Universidade Federal de Uberlândia
Faculdade de Computação

Trabalho de Modelagem e Simulação
Prática 04

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Sumário

Lista de Figuras

Exercício 4.1.7

a) Generate an Exponential(9) random variate sample of size n = 100 and compute the proportion of points in the sample that fall within the intervals $\bar{x}$ $\pm$2s and $\bar{x}$ $\pm$3s. Do this for 10 different rngs streams.

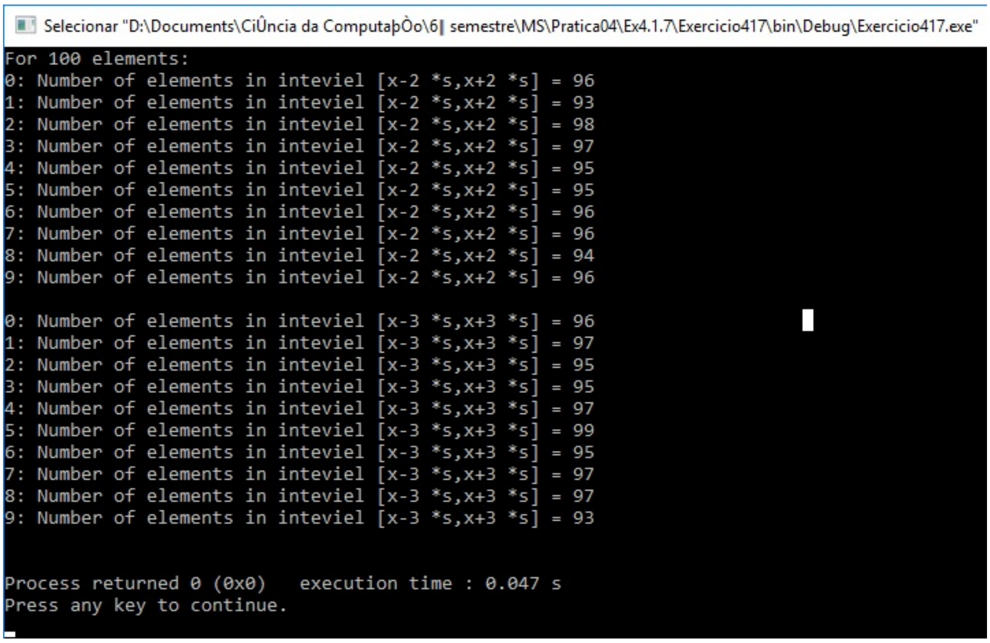


Figura 1: Output 4.1.7 - letra a

b) In each case, compare the results with Chebyshev's inequality.

pk >= 1 - $\frac{1}{k^2}$

p2 >= 1 - $\frac{1}{2^2}$ logo, p2 >= 0.75
p3 >= 1 - $\frac{1}{3^2}$ logo, p3 >= 0.11

c)Comment.

Os valores sempre tendem a cair dentro da faixa de $\bar{x}$ $\pm$ ks.

Exercício 4.1.8

Generate a plot similar to that in Figure 4.1.2 with calls to Exponential(17), rather than Random to generate the variates. Indicate the values to which the sample mean and sample standard deviation will converge.

