Your Name

Your Instructor

Your Section

Problem	Total Points	Score
1	9	
2	8	
3	6	
4	9	
5	9	
6	12	
7	6	
8	8	
9	10	
10	15	
Extra Credit	(8)	
Total	92	
Percent	100 %	

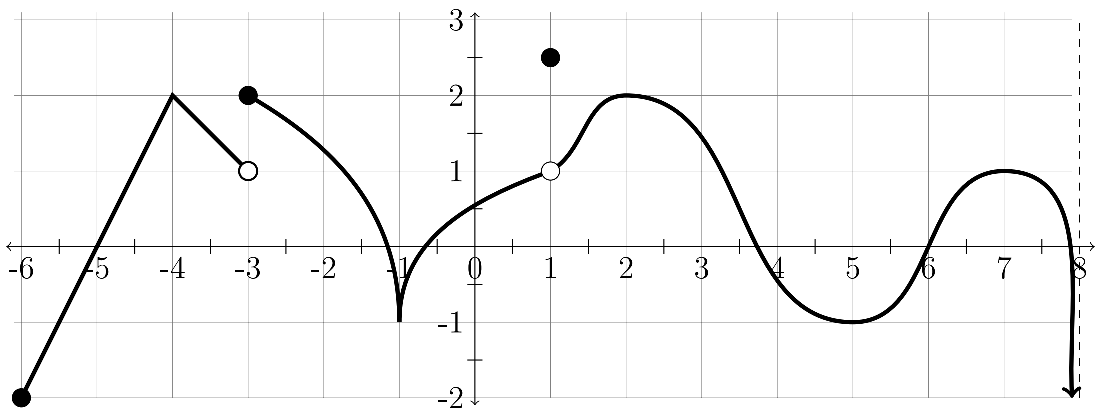
Instructions and information:

- You may use one 3" $\times$ 5" notecard.
- In order to receive credit, you must show your work.

  Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- If you need more room, use the backs of the pages and indicate to the grader where to look.
- Raise your hand or come up to the front if you have a question.
- Calculators are NOT allowed.
- Failure to follow exam instructions may result in point reductions or exam disqualification.

$\boxed{1}$ (9 points)

Using the graph of a function $g(x)$ given below, whose domain is $[-6, 8)$, determine the following. If you are asked to determine a limit, find the limit or one-sided limit as directed. Use ∞ and $-\infty$ where appropriate. If the limit does not exist and cannot be described using ∞ or $-\infty$, write "DNE".



(a) $\lim_{x \to -4} g(x) =$ 2

(c) $\lim_{x \to -3^+} g(x) =$ 2

(b) $\lim_{x \to 1} g(x) =$ 1

(d) $\lim_{x \to 8^-} g(x) =$ $-\infty$

(e) At what x-values in its domain is $g(x)$ NOT continuous? If g is continuous everywhere on its domain, write "none".

$-3, 1$

(f) At what x-values in its domain is $g(x)$ NOT differentiable? If g is differentiable everywhere on its domain, write "none".

$-6, -4, -1, 1$

(g) On the interval $[-6, 4]$, what are the x-values corresponding to local maxima of $g(x)$? If there are no local maxima, write "none".

$-4, -3, 1, 2$

(h) On the interval $[-6, 4]$, what are the x-values corresponding to local minima of $g(x)$? If there are no local minima, write "none".

$-6, -1, 4$

(i) On the interval $[-6, 4]$, what are the x-values corresponding to absolute maxima of $g(x)$? If there are no absolute maxima, write "none".

1

(j) On the interval $[-6, 4]$, what are the x-values corresponding to absolute minima of $g(x)$? If there are no absolute minima, write "none".

