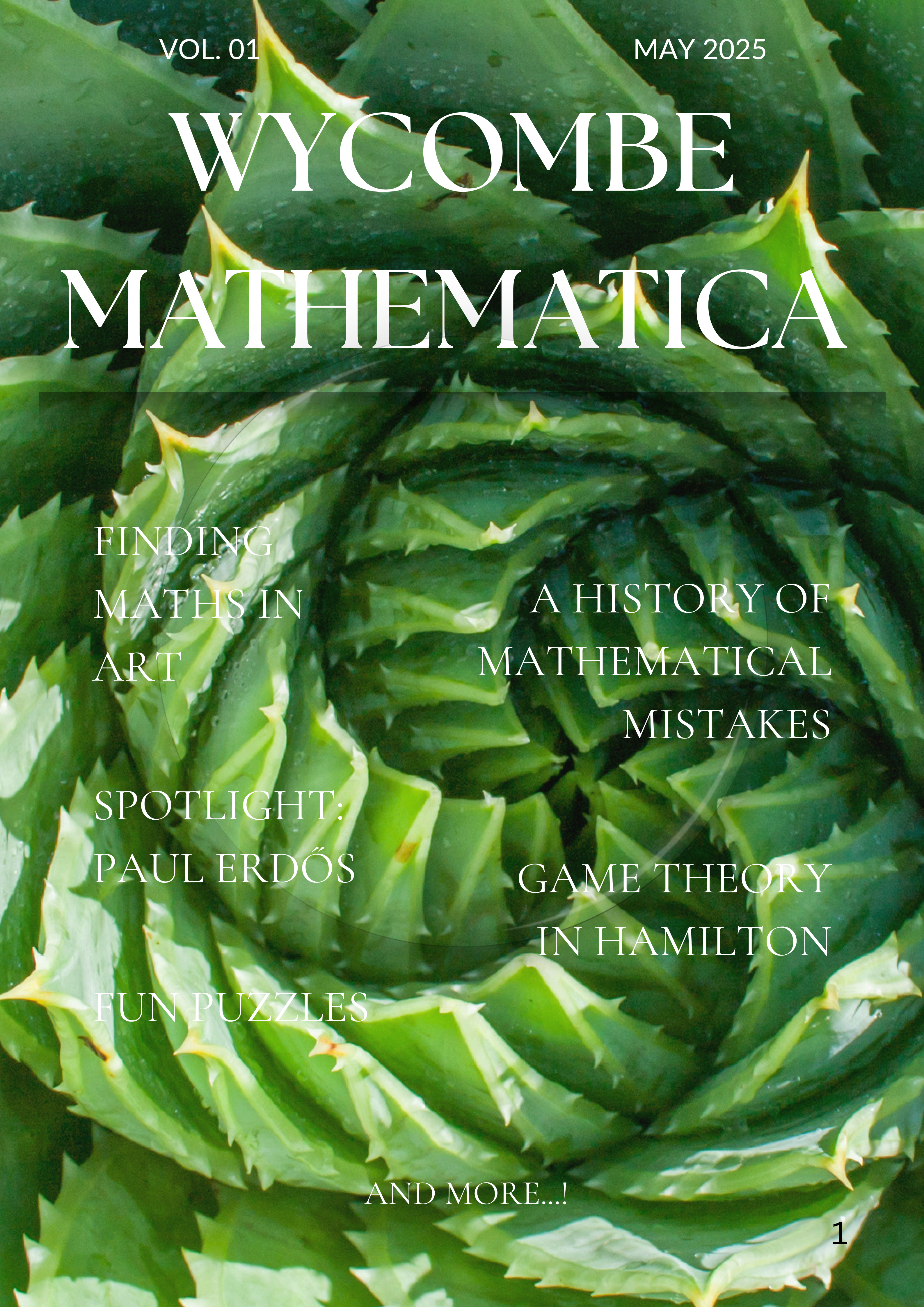

VOL. 01 MAY 2025

WYCOMBE MATHEMATICA

FINDING MATHS IN ART

A HISTORY OF MATHEMATICAL MISTAKES

SPOTLIGHT: PAUL ERDŐS

GAME THEORY IN HAMILTON

FUN PUZZLES

AND MORE...!

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From the Editors

Welcome to the inaugural issue of Wycombe's maths publication: Mathematica!

Our goal with the *Wycombe Abbey Mathematica* publication is to celebrate how fun the world of maths is. We want to show that maths is more than problem statements on paper, be it through articles exploring areas of maths beyond what we learn in class, or engaging puzzles and games that you can try out with your friends.

'Mathematica' is the Latin term for mathematics. Before you say that this is an awfully uninventive name for a maths publication, part of our inspiration actually comes from the famous work *Principia Mathematica* by Bertrand Russell and Alfred North Whitehead, a book on the foundations of maths and logic!

