

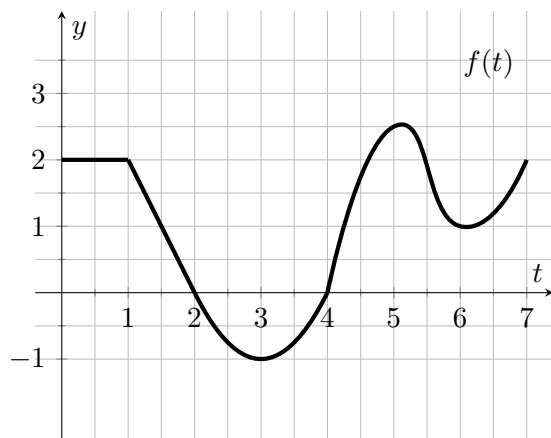
Math 251 Fall 2017

Quiz #11, November 29th

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (3 pts.) Let $g(x) = \int_0^x f(t)dt$ where the graph of $y = f(t)$ is displayed below.



(a) Find $g(2) = 2 \cdot 1 + \frac{1}{2} \cdot 2 \cdot 1 = 3$

(b) In the open interval $(0, 7)$, when does $g(x)$ have a maximum?

$x=2$

(c) When is $g(x)$ increasing?

$(0, 2) \cup (4, 7)$

Exercise 2. (5 pts.) Find the derivative of the function.

(a) $g(x) = \int_x^2 \sec^2 t dt = - \int_2^x \sec^2 t dt$

$\Rightarrow \frac{d}{dx} g(x) = -\sec^2(x)$

(b) $F(x) = \int_0^{x^4} \sqrt{1+t^2} dt$

$F'(x) = \sqrt{1+x^8} \cdot 4x^3$
 $= 4x^3 \sqrt{1+x^8}$

Exercise 3. (3 pts.) What, if anything, is wrong with the following calculation?

$$\int_0^6 \frac{1}{x-4} dx = \ln|x-4| \Big|_0^6 = \ln 2 - \ln 4 = \ln\left(\frac{2}{4}\right) = \ln\left(\frac{1}{2}\right)$$

$\frac{1}{x-4}$ is not continuous on $[0, 6]$, so FTC does not apply.

Exercise 4. (6 pts.) Evaluate the following integrals.

(a) $\int_0^{\pi/4} (2 \sec^2 t - e^t) dt$

$$\begin{aligned} &= 2 \tan t - e^t \Big|_0^{\pi/4} \\ &= 2 \cdot 1 - e^{\pi/4} - (2 \cdot 0 - 1) \\ &= 3 - e^{\pi/4} \end{aligned}$$

(b) $\int_0^1 \frac{3}{\sqrt{1-x^2}} dx = 3 \arcsin(x) \Big|_0^1$

$$\begin{aligned} &= 3 \arcsin(1) - 3 \arcsin(0) \\ &= 3 \cdot \frac{\pi}{2} - 3 \cdot 0 = \frac{3\pi}{2} \end{aligned}$$

Exercise 5. (8 pts.) Evaluate the following integrals.

(a) $\int_0^1 (v^2 + 1)^2 dv$

$$\begin{aligned} &= \int_0^1 v^4 + 2v^2 + 1 \, dv \\ &= \frac{v^5}{5} + \frac{2}{3} v^3 + v \Big|_0^1 \\ &= \frac{1}{5} + \frac{2}{3} + 1 = \frac{3+10+15}{15} \\ &= \frac{28}{15} \end{aligned}$$

(b) $\int_1^4 \frac{(2-t)}{\sqrt{t}} dt = \int_1^4 2t^{-1/2} - t^{1/2} dt$

$$\begin{aligned} &= \frac{2}{1/2} t^{1/2} - \frac{2}{3} t^{3/2} \Big|_1^4 \\ &= 4t^{1/2} - \frac{2}{3} t^{3/2} \Big|_1^4 \\ &= 8 - \frac{16}{3} - \left(4 - \frac{2}{3} \right) \\ &= 4 - \frac{14}{3} = -\frac{2}{3} \end{aligned}$$