1. State, formally, the definition of the derivative of a function $f(x)$ at $x = a$.

$$\lim_{a \to x} \frac{f(x)-f(a)}{x-a} \quad \underline{\text{or}} \quad \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

2. Let $f(x) = 5x^2 - 3x$.

1. Use the definition to find the derivative of $f(x)$.

$$f'(x) = \lim_{a \to x} \frac{f(x)-f(a)}{} = \lim_{a \to x} \frac{5x^2-3x-(5a^2-3a)}{x-a}$$

$$= \lim_{a \to x} \frac{5(x^2-a^2)-3(x-a)}{x-a}$$

$$= \lim_{a \to x} \frac{5(x-a)(x+a)-3(x-a)}{x-a}$$

$$= \lim_{a \to x} 5(x+a)-3 = 5(a+a)-3$$

2. Find the slope of the tangent line to $f(x)$ when $x = -3$.

$$= \boxed{10a-3}$$

$$f'(-3) = -33$$

3. Write the equation of the line tangent to $f(x)$ when $x = -3$.

$$y = f(-3) + f'(-3)(x-(-3))$$

$$= 54 - 33(x+3)$$

3. Suppose N represents the number of people in the United States who travel by car to another state for a vacation this Memorial Day weekend when the average price of gasoline is p dollars per gallon.

1. What are the units of dN/dp?

people / dollar

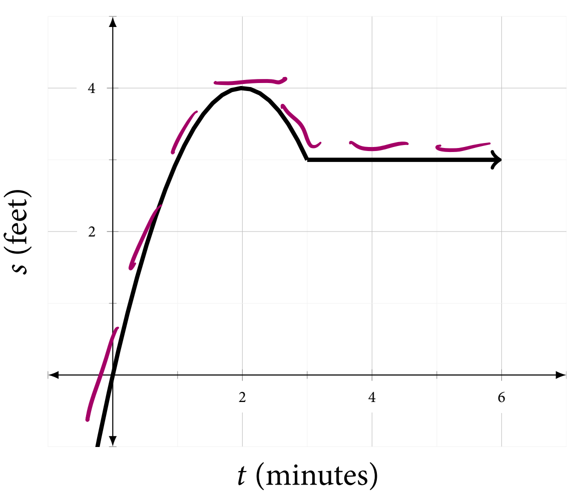
2. In the context of the problem, interpret $\frac{dN}{dp}$.

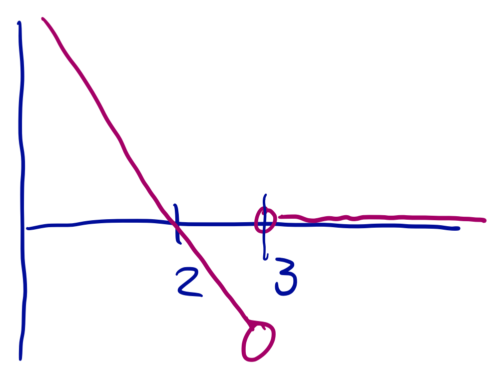
This is the rate at which the number of travellers changes as the price of gas increases.

3. Would you expect dN/dp to be positive or negative? Explain your answer.

Negative. The number of travelers should decrease as the price of gas goes up.

4. The graph of $f(x)$ is sketched below. On a separate set of axes, give a rough sketch $f'(x)$.





5. Find the domain of each function.

1. $f(x) = \sqrt{x^2 - x - 6}$

Need $x^2 - x - 6 \geq 0$

$(x+2)(x-3) \geq 0$

so $x \leq -2$ or $x \geq 3$

$(-\infty, -2] \cup [3, \infty)$

2. $g(t) = \ln(t+6)$

$t + 6 > 0$

$t > 6$

$(6, \infty)$

6. State the definition of "The function $f(x)$ is continuous at $x = a$".

$$\lim_{x \to a} f(x) = f(a)$$

7. Suppose

$$f(x) = \begin{cases} -\frac{2}{x} & x < 2 \\ \frac{x}{x-3} & x \geq 2 \end{cases}$$

Is $f(x)$ continuous at $x = 0$? At $x = 2$? Justify your answers using the definition of continuity.

At 0? No. The function isn't defined there.

At $x=2$? No. $\lim_{x \to 2^-} \frac{-2}{x} = -1$, $\lim_{x \to 2^+} \frac{x}{x-3} = \frac{2}{-1} = -2$.

Since $-1 \neq -2$, $\lim_{x \to 2} f(x)$ does not exist, much less equal $f(2)$.

8. Find the limit or show that it does not exist. *Make sure you are writing your mathematics correctly and clearly.*

1. $\displaystyle\lim_{x\to\infty} \frac{10^x - 1}{3 - 10^x} = \lim_{x \to \infty} \frac{1 - 10^{-x}}{3 \cdot 10^{-x} - 1} = \frac{1-0}{0-1} = -1$

2. $\displaystyle\lim_{x\to\infty} \frac{\sqrt[3]{8x^3+1}}{2-5x} = \lim_{x \to \infty} \frac{x\sqrt[3]{8 + 1/x^3}}{2 - 5x}$

