

Importando as Bibliotecas Necessárias

In [1]:

```
import numpy as np
import pandas as pd
from scipy.io import loadmat
from numpy.linalg import inv
import matplotlib.pyplot as plt
```

Lendo os Dados que estão no Fomato do Matlab

In [2]:

```
grupo3 = loadmat("GrupoRobo_3.mat")
```

Transformando os Dados em um DataFrame

In [3]:

```
dados = pd.DataFrame(grupo3['z1'])
```

Isolando os Dados de Saída e de Entrada

In [4]:

```
Y = dados[0]
U = dados[1]
```

Organizando a Matriz X

In [5]:

```
X = []
a = 0
a1 = 0
b = 0
b1 = 0
for i in range(len(Y)):
    k=i-2
    if(k== -2 and (k+1) == -1):
        X.append([a,a1,b,b1])
    elif(k== -1):
        a=Y[k+1]
        b=U[k+1]
        X.append([a,a1,b,b1])
    else:
        a=Y[i-1]
        b=U[i-1]
        a1=Y[i-2]
        b1=U[i-1]
        X.append([a,a1,b,b1])
```

Transformando X em um array

In [6]:

```
X = np.array(X)
```

In [7]:

```
print("Array X : ")
print("\n")
print(X)
print("\n")
```

Array X :

```
[ [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
  [2.07273829e-03 0.00000000e+00 1.00000000e+02 0.00000000e+00]
  [3.06437632e-01 2.07273829e-03 1.00000000e+02 1.00000000e+02]
  [1.19984983e+00 3.06437632e-01 1.00000000e+02 1.00000000e+02]
  [2.66354096e+00 1.19984983e+00 1.00000000e+02 1.00000000e+02]
  [4.58120577e+00 2.66354096e+00 1.00000000e+02 1.00000000e+02]
  [6.95541151e+00 4.58120577e+00 1.00000000e+02 1.00000000e+02]
  [9.72158156e+00 6.95541151e+00 1.00000000e+02 1.00000000e+02]
  [1.28211233e+01 9.72158156e+00 1.00000000e+02 1.00000000e+02]
  [1.62235408e+01 1.28211233e+01 1.00000000e+02 1.00000000e+02]
  [1.98845163e+01 1.62235408e+01 1.00000000e+02 1.00000000e+02]
  [2.37565983e+01 1.98845163e+01 1.00000000e+02 1.00000000e+02]
  [2.77879262e+01 2.37565983e+01 1.00000000e+02 1.00000000e+02]
  [3.19740943e+01 2.77879262e+01 1.00000000e+02 1.00000000e+02]
  [3.62518250e+01 3.19740943e+01 1.00000000e+02 1.00000000e+02]
  [4.05889302e+01 3.62518250e+01 1.00000000e+02 1.00000000e+02]
  [4.49426484e+01 4.05889302e+01 1.00000000e+02 1.00000000e+02]
  [4.93452467e+01 4.49426484e+01 1.00000000e+02 1.00000000e+02]
  [5.36755275e+01 4.93452467e+01 1.00000000e+02 1.00000000e+02]
  [5.80088724e+01 5.36755275e+01 1.00000000e+02 1.00000000e+02]
  [6.22149129e+01 5.80088724e+01 1.00000000e+02 1.00000000e+02]
  [6.63595282e+01 6.22149129e+01 1.00000000e+02 1.00000000e+02]
  [7.04025183e+01 6.63595282e+01 1.00000000e+02 1.00000000e+02]
  [7.42813554e+01 7.04025183e+01 1.00000000e+02 1.00000000e+02]
  [7.80620348e+01 7.42813554e+01 1.00000000e+02 1.00000000e+02]