-6

2 (8 points)

(a) Compute the following limit. Show your work.

$$\lim_{x\to 4}\frac{\sqrt{x+12}-4}{x-4} = \lim_{x\to 4}\frac{\sqrt{x+12}-4}{x-4}\cdot\frac{\sqrt{x+12}+4}{\sqrt{x+12}+4}$$

$$= \lim_{x\to 4}\frac{x+12-16}{(x-4)(\sqrt{x+12}+4)} = \lim_{x\to 4}\frac{1}{\sqrt{x+12}+4} = \frac{1}{8}$$

(b) Determine any removable discontinuities, horizontal and vertical asymptotes if they exist of the function

$$k(x) = \frac{2x^2+12x-54}{x^2+2x-15}.$$

Use **limits** to justify your answers.

$$\lim_{x\to\infty}\frac{2x^2+12x-54}{x^2+2x-15} = \lim_{x\to\infty}\frac{2+12/x-54/x^2}{1+2/x-15/x^2} = 2$$

$$\lim_{x\to-\infty}\frac{2x^2+12-54}{x^2+2x-15} = 2$$ Horizontal asymptote $y=2$

$$x^2+2x-15 = (x+5)(x-2)$$

$$\lim_{x\to 2^+}\frac{2x^2+12x-54}{(x+5)(x-2)} = -\infty$$ $\Rightarrow$ vertical asymptote at $x=2$

$$\lim_{x\to 5^+}\frac{2x^2+12x-54}{(x+5)(x-2)} = +\infty$$ $\Rightarrow$ vertical asymptote at $x=2$.

3 (6 points)

Suppose $f(x) = x^2 - 4x$

(a) Use the DEFINITION of the derivative to calculate $f'(3)$. (You should be using limits!)

$$f'(3) = \lim_{h \to 0} \frac{(3+h)^2 - 4(3+h) - [9-12]}{h}$$

$$= \lim_{h \to 0} \frac{9+6h+h^2-12-4h+3}{h}$$

$$= \lim_{h \to 0} \frac{h^2+2h}{h} = \lim_{h \to 0} (h+2) = 2$$

(b) Write the equation for the tangent line to $f(x)$ at the point $(3, f(3))$.

$$y = 2(x-3) - 3 = 2x - 9$$

$\boxed{4}$ (9 points) Find $\frac{dy}{dx}$ for the functions given below. Do not simplify.

(a) $y = x^4 + 7\sqrt[7]{x^4} - \dfrac{2}{x^3}$

$y' = 4x^3 + 7 \cdot \frac{4}{7} x^{-3/7} + 6x^{-4}$

(b) $y = 3\sin(e^{2x} + x)$

$y' = 3\cos(e^{2x} + x) \cdot [2e^{2x} + 1]$

(c) $y = \dfrac{1 + \sin x}{\cos x}$ $= \sec(x + \tan(x)$

$y' = \sec x \tan x + \sec^2(x)$

$\boxed{5}$ (9 points) Find $\frac{dy}{dx}$ for the functions given below. Do not simplify.

(a) $y = 6x \arctan(5x)$

$$y' = 6\arctan(5x) + 6x \cdot \frac{1}{1+(5x)^2} \cdot 5$$

(b) $y = \int_1^{2x^2} \tan(3-t)\,dt$

$$y' = \tan(3-2x^2) \cdot 4x$$

(c) $y = \sin(x)^{\sin(x)}$

$$\ln(y) = \sin(x) \cdot \ln(\sin(x))$$

$$\frac{y'}{y} = \cos(x) \cdot \ln(\sin(x)) + \sin(x) \cdot \frac{\cos(x)}{\sin(x)}$$

$$y' = \sin(x)^{\sin(x)} \left[\cos(x) \cdot \ln(\sin(x)) + \cos(x) \right]$$

6 (12 points) Below you are given data about some unknown function $f(x)$, its first derivative $f'(x)$, and its second derivative $f''(x)$. The indication 'DNE' means a value is not defined. Using the information from the tables, determine the following. Note: not all categories need to have answers; if no values exist, write "none".