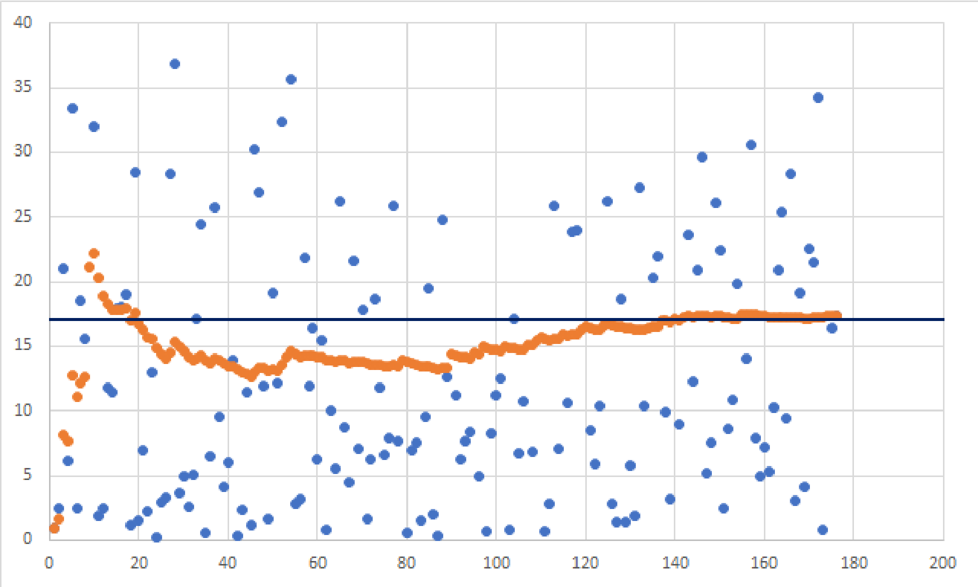


Figura 2: Média e valores randômicos - Exercício 4.1.8

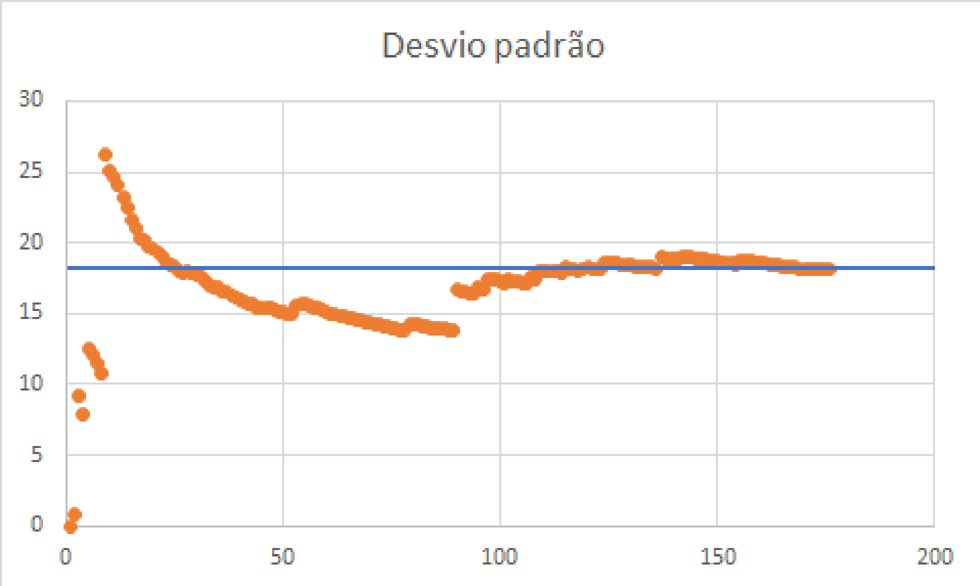


Figura 3: Desvio Padrão - Exercício 4.1.8

Convergiu com erro inferior a 0.0001 em 177.

Exercício 4.1.11

Calculate$\bar{x}$ and s by hand using the 2-pass algorithm, the 1-pass algorithm, and Welford's algorithm in the following two cases.

a) The data based on n = 3 observations: x1 = 1, x2 = 6, and x3 = 2.

n = 3
$x_1 = 1$
$x_2 = 6$
$x_3 = 2$

media = x_1 + x_2 + x_3 / n = (1 + 6 + 2)/3 = 3

Two Pass:

s = $\sqrt[2]{(((x_1 - media)^2 + (x_2 - media)^2 + (x_3 - media)^2)/n)}$

s = $\sqrt(((-2)^2 + (3)^2 + (-1)^2)/3)$

s = 2.1602469

One Pass:

s = $\sqrt[2]{((1/n * (x_1^2 + x_2^2 + x_3^2)) - media^2)}$

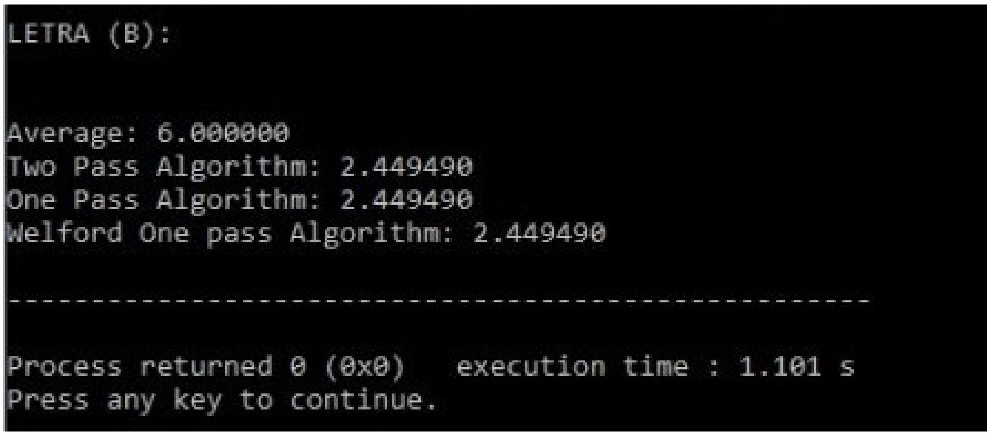
s = $\sqrt(((1/3) * (1^2 + 6^2 + 2^2)) - 3^2)$

s = 2.1602469

Welford:
$media_i = media_{i-1} + 1/i(x_i - media_{i-1})$
$vi = v_{i-1} + ((i-1)/i)(x_i - media_{i-1})^2$
s = $\sqrt[2]{v_i/i}$
$media_1 = media_{1-1} + 1/1(x_1 - media_{1-1})$
$media_1 = 0 + 1/1(1-0) = 0 + 1 = 1$
$v_1 = v_0 + ((1-1)/1)(x_1 - media_{1-1})^2$
$v_1 = 1 + ((1-1)/1) * (1-1)^2$
$v_1 = 0 + 0 = 0$
$media_2 = media_{2-1} + 1/2(x_2 - media_{2-1})$
$media_2 = 1 + 1/2(6-1) = 3.5$
$v_2 = v_1 + ((2-1)/2)(x_2 - media_{2-1})^2$
$v_2 = 0 + ((2-1)/2) * (6-1)^2$

$v_2 = 0 + 1/2 * 25 = 12.5$

b) The sample path x(t) = 3 for 0 < t <= 2, and x(t) = 8 for 2 < t <= 5, over the time interval 0 < t <= 5.



```
LETRA (B):


Average: 6.000000
Two Pass Algorithm: 2.449490
One Pass Algorithm: 2.449490
Welford One pass Algorithm: 2.449490

----------------------------------------------------

Process returned 0 (0x0)   execution time : 1.101 s
Press any key to continue.
```

Figura 4: Dados 4.1.1 - letra b

Exercício 4.1.13

Generate an Exponential(7) random variate sample of size n = 1000 and compute the mean and standard deviation using the Conventional One pass algorithm and the Algorithm 4.1.1.

```
for a sample of size 1000
mean ................. =   7.041
standard deviation ... =   6.915
minimum .............. =   0.002
maximum .............. =  46.059
```

Figura 5: Output usando algoritmo Exemplo 4.1.1

```
for a sample of size 1000
xb ................. =   7.041
s  ................. =   6.915
minimum ............ =   0.002
maximum ............ =  46.059
```

Figura 6: Output usando algoritmo uvs.c

Comment on the results.

Mesmo variando um pouco o codigo, os numeros gerados tem a mesma seed e chamam a função "Exponential"com o mesmo valor, sendo assim, por mais que o uvs.c seja mais robusto, os resultados seram os mesmos.

Exercício 4.2.2

a) Generate the 2000-ball histogram in Example 4.2.2.

