# Modeling of waves and ship motion

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## Introduction

This project is part of the collaboration between the competence center LINK-SIC and UMS Skeldar. The final aim of this project is to better understand the motion of ships caused by waves in order to facilitate the landing of multirotor aerial vehicles on a moving ship. The achieved mathematical models of waves and ship motion together with the resulting simulations are presented.

## Wave model

Waves are modeled as superpositions of distinct harmonic waves. Depending on the type of sea that it is desired to simulate, different *power density spectra* are used. For this project, the *Bretschneider* spectrum was chosen, which is known to describe well the waves of the North Atlantic ocean. The spectrum $S(\omega)$ is defined as

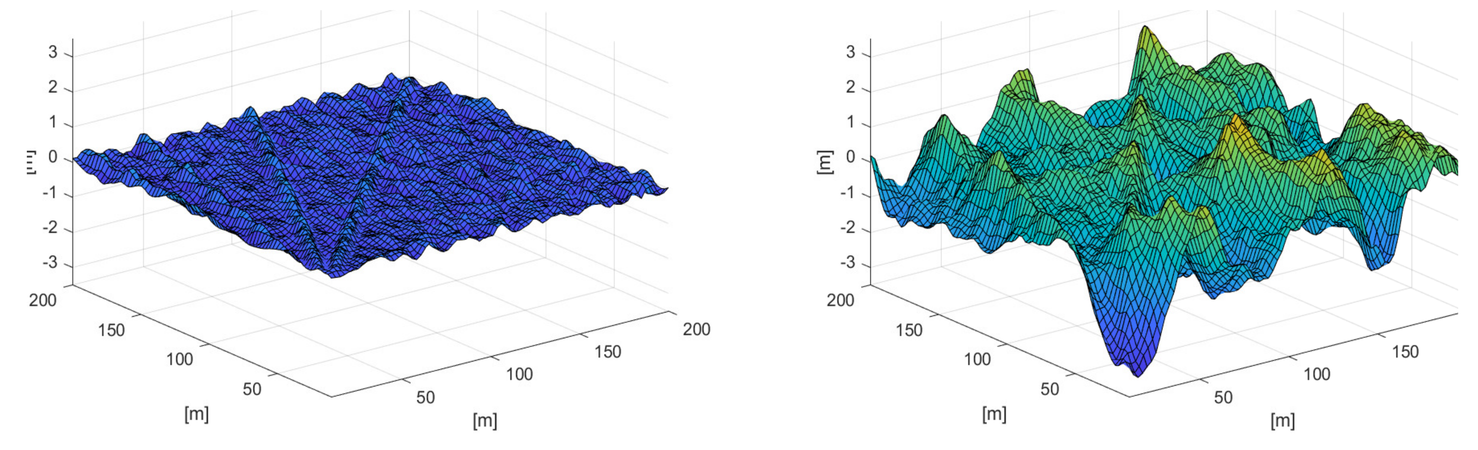
$$S(w) = \frac{0.78}{w^{-5}} \cdot e^{\frac{-B}{w^4}},$$

where $B = \frac{3.11}{H_{1/3}^2}$ and $H_{1/3}$ is a statistical measure of the wave heights depending on the *sea state*. Sea state is a metric of wave heights on a scale from 0 to 9, where 0 is a glassy sea and 9 a phenomenally agitated sea.

The wave elevation in time $t$ for a coordinate $(x, y)$ is given by

$$\xi(x, y, t) = \sum_{k=1}^{N} \sum_{i=1}^{M} A_k \cos\left(\frac{w^2}{g}(x\cos(\alpha) + y\sin(\alpha)) - w_k t + \epsilon_{i,k}\right),$$

where the wave amplitude is $A_k = \sqrt{2S_m(w_k, \alpha)\Delta w \Delta\mu}$. Furthermore, $\alpha = \mu_i - \beta$ which is the angle difference between the $k^{th}$ wave component and the main wave. Simulated waves of sea states 2 and 4 look as follows.