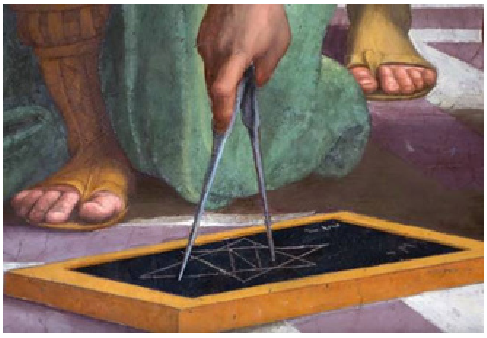
We find that maths tends to be even more fun when enjoyed together, and some parts of our publication are meant to be interactive: if you have any solutions, conjectures or queries, contact us and we'd be *x-cited* to discuss! We are also open to submissions for our next issue. Send us your favourite puzzles, games, fun facts, thoughts on a maths topic, memes, or even overheard quotes from maths class.

We hope you enjoy reading this issue!

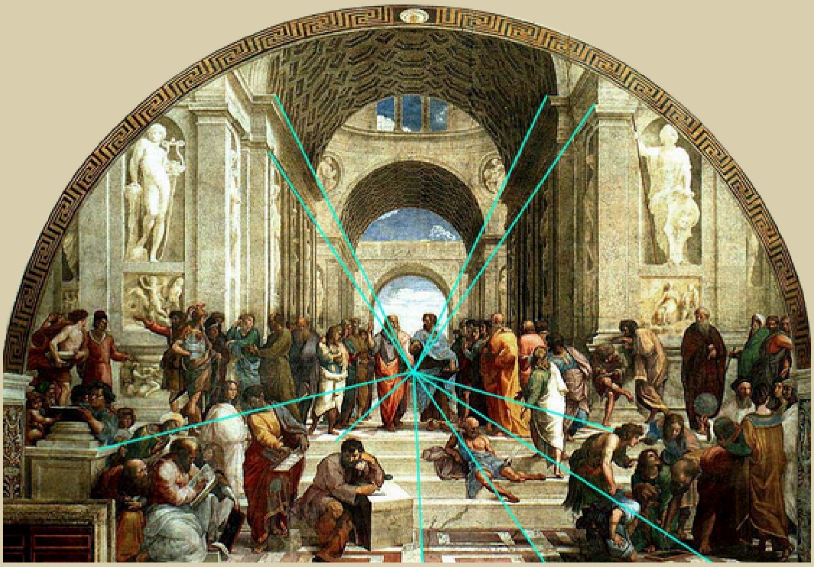
Judy & Cice

Finding Mathematics In Art

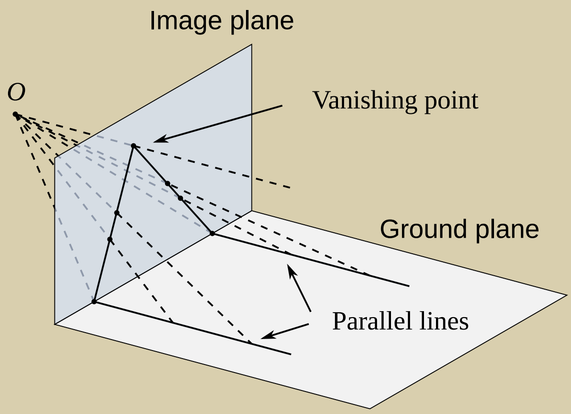
Take a look at the famous Renaissance fresco *The School of Athens* painted by Raphael: Indeed, before finding mathematics in art, we have found some mathematicians in art - on the left, we see Pythagoras scribbling away in a book, and on the right, Euclid is showing his students a geometric construction using his compass.



What is Euclid drawing?



The problem of creating a realistic 2-dimensional image of the 3-dimensional world had troubled artists for a long time. It was during the Italian Renaissance that artists began to use and develop the technique of geometric perspective, and *The School of Athens* is one of the most consummate examples showcasing an accurate and adroit use of this technique. Lines in the architecture—the arches, the tiled floor, the rafters— all converge at a single vanishing point (located between the central figures of Plato and Aristotle), successfully creating the illusion of depth.

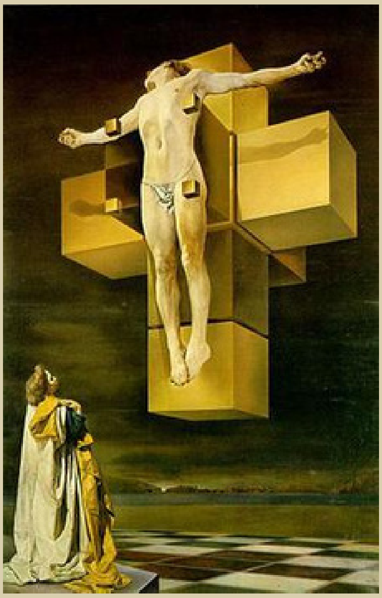


Mathematically, the process of depicting a 3-dimensional object on a 2-dimensional plane is known as projection. In the type of projection that Raphael uses, lines which are parallel in 3D space appear to meet at a point on a 2D plane.

