum_{k=i}^{j-1} \left(p_k^2 - p_{k+1}^2\right) \right)^2$$

- Page 79, Line 13: Finally, we want to note that Theorem 5.21 together with Remark 5.17(i) provides an upper bound for the eigenvalue $\lambda_2(\mathcal{L})$.

  Corollary 5.22. Let $G = (V, E)$ be a graph with $\deg(i) > 0$ for all $i \in V$. Then for the second smallest eigenvalue λ_2 of the normalized Laplace matrix of G the estimate $\lambda_2 \leqslant 2$ holds. □

- Page 105, Zeile -1 and Page 106, Line 1:

$$A = \underbrace{\begin{bmatrix} 0.07 & 0.29 & 0.32 & 0.51 & 0.66 & 0.18 & -0.23 \\ 0.13 & -0.02 & -0.01 & -0.79 & 0.59 & -0.02 & -0.06 \\ 0.68 & -0.11 & -0.05 & -0.05 & -0.24 & 0.56 & -0.35 \\ 0.15 & 0.59 & 0.65 & -0.25 & -0.33 & -0.09 & 0.11 \\ 0.41 & -0.07 & -0.03 & 0.10 & -0.02 & -0.78 & -0.43 \\ 0.07 & 0.73 & -0.67 & 0.00 & -0.00 & 0.00 & 0.00 \\ 0.55 & -0.09 & -0.04 & 0.17 & 0.17 & -0.11 & 0.78 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} 12.4 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 9.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.3 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} 0.56 & 0.09 & 0.56 & 0.09 & 0.59 \\ -0.12 & 0.69 & -0.12 & 0.69 & 0.02 \\ -0.40 & -0.09 & -0.40 & -0.09 & 0.80 \\ \cancel{-0.41} & \cancel{0.09} & \cancel{0.40} & \cancel{0.09} & \cancel{-0.80} \\ 0.51 & 0.48 & -0.51 & -0.48 & -0.00 \\ 0.48 & -0.51 & -0.48 & 0.51 & -0.00 \end{bmatrix}}_{V^{\mathrm{T}}}$$

$$= \begin{bmatrix} 0.07 & 0.29 & 0.32 \\ 0.13 & -0.02 & -0.01 \\ 0.68 & -0.11 & -0.05 \\ 0.15 & 0.59 & 0.65 \\ 0.41 & -0.07 & -0.03 \\ 0.07 & 0.73 & -0.67 \\ 0.55 & -0.09 & -0.04 \end{bmatrix} \begin{bmatrix} 12.4 & & \\ & 9.5 & \\ & & 1.3 \end{bmatrix} \begin{bmatrix} 0.56 & 0.09 & 0.56 & 0.09 & 0.59 \\ -0.12 & 0.69 & -0.12 & 0.69 & 0.02 \\ -0.40 & -0.09 & -0.40 & -0.09 & 0.80 \end{bmatrix}.$$

- Page 106, Zeile 6:

$$\breve{A} = \begin{bmatrix} 0.15 & 1.97 & 0.15 & 1.97 & 0.56 \\ 0.92 & 0.01 & 0.92 & 0.01 & \color{red}{0.94} \\ 4.84 & 0.03 & 4.84 & 0.03 & 4.95 \\ 0.36 & 4.03 & 0.36 & 4.03 & 1.20 \\ 2.92 & \color{red}{-0.00} & 2.92 & \color{red}{-0.00} & 2.98 \\ -0.34 & 4.86 & -0.34 & 4.86 & 0.65 \\ \color{red}{3.92} & \color{red}{0.02} & \color{red}{3.92} & \color{red}{0.02} & \color{red}{4.00} \end{bmatrix} = \begin{bmatrix} 0.07 & 0.29 \\ 0.13 & -0.02 \\ 0.68 & -0.11 \\ 0.15 & 0.59 \\ 0.41 & -0.07 \\ 0.07 & 0.73 \\ 0.55 & \color{red}{-0.09} \end{bmatrix} \begin{bmatrix} 12.4 & \\ & 9.5 \end{bmatrix} \begin{bmatrix} 0.56 & 0.09 & 0.56 & 0.09 & 0.59 \\ -0.12 & 0.69 & -0.12 & 0.69 & 0.02 \end{bmatrix}$$

- Page 107, Line 15:

$$= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0.07 & 0.29 & \cdots & -0.23 \\ 0.13 & -0.02 & & -0.06 \\ 0.68 & -0.11 & & -0.35 \\ 0.15 & 0.59 & & 0.11 \\ 0.41 & -0.07 & & -0.43 \\ 0.07 & 0.73 & & 0.00 \\ 0.55 & \color{red}{-0.09} & \cdots & 0.78 \end{bmatrix} \begin{bmatrix} 12.4 & & & \\ & 9.5 & & \\ & & 1.3 & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} 0.56 & 0.09 & 0.56 & 0.09 & 0.59 \\ -0.12 & 0.69 & -0.12 & 0.69 & 0.02 \\ \vdots & & & & \vdots \\ 0.48 & -0.51 & -0.48 & 0.51 & 0.00 \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