  [8.16740621e+01 7.80620348e+01 1.00000000e+02 1.00000000e+02]
  [8.51178280e+01 8.16740621e+01 1.00000000e+02 1.00000000e+02]
  [8.83787487e+01 8.51178280e+01 1.00000000e+02 1.00000000e+02]
  [9.14850842e+01 8.83787487e+01 1.00000000e+02 1.00000000e+02]
  [9.43809345e+01 9.14850842e+01 1.00000000e+02 1.00000000e+02]
  [9.71158807e+01 9.43809345e+01 1.00000000e+02 1.00000000e+02]
  [9.96392256e+01 9.71158807e+01 1.00000000e+02 1.00000000e+02]
  [1.01987242e+02 9.96392256e+01 1.00000000e+02 1.00000000e+02]
  [1.04126868e+02 1.01987242e+02 1.00000000e+02 1.00000000e+02]
  [1.06099368e+02 1.04126868e+02 1.00000000e+02 1.00000000e+02]
  [1.07863580e+02 1.06099368e+02 1.00000000e+02 1.00000000e+02]
  [1.09449767e+02 1.07863580e+02 1.00000000e+02 1.00000000e+02]
  [1.10846864e+02 1.09449767e+02 1.00000000e+02 1.00000000e+02]
  [1.12110236e+02 1.10846864e+02 1.00000000e+02 1.00000000e+02]
  [1.13191254e+02 1.12110236e+02 1.00000000e+02 1.00000000e+02]
  [1.14074761e+02 1.13191254e+02 1.00000000e+02 1.00000000e+02]
  [1.14846130e+02 1.14074761e+02 1.00000000e+02 1.00000000e+02]
  [1.15433601e+02 1.14846130e+02 1.00000000e+02 1.00000000e+02]
  [1.15890958e+02 1.15433601e+02 1.00000000e+02 1.00000000e+02]
  [1.16231386e+02 1.15890958e+02 1.00000000e+02 1.00000000e+02]
  [1.16433923e+02 1.16231386e+02 1.00000000e+02 1.00000000e+02]
  [1.16536626e+02 1.16433923e+02 1.00000000e+02 1.00000000e+02]
  [1.16505331e+02 1.16536626e+02 1.00000000e+02 1.00000000e+02]
  [1.16400383e+02 1.16505331e+02 1.00000000e+02 1.00000000e+02]
  [1.16196378e+02 1.16400383e+02 1.00000000e+02 1.00000000e+02]
  [1.15906672e+02 1.16196378e+02 1.00000000e+02 1.00000000e+02]
  [1.15545767e+02 1.15906672e+02 1.00000000e+02 1.00000000e+02]
  [1.15109719e+02 1.15545767e+02 1.00000000e+02 1.00000000e+02]
  [1.14611515e+02 1.15109719e+02 1.00000000e+02 1.00000000e+02]
  [1.14090843e+02 1.14611515e+02 1.00000000e+02 1.00000000e+02]
  [1.13495219e+02 1.14090843e+02 1.00000000e+02 1.00000000e+02]
  [1.12896043e+02 1.13495219e+02 1.00000000e+02 1.00000000e+02]
  [1.12231729e+02 1.12896043e+02 1.00000000e+02 1.00000000e+02]
  [1.11569337e+02 1.12231729e+02 1.00000000e+02 1.00000000e+02]
  [1.10885842e+02 1.11569337e+02 1.00000000e+02 1.00000000e+02]
  [1.10160722e+02 1.10885842e+02 1.00000000e+02 1.00000000e+02]
  [1.09446525e+02 1.10160722e+02 1.00000000e+02 1.00000000e+02]
  [1.08749915e+02 1.09446525e+02 1.00000000e+02 1.00000000e+02]
  [1.08030607e+02 1.08749915e+02 1.00000000e+02 1.00000000e+02]
  [1.07323023e+02 1.08030607e+02 1.00000000e+02 1.00000000e+02]
  [1.06629687e+02 1.07323023e+02 1.00000000e+02 1.00000000e+02]
```

