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Order-up-to-level inventory optimization model using time-series demand forecasting with ensemble deep learning

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ABSTRACT

Inventory control aims to meet customer demands at a given service level while minimizing cost. As a result of market volatility, customer demand is generally changing, and ignoring this uncertainty could lead to under or over-estimation of inventories resulting in shortages or inefficiencies. Inventory managers need batch ordering such that the ordered items arrive before the depletion of stocks due to the lead time between the ordering point and delivery. Therefore, to meet demand while optimizing the cost of the inventory system, firms must forecast future demands to address ordering uncertainties. Traditionally, it was challenging to predict such uncertainties with high accuracy. The availability of high volumes of historical data and big data analytics have made it easier to overcome such a challenge. This study aims to predict future demand in the case of an online retail industry using ensemble deep learning-based forecasting methods with a comparison of their performance. Compared to single-model learning, ensemble learning could improve the accuracy of predictions by combining the best performance of each model. Also, the advantages of deep learning and ensemble learning are combined in ensemble deep learning models, allowing the final model to be more generalizable. Finally, safety stocks are estimated using the forecasted demand distribution, optimizing the inventory system under a cycle service level objective.

1. Introduction

Inventory control is associated with decisions on ordering time and quantities of multiple stock-keeping units (SKUs) as well as related materials and parts. Inventory control aims to ensure satisfying customer demand at a determined service level. Reviewing the past literature shows that, by assuming a deterministic demand, most studies seek to minimize the sum of expected ordering and inventory carrying costs [18]. However, in reality, due to the volatility of markets, customer demand could become highly uncertain [19]. Companies are required to establish forecasts of future demand in order to foresee capabilities to satisfy an uncertain demand and optimize costs of the supporting inventory system. In addition, due to the current lead time between the ordering point and the delivery, which necessitates batch-ordering, companies need to order items ahead of demand.

Nowadays, enterprises benefit from the availability of big data and the use of predictive analytics with a growing number of applications reported in the literature [11]. Although achieving accurate demand prediction has been challenging in the presence of market uncertainties, the availability of a large volume of historical data and the use of big data analytics helped improve the accuracy of demand forecasting [47]. Such forecasts contribute to improving customer service and reducing costs resulted from supply-demand mismatches [17]. Despite such an aim, however, there is no clear path from data to a good decision, and as such, failure in determining appropriate forecasting methods can lead to suboptimal solutions. In inventory optimization, to take

Abbreviations: ARIMA, auto-regressive integrated moving average; BMA, Bayesian model averaging; CNN, convolutional neural network; CSL, cycle service level; d, Forecasted demand; DNNs, deep neural networks; EOQ, economic order quantity; ERM, empirical risk minimization; GARCH, generalized autoregressive conditionally heteroscedastic; KDE, kernel density estimators; KO, kernel-weights optimization; KNN, k-nearest neighbor; IID, independent and identically distributed; L, Lead time; LSTM, long short-term memory; MLP, multi-layer perceptron; MSE, mean squared error; MAE, mean absolute error; MAPE, mean absolute percentage error; MA, moving average; NN, neural network; OUTL, order-up-to level; r, Reorder point; R, Review interval; RMSE, root mean square error; q, Order quantity; S, Replenishment level; SAA, sample average approximation; SES, simple exponential smoothing; SKUs, stock-keeping units; SBA, syntetos–boylan approximation; ss, Safety stock; μ, Mean of forecasted demand; σ, Standard deviation of forecasted demand

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advantage of growing availability of data, appropriate methods and algorithms shall be adopted for a data-driven inventory management. This study examines how big data can be utilized to handle demand uncertainty while optimizing supply chain cost.

A wide range of research has been conducted on developing effective methods for demand forecasting [38]. Machine learning algorithms, such as K-nearest neighbor, Gaussian naive Bayes, and decision trees, are used to establish forecasts of future demand based on historical patterns of time-series data [46]. Deep learning approaches are also gaining significant attention, including recurrent neural networks (RNN), long short-term memory (LSTM), gated recurrent units (GRU), autoencoders, and convolutional neural networks (CNN). These approaches achieve higher accuracy in predictions [27]. However, such deep-learning models cannot maintain high forecasting accuracy and robustness in dealing with applications that are subject to dynamic environments.

Ensemble learning is emerging as an approach that can address the above-mentioned challenge by benefiting from combining several models, and benefit from their collective superiorities, to improve forecasting performance [51]. Zhou et al., [54] have demonstrated that combining partial basic predictors can yield comparable or even superior generalization performance compared to combining all predictors simultaneously. These findings underscore the importance of adopting effective ensemble forecasting methods to achieve more accurate demand forecasting and inventory management solutions. Through the aggregation of diverse predictions, ensemble learning can mitigate individual model biases and capture a wider range of patterns, ultimately enhancing the reliability and effectiveness of demand forecasting and inventory management strategies. Reviewing the literature, as reported in the next section, reveals a scarcity of research in utilization of ensemble deep-learning methods for demand forecasting in supply chain management and inventory optimization stocks [46]. Moreover, the majority of ensemble deep learning models primarily concentrate on ensembling homogeneous deep neural networks (DNNs), while disregarding the benefits offered by heterogeneous DNNs due to their intricate architecture. This has created a compromise between forecast accuracy and model complexity, as noted by Zhang et al. [51].

This study advocates using heterogeneous DNNs in inventory optimization. In this sense, prediction algorithms of Multilayer Perceptron (MLP), CNN, and LSTM are investigated and combined in forming an ensemble learning approach that enhances the forecasting accuracy of multivariate time series data used for safety stock optimization achieving minimal total inventory cost. This research builds upon recent progress on DL-based forecasting approaches that 1) highlight the superior efficiency of LSTM over RNN in handling vanishing gradient issues [15], 2) superiority of LSTM and CNN in time-series and spatial features modeling, respectively, and 3) MLP capability in extracting global features [9]. Furthermore, the proposed ensemble approach leverages distinct features extracted from two real-world supply chain time series datasets by employing these different types of DNNs.

The rest of the paper is organized as follows: Section 2 presents a review of the literature as relates to demand forecasting and inventory optimization. Section 3 presents the proposed framework to incorporate an ensemble deep learning approach in demand forecasting for inventory optimization. Section 4 presents the results. Discussions and further analysis are presented in Section 5. Finally, paper concludes in Section 6, providing a summary of the proposed approach, key assumptions and limitations as well as directions for future research.

2. Literature review

The demand uncertainty can be handled using three categories of approaches. The first category of approaches assumes that demand distribution and its parameters are determined at the point of decision-making in each time period [4,18]. According to [37], this is not mostly the case in real-world applications. The decision-maker does not have prior knowledge about demand distribution and how the parameters could change over time (i.e. deterministic scenarios). The second category of methods in addressing the uncertainties for demand forecasting in inventory problems can be decomposed into two phases: (a) an estimation/forecasting phase and (b) an optimization phase [5,35,36]. In these studies, some distributional assumptions are initially made about the historical data, and then statistical methods are used to estimate the parameters. The estimated values for the parameters are then considered in establishing the optimal values of decision variables, such as order quantity and safety stock.

However, in the case of an existing specific noise in data or failure in determining the appropriate forecasting method, the solution of the optimization phase might be sub-optimal [29]. In the third category, the focus is on distributional or parametric assumptions related to the historical demand data that can be estimated using machine learning and deep learning-based methods [47]. These methods can be parametric or nonparametric. In this regard, the methods that directly use data without relying on an assumption of normality or any other distribution forms are referred to as nonparametric forecasting methods [18].

