

# Semantics in Society

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Presentation for the ICS691C Seminar: Language Models and Computation

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# Netflix Problem from the ICS691C lecture notes [1]

$L$	Interstellar	Juno	Kagemusha	Legend
Alice	★ ★ ★★	★★	★ ★ ★★	★
Bob	★	★ ★ ★	★★	
Carol	★★	★	★ ★ ★ ★ ★	
Dave	★	★ ★ ★★	★ ★ ★	
Ed		★★	★ ★ ★ ★ ★	★★

**Table:** The raw star rating induces the normalized rating [1]

Problem: The matrix is missing entries, which are to be predicted.

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$L$	Interstellar	Juno	Kagemusha	Legend
Alice	1.25	0.83	0	-0.12
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Solution: Latent Semantic Analysis

# Outline

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(Conjugate) Transpose of  $A = (a_{ij})_{(m,n)} : \mathbb{C}^n \rightarrow \mathbb{C}^m$

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“Reflect” matrix  $A$  so its columns are rows of  $A^* = (a_{ji})_{(n,m)} : \mathbb{C}^m \rightarrow \mathbb{C}^n$

$$\begin{pmatrix} 1.25 & 0.83 & 0 & -0.12 \\ 1.05 & 1.13 & 0.35 & \\ 1.12 & 1.02 & 0.21 & \\ 1.57 & 0.35 & -0.56 & \\ & 0.18 & 1.012 & 0.98 \end{pmatrix}^* = \begin{pmatrix} 1.25 & 1.05 & 1.12 & 1.57 & \\ 0.83 & 1.13 & 1.02 & 0.35 & 0.18 \\ 0 & 0.35 & 0.21 & -0.56 & 1.02 \\ -0.12 & & & & 0.98 \end{pmatrix} \quad (1)$$

(replacing each entry  $z = a + ib$  with its complex conjugate  $\bar{z} = a - ib$ ).

# Singular Value Decomposition

## Theorem 0 (Singular Value Decomposition (S. V. D.))

*The matrix  $A : \mathbb{C}^n \rightarrow \mathbb{C}^m$  has an SVD, i.e. a triple  $\{U, \Sigma, V\}$  satisfying*

$$L = U \cdot \Sigma \cdot V^* \quad (2)$$

*where  $U, V$  are unitary matrices satisfying  $U^*U = UU^* = I$ ,  $V^*V = I$ , and  $\Sigma$  is a diagonal matrix whose entries are real.*

$$\begin{pmatrix} 1.25 & 1.05 & 1.12 & 1.57 \\ 0.83 & 1.13 & 1.02 & 0.35 \\ 0 & 0.35 & 0.21 & -0.56 \end{pmatrix} = \begin{pmatrix} 0.83 & -0.4 \\ 0.55 & 0.6 \\ 0 & 0.7 \end{pmatrix} \cdot \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0 & 0.5 & 0.3 & -0.8 \end{pmatrix} \quad (3)$$

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The (conjugate) transpose  $L^* : \mathbb{C}^m \rightarrow \mathbb{C}^n$  has the following SVD.

$$L^* = (U\Sigma V^*)^* = (V^*)^* \Sigma^* U^* = V\Sigma U^* \quad (5)$$

# Singular Value Decomposition

The compositions  $L^*L : \mathbb{C}^n \rightarrow \mathbb{C}^n$ ,  $LL^* : \mathbb{C}^m \rightarrow \mathbb{C}^m$  defined by

$$L^*L = V\Sigma(U^*U)\Sigma V^* = V\Sigma^2V^* \quad (6)$$

$$LL^* = U\Sigma(V^*V)\Sigma U^* = U\Sigma^2U^* \quad (7)$$

are such that

- 1 they are symmetric;

$$(L^*L)^* = L^*(L^*)^* = L^*L \quad (8)$$

$$(LL^*)^* = (L^*)^*L^* = LL^*; \quad (9)$$

- 2 the eigenvalues of both are the non-zero entries on the diagonal of  $\Sigma^2$ ;
- 3 the eigenvectors of  $LL^*$  are the columns of  $U$ ;
- 4 the eigenvectors of  $L^*L$  are the columns of  $V$ , and we call them the right singular vectors of  $L$ .