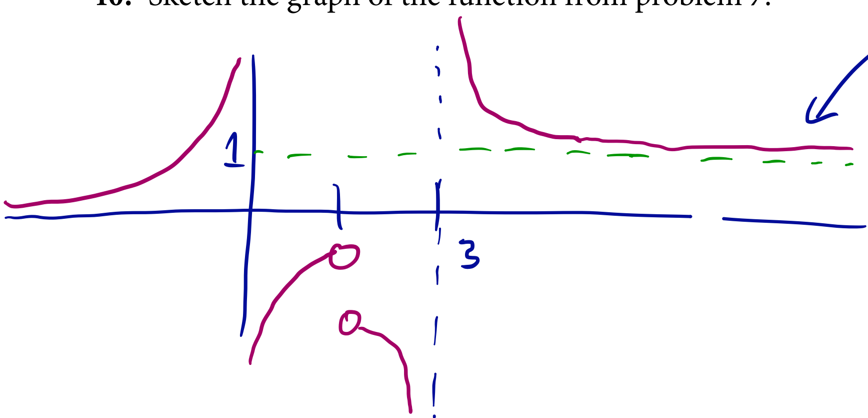
$$= \lim_{x \to \infty} \frac{\sqrt[3]{8 - 1/x^3}}{2/x - 5}$$

$$= \frac{\sqrt[3]{8}}{-5} = \frac{-2}{5}$$

9. Write the formula for a function with vertical asymptotes at $x = -1$ and $x = 3$ and a horizontal asymptote at $y = 4/3$.

$$f(x) = \frac{1}{(x+1)(x-3)} + \frac{4}{3}$$

10. Sketch the graph of the function from problem 7.



$$\lim_{x\to\infty} f(x) = 1$$

Note:

$$\frac{x}{x-3} = \frac{x-3}{x-3} + \frac{3}{x-3} = 1 + \frac{3}{x-3}$$

11. Solve for x.

1. $e^{x-3} + 2 = 6$

$e^{x-3} = 4$

$x - 3 = \ln(4)$

$x = 3 + \ln(4)$

2. $\ln(x+5) - 3 = 7$

$\ln(x+5) = 10$

$x + 5 = e^{10}$

$x = e^{10} - 5$

3. $\ln x + \ln(x-1) = 0$

$\ln(x(x-1)) = 0$

$x(x-1) = e^0 = 1$

$x^2 - x - 1 = 0$

$x = \dfrac{1 \pm \sqrt{1+4}}{2} = \dfrac{1 \pm \sqrt{5}}{2}$

But $\dfrac{1-\sqrt{5}}{2}$ is not in the domain.

4. $\cos(8x) = 0$

$8x = \frac{\pi}{2} + k\pi$

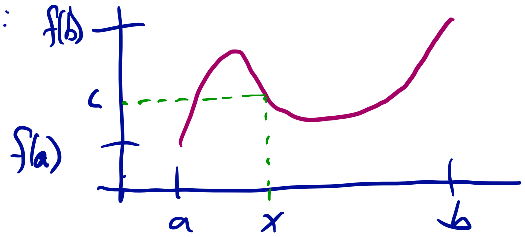
$x = \frac{\pi}{16} + k\frac{\pi}{8}$

$k \in \mathbb{Z}$

12.

1. What does the Intermediate Value Theorem say? You may want to include a picture with your explanation.

If $f(x)$ is continuous on $[a,b]$ and c is a number between $f(a)$ and $f(b)$ then there is x in $[a,b]$ with $f(x)=c$:



2. Use the Intermediate Value Theorem to show $\ln x = x - 5$ has a solution. (Hint: Show there is a solution in the interval $[1, e^5]$.)

Let $f(x) = \ln(x) - x + 5$. Notice $f(x)$ is continuous on $(0,\infty)$ and so also on $[1, e^5]$. Moreover,

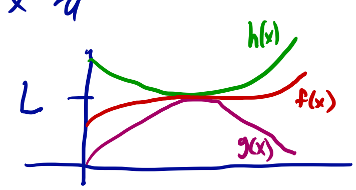
$f(1) = 0 - 1 - 5 = -6 < 0$ and

$f(e^5) = \ln(e^5) + e^5 - 5 = e^5 > 0.$

13. So there is x in $[1, e^5]$ with $f(x) = 0$.

1. What does the Squeeze Theorem say? You may want to include a picture with your explanation.

If $g(x) \le f(x) \le h(x)$ near $x=a$, but maybe not at $x=a$, and if $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} f(x) = L$ also.



14. Use the Squeeze Theorem to show $\lim_{x\to\infty} \frac{\cos(2x)}{3x^2} = 0$.

Since $-1 \le \cos(2x) \le 1$, $-\frac{1}{3x^2} \le \frac{\cos(2x)}{3x^2} \le \frac{1}{3x^2}$.

Since $\lim_{x\to\infty} -\frac{1}{3x^2} = \lim_{x\to\infty} \frac{1}{3x^2} = 0$, $\lim_{x\to\infty} \frac{\cos(2x)}{3x^2} = 0$.

15. Sketch each of the functions below. Label all x- and y-intercepts and asymptotes. State, in interval notation, the domain and range of each function next to its graph.

1. $y = 6 - x^4$
2. $y = \sin(2x)$
3. $y = \tan x$
4. $y = \tan^{-1} x$
5. $y = e^{x-1} + 2$
6. $y = \ln x$
7. $y = -2/(x+3)$
8. $y = \sqrt{x+5}$