Desenvolvimento de sistemas Cálculo Diferencial e Integral

Lista de Exercícios (2025) Matheus Luiz Massuda

1. Limites de Funções

LISTA (LIMITES DE FUNÇÕES):

1.1. LIMITES DE FUNÇÕES POLINOMIAIS PODEM SER OBTIDOS POR MEIO DE SUBSTITUIÇÃO, DESDE QUE NÃO SEJA UMA INDETERMINAÇÃO.

(a) $\lim_{x \to 3} 2x - 1 = 2(3) - 1 = \boxed{5}$

(b) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \frac{(1)^2 - 1}{1 - 1} = \frac{0}{0}? \Rightarrow \frac{(x+1)\cdot\cancel{(x-1)}}{\cancel{(x-1)}} = \boxed{2}$ Ind.

(c) $\lim_{x \to -2} 3x + 7 = 3(-2) + 7 = \boxed{1}$

(d) $\lim_{x \to a} cte = cte = \boxed{9}$

(e) $\lim_{x \to 2} 4x^2 + 5x - 7 = 16 + 10 - 7 = \boxed{19}$

(f) $\lim_{x \to -2} x^3 + 4x^2 - 3 = (-2)^3 + 4(-2)^2 - 3 = \boxed{5}$

$-8 + 16$

-27

(g) $\lim_{x \to 2} \frac{x^4 + x^2 - 1}{x^2 + 5} = \frac{16 + 4 - 1}{9} = \boxed{\frac{19}{9}}$

(h) $\lim_{x \to 3} \frac{-x^3 + 2x}{x - 1} = \frac{-(3)^3 + 6}{2} = \boxed{-\frac{21}{2}}$

(i) $\lim_{x \to 3} \sqrt[3]{\frac{x^2 + 5x + 3}{x^2 - 1}} = \frac{\sqrt[3]{9 + 15 + 3}}{\sqrt[3]{8}} = \frac{\sqrt[3]{27}}{2} = \boxed{\frac{3}{2}}$

(j) $\lim_{x \to 0} x^2 \, sen \frac{1}{x}$ $\quad 0^2 \cdot sen \frac{1}{0}$? Ind. $\Rightarrow \left(\lim_{x \to 0} x^2\right) \cdot \left(\lim_{x \to 0} sen \frac{1}{x}\right) \Rightarrow$ sen de $\forall$ ângulo sempre está entre -1 e 1

* TEOREMA DO CONFRONTO (SANDWICH)

se, $\lim_{x \to k} g(x) = \lim_{x \to k} h(x) = L$, então $\lim_{x \to k} f(x) = L$, se $g(x) \leq f(x) \leq h(x)$

$\therefore -1 \leq sen \frac{1}{x} \leq 1 \cdot (x^2) \Rightarrow \underbrace{-x^2}_{g(x)} \leq \underbrace{x^2 \cdot sen \frac{1}{x}}_{f(x)} \leq \underbrace{x^2}_{h(x)} \Rightarrow$

$\lim_{x \to 0} -x^2 \leq \lim_{x \to 0} x^2 \cdot sen \frac{1}{x} \leq \lim_{x \to 0} x^2 \Rightarrow \lim_{x \to 0} x^2 \cdot sen \frac{1}{x} = \boxed{0}$

(k) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \frac{(x+2)\cdot\cancel{(x-2)}}{\cancel{(x-2)}} = 2 + 2 = \boxed{4}$

(l) $\lim_{x \to -5} \frac{x^2 + 3x - 10}{x + 5} = \frac{25 - 15 - 10}{0} = \frac{0}{0}?$ Ind.

