

COMPUTATIONAL ADVANCES IN DATA-CONSISTENT INVERSION: MEASURE-THEORETIC
METHODS FOR IMPROVING PREDICTIONS

by

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Computational Advances in Data-Consistent Inversion: Measure-Theoretic Methods for Improving Predictions

Thesis directed by Assistant Professor Dr. Troy Butler

ABSTRACT

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The form and content of this abstract are approved. I recommend its publication.

Approved: Dr. Troy Butler

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ABBREVIATIONS AND NOTATION

A.1 Abbreviations

CNR	Control net reduction.
CPR	Control polygon reduction.
CRAN	The Comprehensive R Archive Network.
DHS	Daily Hormone Study, a sub-study of SWAN.
DLT	Day of Luteal Transition.
PDG	Prognanediol-glucuronid, is the urine metabolite of progesterone.
SWAN	The Study of Women's Health Across the Nation.
TTM	Time-to-menopause

A.2 Mathematic Notation

A.2.1 General Notation

x	italicized, Roman or Greek letter, denotes a scalar values
\boldsymbol{x}	italicized, bold, lowercase Roman or Greek letter, denotes a column vector or set.
\boldsymbol{X}	italicized, bold, uppercase Roman or Greek letter, denotes a matrix or set.
$n(\boldsymbol{x})$	cardinality, number of elements, of the vector or set \boldsymbol{x}
$x \in (a, b)$	the value x is within the interval such that $a < x < b$
$x \in (a, b]$	the value x is within the interval such that $a < x \leq b$
$x \in [a, b)$	the value x is within the interval such that $a \leq x < b$
$x \in [a, b]$	the value x is within the interval such that $a \leq x \leq b$
$1_A(x)$	the indicator function,

$$1_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}.$$

$\mathbf{1}_n$	an column vector of n 1s
\mathbf{I}	the identity matrix
\mathbf{I}_n	the $n \times n$ identity matrix
\mathbf{X}^{-1}	the inverse matrix, that is, $\mathbf{X}^{-1}\mathbf{X} = \mathbf{I}$.
\mathbf{X}^T	transpose
\otimes	Kronecker product
\odot	element-wise multiplication

A.2.2 Sets

$\{x, y, z, \dots\}$	The set comprising the elements of x, y, z, \dots
$\{x, y, z\} \setminus x$	The set comprising the elements of y and z , that is, the backslash removes elements from the set.
$\{x_i\}_{i=1}^n$	The set comprising the elements of $x_1, x_2, x_3, \dots, x_n$.
\mathbb{R}	Set of real numbers
$\mathbf{x} \in \mathbb{R}^n$	\mathbf{x} is a vector with n elements, all of which are real numbers.

A.2.3 Statistical Distributions

$\mathcal{N}(\mu, \sigma^2)$	the uni-variable Gaussian distribution with mean μ and variance σ^2 .
$\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	the multi-variable Gaussian distribution with mean vector $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$.
$\phi(x)$	The standard Gaussian density function.

APPENDIX B

APPENDIX HEADER

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