Raphael's clever mathematical composition in this painting is, in its own way, a fitting tribute to the subject matter - the great ancient Greek thinkers. In fact, the ancient Greeks themselves also used mathematical ideas in their art. For example, the sculptor Polykleitos created *Doryphoros* (*The Spear Bearer*) as a demonstration of perfect bodily proportion, and strikingly, the ratios of the man's fingers, hands, and limbs are either the golden ratio or √2.

Centuries later, artists would begin to challenge traditional rules of space and realism. Yet even in modern art, mathematical ideas remained a powerful tool.

Take Salvador Dalí's *Corpus Hypercubus*, a surreal reimagining of the Crucifixion. Rather than placing Christ on a standard wooden cross, Dalí uses a tesseract net—a 3-dimensional unfolding of a 4-dimensional cube into eight cubes, which is analogous to the unfolding of a cube into six squares. The tesseract transcends 3-dimensional human perception, and we can only attempt to grasp it through lower-dimensional analogies—in this way, it becomes a powerful metaphor for the limits of our understanding, whether of space, divinity, or reality itself.



Mathematical structure can even be found in seemingly chaotic abstract art. Abstract expressionist Jackson Pollock is famous for his paint-splattered canvases, which look like randomness incarnate. But in the 1990s, Benoît Mandelbrot, the pioneer of fractal geometry, analysed Pollock's work and discovered something astonishing: the paintings followed fractal patterns. Pollock, through his signature paint-dripping process and motions around the canvas, created fractals similar to those found in nature's generational processes. As he developed his technique, the fractal dimension of his artworks increased, showing growing complexity and control.

Find the Pollock!

(starting top left and going clockwise): a bush, vegetation, seaweed, trees, spiders web, Pollock!

A Brief History of Mathematical Mistakes

Even the brightest minds (and expensive machines) can mess up. From legendary unsolved problems to space-age slip-ups, mathematical mistakes can be as enlightening as they are entertaining.

One famous instance happened in 1999, when NASA lost a Mars orbiter because one engineering team used imperial units while another used metric. The $125 million Mars Climate Orbiter approached Mars too low and burned up due to this unit mix-up . In other words, someone essentially failed to convert inches to centimetres – with spectacularly disastrous results. As one NASA engineer noted later, this “units thing” became a textbook cautionary tale for students (indeed, always double-check your units in science class!).

Not all mathematical misadventures involve rockets. An ancient problem known as “squaring the circle” puzzled mathematicians for 2,000 years. The challenge: given a circle, construct a square of exactly the same area using only a compass and straightedge. Many people (pros and amateurs alike) claimed to solve it and published false proofs. In fact, in 1882 Ferdinand von Lindemann proved the task was actually impossible – π (pi) turned out to be a special type of number that can’t be obtained with those tools . This didn’t stop a bold attempt in 1897 to legislate a value of π by law! The Indiana state legislature nearly passed a bill declaring π = 3.2, based on someone’s dodgy proof . It sounds like a joke, but it was one vote away from making an Indiana Pi. Thankfully, a professor visiting the statehouse stepped in and convinced the senators to shelve the idea, narrowly avoiding a legal math fiasco .

Finally, even harmless foot traffic can outsmart the best structural engineers. When London's Millennium Footbridge opened in June 2000, the very first crossings set off a slight sway of around 70 mm. As pedestrians felt that movement, they instinctively adjusted their footsteps to stay balanced—and in doing so, they fell into step with each other. Each new group of in-step walkers then amplified the wobble, which drew more people into the rhythm, and so on in a vicious feedback loop until the entire span started swinging uncontrollably. Engineers promptly closed the bridge, added simple dampers to absorb those rhythmic footsteps, and learned a vital lesson—that you must model not just wind or weight, but how people actually move.

These blunders remind us that getting things wrong is an essential part of getting better at maths. Each mistake, from a crashed spacecraft to a quirk in a proof, teaches us something new – even if it's just to admit your mistakes or check your working.

Further Reading:

- Lisa Grossman, "Metric Math Mistake Muffed Mars Meteorology Mission" (WIRED) – The full story of NASA's unit mix-up .
- Jack Murtagh, "Indiana's 1897 Pi Bill" (Scientific American) – How an attempt to square the circle almost changed π by law .
- Wikipedia: "London Millennium Footbridge"
- Heidelberg Laureate Forum, "Why Dividing by Zero Is a Terrible Idea" – Explains classic false proofs like 1 = 2 and the errors behind them .
- Humble Pi (book) by Matt Parker

The Divergence of the Harmonic Series

The harmonic series is the infinite series formed by summing all positive unit fractions, namely,

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots.$$

This series is one of the most well-known series in maths, partly because of the numerous applications that it has. One may easily see its connection with music through the name ‘harmonic’: when we pluck a string at positions corresponding to the unit fractions in the sum, we obtain overtones, also known as harmonics.

