Aproximação Polinomial

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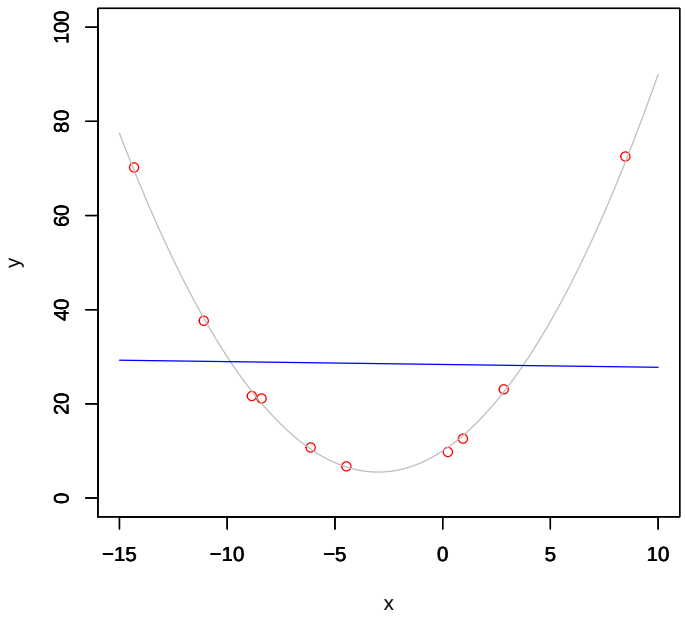
Parametrização:

```
>   rm(list=ls())
>   library('corpcor')
>   xmin<--15
>   xmax<-10
>   xstep<-0.1
>   ymin<-0
>   ymax<-100
>   a1<-0.5
>   a2<-3
>   a3<-10
```

1 Aproximação Polinomial - 10 amostras

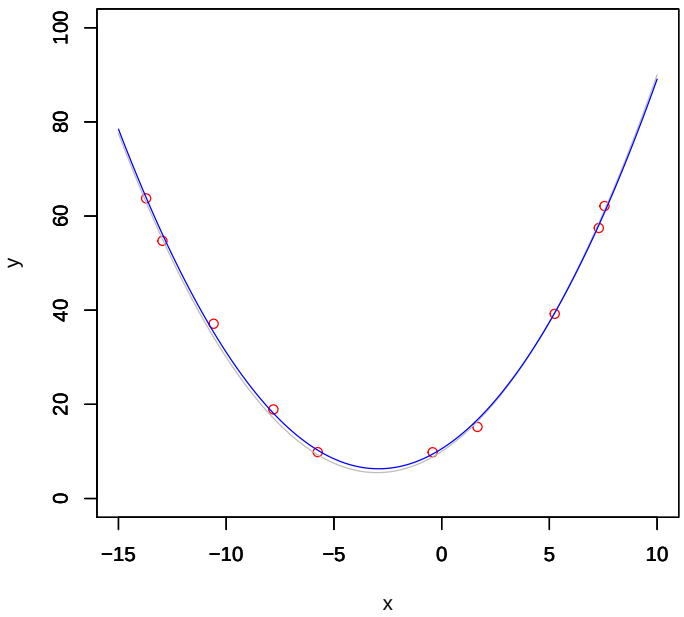
1.1 Polinômio Grau 1 - 10 amostras

```
>   N<-10
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 1
>   H<-cbind(x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x',y
```



