CCO50- Digital Speech Processing

Short Test 5

Description: Design an FIR filter (q[n]) of order M = 5 to cut-off frequencies within the range 2500 Hz ~ 3500 Hz, allowing for all the others to pass through. Assume that the input signal to be filtered (x[n]) was sampled at 10000 samples per second

In order to calculate thge band-stop FIR Q[n], we need to sum H[n] and G[n]. H to be calculated at the 2500Hz and G at 3500Hz

$$h[n] = \frac{\sin\left(\omega_c(n - \frac{M}{2})\right)}{\pi(n - \frac{M}{2})}$$
$$M = 5$$
$$0 \leq n \leq 5$$
$$\pi = 5kHz$$
$$\omega_h = 2,5kHz = \frac{\pi}{2}$$

To high-pass filter, let use the complement

$$\omega_g = 5kHz - 3,5kHz = 1,5kHZ = \frac{3\pi}{10}$$

First, calculate a low-pass filter $h[]$ using the ω_h:

$$h[0] = \frac{\sin\left(\frac{\pi}{2}(0 - \frac{5}{2})\right)}{\pi(0 - \frac{5}{2})} = -0.09003163$$
$$h[1] = \frac{\sin\left(\frac{\pi}{2}(1 - \frac{5}{2})\right)}{\pi(1 - \frac{5}{2})} = 0.15005272$$
$$h[2] = \frac{\sin\left(\frac{\pi}{2}(2 - \frac{5}{2})\right)}{\pi(2 - \frac{5}{2})} = 0.45015816$$
$$h[3] = \frac{\sin\left(\frac{\pi}{2}(3 - \frac{5}{2})\right)}{\pi(3 - \frac{5}{2})} = 0.45015816$$
$$h[4] = \frac{\sin\left(\frac{\pi}{2}(4 - \frac{5}{2})\right)}{\pi(4 - \frac{5}{2})} = 0.15005272$$
$$h[5] = \frac{\sin\left(\frac{\pi}{2}(5 - \frac{5}{2})\right)}{\pi(5 - \frac{5}{2})} = -0.09003163$$

$$h = (-0.09, 0.15, 0.45, 0.45, 0.15, -0.09)$$

Second, calculate a high-pass filter $g[]$ using the ω_g:

$$h[0] = \frac{\sin\left(\frac{3\pi}{10}(0 - \frac{5}{2})\right)}{\pi(0 - \frac{5}{2})} = 0.09003163$$
$$h[1] = \frac{\sin\left(\frac{3\pi}{10}(1 - \frac{5}{2})\right)}{\pi(1 - \frac{5}{2})} = 0.20959398$$
$$h[2] = \frac{\sin\left(\frac{3\pi}{10}(2 - \frac{5}{2})\right)}{\pi(2 - \frac{5}{2})} = 0.28901933$$
$$h[3] = \frac{\sin\left(\frac{3\pi}{10}(3 - \frac{5}{2})\right)}{\pi(3 - \frac{5}{2})} = 0.28901933$$
$$h[4] = \frac{\sin\left(\frac{3\pi}{10}(4 - \frac{5}{2})\right)}{\pi(4 - \frac{5}{2})} = 0.20959398$$
$$h[5] = \frac{\sin\left(\frac{3\pi}{10}(5 - \frac{5}{2})\right)}{\pi(5 - \frac{5}{2})} = 0.09003163$$

$$h = (0.090.2090.2890.2890.2090.09)$$

Now, reverse $h[]$ and invert every odd index to have $g[]$ $$g = (0.09, -0.209, 0.289, -0.289, 0.209, -0.09)

Then, sum $h[]$ and $g[]$

$$q = (-0.09 + 0.09, 0.15 - 0.209, 0.45 + 0.289, 0.45 - 0.289, 0.15 + 0.209, -0.09 - 0.09)$$
$$q = (0, -0.059, 0.739, 0.161, 0.359, -0.18)$$

