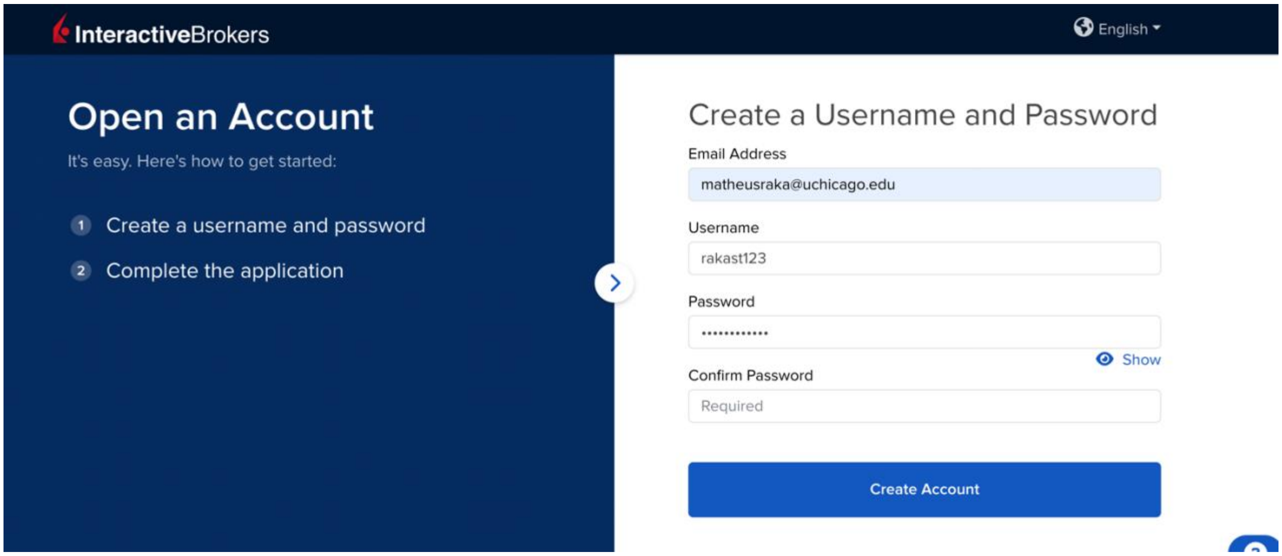
Problem 0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2 Complete the application
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rakast123
Password
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Confirm Password
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Homework 6 Matheus Rakha Prasetyatama

Problem 1

$S_0 = 40,\ K = 30,\ \sigma = 0.22,\ r = 0.05$

$F = Se^{r\cdot(T-t)} = 40\cdot e^{0.05(1)} = 42.0508$

$$d_1 = \frac{\ln(F/K)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2} = \frac{\ln(42.0508/30)}{0.22} + \frac{0.22\sqrt{1}}{2} = 1.6449$$

$$d_2 = \frac{\ln(42.0508/30)}{0.22\sqrt{1}} - \frac{\sigma\sqrt{T-t}}{2} = 1.4249$$

$N(d_1) = N(1.6449) = 0.95$, $N(d_2) = N(1.4249) = 0.9229$

$$c^{BS}(S,t) = e^{-r(T-t)}(F\cdot N(d_1) - K\cdot N(d_2))$$

$$= e^{-0.05(1)}(42.0508 \times 0.95 - 30 \times 0.9229)$$

$c^{BS}(S,t) = 11.6633 \rightarrow$ Call option value

Because we are spending \$3,000 for this call option

$$\text{time-0 value} = \frac{3000}{11.6633} - \frac{3000}{e^{r\cdot T}} =$$

$$= 257.2171 - \frac{3000}{e^{0.05}}$$

$$= -2,596.5$$

Problem 1b)

Going long on a put option will give you the right, not the obligation, to sell the stock at the strike price. Going short on a put option means you have to buy the stock at the strike price.

The $\min(30, 0.75 * S_1)$ mirrors the payoff if you go long on a call option with strike price of $30 and if you go short on a put option with strike price $30.

- You will be able to purchase the stock at $30 = Payoff of a Call Option with Strike $30
- At most, you will buy the stock at $30 = Payoff of a Put Option with Strike $30

To perfectly hedge this risk, we can take the opposite direction of those options. We should go short on the call option with $30-strike price and go long on a call option with a $30-strike price.

And yes, this position needs to be rebalanced over the course of the year because Delta, Gamma, Vega, and Theta of the call and put options, all change as time progresses.

Problem 1c)

Due to the nature of $\min\,(30, 0.75 * S_1)$, we are exposed to the delta of the option.

From the put-call parity:

$$C_0 = P_0 + S_0 - K * Z_0$$
$$\frac{\partial C}{\partial S} = \frac{\partial P}{\partial S} + 1 - 0 = \frac{\partial P}{\partial S} + 1$$
$$\Delta Call = \Delta Put + 1$$
$$\Delta Put = \Delta Call - 1$$

Delta of the stock purchase plan is the leftover delta from the call aspect and put aspect of the purchase plan.

$$\Delta plan = \Delta Call - \Delta Put = \Delta Call - \Delta Call + 1 = 1$$

This means that we need to hold 1 share of Stark stock for every stock purchase plan.

