

Using Pushforward and Pullback Measures for Parameter Identification and Distribution Estimation

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Distribution Estimation

Goal: Obtain the Best Distribution of λ

Bayesian Context: Regular Bayes model is insufficient, Hierarchical Bayes model required.

Data Consistent Context: Use data to construct observed distribution.

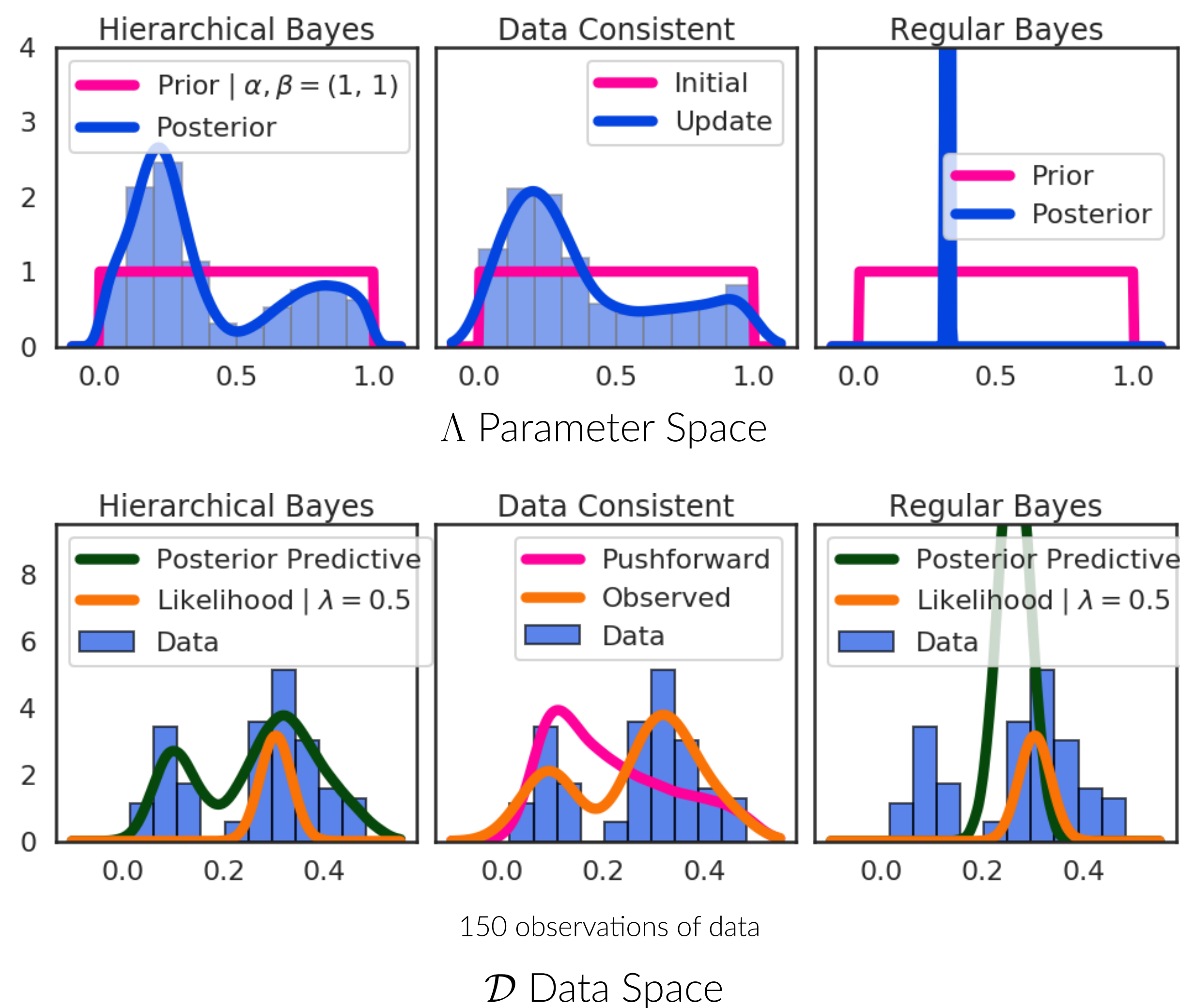
Example

Consider an exponential decay problem with uncertain decay rate:

$$u(t) = u_0 \exp(-\lambda t), \quad u_0 = 0.5, \quad t = 2$$

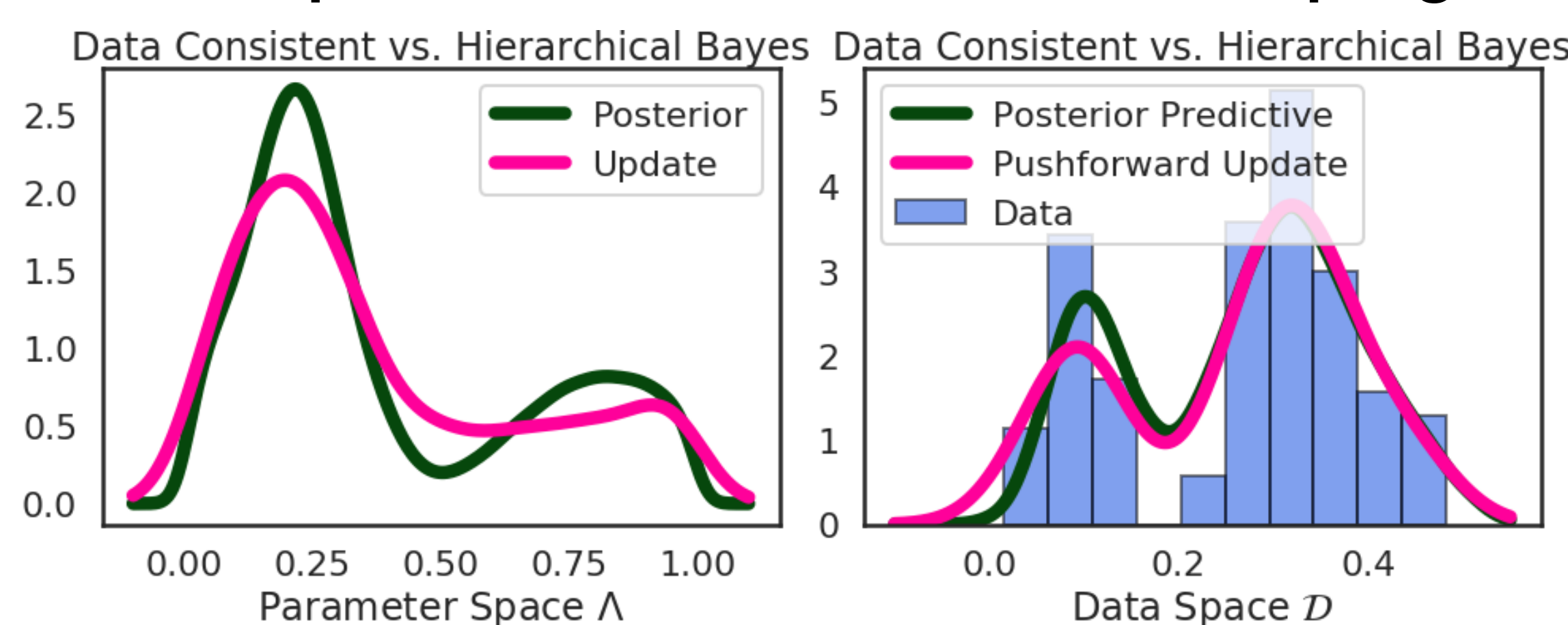
Regular Bayes	$\pi_{\text{prior}} \sim U[0, 1], \quad \pi_L(\mathbf{d} \lambda) \sim N(Q(\lambda), \sigma^2)$ $\pi_{\text{post}}(\lambda \mathbf{d}) \propto \pi_{\text{prior}}(\lambda) \pi_L(\mathbf{d} \lambda)$
Hierarchical Bayes	$\pi_{\text{prior}}(\alpha, \beta) \sim \chi_1^2, \quad \alpha, \beta \in \Omega := [0, \infty) \times [0, \infty)$ $\pi_{\text{prior}}(\lambda \alpha, \beta) \sim \text{Beta}(\alpha, \beta), \quad \pi_L(\mathbf{d} \lambda) \sim N(Q(\lambda), \sigma^2)$ $\pi_{\text{post}}(\lambda \mathbf{d}) \propto \int_{\Omega} \pi_{\text{prior}}(\lambda, \alpha, \beta) \pi_L(\mathbf{d} \lambda, \alpha, \beta) d\Omega$
Data Consistent	$\pi_{\text{up}}(\lambda) = \pi_{\text{in}}(\lambda) \frac{\pi_{\text{obs}}(Q(\lambda))}{\pi_{\text{pre}}(Q(\lambda))}$

