Final Review - Limits

1. Evaluate the following limits.

   (a) $\lim_{x\to 1} e^{x-1}\sin\left(\frac{\pi x}{2}\right)$ $= e^0 \sin(\pi/2) = 1\cdot 1 = \boxed{1}$

   (b) $\lim_{x\to\infty} \frac{x+x^3+3x^5}{1-2x^2+8x^6}$ $\cdot \frac{1/x^6}{1/x^6} = \lim_{x\to\infty} \frac{x^{-5}+x^{-3}+3x^{-1}}{x^{-6}-2x^{-4}+8} = \frac{0}{8} = 0$

   (c) $\lim_{x\to 0} \frac{5x^2}{1-\cos x}$ $\overset{H}{=} \lim_{x\to 0} \frac{10x}{\sin x} \overset{H}{=} \lim_{x\to 0} \frac{10}{\cos x} = 10$

   form $\frac{0}{0}$

   form $\frac{0}{0}$

(d) $\lim_{x\to 5^-} \dfrac{e^x}{(x-5)^3}$ $= -\infty$

as $x \to 5$, $e^x \to e^5$ and $(x-5)^3 \to 0^-$

(e) $\lim_{x\to 0^+} x(\ln x)^2$ $= \lim_{x\to 0^+} \dfrac{(\ln x)^2}{x^{-1}} \overset{H}{=} \lim_{x\to 0^+} \dfrac{2(\ln x)(\frac{1}{x})}{-x^{-2}} = \lim_{x\to 0^+} \dfrac{-2\ln x}{x^{-1}}$

form $0 \cdot -\infty$

form $\dfrac{-\infty}{\infty}$

form $\dfrac{+\infty}{\infty}$

$\overset{H}{=} \lim_{x\to 0^+} \dfrac{-2/x}{-x^{-2}} = \lim_{x\to 0^+} -2x = 0.$

(f) $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x}$ $= \lim_{x\to -4} \dfrac{-x^{-2}}{1} = \dfrac{-1}{16}$

form $\frac{0}{0}$

or algebra $\lim_{x\to -4} \dfrac{1}{4+x} \cdot \dfrac{x+4}{4x} = \lim_{x\to -4} \dfrac{1}{4x} = \dfrac{-1}{16}$

(g) $\lim_{x\to\infty} \dfrac{4x^4+5}{(x^2-2)(2x^2-1)}$ $= \dfrac{4}{2} = 2$

$\lim_{x\to\infty} \dfrac{4x^4+5}{2x^4-4x^2-x^2+2} \cdot \dfrac{1/x^4}{1/x^4} = \lim_{x\to\infty} \dfrac{4+5/x^4}{2-4/x^2+2/x^4}$ $\dfrac{4}{2} = 2$

2. Let $F(t) = \frac{20}{4+e^{-2t}}$ model the population of fish in hundreds over time t measured in years.

   (a) Find and interpret $f(0)$.

   (b) Find and interpret (in language your parents could understand) $\lim_{t\to\infty} F(t)$.

   (c) Find $F'(t)$. (HINT: You can check your answer with the one at the bottom of the page.

   (d) Find and interpret $F'(0)$.

   (e) Find and interpret (in language your parents could understand) $\lim_{t\to\infty} F'(t)$.

   (f) Give a rough sketch the graph of $F(t)$ given the information above.

$F'(t) = \frac{40e^{-2t}}{(4+e^{-2t})^2}$

3. Find the numbers, if any, at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither?

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ e^x & \text{if } 0 \leq x \leq 2 \\ 6x - 7 & \text{if } x > 2 \end{cases}$$