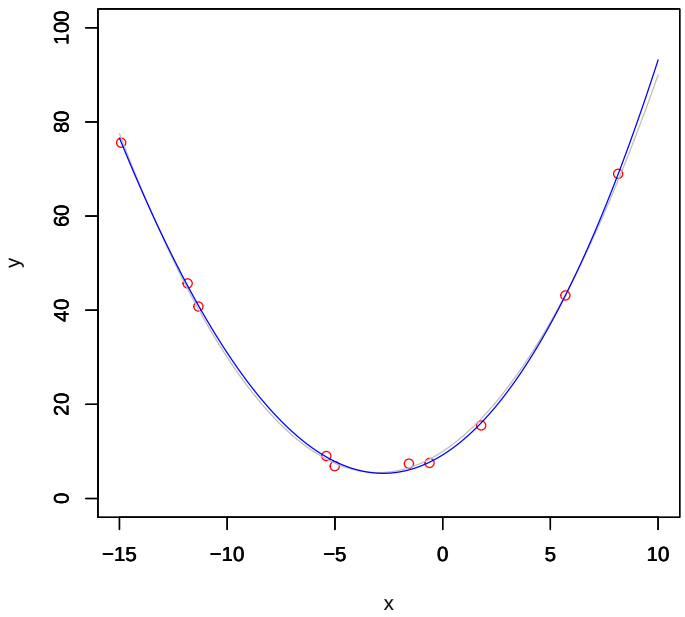
1.2 Polinômio Grau 2 - 10 amostras

```
>   N<-10
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 2
>   H<-cbind(x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x',y
```



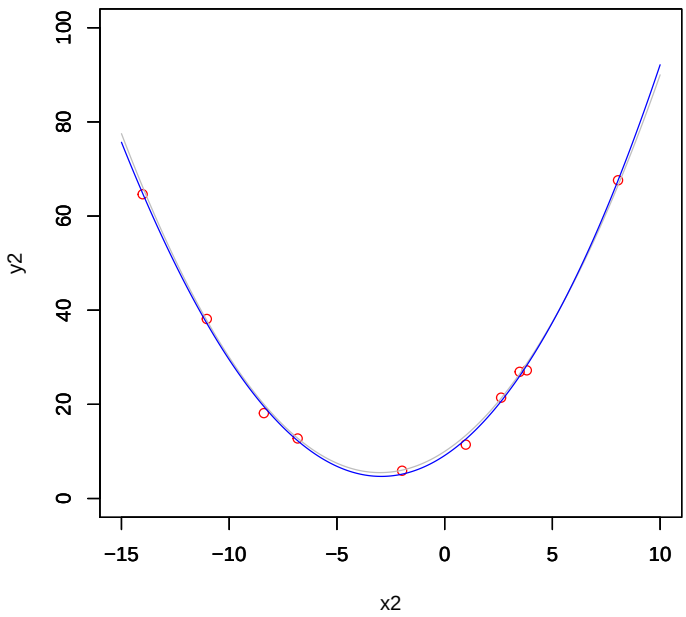
1.3 Polinômio Grau 3 - 10 amostras

```
>   N<-10
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 3
>   H<-cbind(x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x',y
```



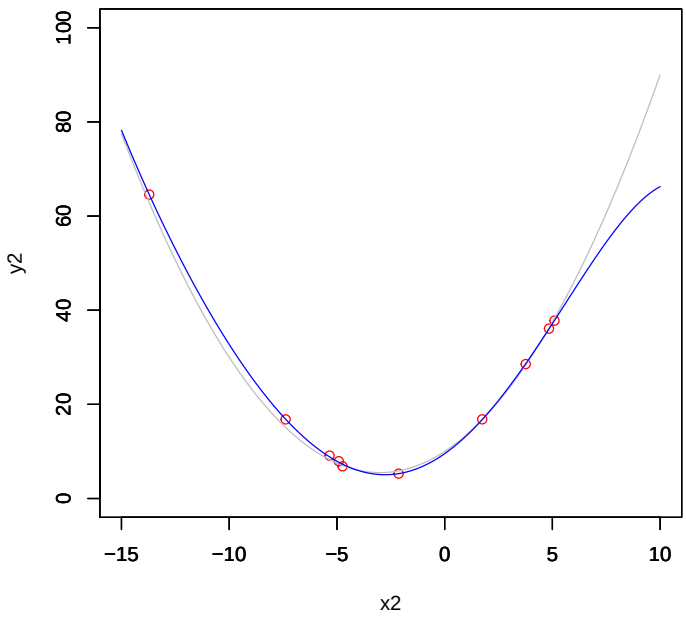
1.4 Polinômio Grau 4 - 10 amostras

```
>   N<-10
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 4
>   H<-cbind(x^4,x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^4,xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x2',
```