Implementing the filter in python, based on the previous implementations

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pyaudio
```

In [2]:
```
def low_pass_filter(cutoff, sample_rate, M=5):
    nyq = sample_rate / 2
    omega = cutoff / nyq
    h = np.zeros(M+1)
    for n in range(M+1):
        if n == (M/2):
            h[n] = 1.0
        else:
            h[n] = (np.sin(np.pi * omega * (n - (M/2)))/(np.pi * (n - (M/2))))
    return h
```

In [3]:
```
def high_pass_filter(cutoff, sample_rate, M=5):
    if M % 2 == 0:
        M -= 1
    cutoff = sample_rate / 2 - cutoff
    g = low_pass_filter(cutoff, sample_rate, M)
    g = np.array([-val if i % 2 != 0 else val for i, val in enumerate(g)])
    return g
```

In [8]:
```
def band_stop_filter(low_cutoff, high_cutoff, sample_rate, M=5):
    low = low_pass_filter(low_cutoff, sample_rate, M)
    high = high_pass_filter(high_cutoff, sample_rate, M)
    return low + high
```

In [10]:
```
q = band_stop_filter(2500, 3500, 10000, M=5)
print([f"{val:.6f}" for val in q])
```

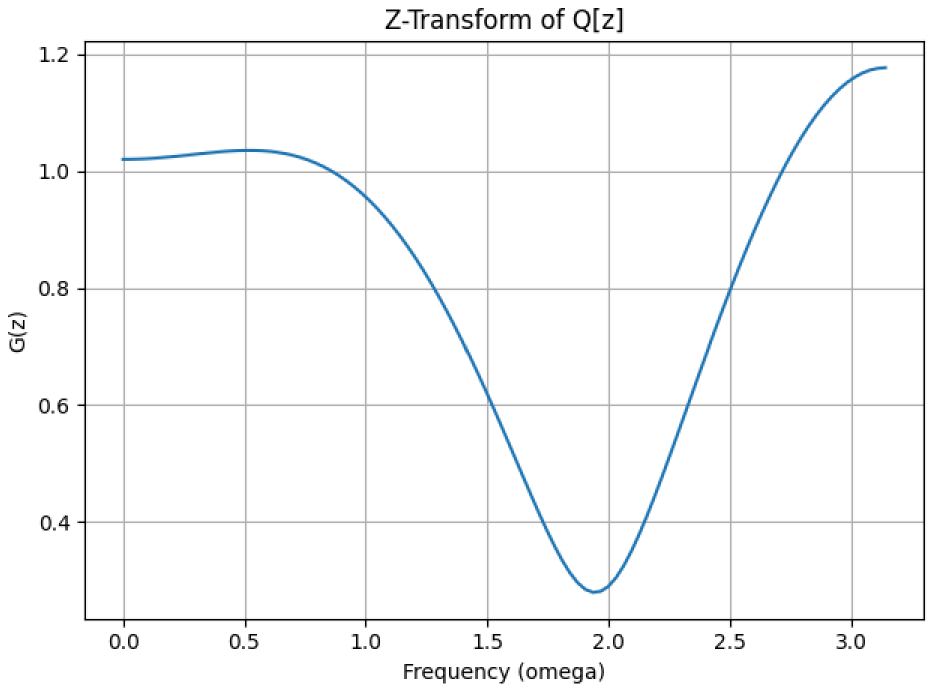
```
['0.000000', '-0.059541', '0.739177', '0.161139', '0.359647', '-0.180063']
```

Using the Z-tranform to check the filter

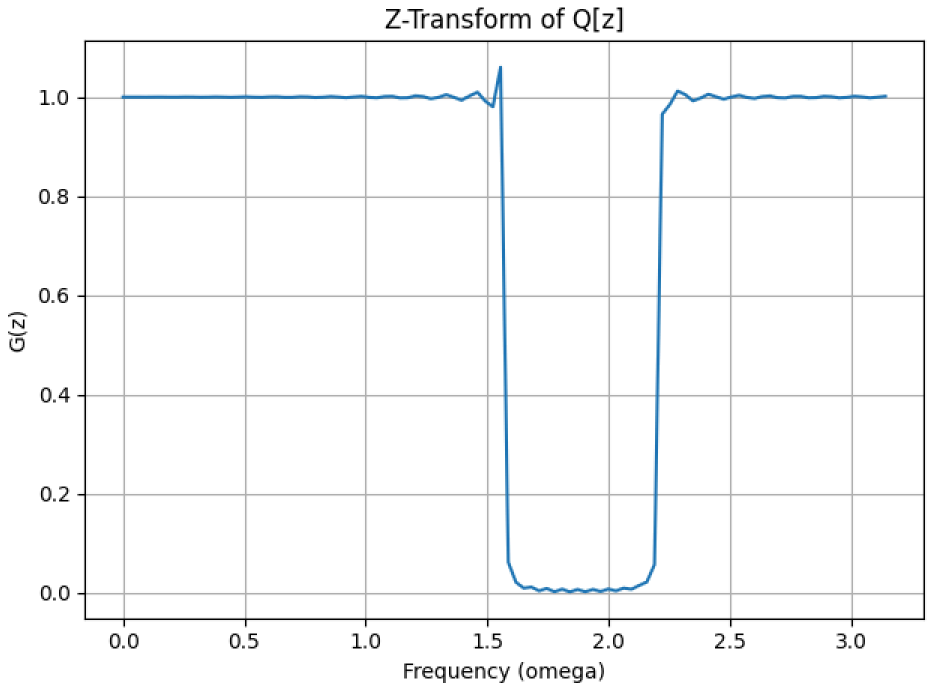
In [11]:
```
def z_transform(x, num_points=1000):
    omega = np.linspace(0, np.pi, num_points)
    z = np.exp(1j * omega)
    X_z = np.zeros_like(z, dtype=complex)
    N = len(x)
    for k in range(N):
        X_z += x[k] * z**(-k)
    return omega, X_z
```

In [12]:
```
def plot_z_transform(omega, G_z, name = "G[z]"):
    plt.plot(omega, np.abs(G_z))