A key property of the harmonic series is that it diverges, i.e., the infinite sequence of its partial sums does not have a finite limit. An elegant and concise proof of its divergence is as follows:

Assume for contradiction that the series converges to some H. Then

$$\begin{aligned} H &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \cdots \\ &\geq 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{6} + \frac{1}{6} + \frac{1}{8} + \frac{1}{8} + \cdots \\ H &\geq 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \\ H &\geq \frac{1}{2} + H, \end{aligned}$$

giving us a contradiction.

Consider an ant that starts to crawl along a 1 km long rubber band at a speed of 1 cm per second (relative to the rubber it is crawling on). At the same time, the band starts to stretch uniformly at a constant rate of 1 km per second, so that after 1 second it is 2 km long, after 2 seconds it is 3 km long, etc. Will the ant ever reach the end of the rubber band?

We see that

- in the 1st second, the ant crawls over $\dfrac{1}{10^5}$ of the band.
- in the 2nd second, as the band stretched, the ant crawls over $\dfrac{1}{2\times10^5}$ of it, and so on.

Hmm... Do these fractions ring a bell? After a long, long time, the ant will cover

$$\frac{1}{10^5}+\frac{1}{2\times10^5}+\frac{1}{3\times10^5}+\cdots=\frac{1}{10^5}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots\right)$$

of the band. We know that the series in the bracket diverges, so at some point, our expression will exceed 1, which is when our ant has reached the end of the band!

Game Theory in Hamilton's Duel: Don't Throw Away Your Shot?

In Lin-Manuel Miranda's *Hamilton*, the infamous duel between Alexander Hamilton and Aaron Burr isn't just a dramatic plot point—it's a great case study in game theory, the mathematics of strategic decision-making. Two opponents, pistols raised at dawn, each thinking: 'What should I do to survive, and what do I expect my opponent to do?' It's a deadly serious example of the kind of dilemmas game theory loves to explore. Apparently, Hamilton decided to "delope," deliberately firing into the sky—essentially saying: "I'm not going to shoot you; let's not kill each other today." But Burr didn't trust the gesture and shot back.

So, can we analyse this scene with game theory? It is likely that Hamilton and Burr viewed the duel under different game theoretic models. To see this, let's break the outcomes down into a clear payoff matrix.

	Burr delopes	**Burr shoots**
Hamilton delopes	R: the world was wide enough	S: Hamilton dies
Hamilton shoots	T: Hamilton lives	P: mutual destruction

(Subtly different from the more famous Prisoner's Dilemma, which requires that T > R > P > S, the classic game of Chicken ranks T > R > S > P, and Stag Hunt ranks R > T ≥ P > S.)

Hamilton likely wanted to treat the duel like a Stag-Hunt, as he advised his son in *Blow Us All Away*— "When the time comes, fire your weapon in the air... He'll follow suit if he's truly a man of honour." Likewise here, he invites Burr to a shared gesture of restraint, probably expecting that he will reciprocate the sentiment as he later sings, "if I throw away my shot, is this how you remember me?". However, Burr's vow— "this man will not make an orphan of my daughter" reveals that he is playing Chicken. Beacuse he plans to fire, Hamilton's strategy is strictly dominated.

Unlike Prisoner's Dilemma and Stag Hunt, under Chicken no one has a "always-shoot" or "always-delope" dominant strategy. Instead, once Hamilton chose to delope, Burr's best response was to shoot, which is exactly what happened.

Since the two shots weren't completely simultaneous, we can also see other factors at play. Hamilton fires first—his sky-shot is a costly signal, sacrificing any chance to harm Burr in proof of peace. Burr's instant return shot shows imperfect information and pessimistic updating: he can't tell a deliberate miss from a misfire, so he fires to be safe. A sequential-move analysis via backward induction then predicts Hamilton would never have a credible delope at all. Finally, risk dominance seals the verdict: when both fear being the lone cooperator, shooting first becomes the de facto rational play.

Modern researchers are still modelling historical duels with game theory to understand why people dueled at all-and the truth is, real duels are often far more complex. They followed certain unwritten rules, could have multiple rounds, and social contexts of honour and reputation have to be taken into account. But for now, let's just enjoy the music.

Further Reading:

- Wikipedia: Burr–Hamilton Duel – Detailed history and various theories about what happened and why .
- Scholarly Article: "Lethality and Deterrence in Affairs of Honor" by P. Shea et al. – A game-theoretic analysis of why dueling persisted .