[1.05940358e+02	1.06629687e+02	1.00000000e+02	1.00000000e+02]
[1.05294717e+02	1.05940358e+02	1.00000000e+02	1.00000000e+02]
[1.04635849e+02	1.05294717e+02	1.00000000e+02	1.00000000e+02]
[1.04029978e+02	1.04635849e+02	1.00000000e+02	1.00000000e+02]
[1.03416756e+02	1.04029978e+02	1.00000000e+02	1.00000000e+02]
[1.02865032e+02	1.03416756e+02	1.00000000e+02	1.00000000e+02]
[1.02309918e+02	1.02865032e+02	1.00000000e+02	1.00000000e+02]
[1.01761229e+02	1.02309918e+02	1.00000000e+02	1.00000000e+02]
[1.01305523e+02	1.01761229e+02	1.00000000e+02	1.00000000e+02]
[1.00865515e+02	1.01305523e+02	1.00000000e+02	1.00000000e+02]
[1.00426219e+02	1.00865515e+02	1.00000000e+02	1.00000000e+02]
[1.00030543e+02	1.00426219e+02	1.00000000e+02	1.00000000e+02]
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[9.75285175e+01	9.75566577e+01	1.00000000e+02	1.00000000e+02]
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[9.75280291e+01	9.75384638e+01	1.00000000e+02	1.00000000e+02]
[9.75781920e+01	9.75280291e+01	1.00000000e+02	1.00000000e+02]
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[9.78194510e+01	9.77573703e+01	1.00000000e+02	1.00000000e+02]
[9.78993251e+01	9.78194510e+01	1.00000000e+02	1.00000000e+02]
[9.79790282e+01	9.78993251e+01	1.00000000e+02	1.00000000e+02]
[9.81228193e+01	9.79790282e+01	1.00000000e+02	1.00000000e+02]
[9.82037730e+01	9.81228193e+01	1.00000000e+02	1.00000000e+02]
[9.83201852e+01	9.82037730e+01	1.00000000e+02	1.00000000e+02]
[9.84268126e+01	9.83201852e+01	1.00000000e+02	1.00000000e+02]
[9.85318456e+01	9.84268126e+01	1.00000000e+02	1.00000000e+02]
[9.86482278e+01	9.85318456e+01	1.00000000e+02	1.00000000e+02]
[9.87425081e+01	9.86482278e+01	1.00000000e+02	1.00000000e+02]
[9.88738803e+01	9.87425081e+01	1.00000000e+02	1.00000000e+02]
[9.89950487e+01	9.88738803e+01	1.00000000e+02	1.00000000e+02]
[9.91065383e+01	9.89950487e+01	1.00000000e+02	1.00000000e+02]
[9.92322134e+01	9.91065383e+01	1.00000000e+02	1.00000000e+02]
[9.93386908e+01	9.92322134e+01	1.00000000e+02	1.00000000e+02]
[9.94351780e+01	9.93386908e+01	1.00000000e+02	1.00000000e+02]
[9.95310204e+01	9.94351780e+01	1.00000000e+02	1.00000000e+02]
[9.96494612e+01	9.95310204e+01	1.00000000e+02	1.00000000e+02]
[9.97354541e+01	9.96494612e+01	1.00000000e+02	1.00000000e+02]
[9.98297907e+01	9.97354541e+01	1.00000000e+02	1.00000000e+02]
[9.99154627e+01	9.98297907e+01	1.00000000e+02	1.00000000e+02]
[9.99915534e+01	9.99154627e+01	1.00000000e+02	1.00000000e+02]
[1.00075722e+02	9.99915534e+01	1.00000000e+02	1.00000000e+02]
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[1.00321681e+02	1.00255063e+02	1.00000000e+02	1.00000000e+02]
[1.00381004e+02	1.00321681e+02	1.00000000e+02	1.00000000e+02]
[1.00413358e+02	1.00381004e+02	1.00000000e+02	1.00000000e+02]
[1.00462223e+02	1.00413358e+02	1.00000000e+02	1.00000000e+02]
[1.00500608e+02	1.00462223e+02	1.00000000e+02	1.00000000e+02]
[1.00559029e+02	1.00500608e+02	1.00000000e+02	1.00000000e+02]
[1.00545582e+02	1.00559029e+02	1.00000000e+02	1.00000000e+02]
[1.00573522e+02	1.00545582e+02	1.00000000e+02	1.00000000e+02]
[1.00589997e+02	1.00573522e+02	1.00000000e+02	1.00000000e+02]
[1.00593837e+02	1.00589997e+02	1.00000000e+02	1.00000000e+02]
[1.00622215e+02	1.00593837e+02	1.00000000e+02	1.00000000e+02]
[1.00631397e+02	1.00622215e+02	1.00000000e+02	1.00000000e+02]
[1.00629055e+02	1.00631397e+02	1.00000000e+02	1.00000000e+02]
[1.00633758e+02	1.00629055e+02	1.00000000e+02	1.00000000e+02]
[1.00631924e+02	1.00633758e+02	1.00000000e+02	1.00000000e+02]
[1.00626102e+02	1.00631924e+02	1.00000000e+02	1.00000000e+02]
[1.00614421e+02	1.00626102e+02	1.00000000e+02	1.00000000e+02]
[1.00607470e+02	1.00614421e+02	1.00000000e+02	1.00000000e+02]
[1.00595287e+02	1.00607470e+02	1.00000000e+02	1.00000000e+02]

```
[1.00599540e+02 1.00595287e+02 1.00000000e+02 1.00000000e+02]
[1.00578887e+02 1.00599540e+02 1.00000000e+02 1.00000000e+02]
[1.00567722e+02 1.00578887e+02 1.00000000e+02 1.00000000e+02]
[1.00541616e+02 1.00567722e+02 1.00000000e+02 1.00000000e+02]
[1.00510222e+02 1.00541616e+02 1.00000000e+02 1.00000000e+02]
[1.00514888e+02 1.00510222e+02 1.00000000e+02 1.00000000e+02]
[1.00502569e+02 1.00514888e+02 1.00000000e+02 1.00000000e+02]]
```

