Simulating seizure EEG with a heavy-tail factor

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1 Latent process

Let $\{Z_k(t)\}_{t=1}^{n}$ be a casual AR(2) process characterized by the coefficients $(\phi_{k,1}, \phi_{k,2})$.

The coefficients $\phi_{k,1}$ and $\phi_{k,2}$ can be expressed in terms of two features from the SDF, the *peak–location*$_k$ and its *sharpness*$_k$, in the following way:

$$\phi_{k,1} = 2\cos\left(2\pi\Psi_k\right)\exp\left(-\text{shrapness}_k\right) \quad \text{and} \quad \phi_{k,2} = -\exp\left(-2 \times \text{shrapness}_k\right),$$

where $\Psi_k = 2 \times \dfrac{\text{peak–location}_k}{\text{sampling–rate}}$

In order to simulate EEG data from a patient i, we create a latent process based on AR(2) rhythms:

$$\boldsymbol{Z}^i(t) = \left[Z^i_\delta(t),\ \ Z^i_\theta(t),\ \ Z^i_\alpha(t),\ \ Z^i_\beta(t),\ \ Z^i_\gamma(t)\right]^T,$$

where **the coefficients** $\phi_{k,1}$ and $\phi_{k,2}$, $k \in \{\delta,\ \theta,\ \alpha,\ \beta,\ \gamma\}$ **are shared by all patients**.

Obs.: Note that the frequency bands δ, θ, α, β, and γ are generated independently.

2 Weights: controlling the importance of each rhythm

Consider we want to generate a brain network with $J = 6$ EEG channels.

Let $\left\{\boldsymbol{X}^i(t) = [X^i_1(t), \ldots, X^i_{J=6}(t)]^T\right\}_{t=1}^{n}$ be the EEG signal to be simulated from $\{\boldsymbol{Z}^i(t)\}_{t=1}^{n}$.

We want some rhythms of $\boldsymbol{Z}^i$ to have more relevance in explaining the oscillations of $\boldsymbol{X}^i$. So, we use different weights for each rhythm of $\boldsymbol{Z}^i$ through a matrix $\boldsymbol{A}^i$:

$$X_j^i(t) = \boldsymbol{A}^i(t)[\boldsymbol{Z}^i(t)]^T, \forall j = 1, 2, \dots, J = 6,$$

where $\boldsymbol{A}^i(t) = \left[A_\delta^i(t),\ \ A_\theta^i(t),\ \ A_\alpha^i(t),\ \ A_\beta^i(t),\ \ A_\gamma^i(t)\right]^T$, $\forall t = 1, \dots, n$.

For simplicity, we keep:

- $A_k^i(t) = c_k$, $\forall t = 1, \dots, n$: in the case of non-seizure patients;
- $A_k^i(t) = c_{b,k}\mathbb{1}_{\{1,\dots,n'-1\}}(t) + c_{d,k}\mathbb{1}_{\{n',\dots,n''\}}(t) + c_{a,k}\mathbb{1}_{\{n''+1,\dots,n\}}(t)$, $\forall t = 1, \dots, n$: in the case of a patient facing a seizure event from n' to n'',

where c_k, $c_{b,k}$, $c_{d,k}$, $c_{a,k} \in \mathbb{R}^+$ and $\mathbb{1}_{\{C\}}(x)$ is the indicator function.

Obs.: 1. If $\sum_{k\in\{\delta,\theta,\alpha,\beta,\gamma\}} A_{j,k}^i(t) = 1, \forall t = 1, \dots, n$, then, the weights have better interpretability. However, we lose the "seizure behavior" we want in the EEG signal. 2. So far, there is no difference between $X_j(t)$ and $X_i(t)$. We correct this in the next section.

3 Dependence structure in the bulk

In order to introduce a dependence structure between the EEG channels of patient i, we generate a zero-mean white noise with covariance matrix $\Sigma_{J=6}^i$:

$$\boldsymbol{W}^i(t) \sim \mathcal{N}_{J=6}\left(\boldsymbol{0}, \Sigma_{J=6}^i\right),$$

where $\left\{\boldsymbol{W}^i(t) = [W_1^i(t), \dots, W_{J=6}^i(t)]^T\right\}_{t=1}^n$.

Hence,

$$X_j^i(t) = \boldsymbol{A}^i(t)[\boldsymbol{Z}^i(t)]^T + W_j^i(t), \forall t = 1, \dots, n, \text{ and } \forall j = 1, 2, \dots, J = 6.$$

4 Dependence structure in the tail

If we want two different channels, $X_k^i(t)$ and $X_s^i(t)$, to be Asymptotic Dependent, we introduce a common heavy-tail factor to these channels using an exponential distribution.

$$X_k^i(t) = \boldsymbol{A}^i(t)[\boldsymbol{Z}^i(t)]^T + W_k^i(t) + F^i(t) \text{ and } X_s^i(t) = \boldsymbol{A}^i(t)[\boldsymbol{Z}^i(t)]^T + W_s^i(t) + F^i(t),$$

where $F^i(t) \sim \text{Exp}(\lambda)$, $\forall t = 1, \ldots, n$, and $\lambda > 0$.

5 Clusters

We want clusters of Asymptotic Dependent channels.

Hence, EEG channels within the same AD cluster C will share:

- the same weight matrix $\boldsymbol{A}_C$, and
- the same heavy-tail factor F_C.