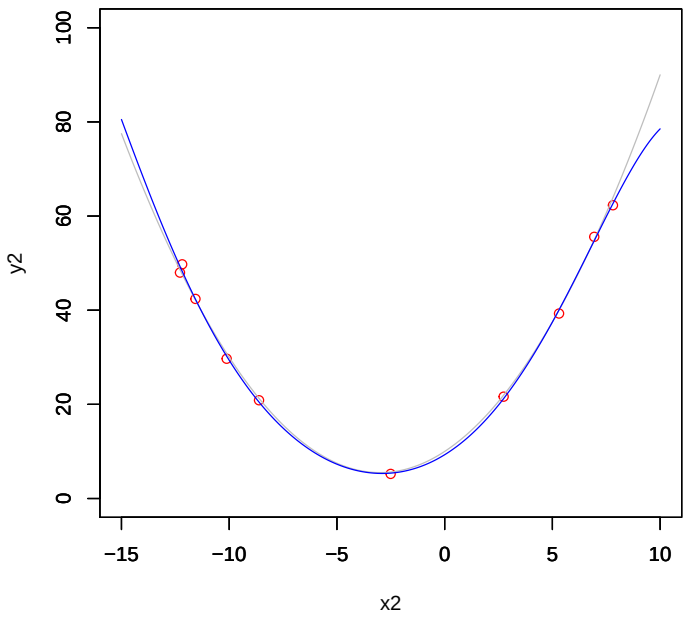
1.5 Polinômio Grau 5 - 10 amostras

```
>   N<-10
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 5
>   H<-cbind(x^5,x^4,x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^5,xgrid^4,xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x2',
```



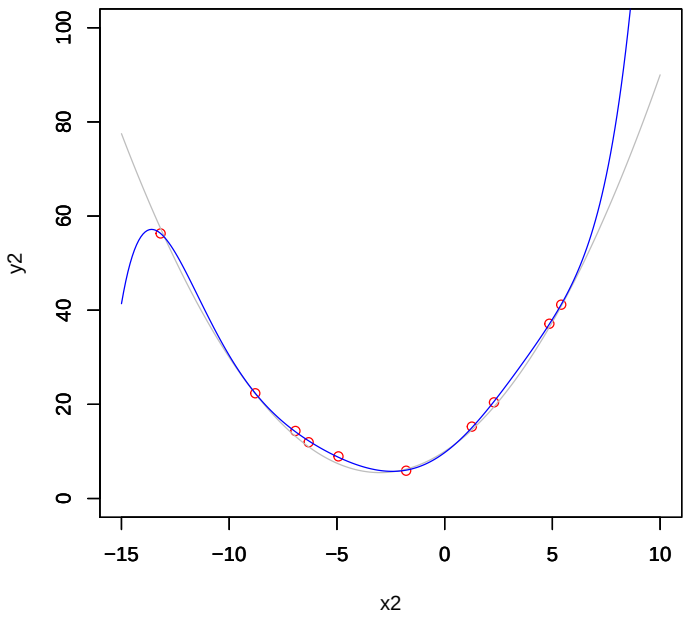
1.6 Polinômio Grau 6 - 10 amostras

```
>   N<-10
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 6
>   H<-cbind(x^6,x^5,x^4,x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^6,xgrid^5,xgrid^4,xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x2',
```