$x \cdot 2 \cdot \sqrt{-10} \neq 3x$, então

se $a = 1$, $S = 3$ e $P = -10$

$x_1 = 5$ e $x_2 = -2$

$\frac{(x + x_1)\cdot(x + x_2)}{(x+5)} = \frac{\cancel{(x+5)}\cdot(x-2)}{\cancel{(x+5)}}$

$-5 - 2 = \boxed{-7}$

S/L	T/M	Q/M	Q/J	S/V	S/S	D/D

m) $\lim_{x\to 0} \frac{(2+x)^2-4}{x} = \frac{(2+x+2)\cdot(\cancel{2}+x\cancel{-2})}{x} = \frac{(x+4)\cdot\cancel{x}}{\cancel{x}}$

$\lim_{x\to 0} x+4 = \boxed{4}$

n) $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} = \frac{\sqrt{2}-\sqrt{2}}{0} = \frac{0}{0}$ Ind. $\Rightarrow \frac{(\sqrt{2+x}-\sqrt{2})}{x} \cdot \frac{(\sqrt{2+x}+\sqrt{2})}{(\sqrt{2+x}+\sqrt{2})} =$

$\frac{(\sqrt{2+x})^2-(\sqrt{2})^2}{x\cdot(\sqrt{2+x}+\sqrt{2})} = \frac{\cancel{2}+x\cancel{-2}}{x(\sqrt{2+x}+\sqrt{2})} = \boxed{\frac{1}{\sqrt{2+x}+\sqrt{2}}} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$= \frac{\sqrt{2}}{2\cdot 2} = \boxed{\frac{\sqrt{2}}{4}}$

o) $\lim_{x\to 1} \frac{\sqrt{x^2+8}-3}{x-1} = \frac{\sqrt{9}-3}{0} = \frac{0}{0}$ Ind. $\Rightarrow \frac{\sqrt{x^2+8}-3}{x-1} \cdot \left(\frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}\right) =$

$\frac{(\sqrt{x^2+8})^2-3^2}{(x-1)\cdot(\sqrt{x^2+8}+3)} = \frac{x^2+8-9}{(x-1)\cdot(\sqrt{x^2+8}+3)} = \frac{(x+1)\cdot\cancel{(x-1)}}{\cancel{(x-1)}\cdot(\sqrt{x^2+8}+3)} = \frac{x+1}{\sqrt{9}+3} = \frac{2}{6} = \boxed{\frac{1}{3}}$

p) $\lim_{x\to -2} \frac{-4x-8}{x^3+2x^2} = \frac{8+8}{-8+8} = \frac{0}{0}$ Ind. $\Rightarrow \frac{-4\cancel{(x+2)}}{x^2\cancel{(x+2)}} = \frac{-4}{4} = \boxed{-1}$

q) $\lim_{x\to 7} \frac{x-7}{x^2-49} = \frac{7-7}{49-49} = \frac{0}{0}$ Ind. $\Rightarrow \frac{\cancel{(x-7)}}{(x+7)\cdot\cancel{(x-7)}} = \boxed{\frac{1}{14}}$

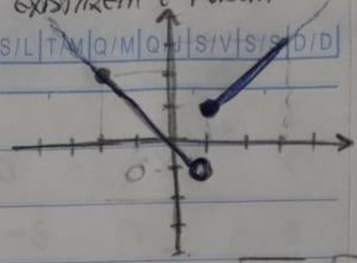
R) $\lim_{x\to 0} \frac{(3+x)^2-9}{x} = \frac{9-9}{0} = \frac{0}{0}$ Ind. $\Rightarrow (3+x)^2-(3)^2$; a, b ; $(a+b)\cdot(a-b)$

$\frac{(\cancel{3}+x\cancel{-3})\cdot(3+x+3)}{x} = \frac{\cancel{x}\cdot(x+6)}{\cancel{x}} = \boxed{6}$

1.2. SE $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$, ENTÃO $\lim_{x \to a} f(x) = L$

EXISTE lim BILATERAL, SE OS lim UNILATERAIS EXISTIREM E FOREM =

(a) $f(x) \begin{cases} x, & \text{se } x \geq 1 \\ -x, & \text{se } x < 1 \end{cases}$ $\lim_{x \to 1^+} x = \boxed{1}$ e $\lim_{x \to 1^-} -x = \boxed{-1}$

