Matrix Methods for Prediction

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Exposition for the ICS691C Seminar: How Large Language Models really work?

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Outline

Singular Value Decomposition

If $L = U\Sigma V^*$ is an SVD for $L : \mathbb{R}^U \to \mathbb{R}^I$, then $L^* : \mathbb{R}^I \to \mathbb{R}^U$

$$L^* = (V^*)^* \Sigma^* U^* = V\Sigma U^* \tag{1}$$

gives $L^*L : \mathbb{R}^U \to \mathbb{R}^U$, $LL^* : \mathbb{R}^I \to \mathbb{R}^I$

$$L^*L = V\Sigma (U^*U) \Sigma V^* = V\Sigma^2 V^* \tag{2}$$
$$LL^* = U\Sigma (V^*V) \Sigma U^* = U\Sigma^2 U^* \tag{3}$$

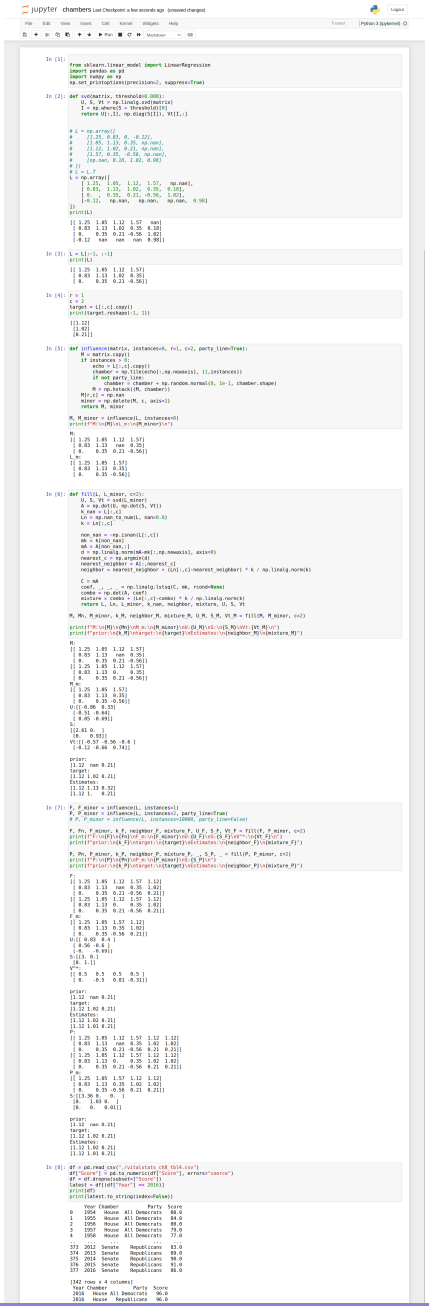
and

$$(L^*L)^* = L^* (L^*)^* = L^*L \tag{4}$$
$$(LL^*)^* = (L^*)^* L^* = LL^*. \tag{5}$$

Questions

1. How do you fill in the missing entries of an incomplete matrix?
    1. Without SVD, with imputation?
    2. Without imputation?
2. Meaning of singular values $\sqrt{\lambda}$
3. Expression of, e. g., user (with missing entries)
    1. as a column of a matrix representation of L?
    2. as a linear combination of such columns?
    3. as a linear combination of singular vectors?

Demo in Jupyter Notebook



Vital Statistics on Congress, Brookings

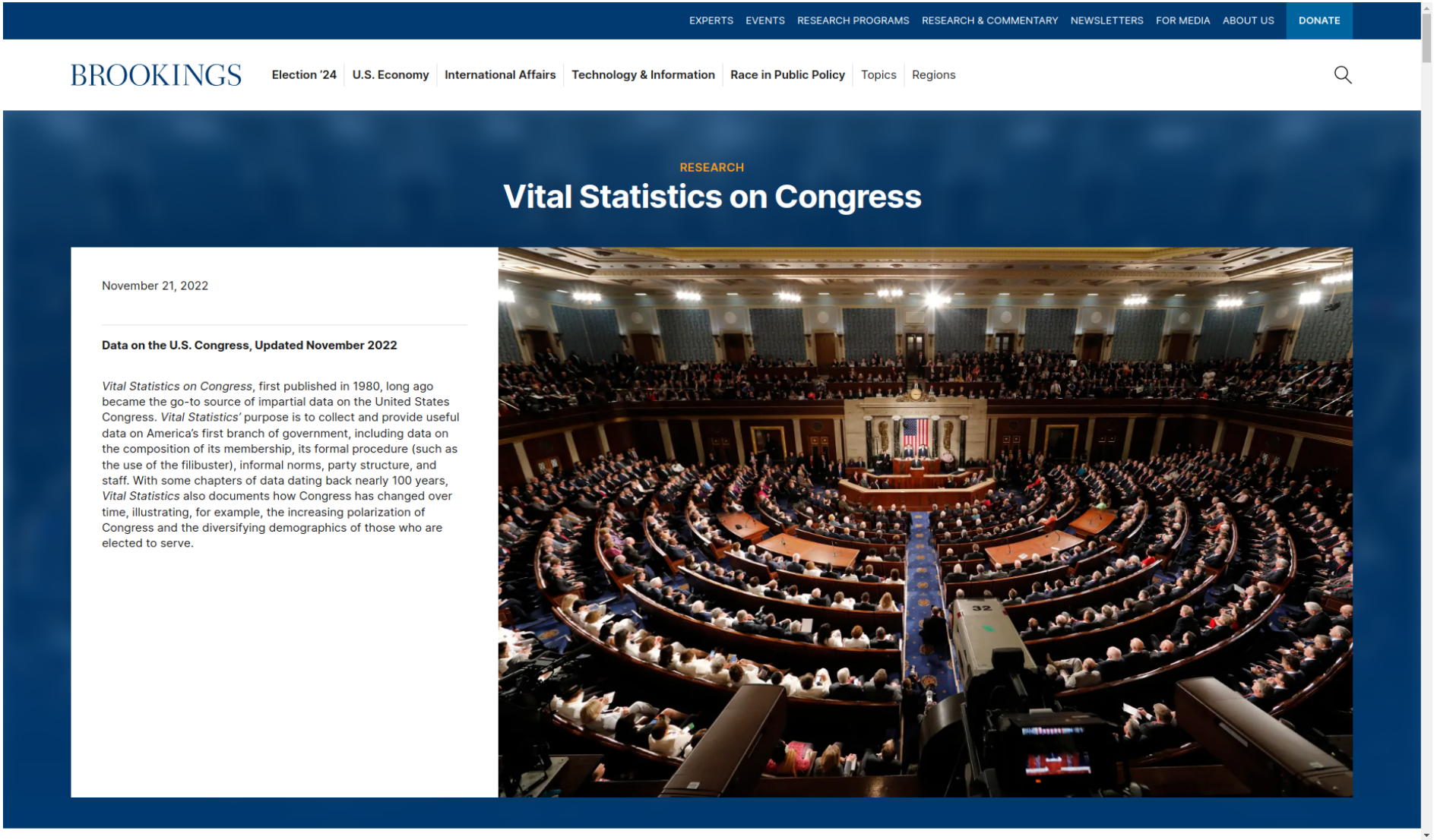


Figure: Party Unity Scores in Congressional Voting, 1945-2016 (percent) [1]

Reference

[1] Molly E. Reynolds et al. Vital statistics on congress. https://www.brookings.edu/articles/vital-statistics-on-congress, November 2022. Accessed: 2024-02-23.