In this context, literature review studies related to the integration of forecasting and inventory policies are provided, with a particular focus on the most relevant and recent papers that employ machine learning forecasting methods. In time-series forecasting, a linear function of the past observations is established following autoregressive (AR), moving average (MA) or auto-regressive integrated moving average (ARIMA) methods. Recent advances in data analytics and artificial intelligence have provided the practitioners with use of machine learning and deep learning approaches in time-series forecasting [47]. Machine learning methods could evaluate the demand with or without assuming distribution or parameters. İfraz et al., [25] investigated demand forecasting of spare parts in bus fleets and compared various forecasting methods, including regression-based, rule-based, tree-based, and artificial neural networks. Their results indicated that the artificial neural network outperformed all others, providing the highest accuracy rate and the least deviation in demand forecasting. Swaminathan & Venkitasubramony, [42] reviewed forecasting techniques in predicting demand for fashion products focusing on advancements in artificial intelligence and machine learning methods considering various combinations of them.

The challenge of incorporating demand uncertainty in inventory models without simplifying assumptions about distribution was investigated by [44]. They questioned the widely used assumption of demand following a normal distribution is questionable. They suggested the use of a nonparametric kernel density approach for short lead times to fill this gap. In doing so, kernel density estimators (KDE), generalized autoregressive conditionally heteroscedastic (GARCH), and SES were considered in the order-up-to-level (OUTL) policy and newsvendor to estimate the safety stock. The authors also investigated the effects of sample size and demand distribution on safety stock. They concluded that as data-driven approaches, nonparametric methods perform less accurate when sample sizes are small, whereas parametric techniques perform well when the distribution is normal. Similarly, [45]) combined two quantile forecasts (KDE and CGARCH) to minimize the loss function and improve the empirical safety stock predictions using backorders to achieve a lower cost in their newsvendor model.

Another study [12] presents a data-driven approach, a double parallel feed-forward network, in determining stock levels for a newsvendor problem and its multiperiodic extension. Demand is assumed time correlated following a normal distribution with a mean of zero and an unknown variance. The proposed approach is shown to be outperforming a number of statistical forecasting methods such as Holt-Winters' triple exponential smoothing. The proposed method captures both stationary and nonstationary time series [12]. Babai, Dai, et al., [6] provided a comprehensive investigation on estimating demand forecast error under stochastic lead times. The findings highlighted the limitations of the classical demand forecasting approaches and emphasized the importance of considering demand autocorrelation and lead-time variability to enhance the accuracy when selecting a forecasting strategy for inventory control.

Ban & Rudin, [8] proposed a solution approach for a big data newsvendor problem using single-step machine-learning algorithms. Specifically, they presented two algorithms. The first was based on the empirical risk minimization (ERM) principle, with and without regularization; the second was based on kernel-weights optimization (KO). Furthermore, the authors showed several algorithm properties and tested them with empirical data in a newsvendor-type nurse staffing problem. A data-driven newsvendor problem to approximate demand for retailer's perishable items was proposed [24]. They used and compared the performance of a number of machine learning-based approaches including a gradient-boosted decision tree, a single-layer neural network, and a nonparametric sample average approximation (SAA) method. Similarly, the application of deep neural networks in identifying the solutions for a newsvendor problem has been investigated [33]. Various features of demand data were investigated in terms of their effect on the optimal order quantities per product. Then, the algorithms were extended for (r, Q) policy. The authors validated their results by comparing the proposed methodology with other nonparametric machine learning algorithms including empirical quantile, quantile regression, kernel regression, k-nearest neighbor (KNN), and random forest.

To deal with forecasting intermittent characteristics of demand, bootstrapping methods, WSS (for Willemain, Smart and Schwarz) [49], and VZ (for Viswanathan and Zhou) (C. [52]) was applied to identify lead time demand distribution parameters. Meanwhile, SES, syntetos–boylan approximation (SBA), Croston, and neural network (NN) methods were applied to estimate lead time demand and variance of lead time demand calculated through the MSE. The results were compared through an OUTL policy that operates under a cycle service level (CSL) objective. Babai et al. [7] proposed NN approach with good training and learning processes that could achieve higher inventory efficiency than the bootstrapping techniques. Inventory holding and back ordering volumes are compared to validate proposed NN approach [7]. Omar et al. [32] introduced a novel forecasting approach that leverages customer shopping basket data to predict both online and store sales. Their approach outperforms traditional benchmark methods, such as ARIMA, particularly for products with sporadic demand. Additionally, they demonstrated the advantages of joint forecasting and shared inventory in an omnichannel context, leading to reduced inventory shortages and improved service levels.

Recent studies have also highlighted the effectiveness of ensemble learning in improving demand forecasting and inventory management. Ensemble learning plays a crucial role in demand forecasting for inventory management by combining multiple forecasting models to improve accuracy and robustness. Zhang et al., [51] emphasize the advantage of ensemble learning in combining multiple models to enhance forecasting performance. By aggregating diverse predictions, ensemble learning can address individual model biases and capture a wider range of patterns, leading to more reliable and effective demand forecasting and inventory management strategies. Additionally, Zhou et al., [54] have demonstrated that combining predictors partially can yield comparable or even superior generalization performance compared to combining all predictors simultaneously. Yang et al., [50] explored the challenges and developments of ensemble deep learning in the era of deep learning, highlighting the need for more efficient methods to deploy ensemble learning in specific fields while reducing the associated time and space overheads. Mohammed & Kora [31] discussed the potential of integrating ideas from traditional ensemble learning into deep learning to overcome the challenge of tuning optimal hyper-parameters, emphasizing the benefits of cost-effective ensemble deep learning approaches. These findings underscore the importance of adopting effective ensemble forecasting methods to enhance the accuracy of inventory optimization solutions. Despite the above benefits, to the best of authors' knowledge, very limited research has been reported in the literature focusing on the use of ensemble deep learning methods for demand forecasting in supply chain management and inventory optimization stocks [46].

In ensemble learning, combining basic predictors is an essential task [16]. The simple average [53], weighted average [23], Bayesian model averaging (BMA) [39], and meta-learning methods [41] are some of the methods suggested to conduct such combinations. In particular, in many classification and regression tasks, the combinations using meta-learning methods using a stacking generalization technology have demonstrated satisfactory performance ([20,40]; Q. [48,51]). The study of Andrade & Cunha [3] proposed a methodology for disaggregated demand forecasting in the retail industry. The authors utilized XGBoost, a non-linear non-parametric ensemble-based model, and incorporated a structural change correction method to account for sudden changes in consumer behavior. Their approach demonstrated superior accuracy metrics, reduced stockouts and inventory costs, and a high degree of automation. Furthermore, most ensemble deep learning models focus on the ensemble of homogeneous DNNs and neglect the advantages of heterogeneous DNNs due to their rather complex architecture; reflecting a trade-off between forecast accuracy and model complexity [51].