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = \infty, \qquad \lim_{x\to\infty} f(x) = 3, \qquad \lim_{x\to-\infty} f(x) = -\infty,$$

$$f(-6) = 0, \qquad f(-4) = 3, \qquad f(-1) = 1, \qquad f(x) \text{ is undefined at } x = 2$$

x	$x < -4$	-4	$-4 < x < -1$	-1	$-1 < x < 2$	2	$2 < x$
sign of $f'(x)$	$+$	0	$-$	0	$+$	DNE	$-$

x	$x < -3$	-3	$-3 < x < 2$	2	$2 < x$
sign of $f''(x)$	$-$	0	$+$	DNE	$+$

The following questions are about the behavior of the original function f.

(a) Critical numbers of f: $x =$ $-4, -1$

(b) interval(s) on which f is increasing $(-\infty, -4) \cup (-1, 2)$

(c) x-values at which f attains a local maximum, if any $x = -4$

Justification? $f'(x)$ goes from positive to negative.

(d) x-values at which f attains a local minimum, if any $x = -1$

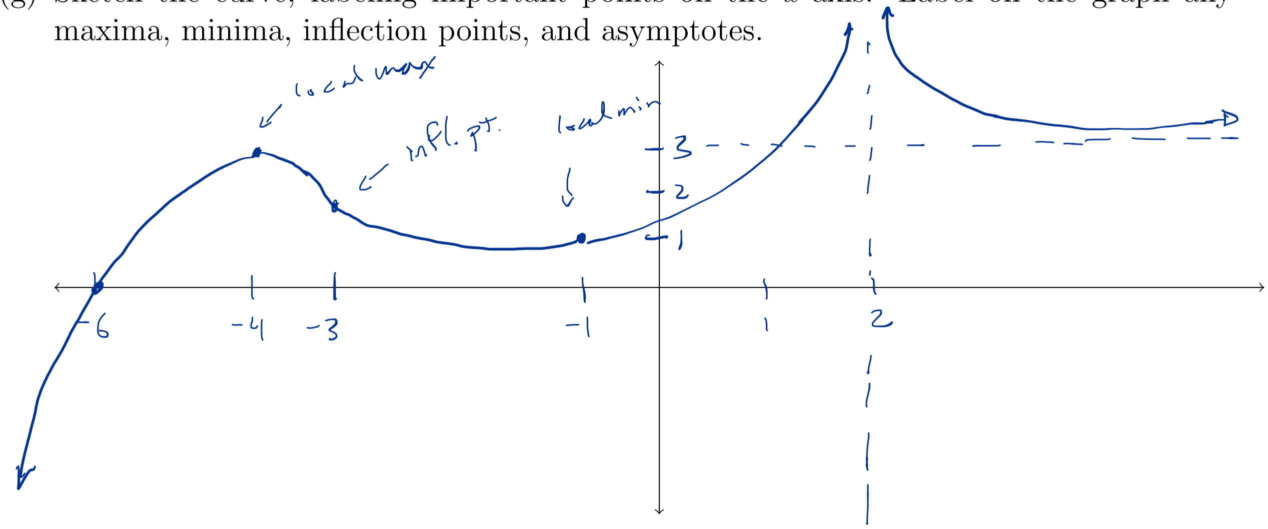
Justification? $f'(x)$ goes from negative to positive

(e) interval(s) on which f is concave down $(-\infty, -3)$

(f) inflection point(s) of f $x = -3$

Justification? $f''(x)$ changes sign there

(g) Sketch the curve, labeling important points on the x-axis. Label on the graph any maxima, minima, inflection points, and asymptotes.

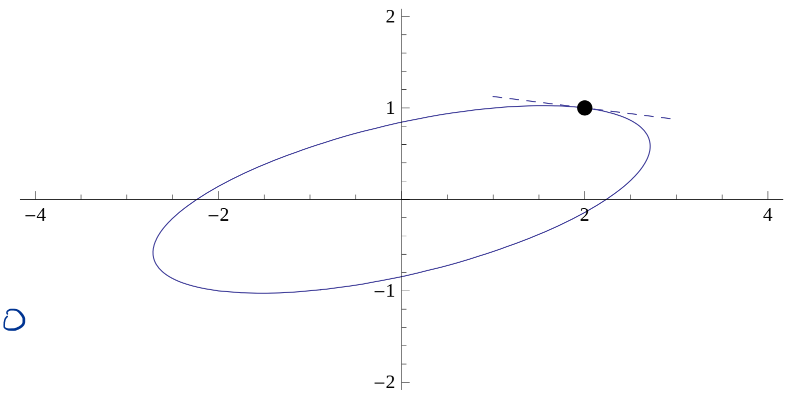


7 (6 points)