Since we have $3,000 to be spent on the Stark stock,

Position to perfectly hedge risk = $\Delta plan * 3000 = 1 * 3000 = \$3{,}000$

A 3,000 long position on the Stark stock to perfectly hedge the risk.

Problem 2

a) V_S = Value of all stocks being held in the portfolio

$$V_S = \left(\beta \frac{L_t}{S_t}\right) \cdot S_t = \beta L_t$$

V_B = Value of portfolio being held in the bank account

$$V_B = L_t - V_S = L_t - \beta L_t = (1-\beta) L_t$$

$$V_B = (\text{units of bank account}) \cdot B_t$$

$$\text{units of bank account} = \frac{V_B}{B_t} = \frac{(1-\beta)L_t}{e^{r \cdot t}}$$

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b) Self financing: dL_t = (change in stock value) + (change in bank account value)

$$\text{Change in stock value} = \left(\beta \cdot \frac{L_t}{S_t}\right) dS_t = \beta \cdot \frac{L_t}{S_t}\left(\mu S_t \cdot dt + \sigma S_t \, dW_t\right)$$

$$= \beta L_t \cdot \mu dt + \beta \cdot L_t \cdot \sigma dW_t$$

$$\text{Change in bank account value} = \left(\frac{(1-\beta)L_t}{e^{r \cdot t}}\right) dB_t = \frac{(1-\beta)L_t}{e^{r \cdot t}} \cdot r \cdot e^{r \cdot t} \cdot dt$$

$$= (1-\beta)L_t \cdot r \, dt$$

$$dL_t = \beta L_t \cdot \mu dt + \beta L_t \cdot \sigma dW_t + (1-\beta)L_t \cdot r dt$$

$$dL_t = L_t\{\beta \cdot \mu + (1-\beta) r\} dt + \beta L_t \cdot \sigma dW_t$$

$dL_t = \mu_L L_t \, dt + \sigma_L L_t \cdot dW_t \rightarrow L$ is a geometric Brownian motion

drift term: $\mu_L = \beta\mu + (1-\beta) r$ //

Volatility: $\beta\sigma$ //

Problem 3

a) $C_t = P_t + S_t - k \cdot z_0$, $z_0 = e^{-r(T-t)}$ (4.19) (4.31)

$C_t = P_t + S_t - k \cdot e^{-r(T-t)}$

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b) $N(x) + N(-x) = 1$

$N(x) = 1 - N(-x)$

$N(x) - 1 = -N(-x)$

$C_t = S_t N(d_1) - k e^{-r(T-t)} N(d_2)$

$P_t + S_t - k e^{-r(T-t)} = S_t \cdot N(d_1) - k e^{-r(T-t)} N(d_2)$

$P_t = S_t \cdot N(d_1) - S_t - k e^{-r(T-t)} \{N(d_2) - 1\}$

$= \{N(d_1) - 1\} S_t - k e^{-r(T-t)} \{N(d_2) - 1\}$

$P_t = -N(-d_1) \cdot S_t + k e^{-r(T-t)} \cdot N(-d_2)$

$P_t = k e^{-r(T-t)} N(-d_2) - S_t \cdot N(-d_1)$

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Problem 3 ~~b~~ c)

$$\frac{\partial P^{BS}}{\partial S} = Ke^{-r(T-t)}\cancel{N'(-d_2)\frac{\partial(-d_2)}{\partial S}} - \cancel{S_t \cdot N'(-d_1)\cdot\frac{\partial(-d_1)}{\partial S}} - N(-d_1)$$

$$\frac{\partial P^{BS}}{\partial S} = -N(-d_1)$$ $\rightsquigarrow$ time-t delta of that put

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Problem 3d)

$$d_1 = \frac{\log(S e^{r(T-t)}/K)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2} = \frac{\log(S)}{\sigma\sqrt{T-t}} + \frac{\log(e^{r(T-t)}/K)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2}$$

$$\frac{\partial(d_1)}{\partial S} = \frac{1}{S_t\sigma\sqrt{T-t}} \quad \left\{ \frac{\partial(-d_1)}{\partial S} = -\frac{\partial(d_1)}{\partial S} = -\frac{1}{S_t\sigma\sqrt{T-t}}\right.$$

$$\frac{\partial^2 P}{\partial S} = -N'(-d_1)\cdot\frac{\partial(-d_1)}{\partial S} = -N'(-d_1)\cdot\left\{-\frac{1}{S_t\sigma\sqrt{T-t}}\right\} = \frac{N'(-d_1)}{S_t\sigma\sqrt{T-t}}$$

Since $N'(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$, $N'(d_1) = N'(-d_1)$

$\hookrightarrow$ Gamma of the put $= \dfrac{N'(d_1)}{S_t\sigma\sqrt{T-t}} = \dfrac{1}{S_t\sigma\sqrt{T-t}}\cdot\dfrac{1}{\sqrt{2\pi}}e^{-(d_1)^2/2}$

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