2.1. Statement of novelty

Table 1 presents a summary of research in applications of machine learning methods in data-driven inventory optimization under demand uncertainty. Various forecasted variables are considered in these studies, including the amount of future demand [7,12], variance of lead time [6] and the order quantity [8,24,33]. The scarcity of papers included in the table, which specifically explore the application of machine learning methods in data-driven inventory optimization, highlights the relatively limited research efforts in this domain. In this regard, the current study (and its contribution) is also listed in Table 1 to provide a comparison with previous research. In this context, the key contributions of this research can be condensed as follows:

- Developing a data-driven inventory optimization process to predict daily demand and optimize stock levels for online retailers.
- Combining heterogeneous DNNs to capture diverse features and improve forecasting accuracy.
- Proposing an ensemble deep learning approach for multivariate time series forecasting, combining the strengths of MLP, LSTM, and 1D-CNN models to capture global, temporal, and local patterns in the data.
- Utilizing an OUTL (Order-Up-To-Level) policy for inventory management, aiming to avoid excessive inventories and minimize opportunity costs while considering demand variations.
- Evaluating the proposed approach using real-world supply chain time series data of electronic and sports supplies.
- Comparing the proposed ensemble deep learning model with individual base models (MLP, LSTM, and 1D-CNN) and showing its superior performance in terms of accuracy metrics.
- Calculating safety stock, order quantity, and reorder point based on the proposed model and performing comparison studies.
- Performing sensitivity analysis under different lead time situations.

3. Methodology

A typical data-driven inventory optimization process is presented in Fig. 1. First, it is to decide how to handle demand uncertainty. Then, demand distribution is formulated as the outcome of the forecasting component. In doing so, demand is forecasted by developing an ensemble deep learning framework. These predictions will serve as an

Table 1
Literature on inventory models with machine learning forecasting methods.

Authors	Review Interval	Inventory Policy	Decision Variables	Objective Function	Forecasting Machine Learning Methods	Estimated Variable	Demand Probability Distribution
Ban and Rudin [8]	Single periodic	Newsvendor	-	Min total cost	Quantile regression Kernel regression	Order quantity	Normal
Cao and Shen [12]	Single periodic	Newsvendor	Order-up-to level Base-stock quantity	Min total cost	Double parallel feedforward network	Demand	Normal
Huber et al., [24]	Single periodic	Newsvendor	-	Min total cost	Neural networks Gradient-boosted decision trees Linear regression	Order quantity	Normal
Trapero et al., [44]	Periodic Single periodic	(R, S) Newsvendor	Safety stock Order-up-to level Order quantity	Target service level	Kernel density estimation	Variance of lead time demand	Normal, lognormal, gamma
Trapero et al., [45]	Single periodic	Newsvendor	Safety stock Backorders	Target service level	Kernel density estimation	Variance of lead time demand	Normal, lognormal
Oroojlooyjadid et al., [33]	Single periodic Continuous	Newsvendor (r, Q)	-	Min total cost	Deep neural network Empirical quantile Quantile regression Kernel regression k-nearest neighbor Random forest	Order quantity	Normal, beta, lognormal, uniform, exponential
Babai et al., [7]	Periodic	(R, S)	Safety stock Order-up-to level	Min total cost Target service level	Neural network	Variance of lead time demand	Negative binomial distribution
Babai et al., ([6]	Periodic	Is not specified	Order-up-to level	Min total cost	Single exponential smoothing & minimum mean squared error forecasting	Demand autocorrelation and variance of lead-time	ARMA(1,1) demand process
This study	Periodic	(R, S)	Safety stock Reorder Point Order quantity	Min total cost	Ensemble Deep Learning (LSTM/MLP/1-D CNN)	Demand	Mean & Variance are calculated inEq. (1)&(2)

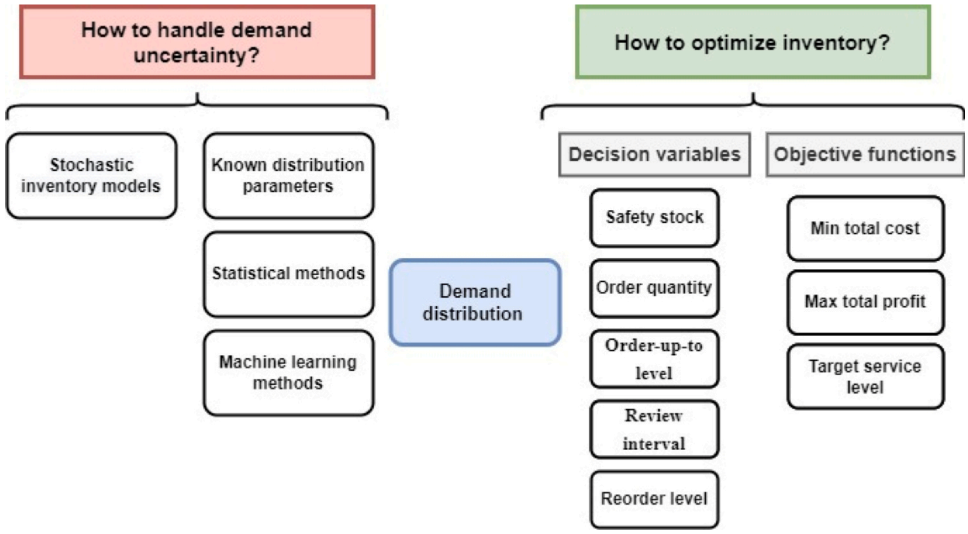

Fig. 1. A schematic of data-driven inventory optimization process.

input for the inventory optimization component with a number of decision variables being adjusted to optimize the total inventory cost.

In doing so, this study contributes to the body of literature by considering ensemble deep learning approaches to predict daily demand and optimizes the stock levels for the case of online retailers. Additionally, in terms of the prediction algorithm, this study advocates addressing a research gap identified in the literature review in having very limited research on using heterogeneous deep neural networks (DNNs) in inventory optimization. The rationale behind the proposed methodology will be further discussed below.

This section aims at proposing an ensemble deep-learning approach to improve multivariate time series forecasting performance. Currently, most ensemble deep learning models focus on the ensemble of homogeneous DNNs and neglect the advantages of heterogeneous DNNs due to their rather complex architecture; reflecting a trade-off between forecast accuracy and model complexity [51]. For example, LSTM and CNN have an advantage in extracting time and space-related features from data, respectively, compared to multi-layer perceptron (MLP) that can only extract global features from data [9]. In this sense, a comparative investigation of MLP, CNN, and LSTM methods (forming the ensemble model) is advocated by leveraging the different features extracted using different types of DNNs.

There are several assumptions in this study. First, the safety stock evaluation will be done through an OUTL policy that operates under a CSL objective. In the prediction section, it is assumed that no unprecedented events will happen in the future (e.g., pandemics, global recession, international shipping problems, crashes in shopping platforms). Furthermore, demand is predicted daily and is incorporated to estimate future order quantity, safety stock, and all related inventory scheduling. In the following subsections, methods adopted in constructing and combining the basic predictors will be described in detail. We further investigate how demand predictions can affect the total cost of inventory.

3.1. Demand forecasting

Fig. 2 presents the proposed ensemble deep learning approach for demand time series forecasting. The aim is to establish forecasts of daily demand using an ensemble deep-learning neural network and then calculate safety stock and order quantity to minimize the total cost. Data preparation, base model construction (MLP, LSTM, and 1D-CNN), and meta-learner combination are the three stages of the proposed approach.

In the first step, data is split into the following three sections [43]: (a) holding the latest 10% of the data as the holdout test set, (b) splitting the remaining 90% into an earlier grid-search cross-validation training set (2/3 *90%), and (c) a later metamodel training set (1/3 *90%). Fig. 3 shows such split data for training and testing. Data preparation involves partitioning the original time series data sets using cross-validation. Cross-validation determines the depth and number of nodes in each hidden layer, as is the best practice when optimizing hyperparameters [13].

