Credit Markets – Homework 5
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Problem 2. a)

For the premium leg,

$$PV_{CDS\_PL} = \sum_{k=1}^{\infty} \frac{c}{4} * e^{-k*\frac{(r+h)}{4}}$$

If $k = 0$,

$$\frac{c}{4} * e^{-k*\frac{(r+h)}{4}} = \frac{c}{4} * e^{-0*\frac{(r+h)}{4}} = \frac{c}{4}$$

Therefore,

$$PV_{CDS\_PL} = \sum_{k=1}^{\infty} \frac{c}{4} * e^{-k*\frac{(r+h)}{4}} = \sum_{k=0}^{\infty} \frac{c}{4} * e^{-k*\frac{(r+h)}{4}} - \frac{c}{4}$$

We have the geometric sum formula:

$$\sum_{k=0}^{\infty} a * b^k = a + ab + ab^2 + \cdots = \frac{a}{1-b}$$

Applying it here:

$$\sum_{k=0}^{\infty} \frac{c}{4} * e^{-k*\frac{(r+h)}{4}} = \frac{c}{4} + \frac{c}{4} e^{-\frac{(r+h)}{4}} + \frac{c}{4} * e^{-2\frac{(r+h)}{4}} + \frac{c}{4} * e^{-3\frac{(r+h)}{4}} + \cdots$$

Therefore,

$$a = \frac{c}{4} \quad , \quad b = e^{-\frac{(r+h)}{4}}$$

$$\sum_{k=0}^{\infty} \frac{c}{4} * e^{-k*\frac{(r+h)}{4}} = \frac{a}{1-r} = \frac{\frac{c}{4}}{1 - e^{-\frac{r+h}{4}}}$$

$$PV_{CDS\_PL} = \frac{\frac{c}{4}}{1 - e^{-\frac{r+h}{4}}} - \frac{c}{4}$$

For the default leg,

$$PV_{CDS\_DL} = \frac{(1-R) * h}{r+h} * \left(1 - e^{-T(r+h)}\right)$$

For $T = \infty$,

$$PV_{CDS\_DL} = \frac{(1-R) * h}{r+h} * \left(1 - \frac{1}{e^{\infty}}\right)$$

$$PV_{CDS\_DL} = \frac{(1-R) * h}{r+h}$$

Problem 2. d)

From Page 22 in Lecture 4

$$SP(t,s) = \exp\left\{-\int_t^s h(t,u)du\right\}$$

$$\int_t^s h(t,u)du = h(s-t)$$

$$SP(t,s) = e^{-h(s-t)}$$

Intuition:

- *If there are originally 100 people in a boat*
- *Only 70 people survived from time 0 to time T* → $SP(0,T) = 70\%$
- *Only 30 people survived from time 0 to time T+10* → $SP(0,T+10) = 30\%$
- *There are 40 people that died from time T to time T+10* → $DP(T,T+10) = 40\%$

We know that:

$$DP(T,T+10) = SP(0,T) - SP(0,T+10)$$

$$DP(T,T+10) = 10\%$$

Therefore,

$$DP(T,T+10) = SP(0,T) - SP(0,T+10)$$

$$DP(T,T+10) = e^{-hT} - e^{-h(T+10)} = e^{-hT}(1-e^{-10h})$$

$$0.1 = e^{-hT}(1-e^{-10h})$$

$$e^{hT} = 10 - 10e^{-10h}$$

$$hT = \ln(10-10e^{-10h})$$

$$T = \frac{1}{h}\ln(10-10e^{-10h})$$