## SECTION 3.4 THE CHAIN RULE

1. For each function H(x) below, write it as a (nontrivial) composition of functions in the form f(g(x)).

(a) 
$$H(x) = \tan(2 - x^4)$$

(b) 
$$H(x) = e^{2-2x}$$
  
 $F(x) = e^{x}$   
 $g(x) = 2-2x$ 

- f(x) = tanx  $g(x) = 2-x^4$ 
  - 2. Complete the Chain Rule (using both types of notation)

• If 
$$F(x) = f(g(x))$$
,  
then  $F'(x) = \left[ f'(g(x)) \right] \left[ g'(x) \right]$ 

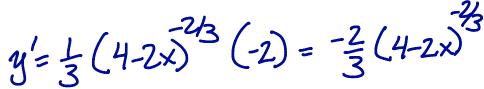
• If 
$$y = f(u)$$
 and  $u = g(x)$ ,

then 
$$\frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

3. Find the derivative of the function. You do not need to simplify your answer.

(a) 
$$y = \sqrt[3]{4 - 2x}$$
 =  $(4 - 2x)^3$ 

$$f = x^{\frac{1}{3}} \qquad g = 4-2x$$



(b) 
$$f(x) = 0.04\sin(3x + e^x)$$

$$f'(x) = (0.04)(\cos(3x+e^{x}))(3+e^{x})$$

(c) 
$$x(t) = \frac{e^{-\pi t/10}}{100}$$
 (Don't use the quotient rule here!)

$$x(t) = \frac{1}{100} e^{(\frac{1}{100}t^2)}$$
 $x'(t) = \frac{1}{100} (e^{-\frac{1}{100}t^2}) (-\frac{1}{100}) = \frac{1}{500} e^{\frac{1}{100}t^2}$ 

(d) 
$$g(x) = \frac{50\sqrt{2}}{x + \tan x}$$
 (**Don't** use the quotient rule here!)

$$g(x) = 50\sqrt{2}(x+\tan x)$$

$$g'(x) = (50\sqrt{2})(-1)(x+\tan x)(1+\sec^2 x)$$

- 4. Suppose that  $f(x) = x^3$ ,  $g(x) = \cos(x)$  and  $h(x) = 7 + e^x$ .
  - (a) Find F(x) = f(x) (g(h(x))), then find its derivative.

$$F(x) = x^{3} \cos(7+e^{x})$$

$$F'(x) = 3x^{2} \cos(7+e^{x}) + x^{3} \left(-\sin(7+e^{x})\right) \left(e^{x}\right)$$

$$= 3x^{2} \cos(7+e^{x}) - x^{3} e^{x} \sin(7+e^{x})$$

(b) Find 
$$G(x) = f(g(x)h(x))$$
, then find its derivative.

$$g(x) h(x) = (cosx)(7+e^{x}) = 7 cosx + e^{x}cosx$$
 $G(x) = (7 cosx + e^{x}cosx)^{3}$ 
 $G(x) = 3(7cosx + e^{x}cosx)^{2}(-7sinx + e^{x}cosx - e^{x}sinx)$ 
 $= 3(e^{x}cosx - (7+e^{x})sinx)(7cosx + e^{x}cosx)^{2}$ 

(c) Find 
$$K(x) = \frac{g(x)}{h(f(x))}$$
, then find its derivative.

So 
$$K(x) = \frac{Cosx}{e^{x^3}}$$

So 
$$K'(x) = \frac{e^{x^3}(-\sin x) - (\cos x)(e^{x^3}(3x^2))}{(e^{x^3})^2}$$

$$= -\frac{e^{x^{3}}(\sin x + 3x^{2}\cos x)}{(e^{x^{3}})^{2}} = -\frac{(\sin x + 3x^{2}\cos x)}{e^{x^{3}}}$$

(d) Find 
$$G(x) = f(g(h(x)))$$
, then find its derivative.

$$G'(x) = 3(\cos(7+e^{x}))^{2}(-\sin(7+e^{x}))(e^{x})$$
  
=  $-3e^{x}\sin(7+e^{x})[\cos(7+e^{x})]^{2}$ 

3-4 The Chain Rule