    plt.title('Z-Transform of {name}'.format(name=name))
    plt.xlabel('Frequency (omega)')
    plt.ylabel('G(z)')
    plt.grid()
    plt.tight_layout()
    plt.show()
```

In [14]:
```
omega, Q_z = z_transform(q, 100)
plot_z_transform(omega, Q_z, name="Q[z]")
```



In [17]:
```
q = band_stop_filter(2500, 3500, 10000, M=501)
omega, Q_z = z_transform(q, 100)
plot_z_transform(omega, Q_z, name = "Q[z]")
```



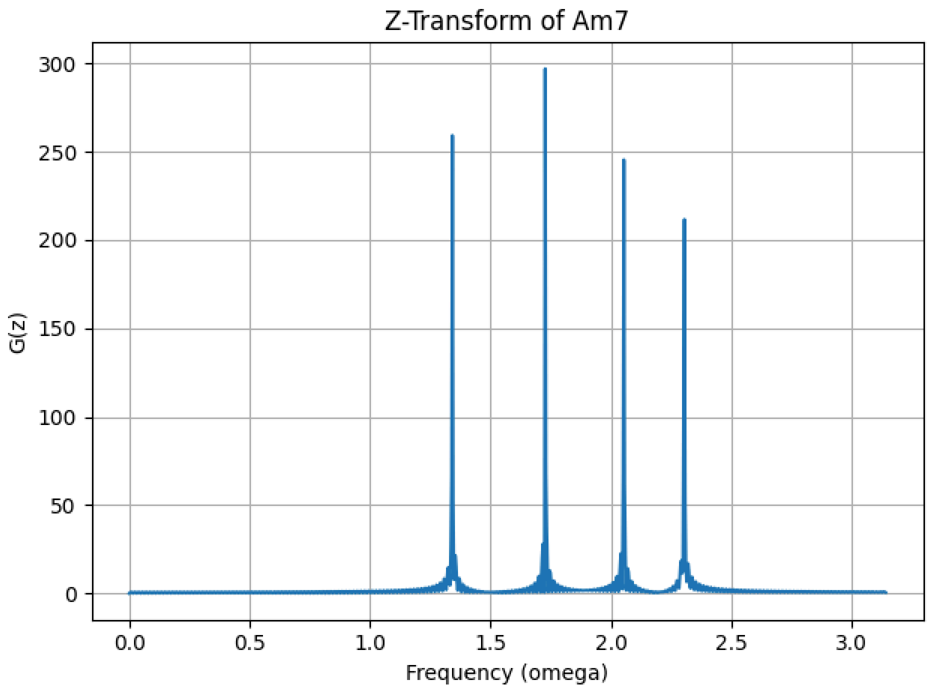
Extra: generating waves and filtering them

In [18]:
```
#defining util methods
def play_signal(signal, sample_rate=44100):
    p = pyaudio.PyAudio()
```

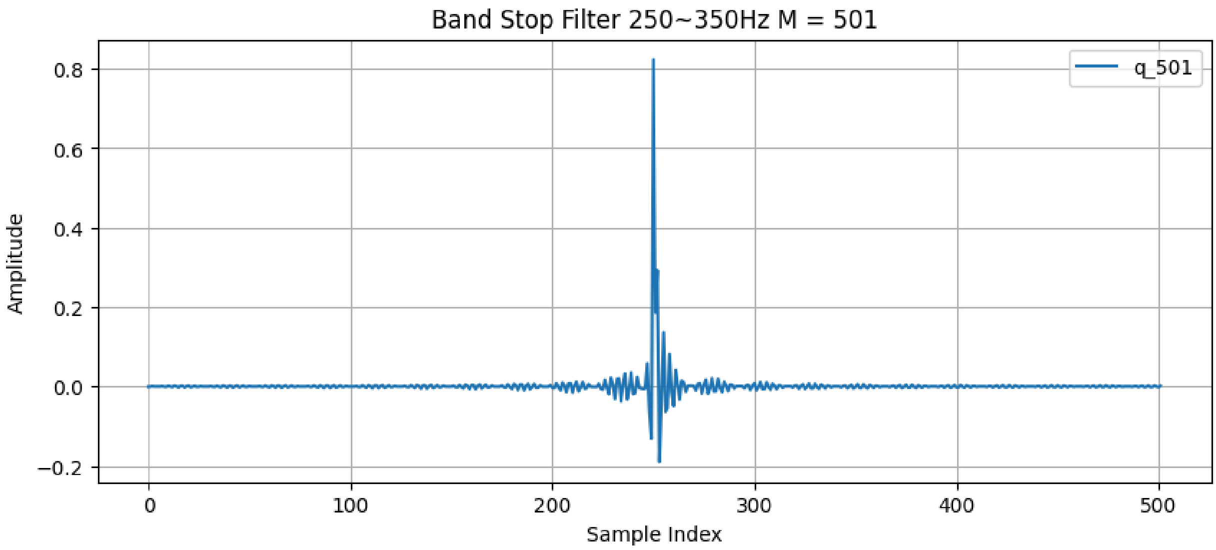
```
    stream = p.open(format=pyaudio.paFloat32,
                    channels=1,
                    rate=sample_rate,
                    output=True)