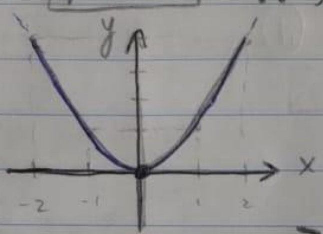


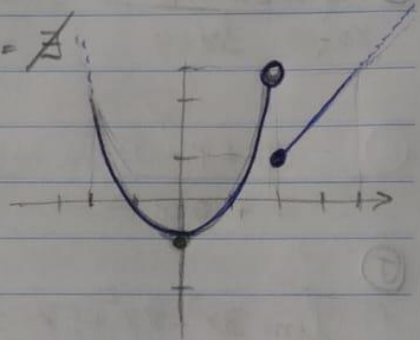
$\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x) \therefore \lim_{x \to 1} f(x) = \boxed{\nexists}$

(b) $\lim_{x \to 2} \sqrt{x-2} \Rightarrow x - 2 \geq 0$ P/ $\in \mathbb{R}$ 1,99 2,001 $\rightarrow 2 \leftarrow$ $\lim_{x \to 2^+} \sqrt{x-2} = \sqrt{>0} = \boxed{0}$

e $\lim_{x \to 2^-} \sqrt{x-2} = \sqrt{0^-} = \boxed{\nexists \text{ p/ } \mathbb{R}}$ $\therefore \lim_{x \to 2} \sqrt{x-2} = \nexists$

(c) $\lim_{x \to 0} x^2 = \boxed{0}$





(d) $f(x) \begin{cases} x^2 - 1, & \text{se } x < 2 \\ 1, & \text{se } x = 2 \\ x - 1, & \text{se } x > 2 \end{cases}$ $\lim_{x \to 2^+} x - 1 = 1$, e $\lim_{x \to 2^-} x^2 - 1 = 4 - 1 = 3$

$\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x) \Rightarrow \lim_{x \to 2} f(x) = \boxed{\nexists}$

2

(e) $\lim_{x \to +\infty} 3 + \frac{1}{x} \Rightarrow \lim_{x \to +\infty} \overset{3}{3} + \lim_{x \to +\infty} \overset{0}{\frac{1}{x}} = 3 + 0 = \boxed{3}$

$\frac{1}{\infty}$ = UM DIVIDIDO POR UM Nº MUITO GRANDE $\cong 0$ EX: $\frac{1}{1.000.000} = 0{,}000001 \cong 0$

(f) $\lim_{x \to -\infty} \left(\frac{\pi \sqrt{2}}{x^3} \right) \Rightarrow \lim_{x \to -\infty} \pi \cdot \lim_{x \to -\infty} \frac{\sqrt{2}}{x^3} = \pi \cdot 0 = \boxed{0}$

$\pi \cdot 0$

(g) $\lim_{x \to +\infty} \frac{3x^2 + 5x - 3}{2x^2 + 1} = \frac{3(\infty)^2 + 5(\infty) - 3}{2(\infty)^2 + 1} = \frac{+\infty + \infty - 3}{+\infty + 1} = \frac{+\infty}{+\infty}$ Ind.

$$\frac{\frac{3x^2 + 5x - 3}{x^2}}{\frac{2x^2 + 1}{x^2}} = \frac{3 + \cancel{\frac{5}{x}}^{\,0} - \cancel{\frac{3}{x^2}}^{\,0}}{2 + \cancel{\frac{1}{x^2}}_{\,\to 0}} = \frac{3 + 0 - 0}{2 + 0} = \boxed{\frac{3}{2}}$$

S/L | T/M | Q/M | Q/J | S/V | S/S | D/D

h) $\lim\limits_{x\to-\infty} \dfrac{2x+3}{3x^2-2} \Rightarrow \dfrac{-\infty}{+\infty}\Big\}$ Ind. $\Rightarrow \dfrac{\dfrac{2x}{x^2}+\dfrac{3}{x^2}}{\dfrac{3x^2}{x^2}-\dfrac{2}{x^2}} =$

$$\frac{\frac{2}{x}\to 0 + \frac{3}{x^2}\to 0}{3-\frac{2}{x^2}\to 0} = \frac{0}{3-0} = \frac{0}{3} = \boxed{0}$$