Plots of Concepts and Results



Takeaways

Non-parametric method with less sampling



Intro to Data Consistent Inversion

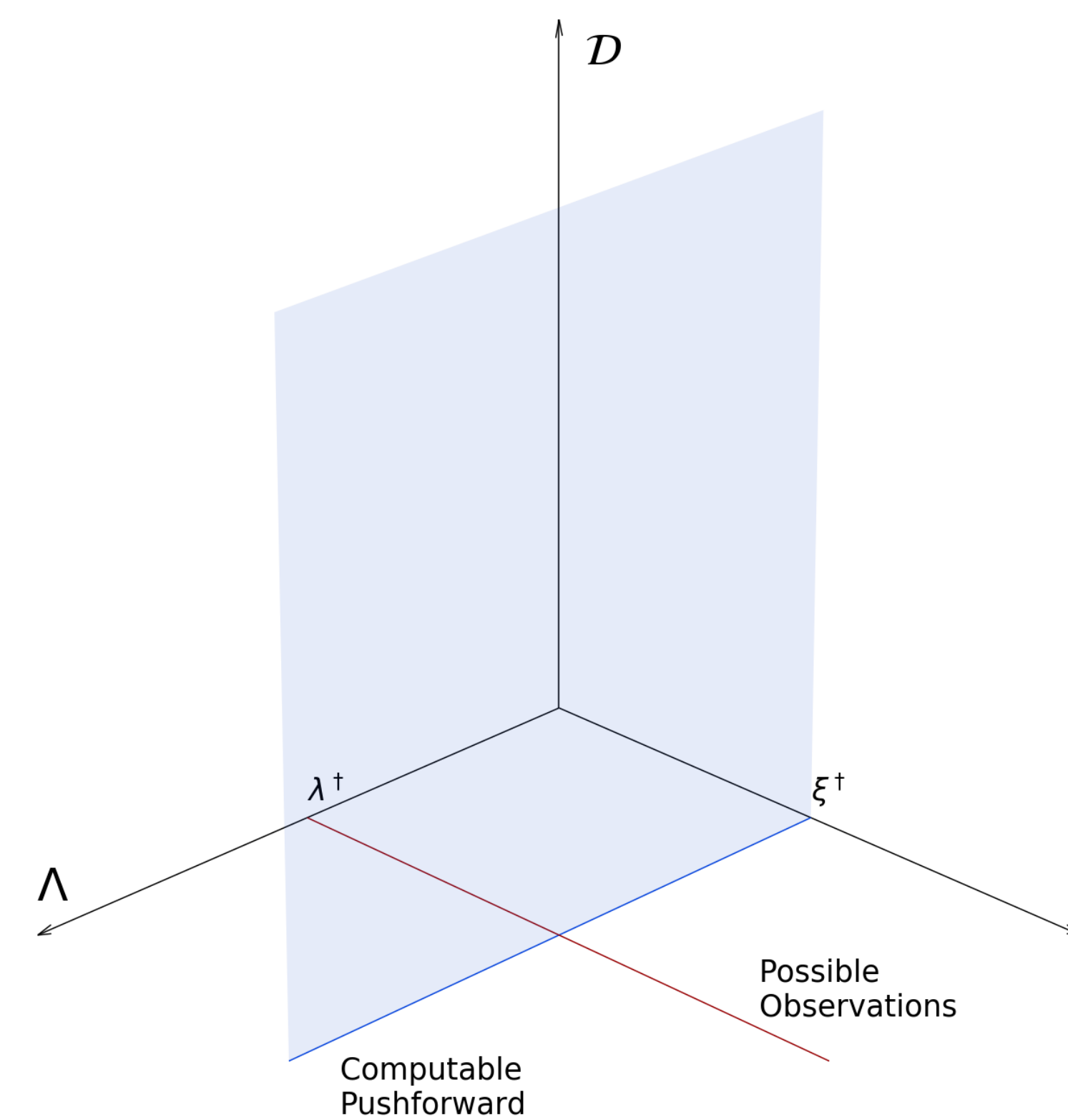
Solving Stochastic Inverse Problems

Data Consistent Inversion is a novel framework that uses pushforward and pullback measures to ensure solutions are consistent with the observed distribution of data.

The Data Consistent Approach

$$\pi_{\text{up}}(\lambda) = \pi_{\text{in}}(\lambda) \frac{\pi_{\text{obs}}(Q(\lambda))}{\pi_{\text{pre}}(Q(\lambda))}$$

Which Stochastic Inverse Problem?



Do you want to solve for a single parameter value or for a parameter distribution?

Notation

- $\lambda \in \Lambda, \xi \in \Xi$ Parameter Space, Noise Space