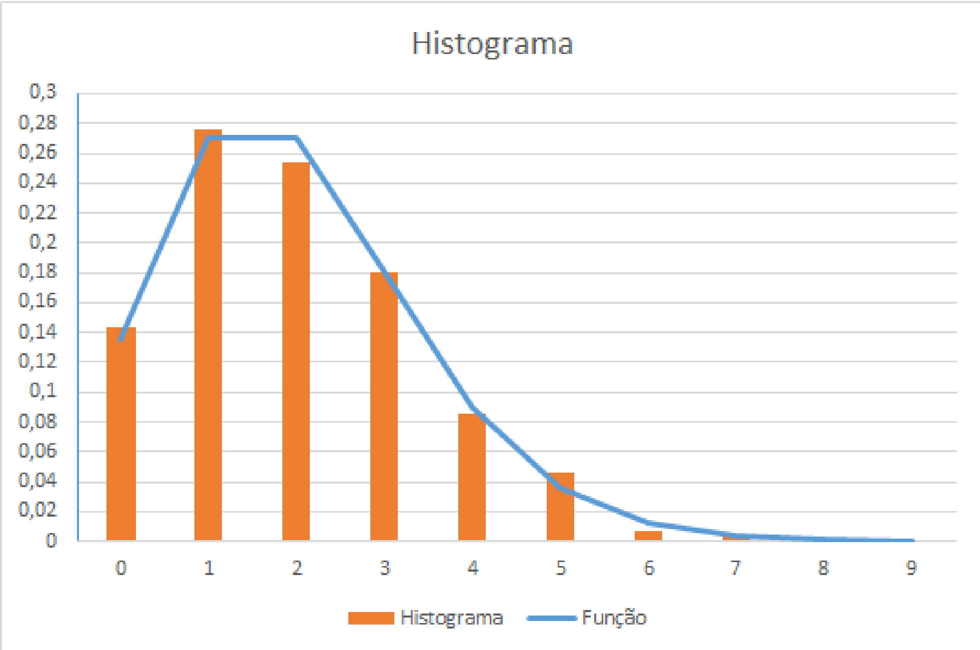


Figura 7: Histograma 4.2.2 - letra a

b) Verify that the resulting relative frequencies $\hat{f}(x)$ satisfy the equation

$\hat{f}(x) \cong \frac{2^x exp(-2)}{x!}$

x = 0,1,2,...

Escolhemos três valores aleatórios para conferir o resultado relativo à $\hat{f}(x)$:

$\hat{f}(2) = 0.270670566$
$\hat{f}(5) = 0.036089409$
$\hat{f}(7) = 0.003437087$

c) Then generate the corresponding histogram if 10 000 balls are placed, at random, in 1000 boxes.