The graph of the tilted ellipse shown below is given by the implicit equation

$$x^2 - 3xy + 7y^2 = 5.$$

The point $(2,1)$ lies as shown on the ellipse. Determine the equation of the tangent line to the ellipse that passes through the point $(2,1)$.



$2x - 3y - 3xy' + 14yy' = 0$

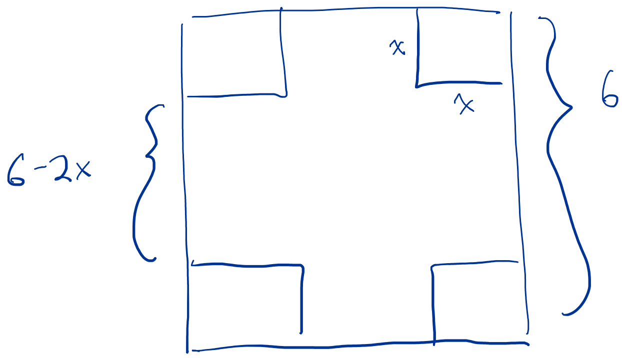
$y'(14y - 3x) = 3y - 2x$

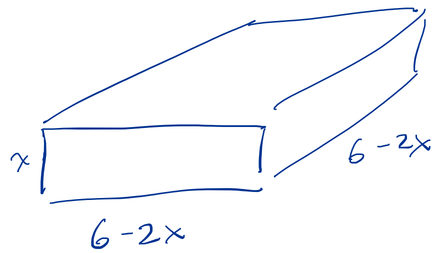
$y' = \dfrac{3y-2x}{14y-3x}$

$y'(2,1) = \dfrac{3\cdot 1 - 2\cdot 2}{14\cdot 1 - 3\cdot 2} = \dfrac{-1}{8}$

$\boxed{y = \frac{-1}{8}(x-2) + 1}$

$\boxed{8}$ (8 points) An open box (sides, bottom, no top) is to be made from a 6×6 rectangular sheet of metal by cutting out squares of equal size from each corner and then folding up the sides. Find the size of the square cut that will yield the maximum volume. What is this maximum volume, and what are the resulting dimensions of the box? Be sure to verify you have obtained the maximum.





$$\text{Volume} = (6-2x)^2 \cdot x$$
$$= 36x - 24x^2 + 4x^3$$
$$\text{Volume}'(x) = 36 - 48x + 12x^2$$
$$0 = 36 - 48x + 12x^2$$
$$0 = 3 - 4x + x^2$$
$$= (x-3)(x-1)$$

note $0 \le x \le 3$

$$\text{Volume}(0) = \text{Volume}(3) = 0$$
$$\text{Volume}(1) = 36 - 24 + 4 = 16$$

maximum volume is 16 when box is

$4 \times 4 \times 1$

9 (10 points)

(a) If $g'(x) = x^2 + (\sec(x))^2$, what is the most general function $g(x)$?

$$g(x) = \frac{x^3}{3} + \tan(x) + C$$

(b) Compute $\int_1^8 (3x - \sqrt[3]{x})\,dx$ $= \int_1^8 3x - x^{1/3}\,dx = \frac{3}{2}x^2 - \frac{3}{4}x^{4/3}\Big|_1^8$

$$= \frac{3}{2}64 - \frac{3}{4}16 - \left(\frac{3}{2} - \frac{3}{4}\right)$$

$$= 96 - 12 - \frac{3}{4}$$

$$= 84 - 3/4 = 83 + 1/4$$

(c) A particle is moving with acceleration

$$a(t) = t + e^{-2t}.$$

You measure that at time $t = 0$, its position is given by $s(0) = 0$ and its velocity is given by $v(0) = 8$.