In order to guarantee the precision of the suggested demand forecasting, a robust process is implemented that involves outlier detection and interpolation for the collected test data. To achieve this, an unsupervised Support Vector Machine (SVM) is utilized. Firstly, the data is appropriately scaled to ensure compatibility with the SVM algorithm. Subsequently, the OneClassSVM technique is employed to map the data

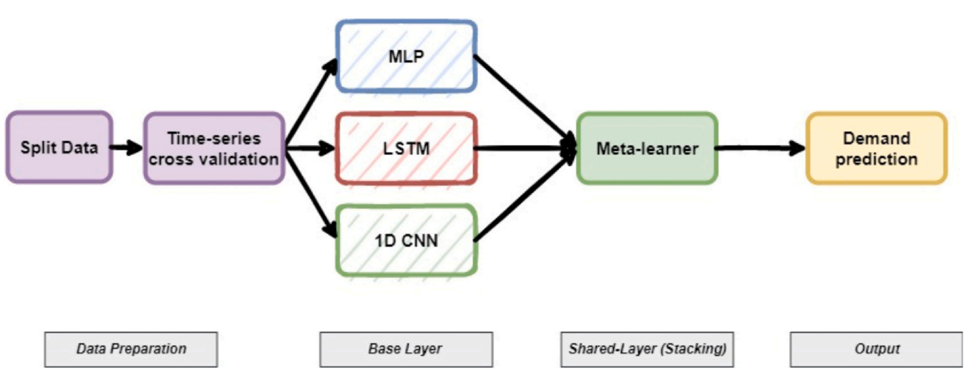

Fig. 2. Proposed time-series demand forecasting approach using ensemble deep learning.

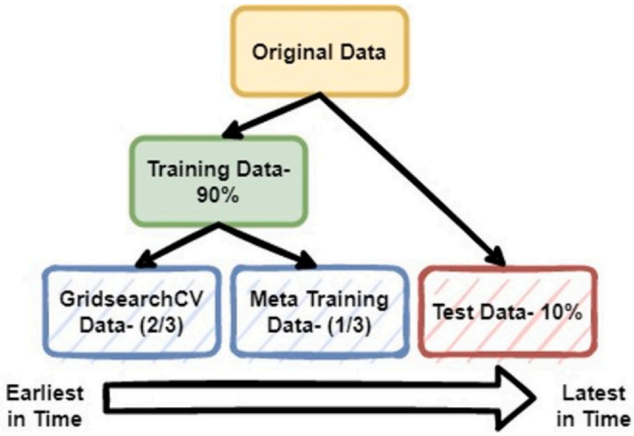

Fig. 3. Data pipeline for different stages of ensemble methodology [43].

into two labels: 0 for normal data points and 1 for abnormal data points [34]. This classification enables the identification of outliers within the dataset. Once the outliers have been detected, a strategy of replacing these anomalous values with more accurate estimations is implemented. Specifically, linear interpolation is employed to estimate the missing or erroneous values caused by the outliers. By leveraging this interpolation technique, the gaps resulting from the outliers can be managed, ensuring a more complete and reliable dataset for demand forecasting purposes.

In the second step, the feature extraction process involves utilizing three different deep learning models: Multilayer Perceptron (MLP), Long Short-Term Memory (LSTM), and 1D Convolutional Neural Network (1D-CNN). These models run in parallel to extract informative features from the input time-series data. Each model specializes in capturing different aspects of data.

The first model, MLP, is a feedforward neural network with fully connected layers. It operates on (input) time series data and leverages its ability to learn complex relationships between the input features and the target variable. The MLP extracts feature from a global perspective using a nonlinear activation function [10,22]:

$$h_i = \varphi[W_i \times h_{i-1} + b_i] \tag{1}$$

Where:

h_i is the output of the i-th layer.

φ represents the activation function applied element-wise.

W_i is the weight matrix of the i-th layer.

h_{i-1} is the input to the i-th layer.

b_i is the bias vector of the i-th layer.

The second model, LSTM, utilizes recurrent neural networks with memory units and loops. This architecture allows the LSTM to capture long-term dependencies and retain information over time. It consists of input gates, forget gates, internal states, and output gates, which help in controlling the flow of information and preserving relevant temporal patterns [2,21].

$$a(t_i) = \sigma[w_a x(t_i) + w_{ha} h(t_{i-1}) + b_a)] \tag{2}$$

$$f(t_i) = \sigma[w_f x(t_i) + w_{hf} h(t_{i-1}) + b_f)] \tag{3}$$

$$c(t_i) = f_t \times c(t_{i-1}) + a_t \times \tanh(w_c.\, x(t_i)) + w_{hc}[h(t_{i-1}) + b_c)] \tag{4}$$

$$o(t_i) = \sigma[w_o x(t_i) + w_{ho} h(t_{i-1}) + b_o] \tag{5}$$

$$h(t_i) = o(t_i) \times \tanh(c(t_i)) \tag{6}$$

Where: $x(t_i)$ is the input value at time step i.$h(t_{i-1})$& h(t_i) are the output values at time steps (i-1) and i, respectively.$c(t_{i-1})$& c(t_i) are the cell states at time steps (i-1) and i, respectively.

b_a, b_f, b_c & b_o are the biases of the input gate, forget gate, internal state, and output gate, respectively.

w_a, w_f, w_c & w_o are the weights of the input gate, forget gate, internal state, and output gate, respectively.

w_{ha}, w_{hf}, w_{hc} & w_{ho} are the recurrent weights.

$a(t_i)$, $f(t_i)$, $c(t_i)$& o(t_i) are the output results for the input gate, forget gate, internal state, and output gate, respectively.

σ represents the sigmoid activation function.

tanh represents the hyperbolic tangent activation function.

The third model, 1D-CNN, applies convolutional operations along the temporal dimension of the time series data. By sliding filters over the data, the 1D-CNN captures local patterns and extracts relevant features. Activation functions introduce non-linearity into the network, enabling it to learn complex representations [28].

$$h_i = \varphi[W \times x_{i:i+k-1} + b)] \tag{7}$$

Where: h_i is the output at position i.φ represents the activation function applied element-wise.W is the weight matrix of the i-th layer.$x_{i:i+k-1}$ is the input to the i-th layer.b is the bias vector.

The above models run simultaneously, processing the input data and extracting different sets of features. In the training phase, a grid-search approach can be applied to find the optimal hyperparameters for each of the base models using a subset of the training data. Once the optimal hyperparameters are determined, the base models are trained on the full grid-search training data.

The predictions obtained from the base models on the meta-training set form the explanatory variables for training the metamodel. The base models are essentially stacked or integrated, where their predictions serve as input to the shared layer. This step results in the creation of more information-rich features due to the dense structure of the shared layer [31].

The metamodel, which is an MLP, is then constructed based on the predictions generated by the base models using a holdout test set. These predictions are fed into the metamodel, and a nonlinear activation function is applied to produce the final demand forecasts.

Mathematically, the ensemble process can be represented as follows:

Base Model 1 (MLP): $f_1(X) = MLP(X; \theta_1)$.

Base Model 2 (LSTM): $f_2(X) = LSTM(X; \theta_2)$.