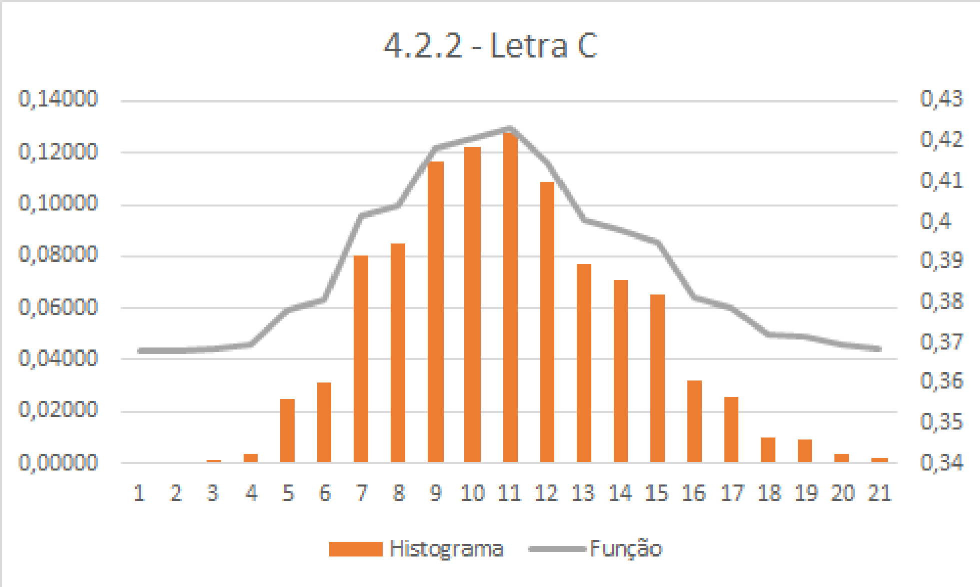


Figura 8: Dados 4.2.2 - letra c e d

d) Find an equation that seems to fit the resulting relative frequencies well and illustrate the quality of the fit.

$\hat{\mathrm{f}}(\mathrm{x}) \cong \frac{3^x exp(-2)}{x!}$

Exercício 4.3.2

Repeat the experiment in Example 4.3.6 with t = 5000 and n = 2000. Do not use a bubble sort.

```
 bin      midpoint      count   proportion     density

   1        0.450        595       0.297        0.331
   2        1.350        412       0.206        0.229
   3        2.250        325       0.163        0.181
   4        3.150        201       0.101        0.112
   5        4.050        153       0.076        0.085
   6        4.950         82       0.041        0.046
   7        5.850         68       0.034        0.038
   8        6.750         54       0.027        0.030
   9        7.650         27       0.014        0.015
  10        8.550         26       0.013        0.014
  11        9.450         17       0.009        0.009
  12       10.350         15       0.007        0.008
  13       11.250          8       0.004        0.004
  14       12.150          6       0.003        0.003
  15       13.050          2       0.001        0.001
  16       13.950          3       0.002        0.002
  17       14.850          2       0.001        0.001
  18       15.750          1       0.001        0.001
  19       16.650          1       0.001        0.001
  20       17.550          1       0.001        0.001

sample size .... =     2000
mean ........... =    2.516
stdev .......... =    2.435

NOTE: there were 1 high outliers

C:\Users\tyr\Documents\MS\code>
```

Figura 9: Output utilizando exemplo 4.3.6 t = 5000, n = 2000

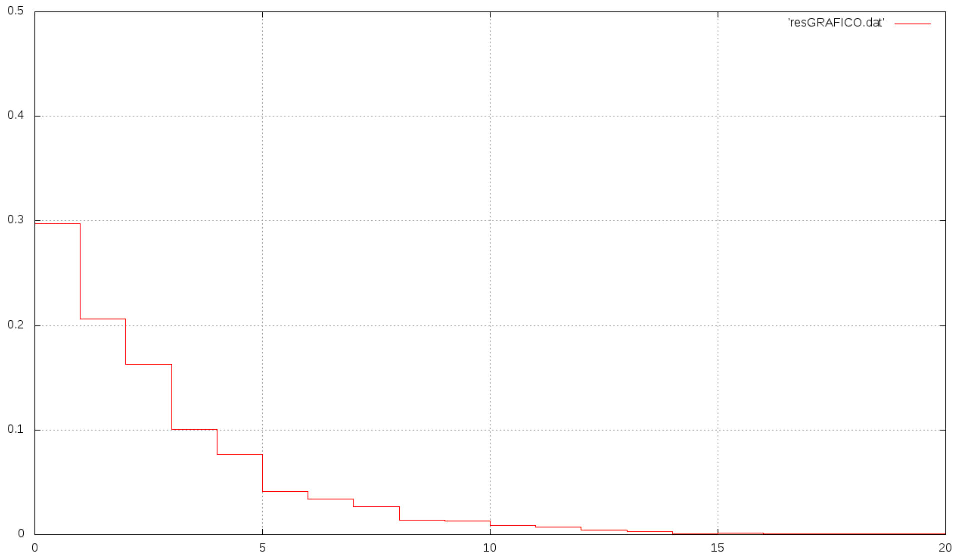


Figura 10: Histograma utilizando exemplo 4.3.6 t = 5000, n = 2000

Exercício 4.3.4

Generate a random variate sample x1, x2, ..., xn of size n = 10000 as fallows:

a) Use program cdh to construct a 20-bin continuous-data histogram

bin	midpoint	count	proportion	density
1	0.050	56	0.006	0.056
2	0.150	140	0.014	0.140
3	0.250	265	0.026	0.265
4	0.350	352	0.035	0.352
5	0.450	403	0.040	0.403
6	0.550	540	0.054	0.540
7	0.650	658	0.066	0.658
8	0.750	733	0.073	0.733
9	0.850	901	0.090	0.901
10	0.950	1009	0.101	1.009
11	1.050	970	0.097	0.970
12	1.150	804	0.080	0.804
13	1.250	717	0.072	0.717
14	1.350	625	0.063	0.625
15	1.450	548	0.055	0.548
16	1.550	467	0.047	0.467
17	1.650	369	0.037	0.369
18	1.750	230	0.023	0.230
19	1.850	168	0.017	0.168
20	1.950	45	0.004	0.045

```
sample size .... =  10000
mean ........... =  1.000
stdev .......... =  0.409


Process returned 0 (0x0)   execution time : 0.043 s
Press any key to continue.
```

Figura 11: Output cdh

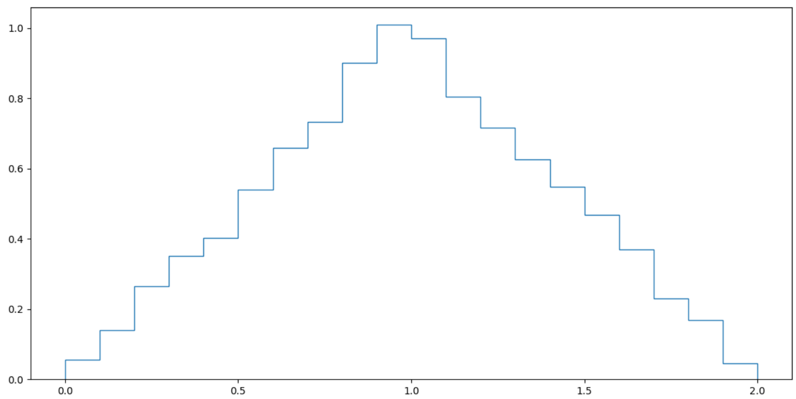


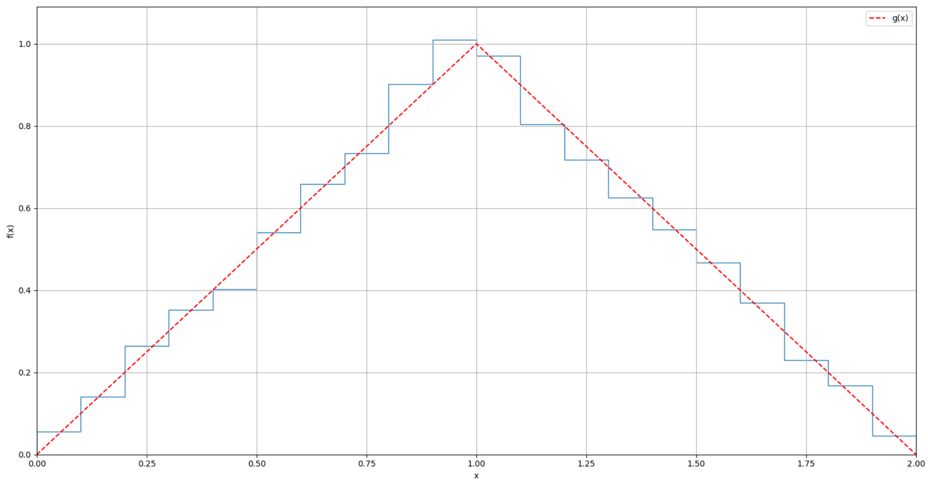
Figura 12: histograma para as 10000 amostras

b) Can you find an equation that seems to fit the histogram density well?

