COMPUTATIONAL ADVANCES IN DATA-CONSISTENT INVERSION: MEASURE-THEORETIC METHODS FOR IMPROVING PREDICTIONS

by

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A thesis submitted to the

Faculty of the Graduate School of the

University of Colorado in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

Applied Mathematics Program

2019

This thesis for the Doctor of Philosophy degree by

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Date 21 Sep 2019

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Computational Advances in Data-Consistent Inversion: Measure-Theoretic Methods for Improving Predictions

Thesis directed by Assistant Professor Dr. Troy Butler

ABSTRACT

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The form and content of this abstract are approved. I recommend its publication.

Approved: Dr. Troy Butler

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ACKNOWLEDGEMENTS

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ABBREVIATIONS AND NOTATION

A.1 Abbreviations

CNR Control net reduction.

CPR Control polygon reduction.

CRAN The Comprehensive R Archive Network.

DHS Daily Hormone Study, a sub-study of SWAN.

DLT Day of Luteal Transition.

PDG Prognanediol-glucuronid, is the urine metabolite of progesterone.

SWAN The Study of Women's Health Across the Nation.

TTM Time-to-menopause

A.2 Mathematic Notation

A.2.1 General Notation

x italicized, Roman or Greek letter, denotes a scalar values

$\boldsymbol{x}$ italicized, bold, lowercase Roman or Greek letter, denotes a column vector or set.

$\boldsymbol{X}$ italicized, bold, uppercase Roman or Greek letter, denotes a matrix or set.

$n(\boldsymbol{x})$ cardinality, number of elements, of the vector or set $\boldsymbol{x}$

$x \in (a, b)$ the value x is within the interval such that $a < x < b$

$x \in (a, b]$ the value x is within the interval such that $a < x \leq b$

$x \in [a, b)$ the value x is within the interval such that $a \leq x < b$

$x \in [a, b]$ the value x is within the interval such that $a \leq x \leq b$

$1_A(x)$ the indicator function,

$$1_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}.$$

$\mathbf{1}_n$	an column vector of n 1s
$\boldsymbol{I}$	the identity matrix
$\boldsymbol{I}_n$	the $n \times n$ identity matrix
$\boldsymbol{X}^{-1}$	the inverse matrix, that is, $\boldsymbol{X}^{-1}\boldsymbol{X} = \boldsymbol{I}$.
$\boldsymbol{X}^T$	transpose
$\otimes$	Kronecker product
$\odot$	element-wise multiplication

A.2.2 Sets

$\{x, y, z, \ldots\}$	The set comprising the elements of x, y, z, ...
$\{x, y, z\} \backslash x$	The set comprising the elements of y and z, that is, the backslash removes elements from the set.
$\{x_i\}_{i=1}^n$	The set comprising the elements of $x_1, x_2, x_3, \ldots, x_n$.
$\mathbb{R}$	Set of real numbers
$\boldsymbol{x} \in \mathbb{R}^n$	$\boldsymbol{x}$ is a vector with n elements, all of which are real numbers.

A.2.3 Statistical Distributions

$\mathcal{N}\left(\mu, \sigma^2\right)$	the uni-variable Gaussian distribution with mean μ and variance σ^2.
$\mathcal{N}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$	the multi-variable Gaussian distribution with mean vector $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$.
$\phi(x)$	The standard Gaussian density function.

APPENDIX B
APPENDIX HEADER

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