## Ship motion model

The suggested model is $M_{RB}\dot{v} + (C_{RB} + D_v)v = \tau$, which is a simplified version of the model suggested by Thor I. Fossen, where

| | |
|---|---|
| $M_{RB}$ | Mass and inertia rigid body matrix; |
| $v$ | Body-fixed velocities and angular velocities; |
| $C_{RB}$ | Coriolis rigid body matrix; |
| $D_v$ | Linear viscous damping; |
| $\tau$ | Forces and moments, including restoring forces. |

The state space model of the ship contains 12 states, 6 for translation and 6 for rotation. Furthermore, two coordinate systems are defined, one ship-fixed $(u, v, w)$ and one earth-fixed $(x, y, z)$. See the motion model:

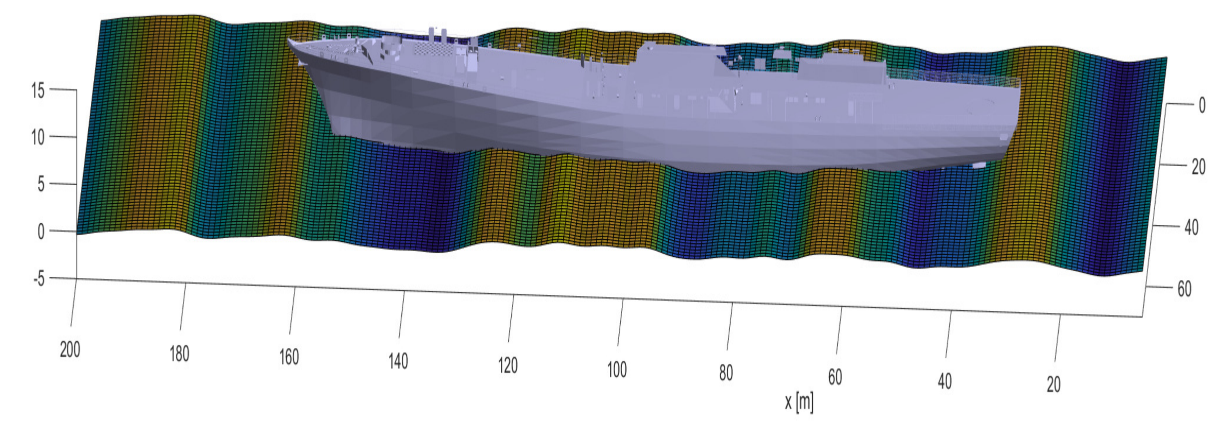
$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_u \\ \dot{v}_v \\ \dot{v}_w \end{pmatrix} = \begin{pmatrix} \mathbf{R}^T \begin{pmatrix} v_u \\ v_v \\ v_w \end{pmatrix} \\ -\frac{c_u}{m}v_u + \frac{1}{m}F_u \\ -\frac{c_v}{m}v_v + \frac{1}{m}F_v \\ -\frac{c_w}{m}v_w + \frac{1}{m}F_w \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{\omega}_\phi \\ \dot{\omega}_\theta \\ \dot{\omega}_\psi \end{pmatrix} = \begin{pmatrix} \mathbf{T} \begin{pmatrix} \omega_\phi \\ \omega_\theta \\ \omega_\psi \end{pmatrix} \\ -\frac{B_u}{I_u}\omega_\phi + \frac{1}{I_u}\tau_u \\ -\frac{B_v}{I_v}\omega_\theta + \frac{1}{I_v}\tau_v \\ -\frac{B_w}{I_w}\omega_\psi + \frac{1}{I_w}\tau_w \end{pmatrix}.$$