Sum Up: Maths Soc Activities This Spring

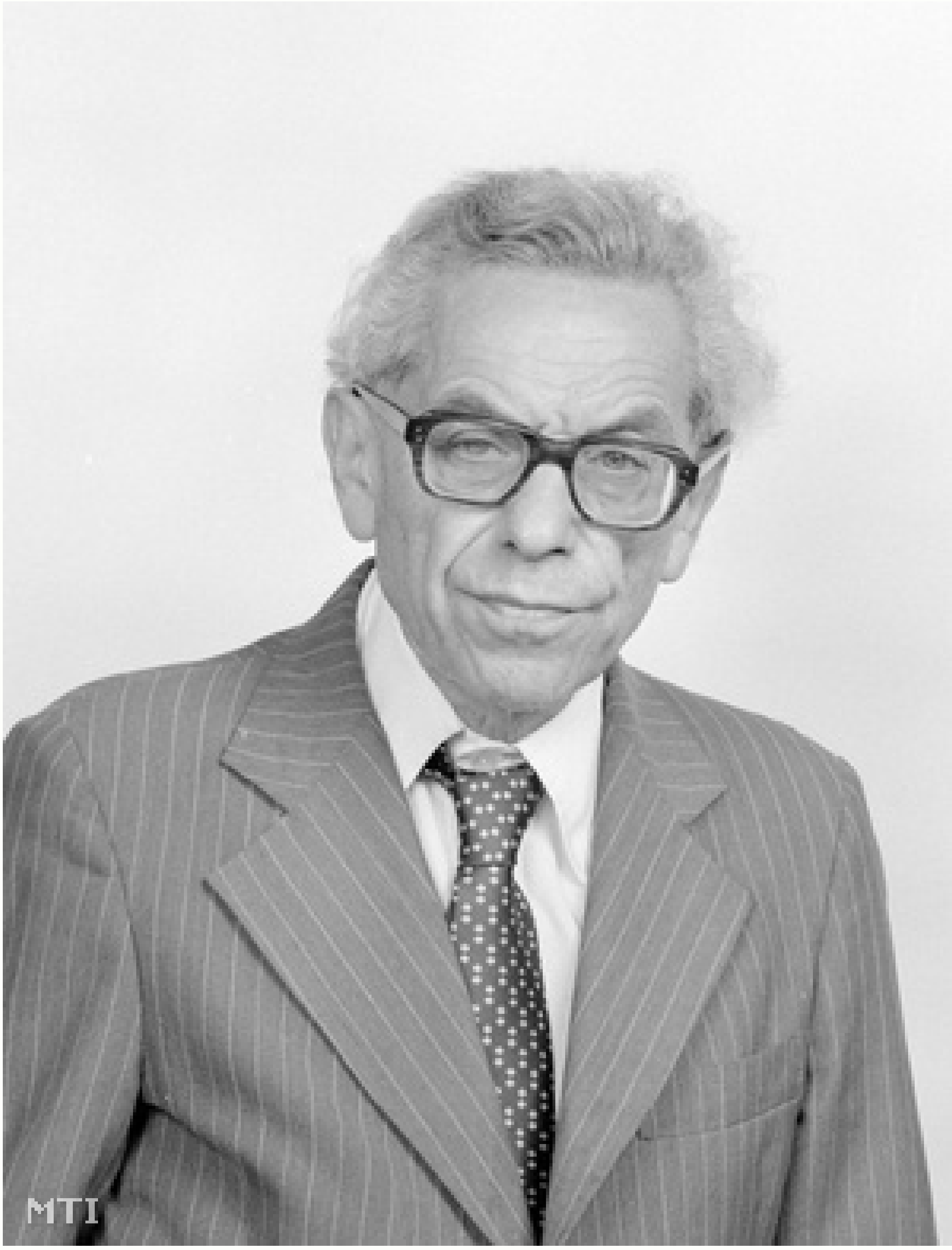
Spring term saw a busy schedule in Maths Society's activities, ranging from competition preparation, presentations, to Pi recitations.

To kick things off, we looked at some past AMC8 and AIME questions in the first session in preparation for the American maths (or should I say, math) competitions. This was followed by a Chinese New Year themed problem-solving session led by Judy, featuring Lo Shu magic squares, Chinese Zodiac probabilities, and firecracker sequences. Some of us also enjoyed learning how to perform basic arithmetic using Chinese numbers. In addition, Cice led a geometry presentation on the Fermat point, which is the point in a triangle that minimizes the sum of the distances from that point to the three vertices.

The highlight of the term was all the Pi day fun. The Guess the Circumference challenge proved to be popular, receiving more than 40 entries. A variety of creative approaches were used for the estimation, ranging from walking around the circle to more mathematical methods featuring geometric constructions. The runners-up are Mia (UIV) and Lara (UIV). The winner was Mrs Compton's LVI Further Maths Tuesday class, whose estimate was within an impressive 2cm of the actual answer. Everyone who participated had lots of fun in this outdoor activity.

Maths Society also featured a stimulating proof of Pi's irrationality, presented by Mr Graham, and an exciting Pi recital contest challenged pupils to recite as many digits of Pi as possible from memory. A huge congratulations to Freya (UIII) for reciting 110 digits and Yolanda (LV) for an impressive 260 digits!

Spotlight: Paul Erdős



Meet Paul Erdős, one of the most prolific and eccentric mathematicians ever to grace the planet. Erdős (pronounced “Air-dish”) published over 1,500 papers in his lifetime – more than any other mathematician in history – and had more than 500 collaborators . But more than that— he was a real character. Nicknamed “The Oddball’s Oddball” by Time magazine, he truly earned that title in the most delightful way.