Determine the position function $s(t)$.

$$s(t) = \int t + e^{-2t}\,dt = \frac{t^2}{2} - \frac{1}{2}e^{-2t} + C$$

$$s(0) = 0 - \frac{1}{2} + C \Rightarrow C = 1/2$$

So $$\boxed{s(t) = \frac{t^2}{2} - \frac{1}{2}e^{-2t} + \frac{1}{2}}$$

$\boxed{10}$ (15 points) Compute the following integrals.

(a) $\displaystyle\int \frac{e^{4t}}{e^{4t}+2}\,dt = \int \frac{1}{4}\frac{1}{u}\,du = \frac{1}{4}\ln|u| + C$

$u = e^{4t}+2$

$du = 4e^{4t}\,dt$

$= \frac{1}{4}\ln|e^{4t}+2| + C$

(b) $\displaystyle\int \frac{x+8}{\sqrt{x}}\,dx = \int x^{1/2} + 8x^{-1/2}\,dx = \frac{2}{3}x^{3/2} + 16x^{1/2} + C$

(c) $\displaystyle\int_{-6}^{-3} x\sqrt{x+7}\,dx = \int_1^4 (u-7)\sqrt{u}\,du = \int_1^4 u^{3/2} - 7u^{1/2}\,du$

$u = x+7$

$du = dx$

$= \frac{2}{5}u^{5/2} - \frac{14}{3}u^{3/2}\Big|_1^4$

$= \frac{64}{5} - \frac{14}{3}\cdot 8 - \left(2/5 - \frac{14}{3}\right) = \frac{62}{5} - \frac{108}{3}$

(d) $\displaystyle\int \theta^3\sec(\theta^4)\tan(\theta^4)\,d\theta = \int \frac{1}{4}\sec(u)\tan(u)\,du$

$u = \theta^4 \quad du = 4\theta^3\,d\theta$

$= \frac{1}{4}\sec(u) + C$

$= \frac{1}{4}\sec(\theta^4) + C$

(e) $\displaystyle\int \frac{7}{\sqrt{16-y^2}}\,dy = \int \frac{7}{4}\frac{1}{\sqrt{1-(y/4)^2}}\,dy = \int 7\cdot\frac{1}{\sqrt{1-u^2}}\,du$

$y/4 = u \quad du = \frac{dy}{4}$

$= 7\arcsin(u) + C = 7\arcsin(y/4) + C$

11 (Extra Credit: 8 points)

Recall that the error bound for Simpson's Rule is given by

$$|E| \leq \frac{(b-a)^5}{180n^4} \max_{a \leq x \leq b} |f^{(4)}(x)|.$$

(a) Write out the terms in an estimate of $\int_1^7 \frac{1}{x^2}\, dx$ using Simpson's rule and $n = 6$ subintervals (do not simplify your answer).

$$\frac{6}{6}\left(\frac{1}{1^2} + 4 \cdot \frac{1}{2^2} + 2\frac{1}{3^2} + 4 \cdot \frac{1}{4^2} + 2\frac{1}{5^2} + 4\frac{1}{6^2} + \frac{1}{7^2} \right)$$

(b) We wish to estimate the value of $\int_0^2 \frac{1}{18}\sin(6t)\, dt$ to within $\frac{1}{20}$ of the correct answer. (Hint: $\sin(t)$ is a bounded function.)

(i) How many subintervals are required?

$f'(x) = \frac{1}{3}\cos(6t)$

$f''(x) = -2\sin(6t)$

$f'''(x) = -12\cos(6t)$

$f^{(4)}(x) = +72\sin(6t)$

$$\frac{1}{20} \geq \frac{(2-0)^5}{180n^4} \cdot 72$$

$$9n^4 \geq 32 \cdot 72$$

$$n^4 \geq 32 \cdot 8 = 2^5 \cdot 2^3 = 2^8$$

$$n \geq 2^2 = 4$$

(ii) How many parabolas are used in the estimate? 2