Base Model 3 (1D-CNN): $f_3(X) = 1D - CNN(X; \theta_3)$.

Meta Model (MLP): $Meta(X) = MLP([f_1(X), f_2(X), f_3(X)]; \theta_{meta})$.

Ensemble Prediction: Ensemble(X) = Meta(X).

Where X represents the input time series data, θ_1, θ_2, and θ_3 are the parameters of the base models, and θ_{meta} represents the parameters of the metamodel.

The choice of the above-mentioned three models for ensemble is based on their complementary strengths. MLP excels in capturing global patterns and complex relationships, LSTM is proficient in modelling temporal dependencies, and 1D-CNN is effective in capturing local patterns. By combining their individual strengths, the ensemble can benefit from a broader range of features and a more comprehensive understanding of the time series data. The ensemble process allows for the nonlinear mapping of the outputs of the base models using the metamodel. The ensembling helps reduce bias, leveraging diverse model architectures, and improving the overall predictive performance by combining the knowledge and insights from the base models.

In summary, the ensemble deep learning model combines the predictions of the base models (MLP, LSTM, and 1D-CNN) through a metamodel (MLP) to generate demand forecasts. The chosen models provide complementary capabilities in capturing different aspects of the time series data, leading to a more comprehensive representation and improved predictive accuracy.

3.2. Inventory optimization

Most comparative demand forecasting studies focus on comparing accuracy of various methods rather than comparing their implications for inventory control [7]. In this study a dynamic multiple-period inventory management problem is examined by adopting an OUTL

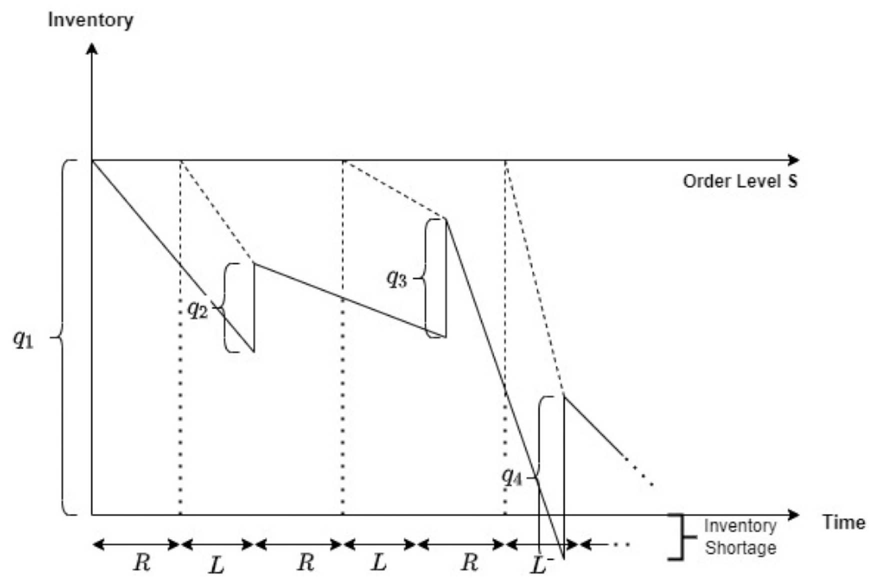

Fig. 4. OULP - Order-up-to level policy [26].

inventory policy. We assume a nonperishable product with a starting inventory each period. Inventory replenishment may occur at each period. The decision variable will be the number of units that are to be ordered each time subject to demand variations. To incorporate such uncertainty, an inventory system with stochastic demand is formulated. An OUTL policy is used to control the stock based on the forecasted demand resulting from the proposed approach in Section 3.1.

Fig. 4 presents the details of an OUTL policy with a variable order quantity (q) placed at a fixed time period(R). To reach the desired quantity(S), a number of units as inventory are needed and ordered subject to a lead time(L). This policy avoids accumulating excessive inventories and therefore could contribute to opportunity costs (i.e. resulting from unmet demand). In case of higher average inventories, the OUTL policy could lead to relatively higher capital commitments including holding costs [30]. In such a case, large quantities could only be acquired by placing several orders leading to higher ordering costs. In addition, shortages could happen if there are long delays in fulfilling the target inventory. As such, companies with cycled replenishment are recommended to use OUTL policies [26].

In the realm of inventory control, one of the most widely used methods for determining the order quantity is the economic order quantity (EOQ) model. The proposed model provides a framework for estimating the optimal order quantity minimizing the total inventory costs. Traditionally, an EOQ model assumes a constant demand rate. However, in the context of demand forecasting, it is more realistic to replace the constant demand with an average forecasted demand. By incorporating the average forecasted demand into EOQ, businesses can better align their order quantities in line with expected demand variations, thereby avoiding excess inventory or stockouts.

When implementing EOQ models, items within the same inventory control group are usually considered to have similar ordering costs. This grouping allows businesses to streamline their procurement processes and benefit from economies of scale. By consolidating the ordering process for similar items, companies can achieve cost savings and operational efficiency. In contrast, the holding cost component in EOQ models is primarily calculated as a percentage of the value of items. This approach takes into account the carrying costs associated with storing and maintaining inventory over time. Holding costs include expenses such as warehousing, insurance, obsolescence, and the opportunity cost of tying up capital in inventory. By considering the item's value as a factor in calculating the holding cost, businesses can reflect the financial implications of carrying inventory.

As a rule of thumb, the carrying charge, which represents the sum of capital and holding costs, is commonly estimated to range from 10% to 15% of the total annual costs [26]. This percentage serves as a guideline for businesses to evaluate the overall cost impact of carrying inventory and make informed decisions regarding inventory control strategies. By understanding the carrying charge, companies can assess the trade-offs between holding costs and ordering costs and optimize their inventory management practices accordingly. In doing so, safety stock (ss), reorder point (r) and replenishment level (S) are defined as follows:

$$ss = z.\ \sigma\sqrt{(R + L)} \tag{8}$$

$$r = d(R + L) + z.\ \sigma\sqrt{(R + L)} \tag{9}$$

$$S = r + q \tag{10}$$

Estimating the distribution of lead time also plays a crucial role in determining the appropriate reorder and order-up-to levels in inventory management. The lead time refers to the time interval between placing an order and receiving it. By understanding the statistical characteristics of lead time, businesses can make informed decisions about inventory replenishment to ensure a smooth flow of goods. To assess the impact of lead time on inventory control, the mean and variance of the demand distribution need to be estimated. These estimations provide valuable insights into the expected demand levels and the associated uncertainty. Eqs. (11) and (12) presented by Babai et al. [7] outline the methodology for these estimations:

$$\mu_{L+1,t} = (L + 1)^{*}\hat{D}_t \tag{11}$$

$$\sigma^2_{L+1,t} = (L + 1)^{*}MSE_t. \tag{12}$$

To improve the accuracy of demand forecasting, this study proposes utilization of an ensemble deep-learning forecasting approach, as discussed in Section 3.1. This approach combines multiple deep-learning models to generate a more robust and reliable demand forecast. In this context, $\hat{D}_t$ represents the estimated demand forecast at period t.

In order to assess the accuracy of the demand forecasts and estimate the variance of the mean demand per period, this study uses a smoothed mean squared error (MSE_t), which is derived using Eq. (13) as proposed by [7]. This calculation provides a measure of the accuracy and reliability of demand forecasts by considering the squared differences between the forecasted and actual demand values.

$$MSE_t = \alpha(D_t - \hat{S}_t)^2 + (1-\alpha)MSE_{t-1}. \quad (13)$$

However, determining the lead-time distribution presents difficulties, particularly when stochastic variations occur. To overcome this challenge, this study assumes fixed lead times when facing uncertain or varying lead-time durations. This simplification could help address the complexity of lead-time variations.

