In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pyaudio
```

CCO50- Digital Speech Processing

Short Test 6

Description: Design an FIR filter (q[n]), subtype I, of order M = 4, to cut-off frequencies within the range 2500 Hz ⇠ 3500 Hz, allowing for all the others to pass through. Assume that the input signal to be filtered (x[n]) was sampled at 10000 samples per second. Normalize the filters' coefficients in such a way that the filter presents a gain of 0 dB in the pass-band. Lastly, write down the difference equation to filter an input signal x[n] by using q[n]

First, the low-pass filter is calculated for the cut-off frequency of 2500 Hz

$$h[n] = \frac{\sin\left(\omega_c(n - \frac{M}{2})\right)}{\pi(n - \frac{M}{2})}, \quad \text{for } n \neq \frac{M}{2}$$

For $n = \frac{M}{2}$:

$$h[n] = \frac{\omega_c}{\pi}$$
$$\omega_c = \frac{\pi}{2}$$
$$h[0] = \frac{\sin\left(\frac{\pi}{2}(0-2)\right)}{\pi(0-2)} = 0$$
$$h[1] = \frac{\sin\left(\frac{\pi}{2}(1-2)\right)}{\pi(1-2)} = 0.318310$$
$$h[2] = \frac{\frac{\pi}{2}}{\pi} = 0.5$$
$$h[3] = \frac{\sin\left(\frac{\pi}{2}(3-2)\right)}{\pi(3-2)} = 0.318310$$
$$h[4] = \frac{\sin\left(\frac{\pi}{2}(4-2)\right)}{\pi(4-2)} = 0$$
$$h_{\text{low}} = [0, 0.318, 0.5, 0.318, 0]$$

To create the high-pass filter, the complement of a low-pass filter calculated for the cut-off frequency of 3500 Hz.

$\omega_c = \frac{7\pi}{10}$

$$h[0] = \frac{\sin\left(\frac{7\pi}{10}(0-2)\right)}{\pi(0-2)} = -0.151365$$
$$h[1] = \frac{\sin\left(\frac{7\pi}{10}(1-2)\right)}{\pi(1-2)} = 0.257518$$
$$h[2] = \frac{\frac{7\pi}{10}}{\pi} = 0.7$$
$$h[3] = \frac{\sin\left(\frac{7\pi}{10}(3-2)\right)}{\pi(3-2)} = 0.257518$$
$$h[4] = \frac{\sin\left(\frac{7\pi}{10}(4-2)\right)}{\pi(4-2)} = -0.151365$$
$$h_{\text{low}} = [-0.151, 0.257, 0.7, 0.257, -0.151]$$

The all-pass filter:

$$h_{\text{all}} = [0, 0, 1, 0, 0]$$

Then subtracting the all pass and the low pass

$$h_{\text{high}} = h_{\text{all}} - h_{\text{low}}$$
$$h_{\text{high}} = [0.151, -0.257, 0.3, -0.257, 0.151]$$

Finally, the band stop is created with the sum of the low and high pass filters

$$q[n] = h_{\text{low}} + h_{\text{high}}$$
$$q = [0.151, 0.0607, 0.8, 0.0607, 0.151]$$

Code implementation

In [155…

```
def low_pass_filter(cutoff, sample_rate, M=5):
    nyq = sample_rate / 2
    omega = cutoff / nyq
    h = np.zeros(M+1)
    for n in range(M+1):
        if n == (M/2):
            h[n] = cutoff / nyq
        else:
            h[n] = (np.sin(np.pi * omega * (n - (M/2)))/(np.pi * (n - (M/2))))
    return h

def all_pass_filter(M):
    h = np.zeros(M+1)
    for n in range(M+1):
        if n == (M/2):
            h[n] = 1
        else:
            h[n] = (np.sin(np.pi * (n - (M/2)))/(np.pi * (n - (M/2))))
    return h

def high_pass_filter(cutoff_freq, sample_rate, M):
    lp = low_pass_filter(cutoff_freq, sample_rate, M)
    all_pass = all_pass_filter(M)
    return all_pass - lp
```

In [136…

```
def band_pass_filter(low_cutoff, high_cutoff, sample_rate, M):
    lp = low_pass_filter(high_cutoff, sample_rate, M)
    hp = high_pass_filter(low_cutoff, sample_rate, M)
    return lp - hp

def band_stop_filter(low_cutoff, high_cutoff, sample_rate, M):
    lp = low_pass_filter(low_cutoff, sample_rate, M)
    hp = high_pass_filter(high_cutoff, sample_rate, M)
    return lp + hp
```

In [156…

```
q = band_stop_filter(2500, 3500, 10000, M=4)
print([f"{val:.6f}" for val in q])
```

```
['0.151365', '0.060792', '0.800000', '0.060792', '0.151365']
```

Normalizing the filter so the coefficients sum up to 1

In [157…

```
#normalizing q
q /= np.sum(q)
print([f"{val:.6f}" for val in q])
print(f"Sum of the normalized q: {np.sum(q)}")
```