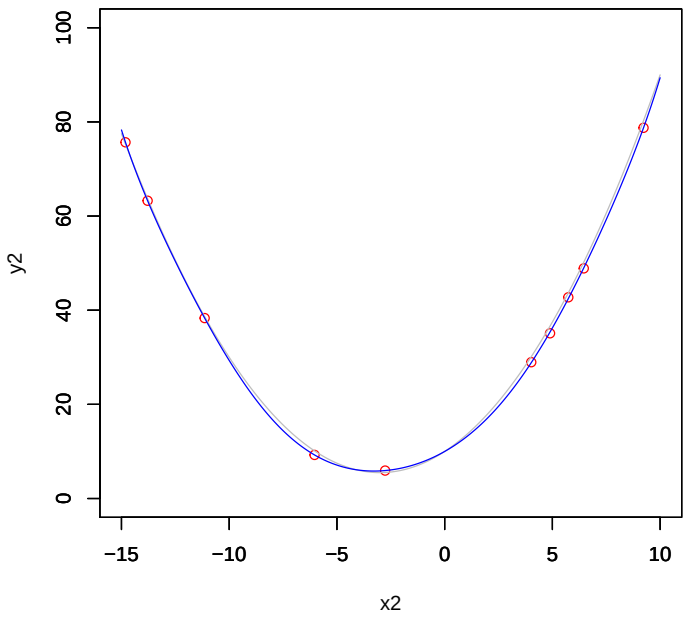
1.7 Polinômio Grau 7 - 10 amostras

```
>   N<-10
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 7
>   H<-cbind(x^7,x^6,x^5,x^4,x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^7,xgrid^6,xgrid^5,xgrid^4,xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x2',
```



1.8 Polinômio Grau 8 - 10 amostras

```
>   N<-10
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 7
>   H<-cbind(x^8,x^7,x^6,x^5,x^4,x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^8,xgrid^7,xgrid^6,xgrid^5,xgrid^4,xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x2',
```



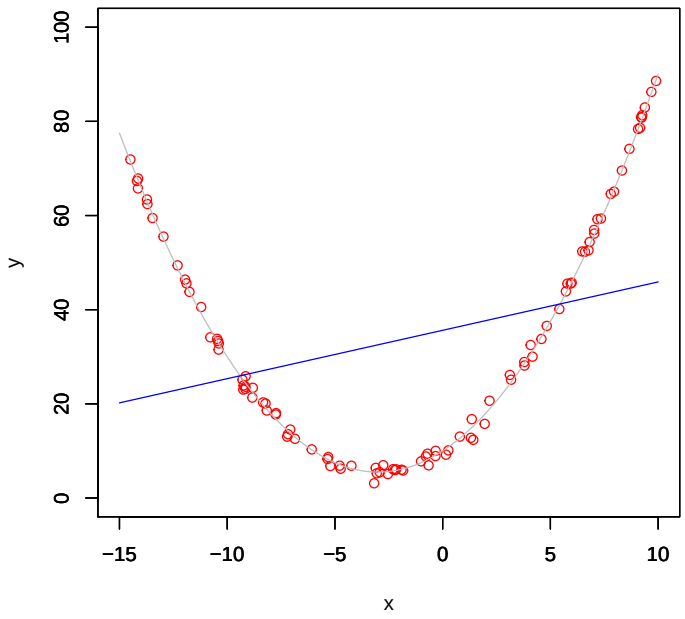
2 Overfitting e Underfitting

1. *Underfitting* ocorreu no polinômio de grau 1.
2. *Overfitting* occorreu nos polinômios de grau 7 e 8, principalmente no de grau 8.

3 Aproximação Polinomial - 100 amostras

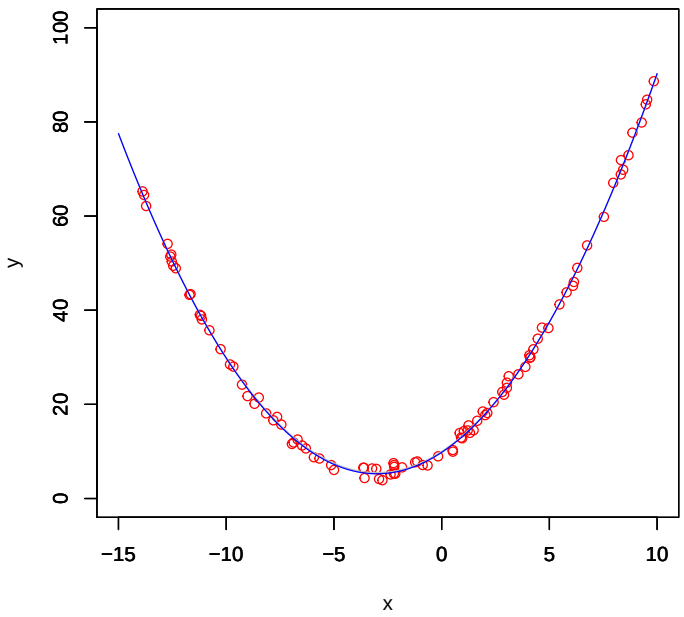
3.1 Polinômio Grau 1 - 100 amostras

```
>   N<-100
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 1
>   H<-cbind(x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x',y
```



