FINM 33000: Homework 1

Due Thursday October 10, 2024

Problem 1

(a) Pay the lower interest rate and collect the higher interest rate:

Portfolio $(-1B, +1B^*)$ has time-0 price $-1 + 1 = 0$ and time-T price $e^{r^*T} - e^{rT} > 0$, and is therefore a type-1 arb.

(b) Call price is too low, violating L1.27, so buy the call and sell the synthetic forward contract:

Portfolio $(+1C, -1S, +110Z)$ has time-0 price $0.5 - 100 + 0.9 \times 110 = -0.5 < 0$ and time-T payoff ≥ 0, and is therefore a type-2 arb.

(c) G's payoff is the same as a the payoff from "writing a covered call" (long S, short C), but G is priced too high.

Portfolio $(-1G, -1C, +1S)$ has time-0 price $-85 - 20 + 100 < 0$ and time-T payoff $= 0$, and is therefore a type-2 arb.

(d) The $(20, 22.5)$ call spread violates the upper bound in L1.28.

Portfolio $(-1C(20), +1C(22.5), 2.5Z)$ has time-0 price $-6.4 + 3.1 + 2.5 \times 0.9 < 0$ and time-T payoff ≥ 0, and is therefore a type-2 arb.

(e) For all S_T we have $-2\log(S_T/100) \geq -0.02(S_T - 100)$ because these two payoff functions are tangent to each other at $S_T = 100$ and the log payoff is strictly greater everywhere else.

Portfolio $(+1X, +0.02Y)$ has time-0 price $0.2 + 0.02 \times (-10) = 0$ but time-T payoff ≥ 0 with positive probability of being > 0, and is therefore a type-1 arbitrage.

Problem 2

(a) `PA.Trump` pays 1 in every scenario where `US.Trump` pays 1 (and also in some scenarios where `US.Trump` pays 0). So the `PA.Trump` payoff dominates the `US.Trump` payoff, but the time-0 prices are inconsistent with the payoff relationship. Specifically

$$(+1 \text{ unit of } \texttt{PA.Trump},\ -1 \text{ unit of } \texttt{US.Trump})$$

has time-0 price $0.16 - 0.17 = -0.01$ but time-T payoff ≥ 0 and is therefore a type-2 arb.

Alternatively, could use US.Biden contracts: $(-1$ unit of `PA.Biden`, $+1$ unit of `US.Biden`$)$

(b) `US.Trump` + `US.Biden` $= 1$ at times 0 and T. So modify the (a) answer by replacing the disallowed -1 `US.Trump` with the equivalent combination $+1$ `US.Biden` and -1 bank:

$$(+1 \text{ unit of } \texttt{PA.Trump},\ +1 \text{ unit of } \texttt{US.Biden},\ -1 \text{ unit of bank account})$$