- $\mathbf{d} \in \mathcal{D}$ Observables
- $Q: \Lambda \rightarrow \mathcal{D}$ Quantity of Interest Map
- $\pi_{\text{prior}}, \pi_L$ Prior, Likelihood
- $\pi_{\text{in}}, \pi_{\text{obs}}, \pi_{\text{pre}}$ Initial, Observed, Predicted (pushforward)
- $\pi_{\text{post}}, \pi_{\text{up}}$ Posterior, Update (pullback)

References & Attribution

Advisor: Dr. Troy Butler
University of Colorado: Denver



Left to Right: Theory, Stability, BET, Poster, Website. Funding: NSF DMS-1818941.

Parameter Identification

Goal: Obtain the Best Value of λ

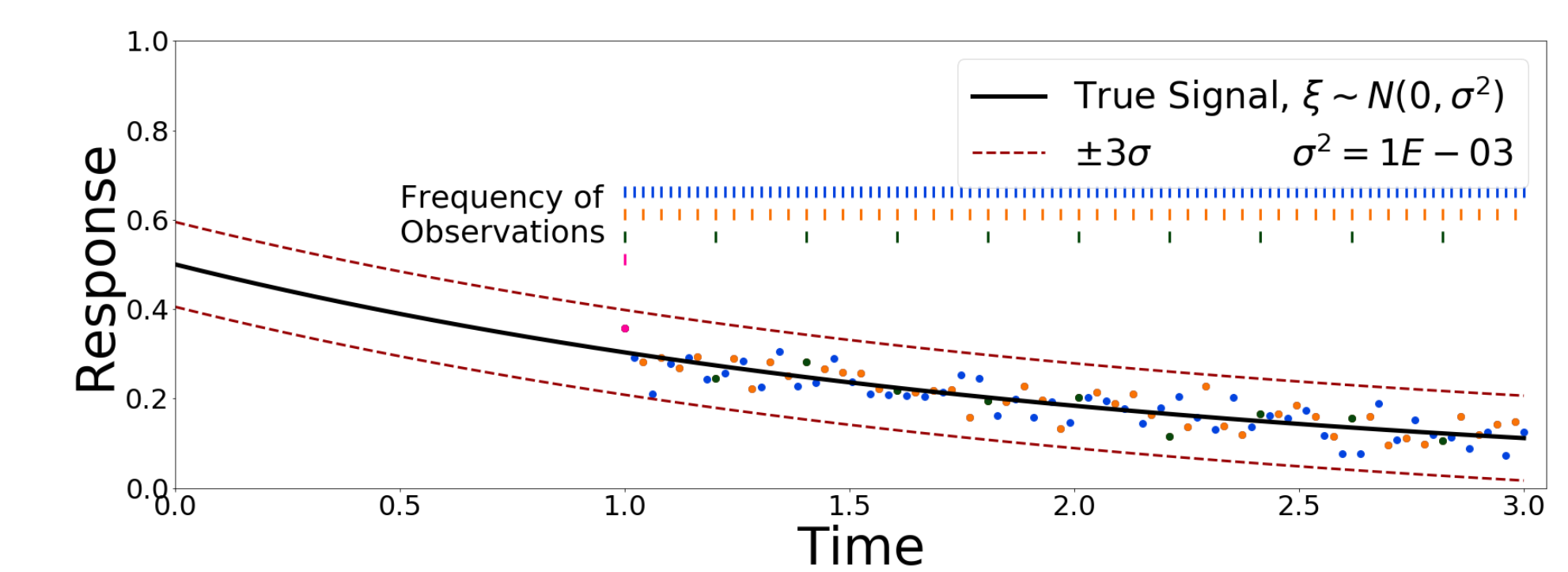
Bayesian Context: Regular Bayes model uses assumed likelihood function of data given λ .