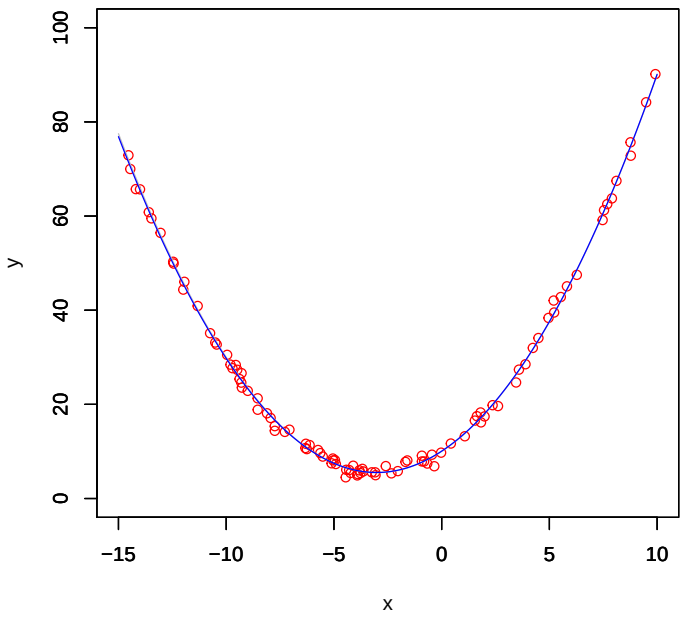
3.2 Polinômio Grau 2 - 100 amostras

```
>   N<-100
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 2
>   H<-cbind(x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x',y
```



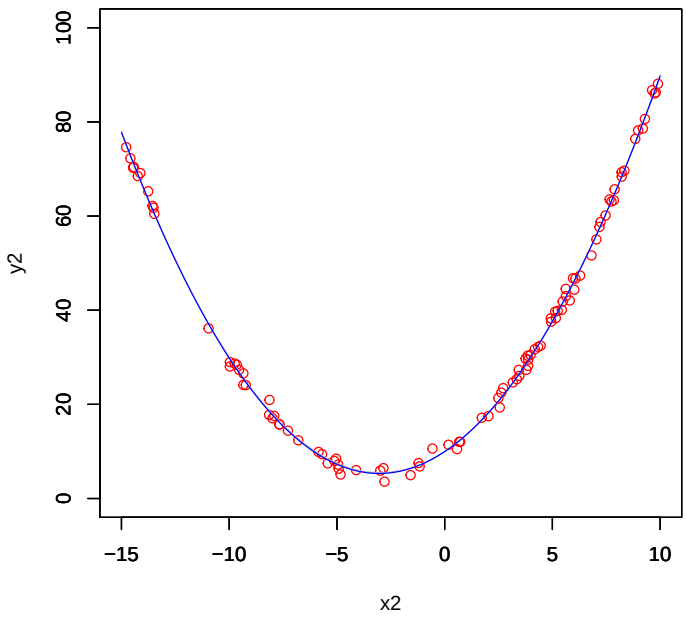
3.3 Polinômio Grau 3 - 100 amostras

```
>   N<-100
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 3
>   H<-cbind(x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x',y
```



