

SOLUTIONS

Name: _____

_____/12

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- Circle your final answer.

Compute the derivatives of the following functions.

1. $f(x) = \frac{x - \ln 3}{5} - \sqrt[3]{x}$

$$f'(x) = \frac{1}{5} - \frac{1}{3}x^{-2/3}$$

2. $h(x) = e^{-x/3} \cos(x)$

$$h'(x) = -\frac{1}{3}e^{-x/3} \cos(x) + e^{-x/3}(-\sin(x))$$

3. $f(t) = \frac{1 - 4t^{1/2} + t^3}{t} = t^{-1} - 4t^{-1/2} + t^2$

$$f'(t) = -t^{-2} + 2t^{-3/2} + 2t$$

4. $g(x) = \frac{1}{\sin(x)} = \csc(x)$

$$g'(x) = -\csc(x) \cot(x)$$

5. $y = \arccos(2x^{1/4} + \sqrt{6})$

$$y' = \frac{-1}{\sqrt{1 - (2x^{1/4} + \sqrt{6})^2}} \left(\frac{1}{2} x^{-3/4} \right)$$

6. $f(x) = x^k + e^{-kx}$, where k is a fixed constant

$$f'(x) = kx^{k-1} - ke^{-kx}$$

7. $y = \frac{\tan(x)}{1 + \ln(x)}$

$$\frac{dy}{dx} = \frac{\sec^2(x)(1 + \ln(x)) - \tan(x)\left(\frac{1}{x}\right)}{(1 + \ln(x))^2}$$

8. $y = e^x \ln(2x) \sec(x)$

$$\begin{aligned} y' &= e^x \cdot \ln(2x) \cdot \sec(x) \\ &\quad + e^x \cdot \frac{1}{2x} \cdot 2 \cdot \sec(x) \\ &\quad + e^x \cdot \ln(2x) \cdot \sec(x) \tan(x) \end{aligned}$$

9. $y = \sin^2(x - \sqrt{x})$

$$\frac{dy}{dx} = 2 \sin(x - \sqrt{x}) \cos(x - \sqrt{x}) \left(1 - \frac{1}{2} x^{-1/2}\right)$$

10. $h(x) = \frac{\pi}{x^2} + \left(\frac{x-1}{4}\right)^3$

$$h'(x) = -\pi x^{-3} + 3\left(\frac{x-1}{4}\right)^2 \left(\frac{1}{4}\right)$$

11. $g(x) = \frac{\cos(2x)}{x^3+x}$

$$g'(x) = \frac{-\sin(2x) \cdot 2(x^3+x) - \cos(2x)(3x^2+1)}{(x^3+x)^2}$$

12. Compute dy/dt if $ye^y + 5 = 2\sin(y)t^3$. You must solve for dy/dt .

$$y'e^y + ye^y y' = 2\cos(y)y't^3 + 2\sin(y)3t^2$$

$$y'(e^y + ye^y - 2t^3 \cos(y)) = 6t^2 \sin(y)$$

$$\frac{dy}{dt} = \frac{6t^2 \sin(y)}{e^y(1+y) - 2t^3 \cos(y)}$$