i) $\lim\limits_{x\to-\infty} \dfrac{2x^2+4x-5}{3x+4} = \dfrac{+\infty-\infty-5}{-\infty+4} = \dfrac{+\infty}{-\infty}\Big\}$ Ind. $\dfrac{\dfrac{2x^2}{x}+\dfrac{4x}{x}-\dfrac{5}{x}}{\dfrac{3x}{x}+\dfrac{4}{x}} = \dfrac{2x+4-\frac{5}{x}\to 0}{3+\frac{4}{x}}$

$$\frac{-\infty+4-0}{3+0} = \frac{-\infty}{3} = \boxed{-\infty}$$

j) $\lim\limits_{x\to+\infty} \dfrac{3x^5-2x^2+5x-1}{2x^7+5x^3-2x^2} = \dfrac{+\infty}{+\infty}\Big\}$ Ind. $\Rightarrow \dfrac{\dfrac{3x^5-2x^2+5x-1}{x^7}}{\dfrac{2x^7+5x^3-2x^2}{x^7}} = \dfrac{\frac{3}{x^2}-\frac{2}{x^5}+\frac{5}{x^6}-\frac{1}{x^7}}{2+\frac{5}{x^4}-\frac{2}{x^5}}$

$$\lim_{x\to+\infty} f(x) = \frac{0}{2} = \boxed{0}$$

k) $\lim\limits_{x\to 0}\left(x^3+2\sqrt[3]{x}+\dfrac{1}{x^2}\right) = 0+2(0)+\dfrac{1}{0}\Big\}$ Ind. $\Rightarrow \lim\limits_{x\to 0} x^3 + \lim\limits_{x\to 0} 2\sqrt[3]{x} + \lim\limits_{x\to 0}\dfrac{1}{x^2} =$

$\lim\limits_{x\to 0}\dfrac{1}{x^2} = \dfrac{1}{0} = \boxed{+\infty}$

COMO O EXPOENTE É PAR, ∀ QUE SEJA VALOR DE X SEMPRE SERÁ UM NÚMERO MUITO PEQUENO, PORÉM POSITIVO $1/0^+$
UM DIVIDIDO POR Nº PEQUENO e ⊕ = $+\infty$

l) $\lim\limits_{x\to 3}\dfrac{x}{x-3} = \dfrac{3}{0}\Big\}$ Ind. $\Rightarrow$ (ESTUDO lim. UNILATERAIS) $= \lim\limits_{x\to 3^+}\dfrac{x}{x-3} = \dfrac{3}{0^+} = +\infty$

2,99 → 3 ← 3,001

UNILATERAL $\neq$ então BILATERAL $\nexists$

$\lim\limits_{x\to 3^-}\dfrac{x}{x-3} = \dfrac{3}{0^-} = -\infty$ $\Big\}\ \boxed{\nexists}$

m) $\lim\limits_{x\to 4}\dfrac{2x^3+x^2-5}{x^2-2x-8} = \dfrac{128+16-5}{16-8-8} = \dfrac{139}{0}\Big\}$ Ind. (ESTUDO lim. UNILATERAIS) 3,99 → 4 ← 4,001

$\lim\limits_{x\to 4^+} = \dfrac{139}{0^+} = +\infty$

$\lim\limits_{x\to 4^-} = \dfrac{139}{0^-} = -\infty$

$\Big\}\ \lim\limits_{x\to 4^+} f(x) \neq \lim\limits_{x\to 4^-} f(x) \therefore \lim\limits_{x\to 4} f(x) = \boxed{\nexists}$

(m) $\lim_{x \to 2} \frac{1}{(x-2)^2} = \frac{1}{0^2}$ = Ind. (ESTUDO LIM UNILATERAL) $\Rightarrow \lim_{x \to 2^+} \frac{1}{(x-2)^2} = \frac{1}{0^+} = +\infty$

EXPOENTE PAR $= 0^+$

S/L T/M Q/M Q/J S/V S/S D/D

$\lim_{x \to 2^-} f(x) = \frac{1}{(0)^2} = \frac{1}{0^+} = +\infty$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2} f(x) = \boxed{+\infty}$

(o)

$\lim_{x \to -\infty} 2x^5 - 5x^3 + 1 = 2(-\infty)^5 - 5(-\infty)^3 + 1 = -\infty + \infty + 1 = \boxed{-\infty}$