Uma equação que se encaixa razoavelmente bem ao histograma é:

$$g(x) = \begin{cases} x, & \text{if } 0 \leq x \leq 1 \\ 1 - x, & \text{if } 1 < x \leq 2 \end{cases}$$

Uma sobreposição da equação junto ao histograma foi gerada:



Exercício 4.3.5

a) As a continuation of Exercise 1.2.6, construct a continuous-data histogram of the service times.

Foram geradas 500 amostras a partir do tempos de serviço do exemplo 1.2.6. Para a construção do histograma no cdh, após analise dos dados foi concluido que os paramentros seriam $a = 0$ e $b = 16$ e $k = 15$. Para a plotagem do histograma b foi reduzido para 9 para obter uma imagem mais condizente.

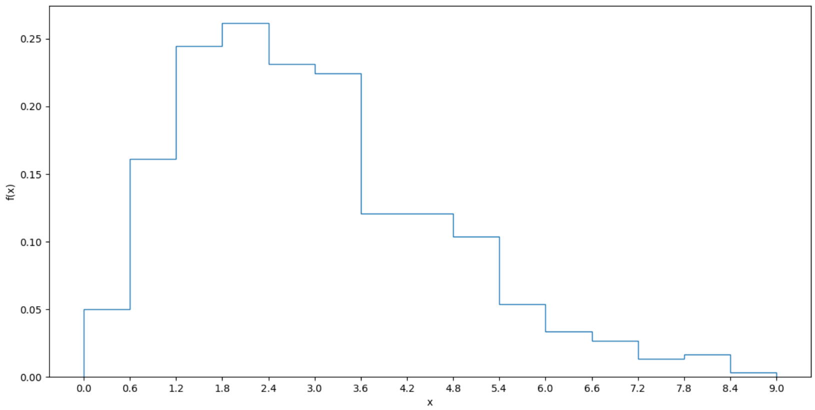


Figura 13: histograma para as 10000 amostras

b) Compare the histogram mean and standard deviation with the corresponding sample mean and standard deviation and justify your choice of the histogram parameters a, b and either k or δ

bin	midpoint	count	proportion	density
1	0.533	46	0.092	0.086
2	1.600	138	0.276	0.259
3	2.667	130	0.260	0.244
4	3.733	75	0.150	0.141
5	4.800	60	0.120	0.113
6	5.867	26	0.052	0.049
7	6.933	12	0.024	0.022
8	8.000	9	0.018	0.017
9	9.067	2	0.004	0.004
10	10.133	0	0.000	0.000
11	11.200	1	0.002	0.002
12	12.267	0	0.000	0.000
13	13.333	0	0.000	0.000
14	14.400	0	0.000	0.000
15	15.467	1	0.002	0.002

```
sample size .... =     500
mean ........... =   3.025
stdev .......... =   1.855
```

Figura 14: Output cdh

A escolha do parâmetros a e b foi feita com base na análise de amostra de forma que todos os valores fossem cobertos.
Já a escolha de $k = 15$ foi com base nos dois guidelines apresentados no livro:

- **Typically $\lfloor \log_2(n) \rfloor \leq k \leq \lfloor \sqrt{n} \rfloor$ with a bias toward $k \cong \lfloor (5/3)\sqrt[3]{n} \rfloor$ (Wand, 1997).**
- **Sturges's rule (Law and Kelton, 2000, page 336) suggests $k \cong \lfloor 1 + \log_2 n \rfloor$.**

Sendo $\lfloor \log_2 500 \rfloor = 8$, $\lfloor \sqrt{500} \rfloor = 22$, $\lfloor (5/3)\sqrt[3]{500} \rfloor = 13$ e $\lfloor 1 + \log_2 500 \rfloor = 9$, seguindo os guidelines do livro temos $8 \leq k \leq 22$, $k \cong 13$ e $k \cong 9$ (Sturge's rule) e portanto $k = 15$ é uma escolha válida para que $\delta = 0{,}6$.

A comparação entre as médias e os desvios padrões amostral e do histograma está feita na tabela abaixo.

	sample	*histogram*
x	3,0318	3,025
s	1,8244	1,855