Transformando Y e U em um array

In [8]:

```
Y = np.array(Y)
U = np.array(U)
print("Array Y : ")
print("\n")
print(Y)
print("\n\n")
print("Array U : ")
print("\n")
print(U)
print("\n")
```

Array Y :

```
[2.07273829e-03 3.06437632e-01 1.19984983e+00 2.66354096e+00
 4.58120577e+00 6.95541151e+00 9.72158156e+00 1.28211233e+01
 1.62235408e+01 1.98845163e+01 2.37565983e+01 2.77879262e+01
 3.19740943e+01 3.62518250e+01 4.05889302e+01 4.49426484e+01
 4.93452467e+01 5.36755275e+01 5.80088724e+01 6.22149129e+01
 6.63595282e+01 7.04025183e+01 7.42813554e+01 7.80620348e+01
 8.16740621e+01 8.51178280e+01 8.83787487e+01 9.14850842e+01
 9.43809345e+01 9.71158807e+01 9.96392256e+01 1.01987242e+02
 1.04126868e+02 1.06099368e+02 1.07863580e+02 1.09449767e+02
 1.10846864e+02 1.12110236e+02 1.13191254e+02 1.14074761e+02
 1.14846130e+02 1.15433601e+02 1.15890958e+02 1.16231386e+02
 1.16433923e+02 1.16536626e+02 1.16505331e+02 1.16400383e+02
 1.16196378e+02 1.15906672e+02 1.15545767e+02 1.15109719e+02
 1.14611515e+02 1.14090843e+02 1.13495219e+02 1.12896043e+02
 1.12231729e+02 1.11569337e+02 1.10885842e+02 1.10160722e+02
 1.09446525e+02 1.08749915e+02 1.08030607e+02 1.07323023e+02
 1.06629687e+02 1.05940358e+02 1.05294717e+02 1.04635849e+02
 1.04029978e+02 1.03416756e+02 1.02865032e+02 1.02309918e+02
 1.01761229e+02 1.01305523e+02 1.00865515e+02 1.00426219e+02
 1.00030543e+02 9.96803477e+01 9.93603009e+01 9.90535205e+01
 9.87777992e+01 9.85418486e+01 9.83305846e+01 9.81400738e+01
 9.79890875e+01 9.78538868e+01 9.77544362e+01 9.76703103e+01
 9.75884971e+01 9.75566577e+01 9.75285175e+01 9.75384638e+01
 9.75280291e+01 9.75781920e+01 9.76324339e+01 9.76812782e+01
 9.77573703e+01 9.78194510e+01 9.78993251e+01 9.79790282e+01
 9.81228193e+01 9.82037730e+01 9.83201852e+01 9.84268126e+01
 9.85318456e+01 9.86482278e+01 9.87425081e+01 9.88738803e+01
 9.89950487e+01 9.91065383e+01 9.92322134e+01 9.93386908e+01
 9.94351780e+01 9.95310204e+01 9.96494612e+01 9.97354541e+01
 9.98297907e+01 9.99154627e+01 9.99915534e+01 1.00075722e+02
 1.00141631e+02 1.00205540e+02 1.00255063e+02 1.00321681e+02
 1.00381004e+02 1.00413358e+02 1.00462223e+02 1.00500608e+02
 1.00559029e+02 1.00545582e+02 1.00573522e+02 1.00589997e+02
 1.00593837e+02 1.00622215e+02 1.00631397e+02 1.00629055e+02
 1.00633758e+02 1.00631924e+02 1.00626102e+02 1.00614421e+02
 1.00607470e+02 1.00595287e+02 1.00599540e+02 1.00578887e+02
 1.00567722e+02 1.00541616e+02 1.00510222e+02 1.00514888e+02
 1.00502569e+02 1.00489229e+02]
```