In the following section, the results and analyses related to the proposed ensemble deep learning forecasting approach and its integration into the inventory control system will be discussed. These results will shed light on performance and effectiveness of the approach in improving demand forecasting accuracy and optimizing inventory management decisions. By leveraging the ensemble deep learning forecasting approach and addressing the challenges associated with lead-time distribution, the proposed methodology aims to enhance the accuracy of demand forecasting and provide more reliable inputs for inventory control decisions. The subsequent section will delve into the empirical findings and provide insights into practical implications of the proposed approach.

4. Results analysis

To analyze accuracy, stability, and generalization ability of the proposed ensemble deep learning approach, two types of products and three widely used evaluation metrics are used to conduct comparison experiments. Same experimental parameters are used for each model to compare their generalization performance on different data sets. Detailed descriptions of data sets, evaluation metrics, and experimental results and analyses are presented in the following subsections.

4.1. Data description & performance criteria

Two real-world supply chain time series data (with records from January 2015 to September 2017) of electronic and sports supplies are used in this study to show the effectiveness and practicality of the proposed model [14]. There are 52 features in the data set, but many of these features are not linked to or can contribute to demand forecasting. The daily demand for all products varied from 250 to 500 depending on time, sales price, and discounts. Forecasts for the target variable are correlated to a small set of these features including sales price, holidays and other timing effects, as well as past demand.

Fig. 5 and Fig. 6 depict specific subsets of data, focusing on Sports and Electronics Products, respectively. These figures provide an insight into the observed patterns within the data, revealing numerous fluctuations characterized by considerable uncertainties associated with seasonality and cyclic behavior. Given the inherent volatility and irregularities observed in the ups and downs of the dataset, traditional predictive approaches may struggle to capture the complexities and intricacies effectively. Ensemble learning, on the other hand, has the potential to handle such challenges by aggregating the predictions from multiple models and capturing diverse patterns and trends within data.

Based on mean absolute percentage error (MAPE), mean absolute error (MAE), and root mean square error (RMSE), the predictions were compared in terms of accuracy. Consequently, a lower RMSE value indicates better predictability (or less inaccuracy). These criteria are calculated as follows:

$$MAE = \frac{1}{n}\sum_{i=1}^{n} |y_i - \hat{y}_i| \quad (14)$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (y_i - \hat{y}_i)^2} \quad (15)$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n} \left|\frac{y_i - \hat{y}_i}{y_i}\right| \quad (16)$$

where y_i and $\hat{y}_i$ show the actual and predicted values at day i, respectively. In this case, n represents the number of days in a testing period (82 days in this example).

4.2. Demand forecasting

To determine the score of holdout test set, a 5-fold cross-validation is performed using modified post-gridsearch hyperparameter settings. During the cross-validation, both batches (base + meta) of the training set are combined to reconstruct the full 90% of training data. Table 2 and Table 3 present cross-validation scores for three base models and the proposed ensemble deep learning model for sports and electronics products, respectively.

The base models are only trained once using the gridsearch training set to predict the target variable's values on the holdout test set. Metamodels are built from predictions made on meta-training sets, which are used to decide what to predict on test sets based on the meta-training model. Fig. 7 and Fig. 8 are forecasted demand versus actual demand in the sports and electronic data sets, respectively. These figures show that the predicted demand and the actual demand have close proximity confirming a good accuracy for predictions. It is shown that the proposed model achieved the lowest MAE, RMSE, and MAPE% scores on the holdout test set.

Moreover, compared to the LSTM (which has the best performance among the three base models presented in Table 4 and Table 5), the proposed ensemble deep learning model achieved an improvement of 22% in the RMSE, 21.7% in the MAE, and 22.3% in the MAPE% and demonstrated improvement in sports products. In other words, the proposed ensemble methodology generates more accurate results than individual base models in the ensemble. Table 4 and Table 5 provide details related to the above-mentioned forecasting performance criteria for sports and electronics data sets, respectively.

Additionally, the RMSE of the proposed model is within the range of the 5-fold cross-validation procedure, while the MAE is below it. Holdout test errors were generally lower than those observed in the cross-validations across all models, indicating that the test set is more predictable. This emphasizes the need to conduct multifold cross-validation to evaluate model performance in such multivariate time-series forecasting.

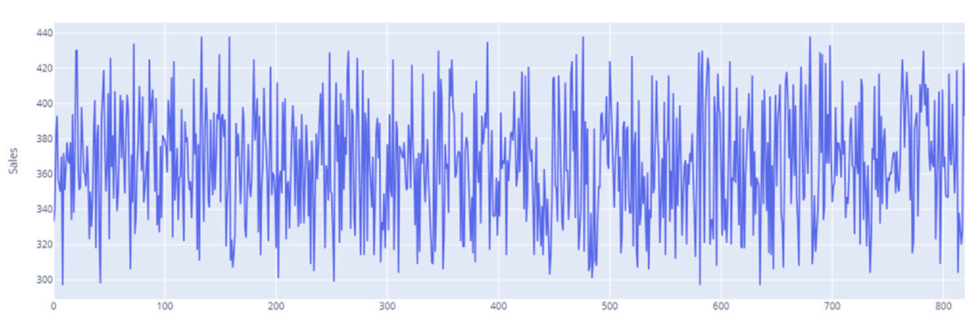

Fig. 5. Daily Sports Products Sales.

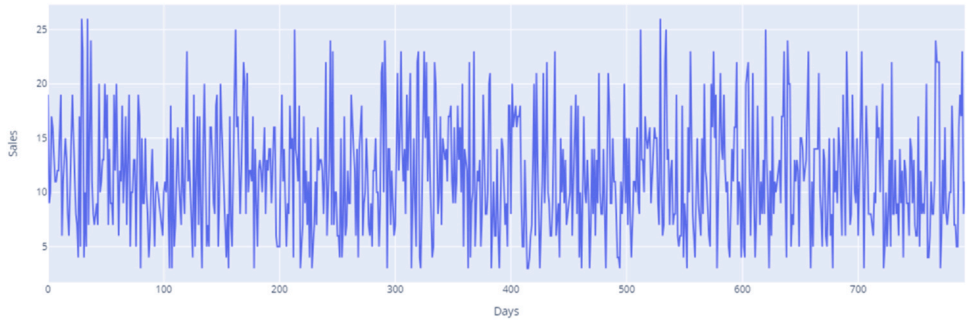

Fig. 6. Daily Electronics Products Sales.

Table 2
Cross-validation scores and statistics for sports products.

