1. For each function $H(x)$ below, write it as a (nontrivial) composition of functions in the form $f(g(x))$.

(a) $H(x) = \tan(2 - x^4)$

$f(x) = \tan x$
$g(x) = 2 - x^4$

(b) $H(x) = e^{2-2x}$

$f(x) = e^x$

$g(x) = 2 - 2x$

2. Complete the Chain Rule (using both types of notation)

- If $F(x) = f(g(x))$,

  then $F'(x) = [f'(g(x))][g'(x)]$

- If $y = f(u)$ and $u = g(x)$,

  then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

3. Find the derivative of the function. You do not need to simplify your answer.

(a) $y = \sqrt[3]{4 - 2x}$ $= (4-2x)^{1/3}$ $f = x^{1/3}$ $g = 4-2x$

$y' = \frac{1}{3}(4-2x)^{-2/3}(-2) = -\frac{2}{3}(4-2x)^{-2/3}$

(b) $f(x) = 0.04 \underset{f}{\sin}(\underset{g}{3x + e^x})$

$f'(x) = (0.04)(\cos(3x+e^x))(3+e^x)$

(c) $x(t) = \dfrac{e^{-\pi t^2/10}}{100}$ (**Don't** use the quotient rule here!)

$$x(t) = \frac{1}{100}\, e^{\left(-\frac{\pi}{10}\right)t^2}$$

$$x'(t) = \frac{1}{100}\left(e^{-\frac{\pi}{10}t^2}\right)\left(-\frac{\pi}{10}\cdot 2\right) = \frac{-\pi}{500}\, e^{-\frac{\pi}{10}t^2}$$

(d) $g(x) = \dfrac{50\sqrt{2}}{x + \tan x}$ (**Don't** use the quotient rule here!)

$$g(x) = 50\sqrt{2}\,(x+\tan x)^{-1}$$

$$g'(x) = (50\sqrt{2})(-1)(x+\tan x)^{-2}(1+\sec^2 x)$$

4. Suppose that $f(x) = x^3,\ g(x) = \cos(x)$ and $h(x) = 7 + e^x$.

(a) Find $F(x) = f(x)\,(g(h(x)))$, then find its derivative.

$$F(x) = x^3\cos(7+e^x)$$

$$F'(x) = 3x^2\cos(7+e^x) + x^3\left(-\sin(7+e^x)\right)(e^x)$$

$$= 3x^2\cos(7+e^x) - x^3e^x\sin(7+e^x)$$

(b) Find $G(x) = f(\ g(x)h(x)\ )$, then find its derivative.

$$g(x)\,h(x) = (\cos x)(7+e^x) = 7\cos x + e^x\cos x$$

$$G(x) = (7\cos x + e^x\cos x)^3$$

$$G'(x) = 3(7\cos x + e^x\cos x)^2(-7\sin x + e^x\cos x - e^x\sin x)$$

$$= 3\left(e^x\cos x - (7+e^x)\sin x\right)(7\cos x + e^x\cos x)^2$$

(c) Find $K(x) = \dfrac{g(x)}{h(f(x))}$, then find its derivative.

$h(f(x)) = e^{x^3}$

So $K(x) = \dfrac{\cos x}{e^{x^3}}$

So $K'(x) = \dfrac{e^{x^3}(-\sin x) - (\cos x)\left(e^{x^3}(3x^2)\right)}{\left(e^{x^3}\right)^2}$

$= \dfrac{-e^{x^3}(\sin x + 3x^2\cos x)}{\left(e^{x^3}\right)^2} = \dfrac{-(\sin x + 3x^2\cos x)}{e^{x^3}}$

(d) Find $G(x) = f(g(h(x)))$, then find its derivative.

$G(x) = \left(\cos(7+e^x)\right)^3$

$G'(x) = 3\left(\cos(7+e^x)\right)^2\left(-\sin(7+e^x)\right)\left(e^x\right)$

$= -3e^x\sin(7+e^x)\left[\cos(7+e^x)\right]^2$