(p) $\lim_{x \to +\infty} \frac{3-x}{\sqrt{5+4x^2}} = \frac{3-\infty}{\sqrt{\infty}}$ = Ind. (Estudo lim. UNILATERAIS) $\Rightarrow \lim_{x \to \infty^+} \frac{3-x}{\sqrt{5+4x^2}}$

→ NUMERADOR $(3-x)$ ⇒ P/ CÁLCULO DO LIMITE, O x é O TERMO DOMINANTE, POIS O "3" é INSIGNIFICANTE em comparação a $-x$, que será substituído por $+\infty$

→ DENOMINADOR $(\sqrt{5+4x^2})$ ⇒ $4x^2$ é o TERMO DOMINANTE, POIS 5 é INSIGNIFICANTE em relação a $4x^2$

↳ SIMPLIFICAR = Dividir TERMOS PELO MAIOR GRAU DE X QUE APARECE DENTRO DA RAIZ = x^2

↳ SIMPLIFICAR = PARA LIDAR COM A RAIZ, PRECISAMOS EXTRAIR x^2 DE DENTRO DA RAIZ

$$\frac{\frac{3-x}{x}}{\sqrt{x^2 \cdot \left(\frac{5}{x^2}+4\right)}} = \frac{\frac{3}{x} - 1}{\sqrt{x^2} \cdot \sqrt{\frac{5}{x^2}+4}} = \frac{\frac{3}{x} - 1}{|x| \cdot \sqrt{\frac{5}{x^2}+4}} = \frac{\frac{3}{x} - 1}{(+x) \cdot \sqrt{\frac{5}{x^2}+4}} \quad (\div x)$$

$x \to +\infty$

PROPRIEDADE: FRAÇÃO DO PRODUTO ⇒ $\frac{\frac{3}{x}-1}{x} \cdot \frac{1}{\sqrt{\frac{5}{x^2}+4}} = \frac{-1}{x} \cdot \frac{1}{\sqrt{4}} = \frac{-1}{\sqrt{4}} =$

$\frac{A}{B \cdot C} = \frac{A}{B} \cdot \frac{1}{C}$

$-1/2 \quad \lim_{x \to +\infty} f(x) = \boxed{\frac{-1}{2}}$

(9) $\lim_{x \to -\infty} \frac{\sqrt{2x^2-7}}{x+3} \Rightarrow \frac{+\infty}{-\infty}$ } Ind. ⇒

|S/L|T/M|Q/M|Q/J|S/V|S/S|D/D|

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 \cdot \left(2 - \frac{7}{x^2}\right)}}{x+3} = \frac{\overbrace{\sqrt{x^2}}^{|x|} \cdot \sqrt{2 - \frac{7}{x^2}}}{x\left(1+\frac{3}{x}\right)} = \frac{-|x| \cdot \sqrt{2 - \frac{7}{x^2}}}{x\left(1+\frac{3}{x}\right)}$$

$$\frac{\cancel{(-x)} \cdot \sqrt{2 - \frac{7}{x^2}}}{\cancel{(x)} \cdot \left(1 + \frac{3}{x}\right)} = -\frac{\sqrt{2 - \cancel{7/x^2}^{\,0}}}{1 + \underbrace{\cancel{3/x}}_{0}} = -\frac{\sqrt{2-0}}{1+0} = -\frac{\sqrt{2}}{1} = \boxed{-\sqrt{2}}$$

(R) $\lim_{x \to -\infty} \frac{3x^5 - x^2 + 7}{2 - x^2} \Rightarrow \frac{-\infty}{-\infty}$ } Ind. ⇒ $\frac{\frac{3x^5 - x^2 + 7}{x^2}}{\frac{2-x^2}{x^2}} = \frac{3x^3 - 1 + \cancel{\frac{7}{x^2}}^{\,0}}{\underbrace{\cancel{\frac{2}{x^2}}}_{0} - 1}$

$$\lim_{x \to -\infty} f(x) = \frac{3(-\infty)^3 - 1 + \cancel{0}}{\cancel{0} - 1} = \frac{-\infty}{-1} = \boxed{+\infty}$$