## Question

- ① How could one predict the missing entries in an incomplete matrix

$$\begin{pmatrix} 1.25 & 0.83 & 0 \\ 1.05 & 1.13 & 0.35 \\ 1.12 & ? & 0.21 \\ 1.57 & 0.35 & -0.56 \end{pmatrix} \quad (10)$$

using data from  $(1.12 \quad ? \quad 0.21)$  and the maximal complete matrix

$$\begin{pmatrix} 1.25 & 0.83 & 0 \\ 1.05 & 1.13 & 0.35 \\ 1.57 & 0.35 & -0.56 \end{pmatrix} ? \quad (11)$$

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- ❷ How would the prediction depend on the other users and their concepts?

# Idea

- 1 Compute an SVD of the maximal complete matrix:

$$\begin{pmatrix} 1.25 & 0.83 & 0 \\ 1.05 & 1.13 & 0.35 \\ 1.57 & 0.35 & -0.56 \end{pmatrix} = U\Sigma V^*. \quad (12)$$

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$$u = (1.02 \quad ? \quad 0.21) \quad (13)$$

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- 3 Obtain  $u_1, V_1^*$  by removing column  $j$  from  $u$  and  $V^*$ .
- 4 Obtain  $u_1$  as a linear combination of columns in  $V_1^*$

$$u_1 = cV_1^* \quad (14)$$

by computing the projection  $c = u_1 (V_1^*)^*$  of  $u_1$  onto  $V_1^*$ .



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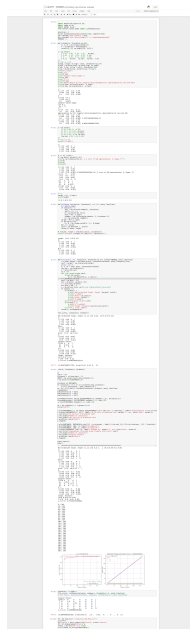
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- 5 Compute  $cV^*$  as an estimate for the direction of  $u$  and, finally, approximate the entries of  $\frac{u}{\|u\|}$  by  $\frac{cV^*}{\|cV^*\|}$ .

# Demo in Jupyter Notebook



# Vital Statistics on Congress, Brookings

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## Vital Statistics on Congress

November 21, 2022

**Data on the U.S. Congress, Updated November 2022**

*Vital Statistics on Congress*, first published in 1990, long ago became the go-to source of impartial data on the United States Congress. *Vital Statistics'* purpose is to collect and provide useful data on America's first branch of government, including data on the composition of its membership, its formal procedure (such as the use of the filibuster), informal norms, party structure, and staff. With some chapters of data dating back nearly 100 years, *Vital Statistics* also documents how Congress has changed over time, illustrating, for example, the increasing polarization of Congress and the diversifying demographics of those who are elected to serve.




Figure: Party Unity Scores in Congressional Voting, 1945-2016 (percent) [2]

*A lonely girl in Washington State — a volunteer Sunday school teacher and part-time babysitter ...*

— Singer & Brooking [3]

# Social Media

*A lonely girl in Washington State — a volunteer Sunday school teacher and part-time babysitter — described how ISIS recruiters gave her the attentive friends she'd always craved. (Only a sharp-eyed grandmother stopped her from boarding a plane to Syria.) ISIS promised adventure and a sense of belonging. "It's a closed community — almost a clique," explained terrorism analyst Seamus Hughes. "They share memes and inside jokes, terms and phrases you'd only know if you were a follower."*

*In each case, recruits to extremist causes are lured by a warmth and camaraderie that seems lacking in their own lonely lives. In each case, such recruits build communities that attract people from across the world but that show almost no diversity of thought.*

— *LikeWar: The Weaponization of Social Media* by Singer & Brooking [3]

# References

- [1] Dusko Pavlovic. Semantics: The meaning of language. 2024.
- [2] Molly E. Reynolds et al. Vital statistics on congress. <https://www.brookings.edu/articles/vital-statistics-on-congress>, November 2022. Accessed: 2024-02-23.
- [3] P.W. Singer and E.T. Brooking. *LikeWar: The Weaponization of Social Media*. Houghton Mifflin Harcourt, 2018.