	MLP			LSTM			1D-CNN			Proposed model		
	MAE	RMSE	MAPE%	MAE	RMSE	MAPE%	MAE	RMSE	MAPE%	MAE	RMSE	MAPE%
1st fold	36.43	45.50	9.66	24.99	30.77	6.83	35.42	42.46	10.69	23.24	23.24	6.28
2nd fold	54.40	66.26	15.64	34.03	40.04	9.14	34.52	42.06	9.60	30.74	30.74	8.32
3rd fold	50.85	61.67	16.08	29.82	36.01	8.17	28.90	36.79	8.24	28.72	28.72	7.90
4th fold	118.78	131.70	13.17	31.85	36.85	8.66	38.33	47.49	11.56	30.14	30.14	8.41
5th fold	125.90	138.37	13.03	31.79	36.93	8.89	43.12	52.45	12.53	28.75	28.75	8.08
Mean	77.27	88.70	13.52	30.49	36.12	8.34	36.06	44.25	10.52	28.32	28.32	7.80
Median	54.40	66.26	13.17	31.79	36.85	8.66	35.42	42.46	10.69	28.75	28.75	8.08
Standard Deviation	41.76	43.06	2.56	3.41	3.35	0.91	5.22	5.94	1.67	2.97	2.97	0.87

Table 3
Cross-validation scores & statistics for electronic products.

	MLP			LSTM			1D-CNN			Proposed model		
	MAE	RMSE	MAPE%	MAE	RMSE	MAPE%	MAE	RMSE	MAPE%	MAE	RMSE	MAPE%
1st fold	5.19	6.18	14.63	4.93	42.03	12.03	4.72	5.77	13.55	5.58	6.38	12.65
2nd fold	3.71	4.71	13.66	4.03	35.53	13.53	4.15	4.94	13.07	4.82	5.61	13.86
3rd fold	4.81	6.10	16.96	4.59	39.78	11.78	4.51	5.60	12.95	4.96	5.90	12.15
4th fold	5.20	6.68	14.07	4.87	42.51	13.51	5.13	6.62	14.18	5.03	6.29	11.12
5th fold	4.40	5.51	15.84	4.03	33.36	14.36	3.80	4.78	11.20	3.87	4.83	13.50
Mean	4.66	5.84	15.03	4.49	38.64	13.04	4.46	5.54	12.99	4.84	5.80	12.65
Median	4.81	6.10	14.04	4.59	39.78	13.51	4.51	5.60	13.07	4.90	5.90	12.66
Standard Deviation	0.62	0.75	1.35	0.43	4.04	1.10	0.51	0.73	1.11	0.61	0.62	1.09

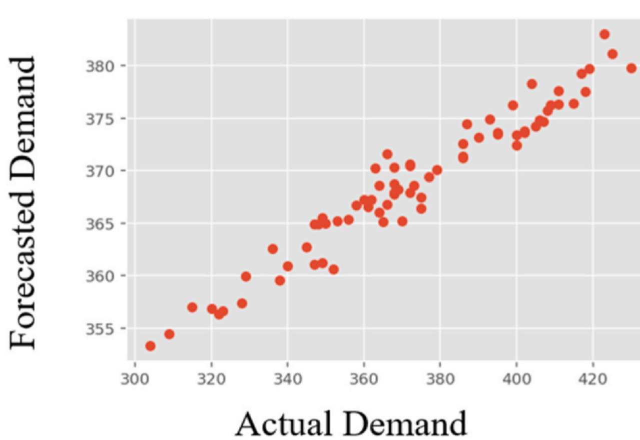

Fig. 7. Forecasted vs. actual sport product demand.

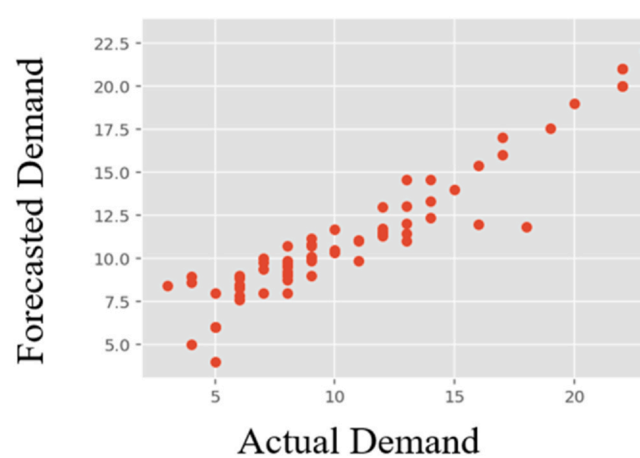

Fig. 8. Forecasted vs. actual electronic product demand.

4.3. Inventory optimization

Total cost includes fixed cost, holding cost, and shortage cost which should be optimized to determine the optimum values of the order-up-to-level and review interval [30]. Sports and electronics products are assumed without a lifetime. A periodic review setting is considered, where reviews are conducted regularly. In OUTL policy, order-up-to level is S and the inventory position is determined after a review (R). Then, an amount equal to the difference between the order-up-to level (S) and on-hand stock is ordered at the beginning of each review period.

Table 4
Test results using holdout data set for sports product.

	MAE	RMSE	MAPE%
MLP	26.26	32.25	7.29
LSTM	24.85	30.78	6.72
1D-CNN	25.24	31.06	6.78
Proposed model	19.46	24.02	5.22

Table 5
Test results using holdout data set for electronic product.

	MAE	RMSE	MAPE%
MLP	5.05	5.93	12.64
LSTM	4.18	5.21	10.12
1D-CNN	6.20	7.96	14.76
Proposed model	3.56	4.99	9.58

The order quantity is received instantly and reflected in the stock level after the review. All of these calculations are based on the distribution of forecasted demand in the review intervals.

Fig. 9 and Fig. 10 represent the total cost in OUTL policy with different base forecasting methods (MLP, LSTM, and 1D-CNN) for sports products and electronic products, respectively. It is shown that the proposed ensemble deep learning model can reach a lower total cost compared to other forecasting methods in a review interval. In this online shopping case study, data shows that demand for sports products are higher than that for electronic products. Lower pricing could be contributed as one of the reasons for this higher daily demand.

5. Discussion

The proposed research utilized an ensemble deep learning approach, comprising three neural network base models (MLP, LSTM, and 1D-CNN), to predict demands for sports and electronic products for one period ahead. A meta-layer was incorporated, modelling a nonlinear relationship with the MLP model. The findings demonstrated that employing a machine learning ensemble stack approach for multivariate time series analysis resulted in more accurate demand predictions. Furthermore, by implementing an OUTL policy, the demand distribution was leveraged to optimize the overall cost.

The application of the proposed forecasting approach to an OUTL policy led to noticeable improvements in inventory performance metrics. The ensemble deep learning approach achieved a mean absolute percentage error (MAPE) of 5.22% and 9.58% for sports and electronic products, respectively, in predicting demand uncertainty. Consequently, when compared to its three individual base models, the ensemble learning approach exhibited enhanced forecasting accuracy.

In Table 6 and Table 7, the changes in order quantity, safety stock, and reorder points for the base models and the proposed ensemble deep learning model are presented. Notably, the ensemble deep-learning forecasting resulted in slightly higher safety stocks. Additionally, both order quantity and reorder points exhibited similar patterns across the datasets, with the proposed model yielding the lowest values compared to the individual base models. This optimization of safety stock led to a reduction in shortage costs and, consequently, a decrease in the total cost.

The results warrant a more detailed discussion to enhance their interpretation and significance. Firstly, in relation to literature, this study contributes to the existing knowledge by showcasing the effectiveness of an ensemble deep learning approach for demand forecasting in the context of sports and electronic products. Evidence of the improved forecasting accuracy was presented through comparing the proposed model with the individual base models. Moreover, the findings highlight the importance of considering an OUTL policy in inventory management. The optimization of safety stock based on demand distribution resulted in cost savings, demonstrating the practical value of leveraging demand insights in decision-making.