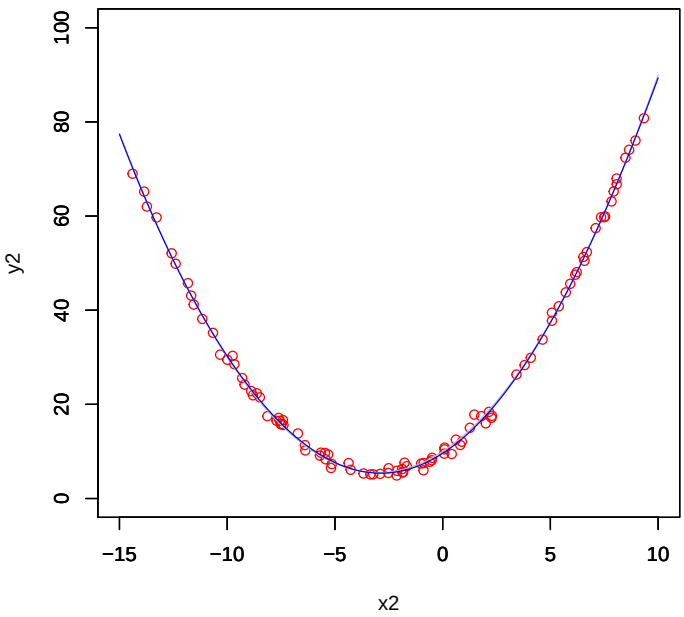
3.4 Polinômio Grau 4 - 100 amostras

```
>   N<-100
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 4
>   H<-cbind(x^4,x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^4,xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x2',
```



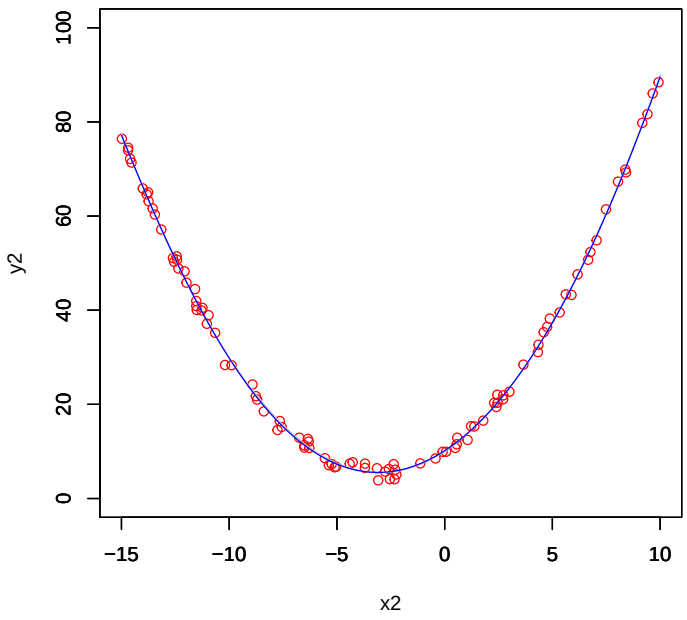
3.5 Polinômio Grau 5 - 100 amostras

```
>   N<-100
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 5
>   H<-cbind(x^5,x^4,x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^5,xgrid^4,xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x2',
```



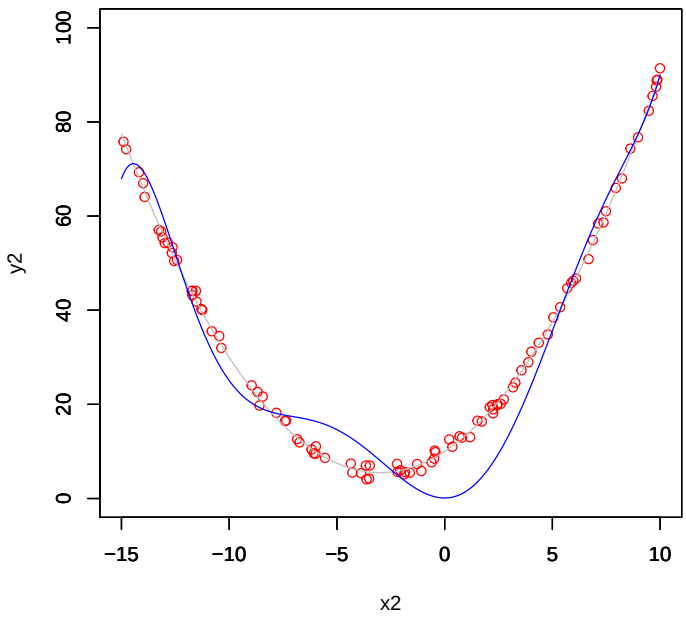
3.6 Polinômio Grau 6 - 100 amostras

```
>   N<-100
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 6
>   H<-cbind(x^6,x^5,x^4,x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^6,xgrid^5,xgrid^4,xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x2',
```