Erdős was born in 1913 in Hungary and was a child prodigy. By the age of four, he could ask you your birthdate and instantly tell you exactly how many seconds you had been alive! He was a whiz with numbers from the start. Fast-forward to adulthood, and Erdős became a kind of mathematical nomad. He would show up unannounced on a colleague's doorstep, mumble "My brain is open," and proceed to collaborate on maths problems day and night. Then, after exhausting the host (and clean out of coffee), he'd move on to the next place, often asking his host to recommend another mathematician he could visit next. He travelled the globe in this fashion for decades. One of his catchphrases was, "Another roof, another proof," reflecting his habit of hopping from house to house, solving a problem or two at each stop. Erdős had minimal possessions – all his worldly goods fit into one suitcase – and he famously couldn't be bothered with everyday tasks. Friends joked that at age 21 he buttered his own bread for the first time. Cooking, cleaning, doing laundry… these mundanities were often kindly handled by his hosts, so Erdős could focus solely on equations. In return, you'd likely end up co-authoring a research paper with him (and probably losing some sleep – he was known to work on math 19 hours a day!).

Despite (or because of) his quirks, Erdős made immense contributions to fields like number theory, combinatorics, and graph theory. One of Erdős's biggest legacies is the concept of the Erdős number. This is a playful way to measure collaborative distance in academic research. Erdős himself has an Erdős number of 0. Anyone who co-authored a paper with Erdős has an Erdős number of 1. If you haven't written with Erdős but you wrote a paper with someone who wrote with Erdős, your Erdős number is 2, and so on . It's like the Kevin Bacon game of mathematics. Having a small Erdős number is a badge of honor in the maths community – a sign that you're "collaboratively close" to the great man himself. Because Erdős collaborated so widely, many thousands of researchers have low Erdős numbers. It's said that virtually every mathematician is at most 5 or 6 steps away from Erdős in that network.

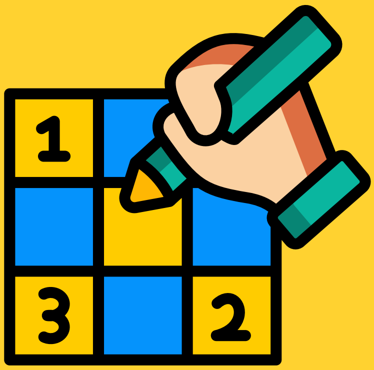
Erdős also had a unique personal vocabulary. He called children "epsilons" (ε is often used in maths to denote a small quantity) , and when someone took a break from mathematics, he would say that they had "died".

Zero was “Line-O”, and he referred to God (despite being an atheist) as the “Supreme Fascist” who kept the most elegant proofs hidden in a book – The Book – until a mathematician earned the right to see them . When Erdős found an exceptionally beautiful proof, he’d exclaim, “This one’s from The Book!”

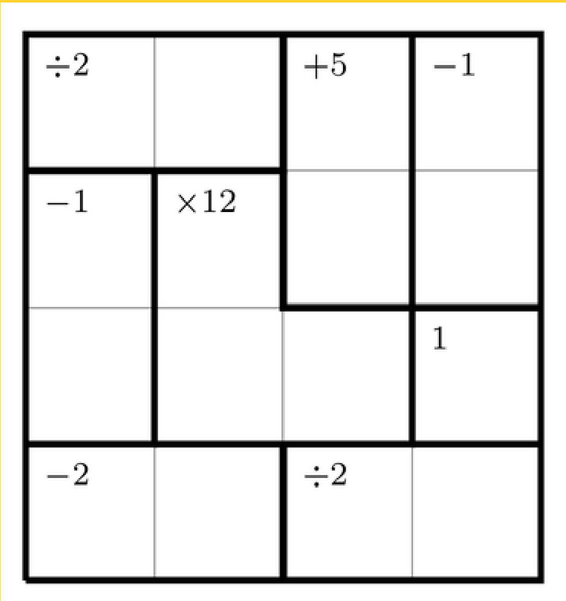
For all his oddities, Paul Erdős was beloved in the mathematics world. He had a childlike simplicity, immense generosity (he often gave away prize money or stipends to help students), and an infectious passion for numbers. He never married and had no family of his own; in a sense, the global community of mathematicians was his family. When he wasn’t working on maths (which was rare), he enjoyed silly jokes and puzzles. In one famous incident, a friend bet Erdős he couldn’t quit caffeine for a month. Erdős succeeded, but then quipped, “You’ve set back mathematics by a month!” and promptly went back to his strong coffee habit.

Paul Erdős passed away in 1996 – fittingly, while attending a maths conference in Warsaw, doing what he loved until the very end . Erdős showed that mathematics can be a profoundly social endeavor, full of camaraderie, fun, and yes, a bit of craziness. I think we can take inspiration from his life in knowing that maths is not just about lone geniuses working in isolation – it can be a collaborative adventure across continents. And if you ever find yourself working on maths with friends into unholy hours of the night, you can tell your housemistresses that you’re living the Erdős spirit!

