LECTURE NOTES 2-2: THE LIMIT OF A FUNCTION

Things to Know:

- The intuitive definitions of a *limit* and a *one-sided limit*.
- How to find a (one-sided) limit using a calculator or the graph of the function, including infinite limits.
- How to find limits for piecewise-defined functions.
- How to distinguish between the various ways a limit may *not* exist.
- Understand how using a calculator can give an incorrect answer when evaluating a limit.

Intuitive Idea and Introductory Examples

(Note that this is motivated by our discussion of tangent lines and instantaneous velocity.)

Say: "the limit of $f(x)$, as x approaches a is L"

Write:

It means:

EXAMPLE 1: Use calculation to guess $\lim_{x\to 2} \dfrac{x-2}{x^2-x-2}$.

What does the table above tell you about the *graph* of $y = \frac{x-2}{x^2-x-2}$?

EXAMPLE 2: [Why do all the calculation? Just pick a number really close to "a," right???!!]

Use calculation to guess $\lim_{t\to 0} \dfrac{\sqrt{t^2+9}-3}{t^2}$.

Let's just pick numbers super-close to $a = 0$, say ± 0.000001 :

t	-0.000001	0	0.000001
$f(t)$		DNE	

Hint: Always be skeptical! Why can't this be right and what went wrong?

EXAMPLE 3: [Sample points may not illustrate the big picture. Theory will be useful.]

Use calculation to guess $\lim_{\theta\to 0} \sin\left(\dfrac{\pi}{\theta}\right)$.

x	-0.1	-0.001	-0.0001	0	0.0001	0.001	0.01
$f(x)$							

Do you believe your answer?

Practice Problems

1. For each problem below, fill out the chart of values, then use the values to *guess* the value of the limit. Finally rate your confidence level on a 0 to 3 scale where (0 = I'm sure this is wrong) and (3 = I'm sure this is right.)

(a) $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = \square$ confidence? _____

x						0					
$f(x)$											

(b) $\lim_{x \to 2} f(x) = \square$ where $\begin{cases} |x-1| & x \leq 2 \\ x+1 & x > 2 \end{cases}$ confidence? _____

x						2					
$f(x)$											

(c) $\lim_{x \to 0} \frac{e^{2x}-1}{x} = \square$ confidence? _____

x	-0.5	-0.1	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01	0.1	0.5
$f(x)$											

DEFINITIONS:

Say: "the limit as x approaches a *on the left* is L"; Write:

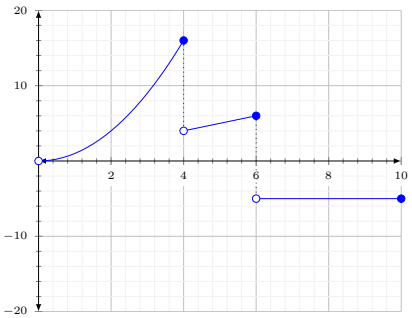
It means

Say: "the limit as x approaches a *on the right* is L"; Write:

It means

Practice Problems

2. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x\to 4^-} f(x) =$ _____

(b) $\lim_{x\to 4^+} f(x) =$ _____

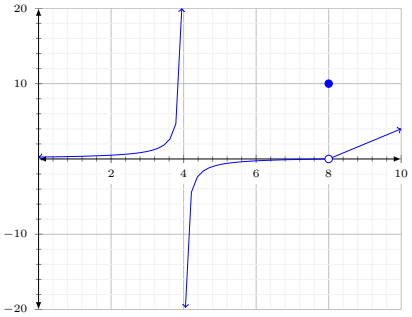
(c) $\lim_{x\to 4} f(x) =$ _____

(d) $f(4) =$ _____

(e) $\lim_{x\to 8} f(x) =$ _____

(f) $f(8) =$ _____

3. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



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(e) $\lim_{x\to 8} f(x) =$ _____

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Write the equation of any vertical asymptote:

4. Sketch the graph of an function that satisfies *all* of the given conditions. Compare your answer with that of your neighbor.

$$\lim_{x\to 0^-} f(x) = 1 \quad \lim_{x\to 0^+} f(x) = -2 \quad \lim_{x\to 4^-} f(x) = 3$$

$$\lim_{x\to 4^+} f(x) = 0 \quad f(0) = -2 \quad f(4) = 1$$

5. Determine the limit. Explain your answer.

(a) $\lim\limits_{x\to 5^+} \dfrac{2+x}{x-5}$

(b) $\lim\limits_{x\to 5^+} \dfrac{2+x}{5-x}$

(c) $\lim\limits_{x\to (\pi/2)^+} \dfrac{\sec x}{x}$