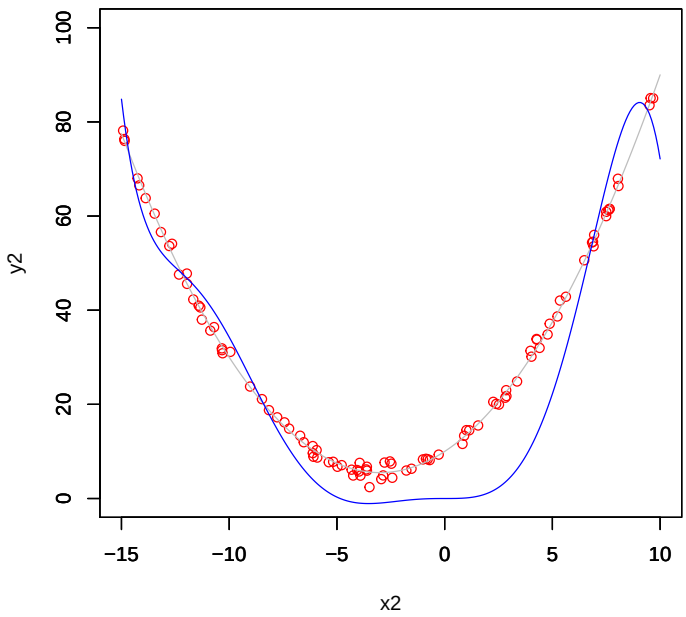
3.7 Polinômio Grau 7 - 100 amostras

```
>   N<-100
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 7
>   H<-cbind(x^7,x^6,x^5,x^4,x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^7,xgrid^6,xgrid^5,xgrid^4,xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x2',
```



3.8 Polinômio Grau 8 - 100 amostras

```
>   N<-100
>   x<-runif(n=N, min=xmin,max=xmax)
>   xgrid<-seq(xmin,xmax,xstep)
>   yr<-(a1*x^2+a2*x+a3)+rnorm(length(x))
>   ygrid<-(a1*xgrid^2+a2*xgrid+a3)
>   plot(x,yr,col='red',type='p', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab='')
>   par(new=T)
>   plot(xgrid,ygrid,col='gray',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='',ylab=
>   par(new=T)
>   #aproximacao de grau 7
>   H<-cbind(x^8,x^7,x^6,x^5,x^4,x^3,x^2,x,1)
>   w<-pseudoinverse(H) %*% yr
>   Hgrid<-cbind(xgrid^8,xgrid^7,xgrid^6,xgrid^5,xgrid^4,xgrid^3,xgrid^2,xgrid,1)
>   yhat<-H%*%w
>   yhatgrid<-Hgrid%*%w
>   plot(xgrid,yhatgrid,col='blue',type='l', xlim=c(xmin,xmax), ylim=c(ymin,ymax),xlab='x2',
```



4 Aproximadores Polinomiais e Redes Neurais Artificiais

Enquanto a aproximação polinomial pode ser representada pela equação matricial:

$$\boldsymbol{w} = \boldsymbol{H}^{+}\boldsymbol{y}$$

Uma Rede Neural Artificial pode ser representada pela equação:

$$f(\mathtt{x}, \mathtt{z}_1, ..., \mathtt{z}_p, w_1, ..., w_p) = \phi(\sum_{i=1}^{p} h_i(\mathtt{x}, \mathtt{z}_i)w_i + \beta)$$