Fun Puzzles

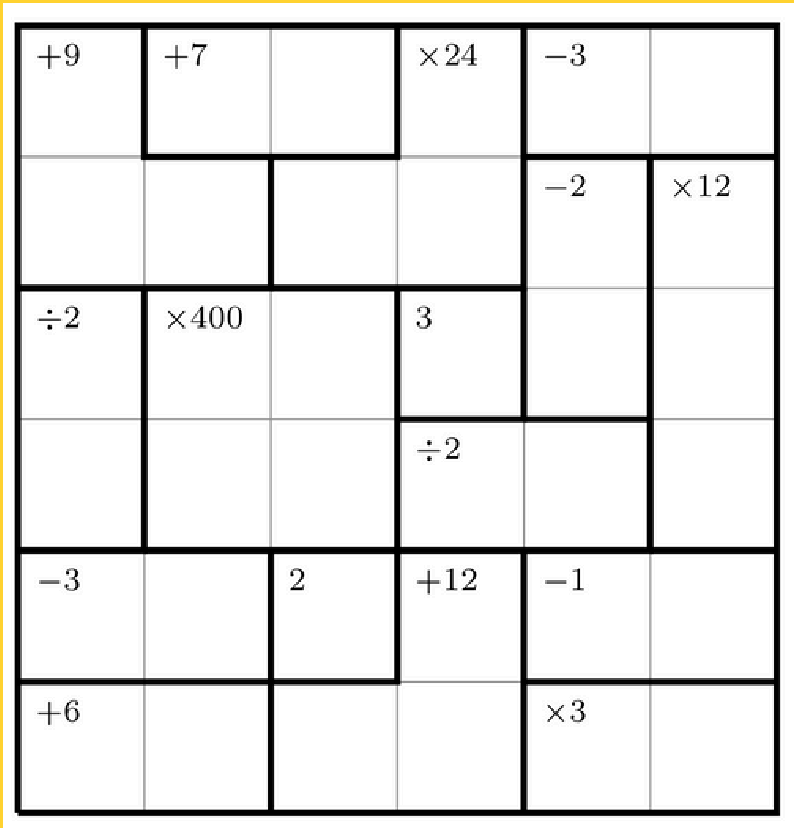


Ken Ken



- Fill in each square with a single number. In a 4 ×4 grid, use the numbers 1–4. In a 6 ×6 grid, use 1–6.
- Do not repeat numbers in any individual row or column. E.g., for the 4 × 4 grid, each column and row should contain the numbers 1,2,3, and 4.
- The numbers in each cage (bold-outlined rectangles) must combine to produce the target number indicated in the cage using the noted math operation. A number may be repeated within a cage as long as it's not in the same row or column.

For the 6x6 grid, there are 4 solutions. Can you find them all?



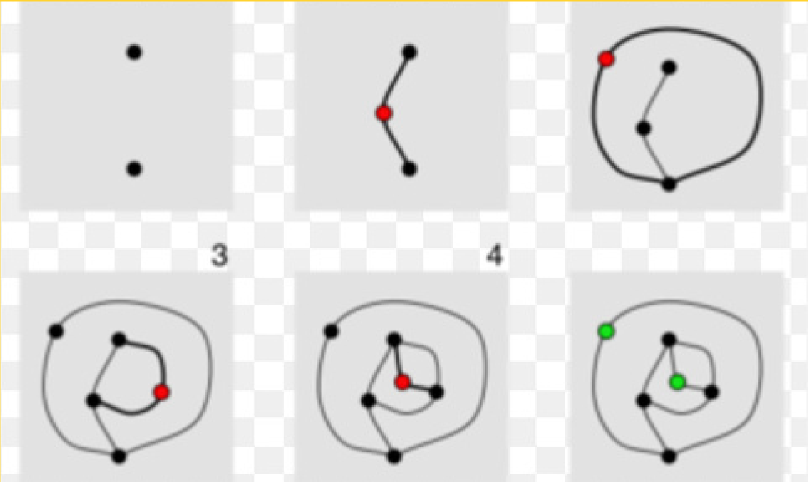
DID YOU KNOW?

KenKen was invented in 2003 by a Japanese maths teacher.
'Ken' means wisdom in Japanese, so 'KenKen' literally means wisdom squared!

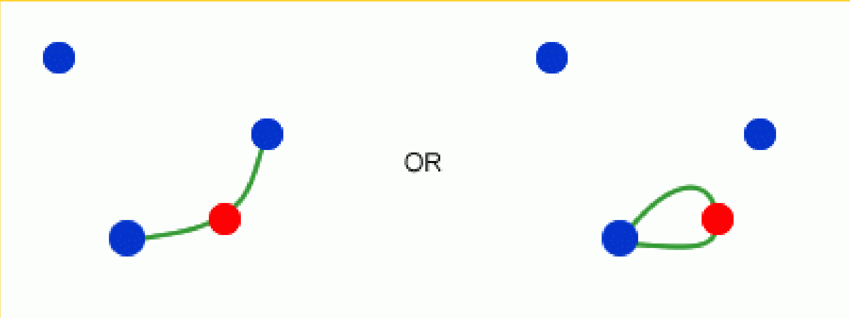
Sprouts

Sprouts is a 2-player game that you can play with pencil and paper. Start by placing a few points randomly on a page. I'd recommend three points to start.

