CCO50- Digital Speech Processing

Short Test 9

Description: Find the transfer function $H[z]$ whose poles are at $z = \frac{1}{3}$ and $z = -\frac{1}{6}$, in addition to one zero at $z = \frac{1}{2}$.

Write down the corresponding difference equation. Is the transfer function stable and causal? Why?

When the tranfer function has poles, then it is IIR.

$$H[z] = \frac{(z - \frac{1}{2})}{(z - \frac{1}{3}) \cdot (z + \frac{1}{6})}$$

$$H[z] = \frac{(z - \frac{1}{2})}{z^2 + \frac{1}{6}z - \frac{1}{3}z - \frac{1}{18}}$$

$$H[z] = \frac{(z - \frac{1}{2})}{z^2 - \frac{1}{6}z - \frac{1}{18}}$$

$$\text{multiplying by } \frac{z^{-2}}{z^{-2}} \text{ to have the negative exponents}$$

$$H[z] = \frac{(z - \frac{1}{2})}{z^2 - \frac{1}{6}z - \frac{1}{18}} \cdot \frac{z^{-2}}{z^{-2}}$$

$$H[z] = \frac{(z^{-1} - \frac{1}{2}z^{-2})}{1 - \frac{1}{6}z^{-1} - \frac{1}{18}z^{-2}}$$

$$H[z] = \frac{Y[z]}{X[z]} = \frac{(z^{-1} - \frac{1}{2}z^{-2})}{1 - \frac{1}{6}z^{-1} - \frac{1}{18}z^{-2}}$$

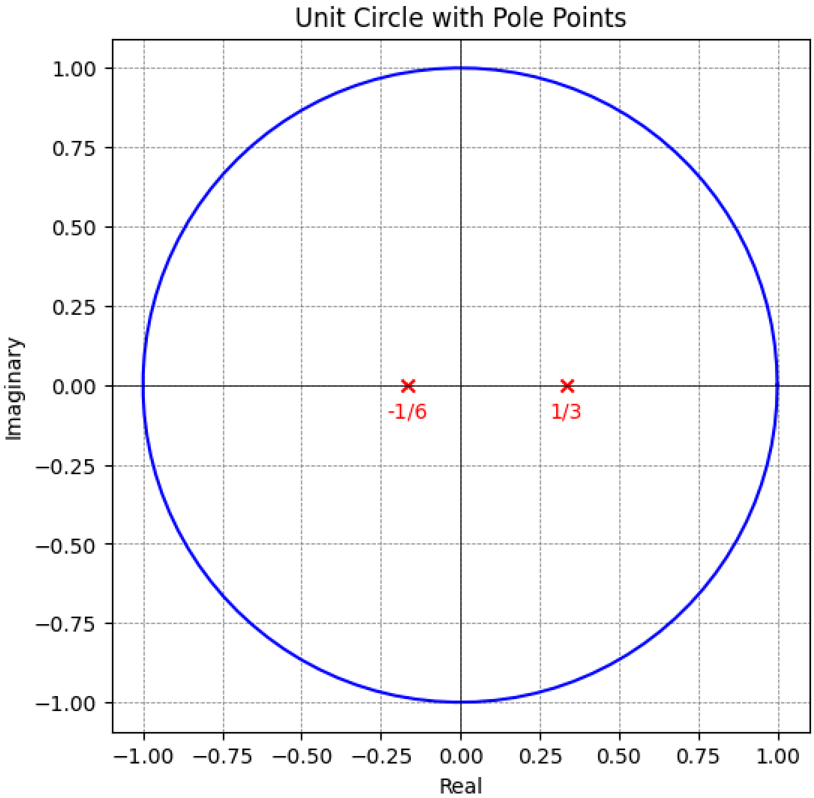
$$Y[z] - \frac{1}{6}Y[z]z^{-1} - \frac{1}{18}Y[z]z^{-2} = X[z]z^{-1} - \frac{1}{2}X[z]z^{-2}$$

$$\text{from frequency domain to time domain:}$$

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{18}y[n-2] = x[n-1] - \frac{1}{2}x[n-2]$$

$$y[n] = x[n-1] - \frac{1}{2}x[n-2] + \frac{1}{6}y[n-1] + \frac{1}{18}y[n-2]$$

In [12]:
```
import numpy as np
import matplotlib.pyplot as plt

theta = np.linspace(0, 2 * np.pi, 100)
x = np.cos(theta)
y = np.sin(theta)

poles = [1/3, -1/6]

plt.figure(figsize=(6, 6))
plt.plot(x, y, label='Unit Circle', color='blue')

for pole in poles:
    plt.scatter(pole, 0, color='red', marker='x', label=f'Pole {pole}')
    fraction = Fraction(pole).limit_denominator()
    plt.text(pole, -0.1, f'{fraction}', color='red', ha='center')


plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.grid(color='gray', linestyle='--', linewidth=0.5)
plt.xlabel('Real')
plt.ylabel('Imaginary')
plt.title('Unit Circle with Pole Points')
plt.axis('equal')
plt.show()
```



So, we can conclude that the function **is stable** because the poles are within the boundaries of the unit circle and **is causal** because the output $y[n]$ depends only on past inputs.

Author: Matheus Sinto Novaes
E-mail: matheus.sinto@unesp.br

Course: Digital Speech Processing
Professor: Dr. Eng. Rodrigo Capobianco Guido
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