```
['0.123633', '0.049654', '0.653427', '0.049654', '0.123633']
Sum of the normalized q: 0.9999999999999999
```

The Difference Equation

Is defined as

$$y[n] = x[n] * q[n]$$
$$Y[z] = X[z] \cdot Q[z]$$
$$Y[z] = X[z] \cdot (a + bz^{-1} + cz^{-2} + \dots)$$
$$Y[z] = aX[z] + bX[z]z^{-1} + cX[z]z^{-2} + \dots$$

if $X[z]$ is the Z Transform of $x[n]$, then $z^{-k}X[z]$ is the Z Transform of $x[n-k]$

$$y[n] = ax[n] + bx[n-1] + cx[n-2] + \dots$$

Applying to the normalized filter we have

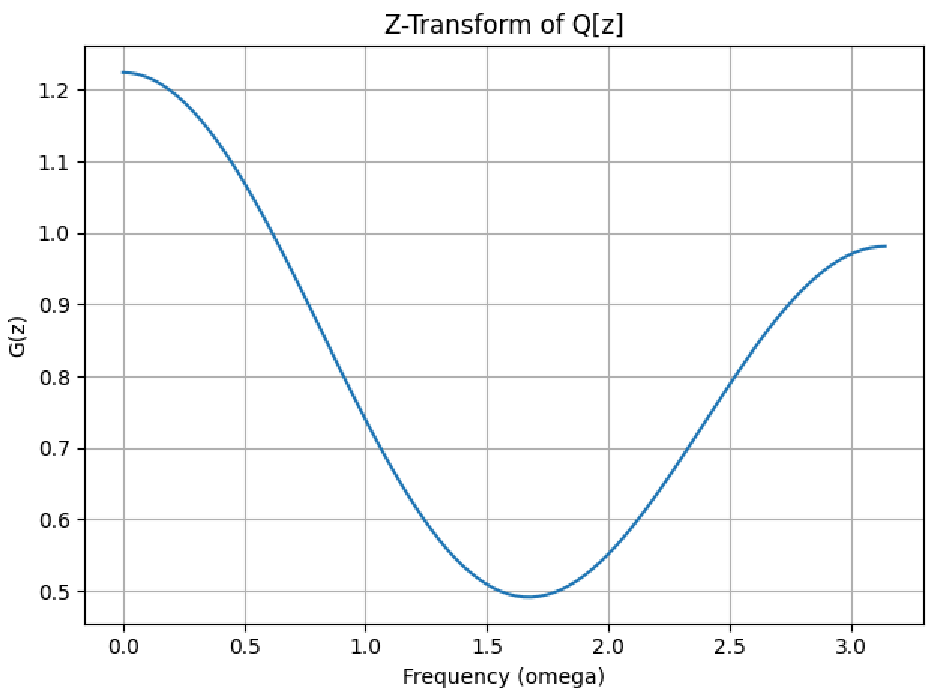
$$y[z] = 0.123x[n] + 0.049x[n-1] + 0.653x[n-2] + 0.049x[n-3] + 0.123x[n-4]$$

Using the Z-tranform to check the filter

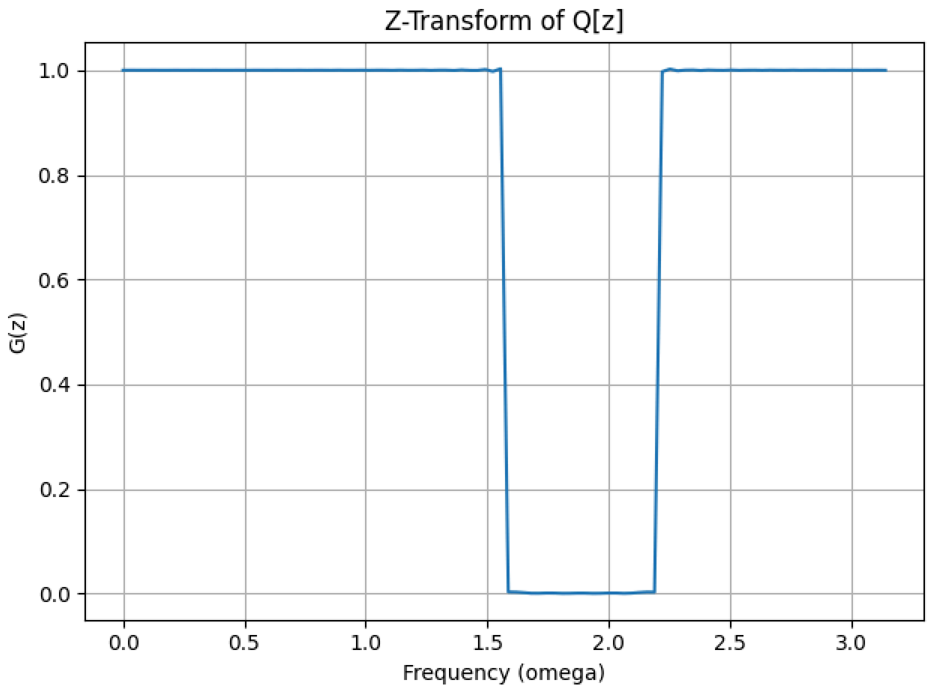
In [19]:
```
def z_transform(x, num_points=1000):
    omega = np.linspace(0, np.pi, num_points)
    z = np.exp(1j * omega)
    X_z = np.zeros_like(z, dtype=complex)
    N = len(x)
    for k in range(N):
        X_z += x[k] * z**(-k)
    return omega, X_z
```

In [20]:
```
def plot_z_transform(omega, G_z, name = "G[z]"):
    plt.plot(omega, np.abs(G_z))
    plt.title('Z-Transform of {name}'.format(name=name))
    plt.xlabel('Frequency (omega)')
    plt.ylabel('G(z)')
    plt.grid()
    plt.tight_layout()
    plt.show()
```

In [140…
```
sample_rate = 10000
omega, Q_z = z_transform(q, 100)
plot_z_transform(omega, Q_z, name="Q[z]")
```



In [143…
```
q = band_stop_filter(2500, 3500, 10000, M=5000)
omega, Q_z = z_transform(q, 100)
plot_z_transform(omega, Q_z, name = "Q[z]")
```



Author: Matheus Sinto Novaes
E-mail: matheus.sinto@unesp.br

Course: Digital Speech Processing
Professor: Dr. Eng. Rodrigo Capobianco Guido
Date: May 8 2025