    stream.write(signal.tobytes())

    stream.stop_stream()
    stream.close()

    p.terminate()

def plot_waveform(wave, label, title):
    plt.figure(figsize=(10, 4))
    plt.plot(wave, label=label)
    plt.title(title)
    plt.xlabel("Sample Index")
    plt.ylabel("Amplitude")
    plt.legend()
    plt.grid()
    plt.show()
```

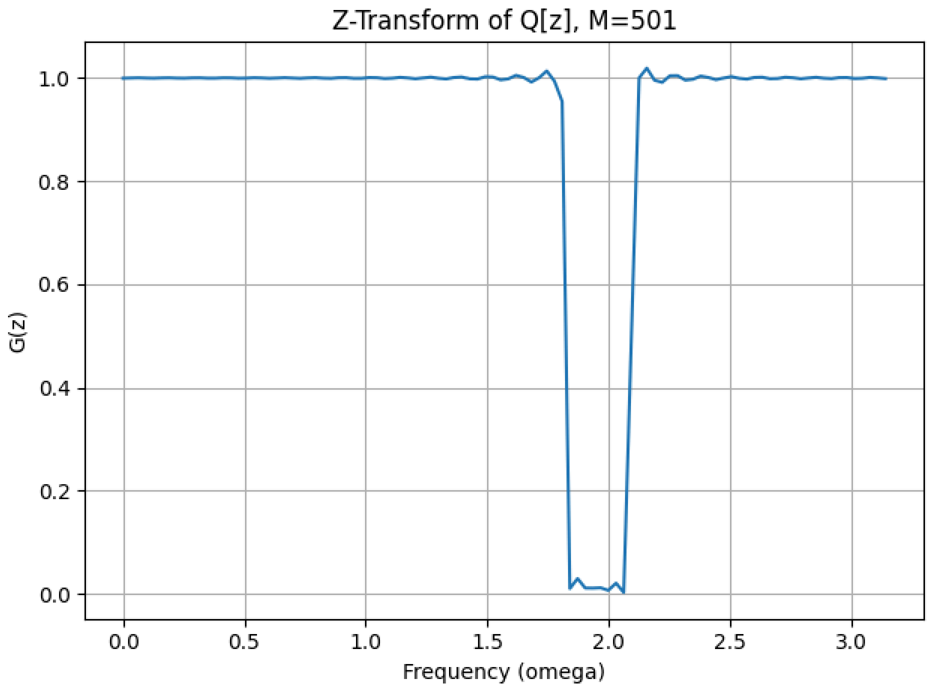
In [19]:
```
# generating an Am7 chord waves
sample_rate = 1200 # using a lower sample rate for less noisy results and more clarity
seconds = 2
octave = 1 # 1 for central octave, to lower pitch use 1/octave
s_A = (np.sin(2 * np.pi * np.arange(sample_rate * seconds) * 440 * octave / sample_rate)).astype(np.float32)
s_C = (np.sin(2 * np.pi * np.arange(sample_rate * seconds) * 256.63 * octave / sample_rate)).astype(np.float32)
s_E = (np.sin(2 * np.pi * np.arange(sample_rate * seconds) * 329.63 * octave / sample_rate)).astype(np.float32)
s_G = (np.sin(2 * np.pi * np.arange(sample_rate * seconds) * 392 * octave / sample_rate)).astype(np.float32)
s_Am7 = s_A + s_C + s_E + s_G
s_Am7 /= np.max(np.abs(s_Am7))  # Normalize the signal
```

In [20]:
```
omega, S_z = z_transform(s_Am7, 1000)
plot_z_transform(omega, S_z, name = "Am7")
play_signal(s_Am7, sample_rate)
```

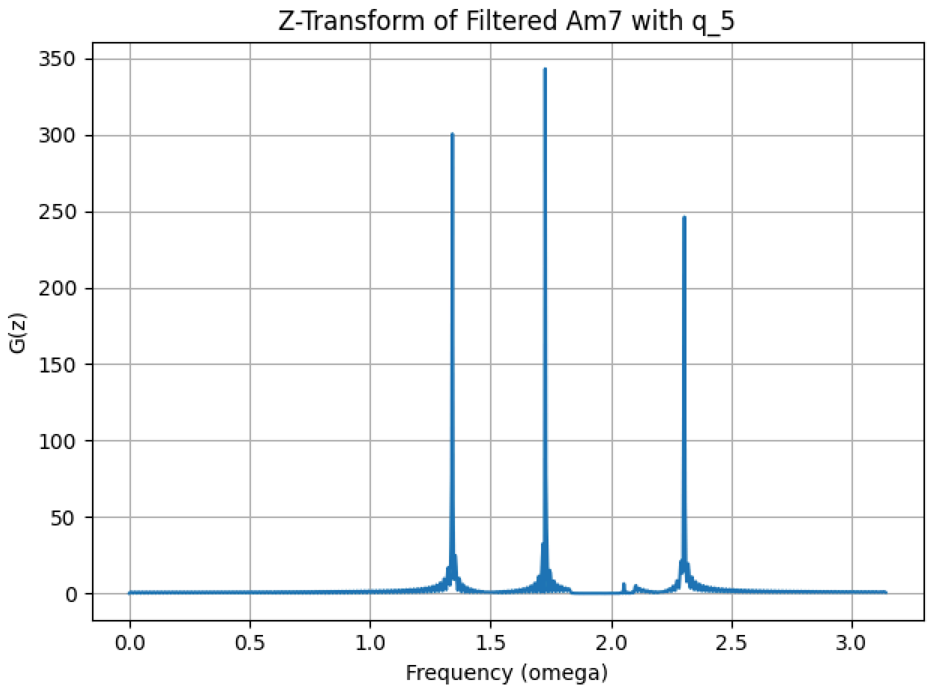


In [23]:
```
q_5 = band_stop_filter(350, 400, sample_rate, 501)
plot_waveform(q_5, "q_501", "Band Stop Filter 250~350Hz M = 501")
omega, G_z = z_transform(q_5, 100)
plot_z_transform(omega, G_z, name = "Q[z], M=501")
```





In [24]:
```
filtered_Am7_g5 = np.convolve(s_Am7, q_5)
filtered_Am7_g5 /= np.max(np.abs(filtered_Am7_g5))  # Normalize the signal
omega, G_z = z_transform(filtered_Am7_g5, 1000)
plot_z_transform(omega, G_z, name = "Filtered Am7 with q_5")
play_signal(filtered_Am7_g5, sample_rate)
```



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Date: May 08 2025