Circle your Instructor:
Faudree, Williams, Zirbes

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Math 251 Fall 2017 **Quiz #6, October 18th**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (4 pts.) Find $\frac{dy}{dx}$ by implicit differentiation for $\sin y = x^3 - y$.

$\frac{d}{dx}\sin y = \frac{d}{dx}[x^3 - y]$

$(\cos y)\cdot y' = 3x^2 - y'$

$y'(1+\cos(y)) = 3x^2$

$y' = \frac{3x^2}{1+\cos(y)}$

Exercise 2. (6 pts.) Find the derivatives of the following functions.

(a) $f(x) = x\arcsin(3x)$

u v

$f'(x) = u'v + uv'$

$= \arcsin(3x) + x\cdot\frac{1}{\sqrt{1-9x^2}}\cdot 3$

$f'(x) = \arcsin(3x) + \frac{3x}{\sqrt{1-9x^2}}$

(b) $g(x) = \arctan(\sqrt{x})$ ← Chain rule

$g'(x) = \frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}}$

$g'(x) = \frac{1}{2\sqrt{x}+2x\sqrt{x}}$

Exercise 3. (3 pts.) Find the derivative of the function $g(x) = \sqrt{\ln x}$. ← chain rule!

$g'(x) = \frac{1}{2\sqrt{\ln x}}\cdot\frac{1}{x} = \frac{1}{2x\sqrt{\ln(x)}}$

Exercise 4. (4 pts.) Use logarithmic differentiation to find the derivative of the function

$$y = (\cos x)^{3x}.$$

$\ln(y) = \ln(\cos(x)^{3x})$

$\ln(y) = 3x \ln(\cos(x))$

$\frac{1}{y} \cdot y' = 3\ln(\cos(x)) + 3x \cdot \frac{1}{\cos(x)} \cdot (-\sin(x))$

$y' = \left[3\ln(\cos(x)) - 3x\tan(x) \right] (\cos(x))^{3x}$

Exercise 5. (8 pts.) The position function of a particle is given by $s = \frac{1}{3}t^3 - 4t^2 + 7t$ where t is measured in seconds and s in meters. Further, assume the first and second derivatives are $s'(t) = t^2 - 8t + 7$ and $s''(t) = 2t - 8$.

a.) What is the velocity function of the particle?

$s'(t) = t^2 - 8t + 7$

b.) What is the acceleration function of the particle?

$s''(t) = 2t - 8$

c.) When is the particle at rest?

want $s'(t) = 0$.

$t^2 - 8t + 7 = 0$ or $(t-7)(t-1) = 0$ so $t = 1$ or 7.

d.) When is the particle moving to the right?

want $s'(t) > 0$, so t in $(-\infty, 1) \cup (7, \infty)$.

e.) At time $t = 5$, is the particle speeding up or slowing down? Explain your answer.

$s'(5) < 0$, $s''(5) = 2 \cdot 5 - 8 = 2 > 0$

Acceleration is opposite of velocity, so speed slowing down.