

Final Review - Limits

1. Evaluate the following limits.

(a) $\lim_{x \rightarrow 1} e^{x-1} \sin\left(\frac{\pi x}{2}\right)$

(b) $\lim_{x \rightarrow \infty} \frac{x + x^3 + 3x^5}{1 - 2x^2 + 8x^6}$

(c) $\lim_{x \rightarrow 0} \frac{5x^2}{1 - \cos x}$

(d) $\lim_{x \rightarrow 5^-} \frac{e^x}{(x-5)^3}$

(e) $\lim_{x \rightarrow 0^+} x(\ln x)^2$

(f) $\lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x}$

(g) $\lim_{x \rightarrow \infty} \frac{4x^4 + 5}{(x^2 - 2)(2x^2 - 1)}$

2. Let $F(t) = \frac{20}{4+e^{-2t}}$ model the population of fish in hundreds over time t measured in years.

(a) Find and interpret $f(0)$.

(b) Find and interpret (in language your parents could understand) $\lim_{t \rightarrow \infty} F(t)$.

(c) Find $F'(t)$. (HINT: You can check your answer with the one at the bottom of the page.

(d) Find and interpret $F'(0)$.

(e) Find and interpret (in language your parents could understand) $\lim_{t \rightarrow \infty} F'(t)$.

(f) Give a rough sketch the graph of $F(t)$ given the information above.

$$F'(t) = \frac{40e^{-2t}}{(4+e^{-2t})^2}$$

3. Find the numbers, if any, at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither?

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ e^x & \text{if } 0 \leq x \leq 2 \\ 6x - 7 & \text{if } x > 2 \end{cases}$$