

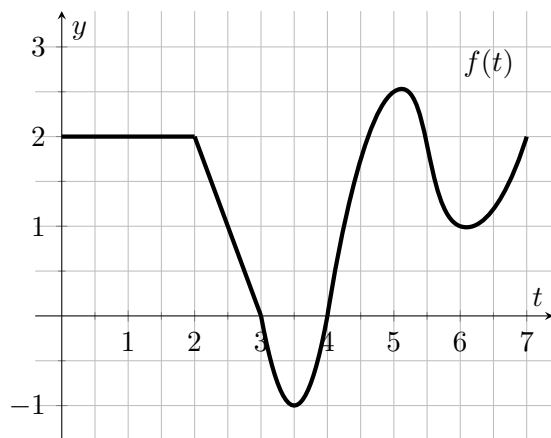
Math 251 Fall 2017

Quiz #11, November 29th

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (3 pts.) Let  $g(x) = \int_0^x f(t) dt$  where the graph of  $y = f(t)$  is displayed below.



(a) Find  $g(3) = 2 \cdot 2 + \frac{1}{2} \cdot 2 \cdot 1 = 5$

(b) In the open interval  $(0, 7)$ , when does  $g(x)$  have a maximum?

$x=3$

(c) When is  $g(x)$  increasing?

$(0, 3) \cup (4, 7)$

Exercise 2. (5 pts.) Find the derivative of the function.

(a)  $g(x) = \int_x^1 \cos t \, dt$

$g'(x) = -\cos(x)$

(b)  $F(x) = \int_0^{x^3} \sqrt{1+t^4} \, dt$

$F'(x) = 3x^2 \sqrt{1+x^{12}}$

Exercise 3. (3 pts.) What, if anything, is wrong with the following calculation?

$$\int_0^5 \frac{1}{x-2} dx = \ln|x-2| \Big|_0^5 = \ln 3 - \ln 2$$

$\frac{1}{x-2}$  is not continuous on  $[0, 5]$  so the FTC doesn't apply.

Exercise 4. (6 pts.) Evaluate the following integrals.

$$\begin{aligned} \text{(a)} \quad & \int_0^{\pi/4} (\sec^2 t - 2e^t) dt \\ &= \tan t - 2e^t \Big|_0^{\pi/4} \\ &= 1 - 2e^{\pi/4} - (0 - 2) \\ &= 3 - 2e^{\pi/4} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \int_0^1 \frac{4}{\sqrt{1-x^2}} dx \\ &= 4 \arcsin(x) \Big|_0^1 \\ &= 4 \cdot \frac{\pi}{2} - 4 \cdot 0 = 2\pi \end{aligned}$$

Exercise 5. (8 pts.) Evaluate the following integrals.

$$\begin{aligned} \text{(a)} \quad & \int_0^1 (v^2 + 2)^2 dv \\ &= \int_0^1 v^4 + 4v^2 + 4 dv \\ &= \frac{v^5}{5} + \frac{4}{3}v^3 + 4v \Big|_0^1 \\ &= \frac{1}{5} + \frac{4}{3} + 4 = \frac{60 + 3 + 20}{15} \\ &= \frac{83}{15} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \int_1^4 \frac{(4-t)}{\sqrt{t}} dt = \int_1^4 4t^{-1/2} - t^{1/2} dt \\ &= 8t^{1/2} - \frac{2}{3}t^{3/2} \Big|_1^4 = 16 - \frac{16}{3} - \left(8 - \frac{2}{3}\right) \\ &= 8 - \frac{14}{3} = \frac{10}{3} \end{aligned}$$