Each player takes turns drawing a line, or curve, to connect one point to another (or back to itself). Then, somewhere along that line, you place a new point, which breaks the line you drew into two smaller segments. The next player then draws a new line to connect two dots and places a new dot somewhere along their line. **You lose when you cannot draw a new line.**



Example of how a game can play out with two starting dots



Potential starting moves from three dots

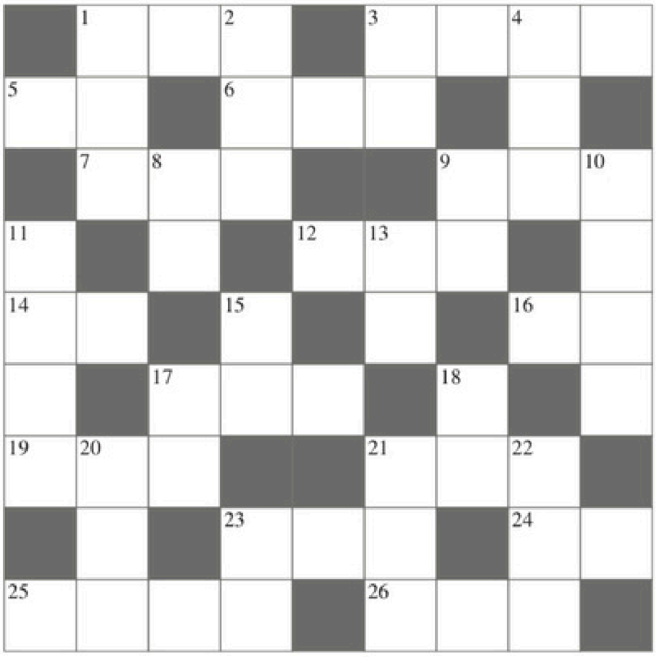
A couple of important rules:

- Once a dot has three connections, it is done. You cannot connect a fourth line segment to that point.
- Lines can curve anywhere you want, but they can never cross each other.
- Lines can also loop back and connect a dot to itself, and this counts as two connections to that dot

A few things to think about:

- Does the person who goes first or second tend to win?
- Can you explain any winning tactics?
- The game must end after a limited number of moves. Explain why.

Try This Crossnumber!



Across

1. The remainder when 11 Down is divided by 19 Across (3)

3. The mean of 25 Across and 10 Down (4)

5. The product of 16 Across and the difference between 1 Down and 20 Down (2)

6. Three less than 4 Down (3)

7. The number of digits in $25^{55} \times 1024^{11}$ (3)

9. A power of 2 (3)

12. A cube (3)

14. A prime number that is the sum of the first few consecutive prime numbers (2)

16. A factor of 10 Down that is a multiple of a square greater than 1 (2)

17. A multiple of 18 Down (3)

19. The square of a prime number, with digits in descending order (3)

21. A Fibonacci number where all adjacent digits differ by one (3)

23. 23 Down increased by 1130% (3)

24. A number with an odd number of factors (2)

25. The product of the first five prime numbers (4)

26. $4x + 14$ where $x = \dfrac{10\,\text{Down}}{22} - 15\,\text{Down}$ (3)

Did you know?

1. Every time you see the little padlock icon in your browser's address bar (HTTPS), your data is being kept safe by prime numbers and modular arithmetic—thanks to RSA encryption.
2. When Google Maps finds the quickest route, it's using graph theory and Dijkstra's algorithm to compute the shortest path through a network of roads.
3. Your Netflix recommendations spring from linear algebra and probability: massive user-item matrices are factorised to predict what you'll love next.

Great Minds Think Alike

"A mathematician is a machine for turning coffee into theorems."
~Paul Erdős
"In mathematics you don't understand things. You just get used to them."
~John von Neumann
"Maths is not even a real subject."
~Your Heads of Maths Society

~~Trivial~~ Trivia Problems

1. Which mathematician was the first to uncover the numerical ratios behind musical harmony (showing that an octave is a 2:1 string-length ratio and a perfect fifth is 3:2)?
2. How many times do the hour and minute hands on an analogue clock overlap in a 24-hour period?
3. I am the smallest positive integer that can be expressed as the sum of two positive cubes in two different ways. What number am I?
4. Hope everyone had a lovely Easter! The Easter Bunny had 5 chocolate eggs, each painted a different colour (Red, Blue, Green, Yellow, Purple). There are 5 doors, each painted in one of those colours. He wanted to deliver one egg to each door, but no egg can go to the door of its matching colour. In how many ways could he have deliver the eggs?
5. Prove that the list of decimal numbers between 0 and 1 is never-ending.

Meme collection

If you don't understand a meme, look forward to unlocking it in your maths journey!