Data Consistent Context: Uses data to construct a predicted distribution of the average residuals given λ .

Example

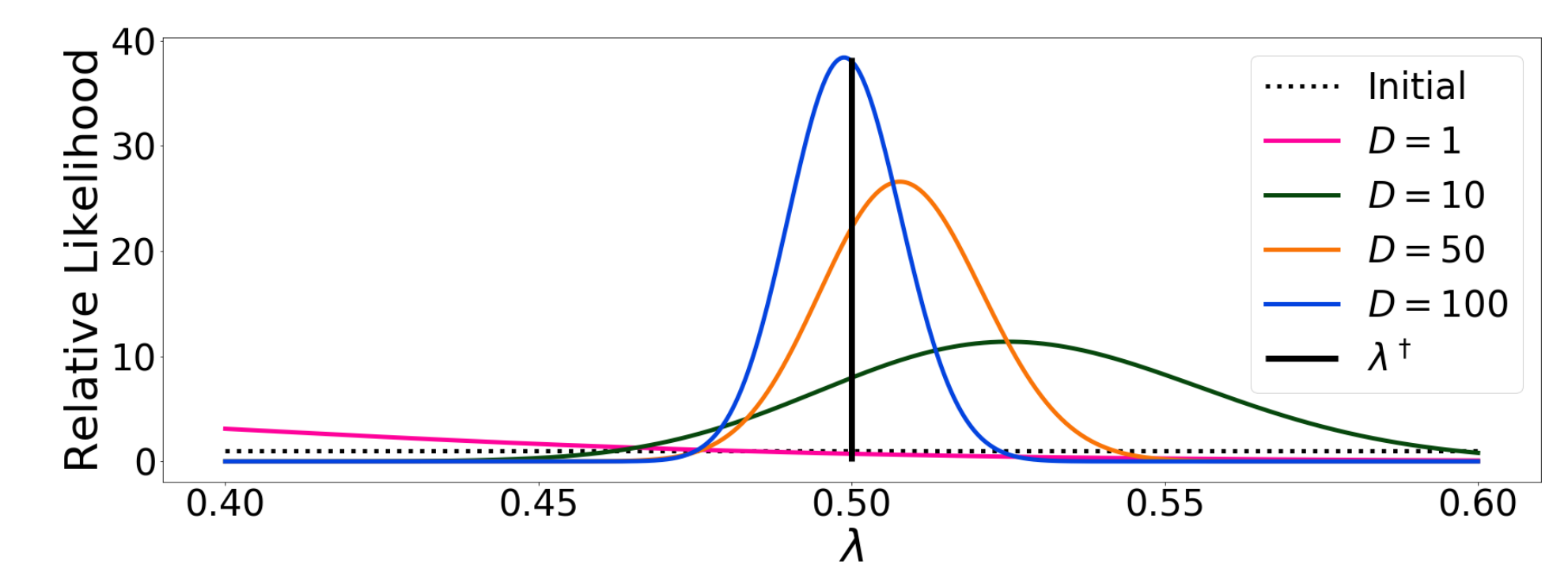
Consider an exponential decay problem with uncertain decay rate:

