Credit Markets – Homework 8
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Problem 2. a) Convertible Bond

Merton Model (L7. Page 19)

$$d_{\pm} = \frac{\ln\left(\frac{A_0}{K}\right) + (r \pm \sigma_A^2) * T}{\sigma_A \sqrt{T}}$$

Fair Value of Risky Bond

$$B_0 = A_0 * \Phi(-d_+) + e^{-rT} * K * \Phi(d_-)$$

At maturity T, the convertible bondholder will receive the greater of:

- Conversion value: $C * A_T$
- Non-conversion payoff, the minimum of the liability or the asset at time T:

$$\min(K, A_T)$$

Therefore, the payoff will be:

$$Payoff(A_T) = \begin{cases} A_T\,, & A_T \leq K \\ K, & K < A_T < \dfrac{K}{C} \\ C * A_T, & A_T C \geq K \end{cases}$$

Only receive $C * A_T$ when

$$A_T C \geq K$$
$$A_T \geq \frac{K}{C}$$

$$CB_0 = B_0 + Conversion\ Option$$

Value of the Conversion Option is similar to a call option with strike:

$$\frac{K}{C},$$

$$\tilde{d}_{\pm} = \frac{\ln\left(\frac{C * A_0}{K}\right) + \left(r \pm \frac{1}{2}\sigma_A^2\right) * T}{\sigma_A \sqrt{T}}$$

$$Conversion\ Option = C * A_0 * \Phi\left(\tilde{d}_+\right) - e^{-rT} * K * \Phi\left(\tilde{d}_-\right)$$

Fair Value of Convertible Bond at time-0

$$CB_0 = A_0 * \Phi(-d_+) + e^{-rT} * K * \Phi(d_-) + C * A_0 * \Phi\left(\tilde{d}_+\right) - e^{-rT} * K * \Phi\left(\tilde{d}_-\right)$$

It is economical for the convertible bond holder to exercise the call option when:

$$A_T \geq \frac{K}{C}$$

Problem 2.b) Convertible Equity Value

When a convertible bondholder exercises the call option, they dilute the existing equity.

If the firm's total asset at time T is A_T, and the bondholders converts $C * A_T$, the remaining portion is $(1 - C) * A_T$ to be enjoyed by the original equity holders.

Convertible Equity Value will be E_0 minus the expected loss from conversion:

$$CE_0 = E_0 - (Expected\ Dilution\ from\ Conversion)$$
$$CE_0 = E_0 - (Conversion\ Option)$$

$$\tilde{d}_{\pm} = \frac{\ln\left(\frac{C * A_0}{K}\right) + \left(r \pm \frac{1}{2}\sigma_A^2\right) * T}{\sigma_A \sqrt{T}}$$

From Lecture 7, page 19:

$$E_0 = A_0 * \Phi(d_+) - e^{-rT} * K * \Phi(d_-)$$

Fair Value of Convertible Equity at time-0

$$EB_0 = E_0 - \left\{C * A_0 * \Phi\left(\tilde{d}_+\right) - e^{-rT} * K * \Phi\left(\tilde{d}_-\right)\right\}$$

$$EB_0 = A_0 * \Phi(d_+) - e^{-rT} * K * \Phi(d_-) - \left\{C * A_0 * \Phi\left(\tilde{d}_+\right) - e^{-rT} * K * \Phi\left(\tilde{d}_-\right)\right\}$$

Problem 4.b)

$$B_0 = 1 + \left(\frac{\frac{C}{2} - \left(e^{\frac{y}{2}} - 1\right)}{\left(e^{\frac{y}{2}} - 1\right)}\right) * (1 - e^{-Ty})$$

For zero-coupon bond,

$$B_0 = 1 + \left(\frac{0 - \left(e^{\frac{y}{2}} - 1\right)}{\left(e^{\frac{y}{2}} - 1\right)}\right) * (1 - e^{-Ty})$$
$$B_0 = 1 + (-1) * (1 - e^{-Ty})$$
$$B_0 = 1 - 1 + e^{-Ty}$$

$$B_0 = e^{-Ty}$$

Problem 4.d)

Geometric Sum formula:

$$\sum_{i=0}^{n} ar^i = a + ar + ar^2 + \cdots = \frac{a(1-r^n)}{1-r}$$

<u>Present Value of Bond</u>

$$B_0 = \sum_{k=1}^{2T} \frac{c}{2} e^{-k*\frac{y}{2}} + e^{-Ty}$$

<u>Present Value of Interest-Only (IO) Bond</u>

$$B_0 = \sum_{k=1}^{2T} \frac{c}{2} e^{-k*\frac{y}{2}}$$

$$a = e^{-\frac{y}{2}} = r$$
$$n = 2T$$

$$\sum_{k=1}^{2T} e^{-k*\frac{y}{2}} = \frac{e^{-\frac{y}{2}}(1-r^n)}{1-r}$$

$$\sum_{k=1}^{2T} e^{-k*\frac{y}{2}} = \frac{e^{-\frac{y}{2}}\left(1-\left(e^{-\frac{y}{2}}\right)^{2T}\right)}{1-e^{-\frac{y}{2}}} = \frac{e^{-\frac{y}{2}}(1-e^{-yT})}{1-e^{-\frac{y}{2}}}$$

$$B_0 = \sum_{k=1}^{2T} \frac{c}{2} e^{-k*\frac{y}{2}}$$

$$B_0 = \frac{c}{2} * \frac{e^{-\frac{y}{2}}(1-e^{-yT})}{1-e^{-\frac{y}{2}}}$$