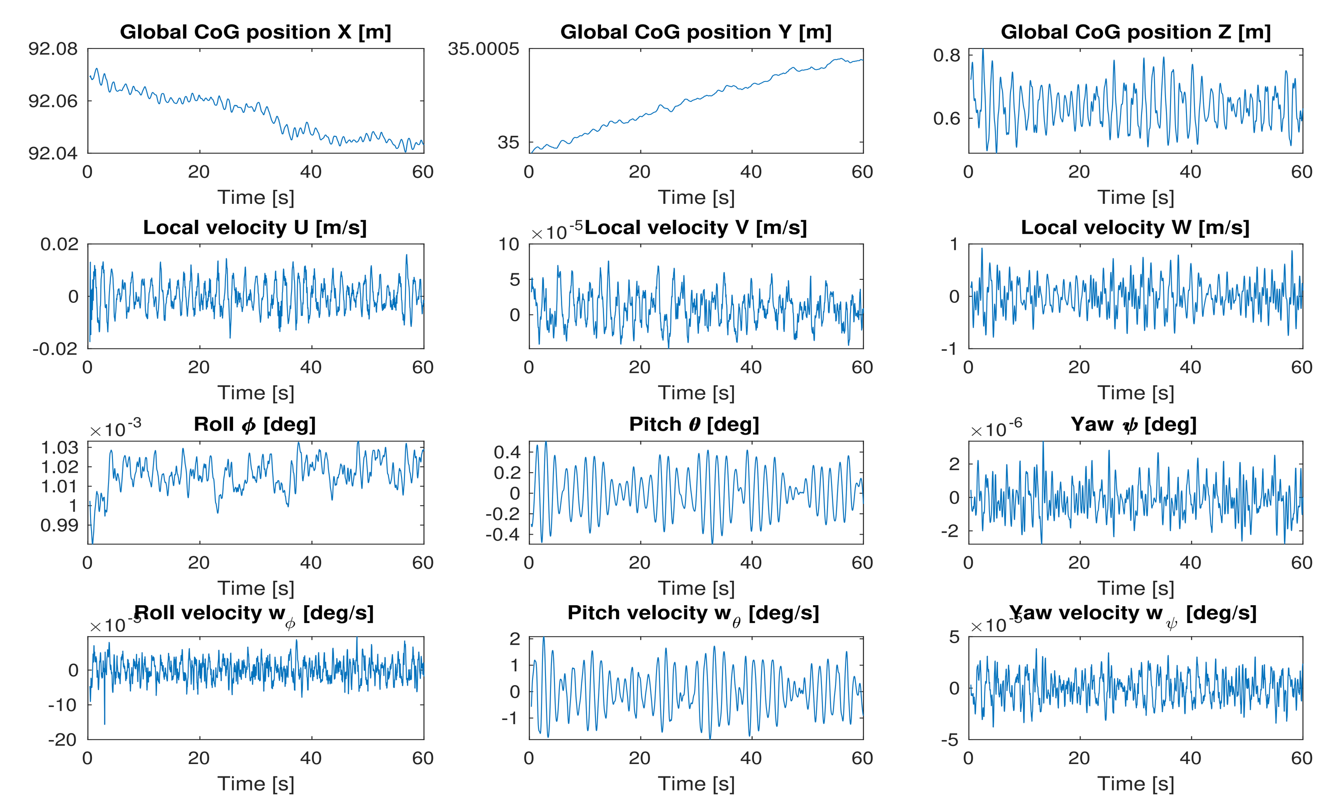
In the equations above, the matrix $\mathbf{R}$ transforms a vector from earth-fixed to ship-fixed coordinates, while $\mathbf{T}$ transforms rotational velocities into angle derivatives.

The forces taken into account are the forces from the hydrostatic pressure of the sea, the viscous damping force, the weight of the ship, and the Coriolis force. Therefore, forces such as the dynamic pressure from the sea, added mass and wind forces are ignored.

## Results

The end result of the research project is a visual time simulation of the motion of a ship when moving on waves of a given sea state. Below is a snapshot of the ship when on waves of sea state 3, and a plot of the 12 states through time.




For more details, access the links of the simulation videos and the code.

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