$$u(t) = u_0 \exp(-\lambda t), \quad u_0 = 0.5, \quad t \in [0, 3]$$



Convergence of Data Consistent Inversion

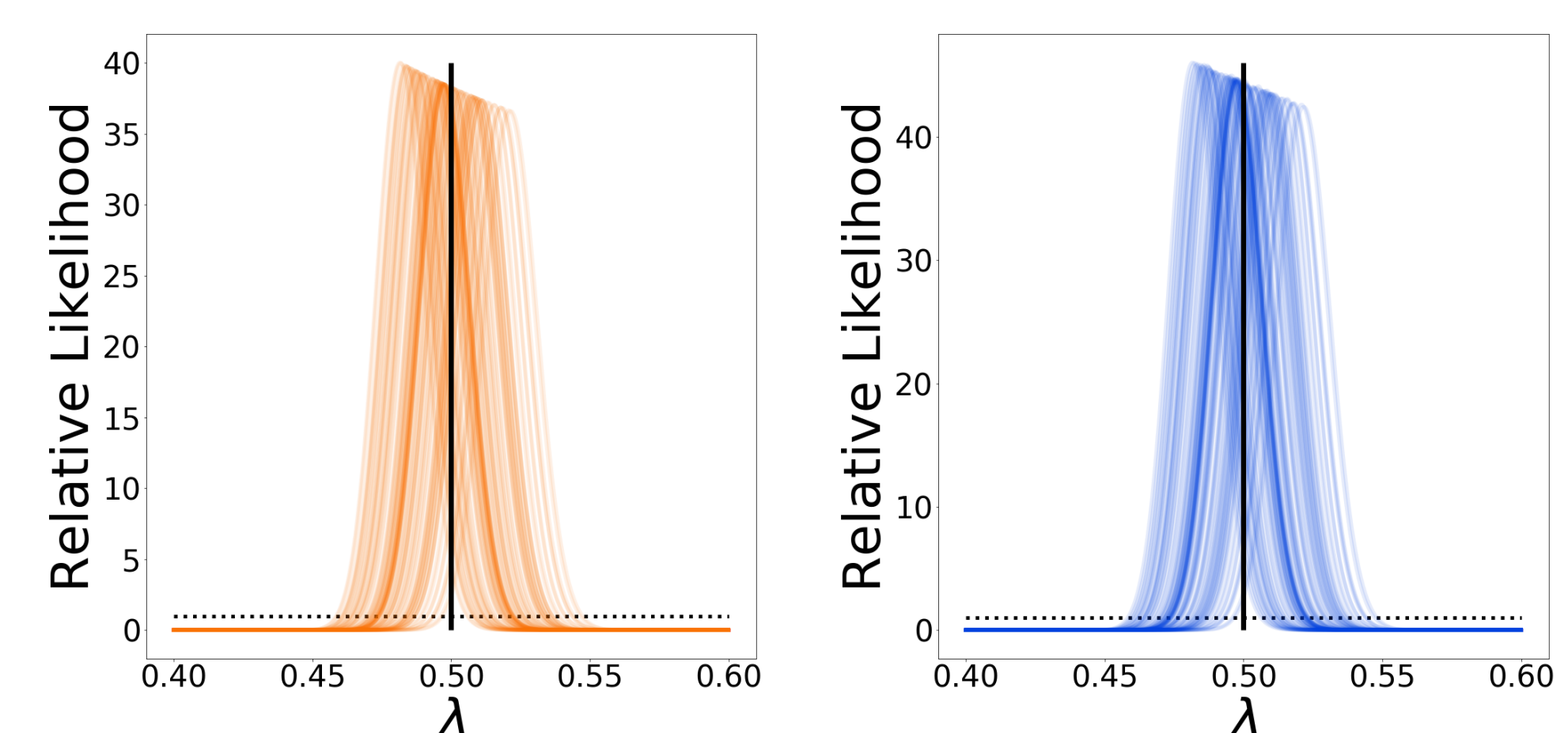
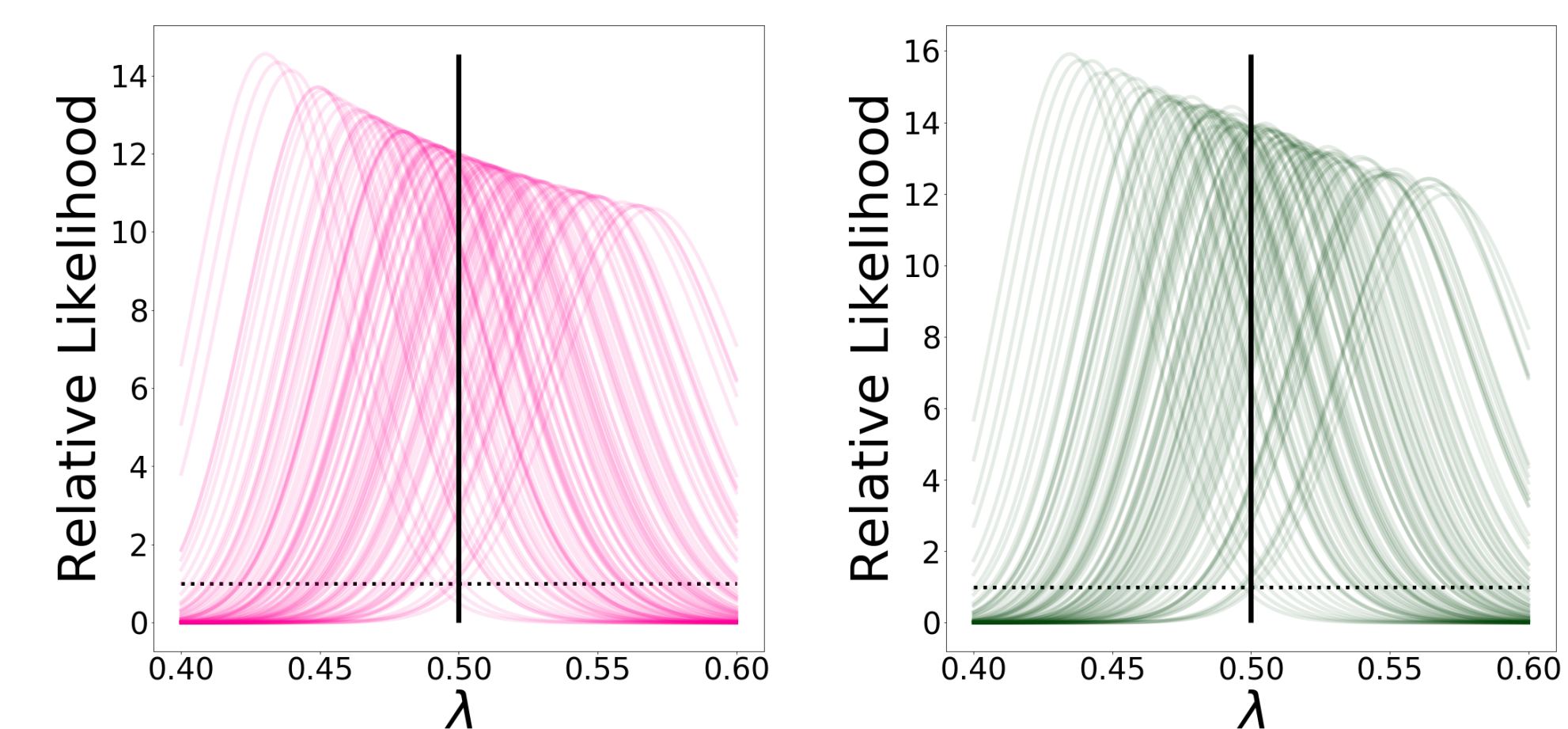
How do solutions change with more data?



λ^\dagger and π_{up} for $D = 1, 10, 50, 100$ for $N = 1000$

Comparison to Regular Bayes

How do solutions on conditionals of Ξ compare?



100 realizations of data for $D = 10, 100$ for Data Consistent Inversion (left) and Regular Bayes (right).