Another analysis has been done to validate the model in both Sports and Electronic products. The following tables provide a sensitivity analysis of the total cost across different lead time scenarios, comparing the proposed model with MLP, LSTM, and CNN models. Table 8 focuses on sports products. Across all lead time scenarios of 7 days, 10 days, and 14 days, the proposed model consistently achieves lower total costs compared to the MLP, LSTM, and CNN models. Specifically, the proposed model demonstrates cost savings of 97 units, 105 units, and 115 units, respectively, compared to the MLP model. Table 9 examines electronic products. In this category, the proposed model also outperforms the other models in terms of cost across all lead time scenarios. Under a 7-day lead time, the proposed model achieves cost savings of 175 units compared to the MLP model. For a 10-day lead time, the savings increase to 90 units, and for a 14-day lead time, the savings amount to 98 units.

In summary, the sensitivity analysis highlights that the proposed model consistently delivers cost savings and outperforms the MLP, LSTM, and CNN models in both sports and electronic product categories. The savings range from approximately 90 to 175 units in electronic products

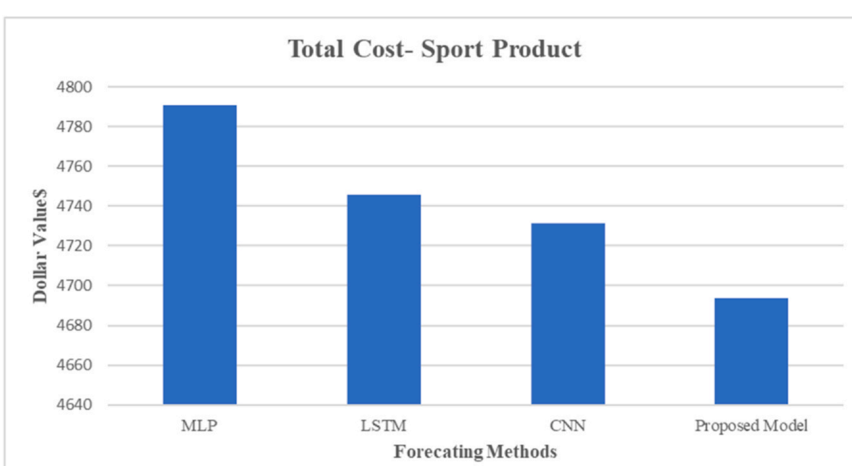

Fig. 9. Comparison of total cost resulted in using different demand forecasting methods for sports products.

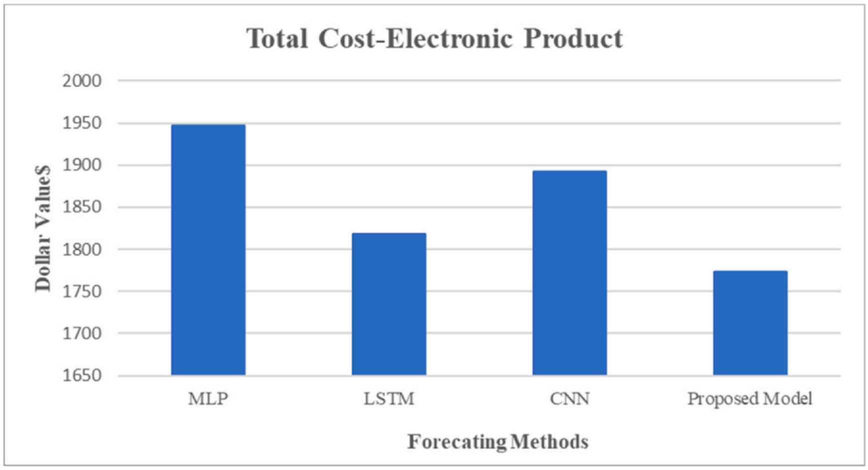

Fig. 10. Comparison of total cost resulted in using different demand forecasting methods for electronic products.

Table 6
Sensitivity analysis of inventory parameters in different forecasting methods for Sports products.

Forecasting method	Order Quantity	Safety Stock	Reorder Point
MLP	623	136	2761
LSTM	618	177	2753
CNN	616	170	2731
Proposed Model	611	197	2717

Table 7
Sensitivity analysis of inventory parameters in different forecasting methods for Electronics products.

Forecasting method	Order Quantity	Safety Stock	Reorder Point
MLP	105	12	108
LSTM	98	10	94
CNN	102	12	103
Proposed Model	96	14	93

and from 97 to 115 units in sports products, depending on the lead time scenario. These findings underscore the effectiveness and competitiveness of the proposed model in optimizing costs across different lead-time situations. By adopting the proposed model, businesses in both sports and electronic industries have the opportunity to achieve significant cost savings and enhance their overall financial performance.

6. Conclusions

The application of multivariate time series forecasting holds great potential for enhancing the efficiency and robustness of inventory systems. In this particular study, a data mining approach was proposed, aiming to analyze supply chain data and extract valuable information for optimizing safety stock and improving inventory management. The study focused on two streams of real-world time series data representing Sports and Electronics products, employing three state-of-the-art deep learning models: Multilayer Perceptron (MLP), Long Short-Term Memory (LSTM), and 1D Convolutional Neural Network (1D-CNN). To demonstrate the superiority of the proposed ensemble forecasting approach, comparative experiments were conducted. The forecasting performance of each individual model was evaluated using commonly used statistical metrics. Additionally, the demand uncertainty, managed under an OUTL (Order-Up-To Level) policy, was addressed by adjusting safety stock and order quantity. Through these experiments, several noteworthy conclusions were drawn:

- The proposed approach successfully leveraged the strengths of both deep learning and ensemble learning techniques, allowing for the extraction of implicit features within the time series data.
- The ensemble model outperformed the baseline models in both the Sports and Electronics product domains, exhibiting superior forecasting accuracy, stability, and generalization capabilities.
- The ensemble deep learning method significantly improved the forecasting accuracy of the underlying base predictors.
- The OUTL policy, coupled with the optimized safety stock and order quantity, resulted in the minimum total cost.

Table 8
Sensitivity analysis under different lead time situations- Sports products.

		MLP	LSTM	CNN	Proposed Model
Total Cost	7 Days Lead Time	4791	4746	4731	4694
	10 Days Lead Time	5207	5158	5143	5102
	14 Days Lead Time	5704	5651	5634	5589

Table 9
Sensitivity analysis under different lead time situations- Electronic products.

		MLP	LSTM	CNN	Proposed Model
Total Cost	7 Days Lead Time	1948	1819	1893	1773
	10 Days Lead Time	998	931	970	908
	14 Days Lead Time	1093	1020	1062	995

While the proposed approach showcased promising results, there are certain limitations that could inspire future research directions. The construction and combination stages of the basic predictors were found to be complex and computationally demanding, requiring significant memory and computational resources. Exploring better strategies for building and pruning ensembles of basic predictors can enhance computational efficiency and scalability.

Moreover, there is room for further exploration and design of more effective ensemble schemes to enhance forecasting accuracy and stability. Researchers have already demonstrated the potential of such approaches in related studies [1,2]. Additionally, it would be valuable to investigate the broader impacts of the proposed approach and the resulting improvements in demand forecasting accuracy on supply chain management. Furthermore, the proposed framework can be tested under different inventory policies, beyond the OUTL policy utilized in this study. Future work may also consider the optimization of inventory for perishable products incorporating constraints related to product life expectancy and associated spoilage costs.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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