- Page 109, Line −2:

$$u_2 = \quad 0.29 \cdot \text{Abbie} - 0.02 \cdot \text{Bailey} + \cdots \color{red}{-0.09} \cdot \text{Gladys},$$

- Page 109, Line 10:

$$\begin{array}{r} \text{Abbie} \searrow \\ \text{Bailey} \searrow \\ \text{Catherine} \searrow \\ \text{Darlene} \searrow \\ \\ \\ \text{Elena} \nearrow \\ \text{Fatima} \rightarrow \\ \text{Gladys} \rightarrow \end{array} \breve{A} = \begin{bmatrix} \boxed{0.07 \quad 0.29} \\ \boxed{0.13 \quad -0.02} \\ \boxed{0.68 \quad -0.11} \\ \boxed{0.15 \quad 0.59} \\ \boxed{0.41 \quad -0.07} \\ \boxed{0.07 \quad 0.73} \\ \boxed{0.55 \quad \color{red}{-0.09}} \end{bmatrix} \begin{bmatrix} 12.4 & \\ & 9.5 \end{bmatrix} \begin{array}{c} \begin{array}{ccccc} \text{Alien} & \text{Casablanca} & \text{Star Wars} & \text{Titanic} & \text{The Matrix} \\ \searrow & \searrow & \swarrow & \swarrow & \swarrow \end{array} \\ \begin{bmatrix} \boxed{\begin{matrix} 0.56 \\ -0.12 \end{matrix}} & \boxed{\begin{matrix} 0.09 \\ 0.69 \end{matrix}} & \boxed{\begin{matrix} 0.56 \\ -0.12 \end{matrix}} & \boxed{\begin{matrix} 0.09 \\ 0.69 \end{matrix}} & \boxed{\begin{matrix} 0.59 \\ 0.02 \end{matrix}} \end{bmatrix} \end{array}$$

- Page 282, Line 6: (ii) For A, $B \in \Sigma$ $\color{red}{\text{with } \mathcal{P}(B) \neq 0}$, $\mathcal{P}(A|B) := \dfrac{\mathcal{P}(A \cap B)}{\mathcal{P}(B)}$ ...
- Page 283, Line 19:

$$\rho(x) = \frac{1}{(2\pi\sigma^2)^{d/2}} \, \mathrm{e}^{-\frac{\|x-\mu\|^2}{2\sigma^2}} \quad \text{respectively} \quad \rho(x) = \frac{1}{\lambda^{\color{red}{d}}(B)} \cdot \mathbb{1}_B(x),$$

- Page 286, Line −5: For $A = A_1 \times \cdots \times A_d \subseteq \mathbb{R}^d$ with $A_i \in \mathcal{B}^{\color{red}{\cancel{d}}}$ we calculate
- Page 288, Line 2:

$$\begin{aligned} (\rho_1 * \rho_2)(s) &= \frac{1}{2\pi\sqrt{ab}} \int_{\mathbb{R}} \exp\Big(-\frac{(s-t)^2}{2a}\Big) \exp\Big(-\frac{t^2}{2b}\Big) \, \mathrm{d}t \\ &= \frac{1}{2\pi\sqrt{ab}} \int_{\mathbb{R}} \exp\Big(-\frac{b(s^2 - 2st + t^2) + at^2}{2ab}\Big) \, \mathrm{d}t \\ &= \frac{1}{2\pi\sqrt{ab}} \int_{\mathbb{R}} \exp\Big(-\frac{t^2(b+a) - 2stb + bs^2}{2ab}\Big) \, \mathrm{d}t \\ &= \frac{1}{\color{red}{2\pi\sqrt{ab}}} \int_{\mathbb{R}} \exp\Big(-\frac{t^2(b+a)/c - 2stb/c + bs^2/c}{2ab/c}\Big) \, \mathrm{d}t \\ &= \frac{1}{\color{red}{\sqrt{2\pi c}\sqrt{2\pi(ab/c)}}} \int_{\mathbb{R}} \exp\Big(-\frac{(t-(bs)/c)^2 - (sb/c)^2 + s^2(b/c)}{2ab/c}\Big) \, \mathrm{d}t \\ &= \frac{1}{\sqrt{2\pi c}} \exp\Big(\color{red}{+}\frac{(sb/c)^2 - s^2(b/c)}{2ab/c}\Big) \frac{1}{\sqrt{2\pi(ab/c)}} \int_{\mathbb{R}} \exp\Big(-\frac{(t-(bs)/c)^2}{2ab/c}\Big) \, \mathrm{d}t \\ &= \frac{1}{\sqrt{2\pi c}} \exp\Big(\color{red}{+}\frac{(sb/c)^2 c^2 - s^2(b/c)c^2}{2abc}\Big) \\ &= \frac{1}{\sqrt{2\pi c}} \exp\Big(\color{red}{+}\frac{s^2(b^2 - bc)}{2abc}\Big) \\ &= \frac{1}{\sqrt{2\pi c}} \exp\Big(-\frac{s^2}{2c}\Big), \end{aligned}$$