Array U :

```
[100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100.
 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100.]
```

```
100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100.
100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100.
100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100.
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100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100.
100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100.
100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100. 100.]
```

Obter os Parâmetros Estimados

In [9]:

```
o = (inv(X.T @ X)@(X.T))@Y
```

In [10]:

```
print("Parâmetros O (Estimados) : ")
print("\n")
print(o)
print("\n")
```

Parâmetros O (Estimados) :

```
[ 1.91655131 -0.9227192   0.00302465  0.00315749]
```

Obter a Soma do Quadrado dos Erros

In [11]:

```
e = (Y-X@(o.T))@((Y-X@(o.T)).T)
```

In [12]:

```
print("Soma do Quadrado dos Erros : ")
print("\n")
print(e)
print("\n")
```

Soma do Quadrado dos Erros :

```
0.07806661461073164
```

Comparando os Dados Preditos com os Valores Corretos

In [13]:

```
Comparar = pd.DataFrame({'Correto':Y, 'Predito':X@(o.T)})
```

In [14]:

```
print("Data Frame para Comparar:")
print("\n")
print(Comparar)
```

Data Frame para Comparar:

	Correto	Predito
0	0.002073	0.000000
1	0.306438	0.306438
2	1.199850	1.203605
3	2.663541	2.635032
4	4.581206	4.615903
5	6.955412	6.940630
6	9.721582	9.721451
7	12.821123	12.832233
8	16.223541	16.220265
9	19.884516	19.881166
10	23.756598	23.758138
11	27.787926	27.801129
12	31.974094	31.954531
13	36.251825	36.257754
14	40.588930	40.593586
15	44.942648	44.958727
16	49.345247	49.300921
17	53.675527	53.721467
18	58.008872	57.958310
19	62.214913	62.267755
20	66.359528	66.330387
21	70.402518	70.392760
22	74.281355	74.317042
23	78.062035	78.020488
24	81.674062	81.687277
25	85.117828	85.121407
26	88.378749	88.388674
27	91.485084	91.460767
28	94.380935	94.405304
29	97.115881	97.089074
..
120	100.141631	100.154345
121	100.205540	100.202998
122	100.255063	100.264667
123	100.321681	100.300612
124	100.381004	100.382591
125	100.413358	100.434818
126	100.462223	100.442088
127	100.500608	100.505887
128	100.559029	100.534363
129	100.545582	100.610912
130	100.573522	100.531234
131	100.589997	100.597191
132	100.593837	100.602985
133	100.622215	100.595142
134	100.631397	100.645987
135	100.629055	100.637401
136	100.633758	100.624439
137	100.631924	100.635613
138	100.626102	100.627760
139	100.614421	100.618294
140	100.607470	100.601277
141	100.595287	100.598735
142	100.599540	100.581800
143	100.578887	100.601191
144	100.567722	100.557686
145	100.541616	100.555343
146	100.510222	100.515613
147	100.514888	100.479531
148	100.502569	100.517443
149	100.489229	100.489527

[150 rows x 2 columns]

Gráfico com os Valores Preditos e Corretos no Decorrer do Tempo

In [15]:

```
T = 0.25
aux = 0
Amostragem = []
```

```

Amostragem = []
for i in range(len(Y)):
    aux += T
    Amostragem.append(aux)

plt.plot(Amostragem,Y,color='orange',label='Correto')
plt.step(Amostragem,X@(0.T),color='blue',label='Predito')
plt.title('Estabilização da Potência')
plt.legend(loc='center right',fontsize=13)
plt.xlabel('Tempo')
plt.ylabel('Potência do Motor')
plt.grid(True)
plt.show()

```

